## Supplement to the paper 'Classification Methods Based on Fitting Logistic Regression to Positive and Unlabeled Data with Some Extensions to Feature Selection and Regularization'

Konrad Furmańczyk, Kacper Paczutkowski, Marcin Dudziński, and Diana-Dziewa Dawidczyk

Institute of Information Technology, Warsaw University of Life Sciences, Warsaw, Poland

#### 1 Explanation to the content of inserted tables

In all of the tables concerning the accuracy, the recall, the precision and the F1-score, the standard deviations are given in parentheses.

### 2 Calibration of parameters for the LassoJoint method

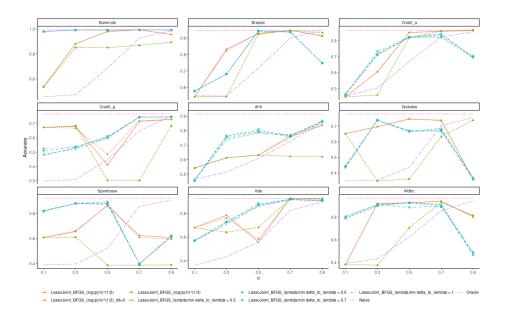


Fig. 1. The accuracy for the low-dimensional datasets

Table 1. Accuracy score on 'Banknote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_(log(p)/n)^(1/2)                        | 0.54(0.05) | 0.88(0.02) | 0.98(0.04) | 0.99(0)    | 0.96(0.08) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.54(0.05) | 0.88(0.02) | 0.98(0.04) | 0.99(0)    | 0.96(0.08) |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.53(0.04) | 0.85(0.02) | 0.85(0.02) | 0.87(0.02) | 0.89(0.02) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.98(0.04) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.98(0.04) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.99(0.01) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.98(0.04) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.98(0.04) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.99(0.01) |
| Naive   | 0.46(0.02) | 0.47(0.02) | 0.69(0.03) | 0.93(0.01) | 0.99(0.01) |
| Oracle  | 0.99(0)    | 0.99(0)    | 0.99(0)    | 0.99(0)    | 0.99(0)    |

Table 2. Accuracy score on 'Breastc' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.54(0.05) | 0.83(0.17) | 0.93(0.03) | 0.95(0.02) | 0.92(0.11) |
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)\_dtl=0$             | 0.54(0.05) | 0.82(0.18) | 0.93(0.03) | 0.95(0.02) | 0.91(0.11) |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.54(0.05) | 0.55(0.05) | 0.92(0.05) | 0.95(0.02) | 0.93(0.02) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.3$      | 0.58(0.08) | 0.68(0.17) | 0.94(0.03) | 0.94(0.02) | 0.75(0.2)  |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.58(0.08) | 0.68(0.17) | 0.94(0.02) | 0.94(0.02) | 0.75(0.2)  |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.58(0.08) | 0.68(0.17) | 0.94(0.03) | 0.94(0.02) | 0.75(0.2)  |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.58(0.08) | 0.68(0.17) | 0.94(0.02) | 0.94(0.03) | 0.75(0.2)  |
| Naive   | 0.55(0.04) | 0.55(0.04) | 0.72(0.06) | 0.9(0.03)  | 0.95(0.02) |
| Oracle  | 0.95(0.02) | 0.95(0.02) | 0.95(0.02) | 0.95(0.02) | 0.95(0.02) |

Table 3. Accuracy score on 'Credit\_a' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                  | 0.45(0.04) | 0.61(0.2)  | 0.85(0.07) | 0.86(0.03) | 0.86(0.03) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.45(0.04) | 0.61(0.2)  | 0.85(0.07) | 0.86(0.03) | 0.86(0.03) |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.45(0.04) | 0.46(0.04) | 0.82(0.13) | 0.86(0.03) | 0.87(0.03) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.46(0.04) | 0.72(0.12) | 0.82(0.09) | 0.82(0.06) | 0.69(0.11) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.46(0.04) | 0.71(0.11) | 0.82(0.09) | 0.83(0.07) | 0.7(0.12)  |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.46(0.04) | 0.73(0.11) | 0.82(0.09) | 0.84(0.04) | 0.7(0.11)  |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.47(0.07) | 0.71(0.11) | 0.82(0.09) | 0.84(0.05) | 0.7(0.11)  |
| Naive   | 0.45(0.03) | 0.5(0.04)  | 0.67(0.04) | 0.82(0.03) | 0.86(0.03) |
| Oracle  | 0.87(0.03) | 0.87(0.03) | 0.87(0.02) | 0.87(0.03) | 0.87(0.02) |

