sympy

March 29, 2022

1 Basic usage

x

x^2

$\sin(x)$

$x^2 + 4x + 3$

$\left(x^2 + 4x + 3\right)^2$

$(x + 1)^2 (x + 3)^2$

$x^4 + 8x^3 + 22x^2 + 24x + 9$

2 Solve equations

`smp.solve(f, x)` will find x that makes $f(x) = 0$ Hence, it's important to rearrange the equation so that $f(x) = 0$

```
[-3, -1]
```

```
[-I, I]
```

```
[]
```

```
[5/2 - sqrt(29)/2, 5/2 + sqrt(29)/2]
```

```
[5/2 + sqrt(29)/2]
```

```
[5/2 - sqrt(29)/2]
```

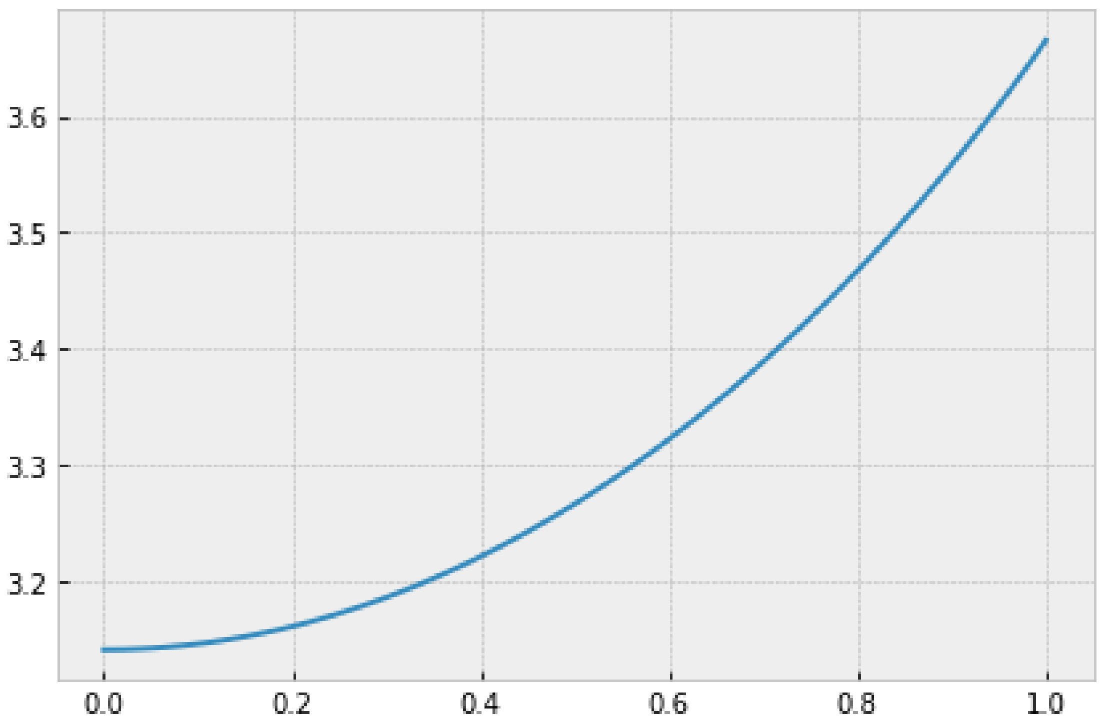
$x^2 + y\sin(z)$

```
x: [-sqrt(-y*sin(z)), sqrt(-y*sin(z))]
y: [-x**2/sin(z)]
z: [asin(x**2/y) + pi, -asin(x**2/y)]
```

$\operatorname{asin}\left(\frac{x^2}{y}\right) + \pi$

```
<function _lambdifygenerated(x, y)>
```

```
3.6651914291880923
```



$x^2 + y\sin(z)$

$x^2 + \sin(y)\cos(y)$

Example given two equations,

$$h_o(t) = h_0 - v_0 t - \frac{1}{2} g t^2$$

$$h_p(t) = v_p t + \frac{1}{2} q t^2$$

solve for t, v_0 when,

$$h_o(t) = h_p(t)$$

$$\frac{dh_o}{dt}(t) = -\frac{dh_p}{dt}(t)$$

use `smp.Rational(x,y)` for fraction

$-\frac{gt^2}{2} + h_0 - tv_0$

$\frac{qt^2}{2} + tv_p$

rearrange so that $f(x)=0$

$$h_o(t) - h_p(t) = 0$$

$$\frac{dh_o}{dt}(t) + \frac{dh_p}{dt}(t) = 0$$

$-\frac{gt^2}{2} + h_0 - \frac{qt^2}{2} - tv_0 - tv_p$

$gt + qt - v_0 + v_p$

```
[((-2*v_p/3 + sqrt(2)*sqrt(3*g*h_0 + 3*h_0*q + 2*v_p**2)/3)/(g + q),
  v_p/3 + sqrt(2)*sqrt(3*g*h_0 + 3*h_0*q + 2*v_p**2)/3)]
```

$\frac{-\frac{2v_p}{3} + \frac{\sqrt{2}\sqrt{3gh_0+3h_0q+2v_p^2}}{3}}{g+q}$

$\frac{v_p}{3} + \frac{\sqrt{2}\sqrt{3gh_0 + 3h_0q + 2v_p^2}}{3}$

use `.simplify()` to simplify the expression

$\frac{g\left(-\frac{2v_p}{3} + \frac{\sqrt{2}\sqrt{3gh_0+3h_0q+2v_p^2}}{3}\right)}{g+q} - \frac{v_p}{3} - \frac{\sqrt{2}\sqrt{3gh_0 + 3h_0q + 2v_p^2}}{3}$

$\frac{-gv_p - \frac{qv_p}{3} - \frac{q\sqrt{6gh_0+6h_0q+4v_p^2}}{3}}{g+q}$

$\frac{gv_p + \frac{qv_p}{3} + \frac{q\sqrt{6gh_0+6h_0q+4v_p^2}}{3}}{g+q}$

3 Calculus

3.1 Limit

$$\lim_{x\to\pi} \sin(\frac{x}{2} + \sin(x))$$

$\sin\left(\frac{x}{2} + \sin(x)\right)$

1

3.2 Derivatives

$$\frac{d}{dx}\left(\frac{1+\sin x}{1-\cos x}\right)^2$$

$\frac{(\sin(x)+1)^2}{(1-\cos(x))^2}$

$\frac{2(\sin(x)+1)\cos(x)}{(1-\cos(x))^2} - \frac{2(\sin(x)+1)^2\sin(x)}{(1-\cos(x))^3}$

$$\frac{d}{dx} f(x + g(x))$$

```
(f, g)
```

```
(f(x + g(x)), g(x))
```

$\left(\frac{d}{dx} g(x) + 1\right) \left.\frac{d}{d\xi_1} f(\xi_1)\right|_{\xi_1 = x + g(x)}$

$\left(\frac{d}{dx} \sin(x) + 1\right) \left.\frac{d}{d\xi_1} f(\xi_1)\right|_{\xi_1 = x + \sin(x)}$

$(\cos(x) + 1) \left.\frac{d}{d\xi_1} f(\xi_1)\right|_{\xi_1 = x + \sin(x)}$

3.3 Integrals

$$\csc(x)\cot(x) dx$$

$\cot(x)\csc(x)$

$-\frac{1}{\sin(x)}$

$$\int_0^{\ln(4)} \frac{e^x}{\sqrt{e^{2x} + 9}} dx$$

$\frac{e^x}{\sqrt{e^{2x} + 9}}$

$-\operatorname{asinh}\left(\frac{1}{3}\right) + \operatorname{asinh}\left(\frac{4}{3}\right)$

$$\int_1^t x^{10} e^x dx$$

$x^{10} e^x$

$\left(t^{10} - 10t^9 + 90t^8 - 720t^7 + 5040t^6 - 30240t^5 + 151200t^4 - 604800t^3 + 1814400t^2 - 3628800t + 3628800\right) e^t - 1334961e$

4 Vector and Matrix

$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$

$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

$$\begin{bmatrix} 2u_1 + v_1 \\ 2u_2 + v_2 \\ 2u_3 + v_3 \end{bmatrix}$$

$u_1 v_1 + u_2 v_2 + u_3 v_3$

$$\begin{bmatrix} u_2 v_3 - u_3 v_2 \\ -u_1 v_3 + u_3 v_1 \\ u_1 v_2 - u_2 v_1 \end{bmatrix}$$

$$\begin{bmatrix} u_1 v_1 & u_1 v_2 & u_1 v_3 \\ u_2 v_1 & u_2 v_2 & u_2 v_3 \\ u_3 v_1 & u_3 v_2 & u_3 v_3 \end{bmatrix}$$

$\left[u_1 v_1 + u_2 v_2 + u_3 v_3\right]$

$\sqrt{|u_1|^2 + |u_2|^2 + |u_3|^2}$

$$proj_v(u) = \frac{u \cdot v}{|v|^2} v$$

proj_v_u = u.dot(v) / (v.norm()**2) * v proj_v_u

$$\begin{bmatrix} 3t \\ \sin(t) \\ t^2 \end{bmatrix}$$

$$\begin{bmatrix} 3 \\ \cos(t) \\ 2t \end{bmatrix}$$

$$\begin{bmatrix} e^t \cos(t) \\ t^4 \\ \frac{1}{t^2+1} \end{bmatrix}$$

$$\begin{bmatrix} \frac{e^t \sin(t)}{2} + \frac{e^t \cos(t)}{2} \\ \frac{t^5}{5} \\ \operatorname{atan}(t) \end{bmatrix}$$

$$\begin{bmatrix} e^{t^2} \cos^3(t) \\ e^{-t^4} \\ \frac{1}{t^2+3} \end{bmatrix}$$

```
array([[0.81549671],
       [0.84483859],
       [0.30229989]])
```

5 Partial Derivatives

$y^2 + \sin(x + y)$

$\cos(x+y)$

$2y + \cos(x+y)$

$$\frac{\partial^3 f}{\partial x y^2}$$

$-\cos(x+y)$

$-\cos(x+y)$

5.1 Chain rule

$w(x(t), y(t), z(t))$

$\frac{d}{dx(t)} w(x(t), y(t), z(t)) \frac{d}{dt} x(t) + \frac{d}{dy(t)} w(x(t), y(t), z(t)) \frac{d}{dt} y(t) + \frac{d}{dz(t)} w(x(t), y(t), z(t)) \frac{d}{dt} z(t)$

6 Multiple Integrals

- rarely works correctly using symbolic method
- usually needs numpy and scipy

this is the rare case that works

$$\int_0^1 \int_0^{1-x^2} \int_3^{4-x^2-y^2} x dz dy dx$$

$\frac{1}{8}$