Table 4. Accuracy score on 'Credit\_g' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                  | 0.67(0.1)  | 0.68(0.09) | 0.41(0.17) | 0.72(0.06) | 0.73(0.04) |
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)\_dtl=0$             | 0.67(0.1)  | 0.67(0.1)  | 0.48(0.19) | 0.72(0.03) | 0.73(0.04) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$                    | 0.67(0.1)  | 0.68(0.08) | 0.3(0.03)  | 0.3(0.06)  | 0.69(0.07) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.3$      | 0.49(0.19) | 0.52(0.15) | 0.6(0.14)  | 0.75(0.03) | 0.75(0.04) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.53(0.19) | 0.54(0.14) | 0.61(0.13) | 0.75(0.03) | 0.75(0.04) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.51(0.19) | 0.54(0.14) | 0.6(0.15)  | 0.75(0.03) | 0.75(0.04) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.48(0.19) | 0.53(0.13) | 0.61(0.13) | 0.75(0.03) | 0.75(0.04) |
| Naive   | 0.3(0.02)  | 0.31(0.03) | 0.45(0.03) | 0.65(0.03) | 0.76(0.03) |
| Oracle  | 0.77(0.02) | 0.77(0.02) | 0.77(0.03) | 0.77(0.03) | 0.77(0.02) |

 ${\bf Table~5.}~{\rm Accuracy~score~on~'dhfr'~dataset}$ 

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                  | 0.54(0.13) | 0.61(0.1)  | 0.63(0.07) | 0.76(0.08) | 0.84(0.04) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.54(0.13) | 0.61(0.1)  | 0.63(0.07) | 0.77(0.07) | 0.84(0.04) |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.54(0.13) | 0.61(0.1)  | 0.63(0.07) | 0.62(0.05) | 0.62(0.05) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.46(0.13) | 0.77(0.18) | 0.8(0.12)  | 0.77(0.1)  | 0.86(0.07) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.5$      | 0.45(0.14) | 0.76(0.18) | 0.81(0.1)  | 0.76(0.1)  | 0.86(0.07) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.46(0.13) | 0.75(0.19) | 0.79(0.12) | 0.76(0.1)  | 0.87(0.05) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.46(0.13) | 0.74(0.2)  | 0.79(0.13) | 0.77(0.09) | 0.85(0.08) |
| Naive   | 0.46(0.06) | 0.51(0.06) | 0.61(0.05) | 0.73(0.05) | 0.86(0.04) |
| Oracle  | 0.92(0.03) | 0.92(0.03) | 0.92(0.03) | 0.92(0.03) | 0.92(0.03) |

Table 6. Accuracy score on 'Diabetes' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_(log(p)/n)^(1/2)                        | 0.65(0.03) | 0.7(0.08)  | 0.75(0.03) | 0.74(0.09) | 0.36(0.05) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.65(0.03) | 0.7(0.08)  | 0.75(0.03) | 0.74(0.1)  | 0.36(0.05) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$                    | 0.65(0.03) | 0.35(0.03) | 0.36(0.07) | 0.63(0.18) | 0.74(0.07) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.44(0.13) | 0.74(0.03) | 0.67(0.13) | 0.69(0.15) | 0.36(0.05) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.44(0.13) | 0.74(0.03) | 0.67(0.13) | 0.68(0.15) | 0.37(0.07) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.44(0.13) | 0.74(0.03) | 0.67(0.12) | 0.67(0.16) | 0.36(0.05) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.43(0.12) | 0.74(0.03) | 0.67(0.12) | 0.68(0.15) | 0.37(0.08) |
| Naive   | 0.35(0.03) | 0.35(0.03) | 0.44(0.04) | 0.7(0.04)  | 0.76(0.03) |
| Oracle  | 0.78(0.02) | 0.78(0.02) | 0.78(0.02) | 0.78(0.03) | 0.78(0.03) |

Table 7. Accuracy score on 'Spambase' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.61(0.01) | 0.66(0.21) | 0.87(0.11) | 0.62(0.26) | 0.61(0.22) |
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)\_dtl=0$             | 0.61(0.01) | 0.66(0.21) | 0.86(0.12) | 0.61(0.25) | 0.59(0.21) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)                   | 0.61(0.01) | 0.61(0.01) | 0.39(0.01) | 0.39(0.01) | 0.39(0.01) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.83(0.02) | 0.88(0.01) | 0.88(0.12) | 0.4(0.01)  | 0.62(0.26) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.5$      | 0.82(0.04) | 0.88(0.02) | 0.88(0.12) | 0.4(0.01)  | 0.62(0.26) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.82(0.04) | 0.88(0.02) | 0.9(0.1)   | 0.4(0.01)  | 0.62(0.27) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.82(0.05) | 0.88(0.02) | 0.87(0.14) | 0.4(0.01)  | 0.62(0.26) |
| Naive   | 0.39(0.01) | 0.4(0.01)  | 0.52(0.02) | 0.86(0.01) | 0.91(0.01) |
| Oracle  | 0.92(0.01) | 0.92(0.01) | 0.92(0.01) | 0.92(0.01) | 0.92(0.01) |

 ${\bf Table~8.}~{\bf Accuracy~score~on~'Vote'~dataset}$ 

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.68(0.06) | 0.79(0.24) | 0.56(0.31) | 0.93(0.07) | 0.92(0.07) |
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)\_dtl=0$             | 0.68(0.06) | 0.76(0.25) | 0.59(0.31) | 0.93(0.08) | 0.93(0.03) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$                    | 0.68(0.06) | 0.64(0.14) | 0.68(0.07) | 0.92(0.1)  | 0.93(0.03) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.3$      | 0.57(0.11) | 0.73(0.24) | 0.87(0.09) | 0.92(0.03) | 0.91(0.06) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 0.57(0.11) | 0.73(0.24) | 0.88(0.06) | 0.92(0.03) | 0.91(0.06) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.57(0.12) | 0.73(0.25) | 0.88(0.09) | 0.92(0.03) | 0.91(0.04) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.57(0.11) | 0.71(0.25) | 0.86(0.12) | 0.92(0.03) | 0.91(0.03) |
| Naive   | 0.36(0.05) | 0.44(0.06) | 0.56(0.06) | 0.83(0.04) | 0.9(0.03)  |
| Oracle  | 0.93(0.03) | 0.93(0.03) | 0.93(0.03) | 0.93(0.03) | 0.93(0.02) |

#### 4 Furmańczyk, K. et al.

Table 9. Accuracy score on 'Wdbc' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.38(0.04) | 0.92(0.02) | 0.93(0.03) | 0.94(0.02) | 0.81(0.22) |
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)\_dtl=0$             | 0.38(0.04) | 0.92(0.02) | 0.93(0.03) | 0.94(0.02) | 0.8(0.23)  |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.38(0.04) | 0.38(0.04) | 0.71(0.26) | 0.93(0.02) | 0.82(0.21) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.3$ | 0.79(0.18) | 0.9(0.12)  | 0.93(0.08) | 0.91(0.12) | 0.48(0.15) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.5$      | 0.79(0.16) | 0.9(0.12)  | 0.93(0.08) | 0.89(0.15) | 0.49(0.13) |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 0.81(0.15) | 0.91(0.08) | 0.89(0.17) | 0.9(0.15)  | 0.47(0.12) |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 0.8(0.17)  | 0.91(0.08) | 0.93(0.09) | 0.9(0.14)  | 0.48(0.14) |
| Naive   | 0.39(0.03) | 0.43(0.03) | 0.63(0.05) | 0.86(0.04) | 0.94(0.02) |
| Oracle  | 0.97(0.01) | 0.97(0.01) | 0.97(0.01) | 0.97(0.01) | 0.97(0.01) |

Table 10. Method avg.rank based on Accuracy metrics

| method  | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|---|------|------|------|------|------|
| Oracle  | 1.00 | 1.44 | 1.44 | 2.22 | 2.00 |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.3$      | 3.89 | 3.56 | 3.33 | 4.56 | 5.00 |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                  | 4.44 | 4.22 | 5.00 | 2.56 | 5.44 |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 0.5$ | 4.89 | 4.11 | 3.44 | 5.78 | 4.78 |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 5.44 | 5.33 | 5.56 | 3.11 | 6.00 |
| LassoJoint_BFGS_lambda.min delta_to_lambda = $0.7$      | 5.00 | 4.33 | 4.56 | 6.22 | 6.11 |
| $LassoJoint\_BFGS\_lambda.min\ delta\_to\_lambda = 1$   | 5.67 | 5.56 | 5.44 | 6.56 | 6.67 |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 6.44 | 7.67 | 7.67 | 6.67 | 5.33 |
| Naive   | 8.22 | 8.78 | 8.56 | 7.33 | 3.67 |

Table 11. No. of features in lambda-fixed scenarios; dataset 'Banknote'

| dataset method                                  | c    | no_features |
|---|------|-------------|
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)       | 0.10 | 0.01        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.30 | 2.00        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.50 | 2.87        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.70 | 3.00        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.90 | 3.00        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)_dtl=0 | 0.10 | 0.01        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)_dtl=0 | 0.30 | 2.00        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)_dtl=0 | 0.50 | 2.88        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)_dtl=0 | 0.70 | 3.00        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/2)_dtl=0 | 0.90 | 3.00        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$   | 0.10 | 0.00        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$   | 0.30 | 1.00        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/3)       | 0.50 | 1.00        |
| Banknote LassoJoint_BFGS_(log(p)/n)^(1/3)       | 0.70 | 1.39        |
| Banknote LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$ | 0.90 | 2.00        |

Table 12. No. of features in lambda-fixed scenarios; dataset 'Breastc'

| dataset method   | c    | no_features |
|--|------|-------------|
| Breastc LassoJoint_BFGS_(log(p)/n)^(1/2)               | 0.10 | 0.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$         | 0.30 | 2.91        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$         | 0.50 | 5.45        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$         | 0.70 | 5.55        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$         | 0.90 | 6.01        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dtl}=0$ | 0.10 | 0.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)_dtl}=0$   | 0.30 | 3.11        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dtl}=0$ | 0.50 | 5.69        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dtl}=0$ | 0.70 | 5.64        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dtl}=0$ | 0.90 | 6.11        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$         | 0.10 | 0.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$         | 0.30 | 0.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$         | 0.50 | 2.60        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$         | 0.70 | 4.07        |
| Breastc LassoJoint_BFGS_( $log(p)/n$ )^(1/3)           | 0.90 | 3.04        |

 ${\bf Table~13.~No.~of~features~in~lambda-fixed~scenarios;~dataset~'Credit\_a'}$ 

| dataset method   | $^{\mathrm{c}}$ | $no\_features$ |
|--|-----------------|----------------|
| $\overline{\text{Credit\_a LassoJoint\_BFGS\_(log(p)/n)^(1/2)}}$ | 0.10            | 0.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.30            | 1.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.50            | 1.01           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.70            | 1.02           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$                    | 0.90            | 2.32           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.10            | 0.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.30            | 1.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.50            | 1.02           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$              | 0.70            | 1.05           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_dtl=0$            | 0.90            | 2.34           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$                    | 0.10            | 0.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.30            | 0.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$                    | 0.50            | 0.93           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.70            | 1.00           |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$                  | 0.90            | 1.00           |

 ${\bf Table~14.~No.~of~features~in~lambda-fixed~scenarios;~dataset~'Credit\_g'}$ 

| dataset method   | c n  | o_features |
|--|------|------------|
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$        | 0.10 | 0.00       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)         | 0.30 | 0.03       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)         | 0.50 | 1.33       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)         | 0.70 | 2.26       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$        | 0.90 | 3.04       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_dtl=0$  | 0.10 | 0.00       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$    | 0.30 | 0.12       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dt}=0$ | 0.50 | 1.80       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_dtl=0$  | 0.70 | 2.41       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_dtl=0$  | 0.90 | 3.10       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$        | 0.10 | 0.00       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)         | 0.30 | 0.00       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)         | 0.50 | 0.00       |
| Credit_g LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)         | 0.70 | 0.00       |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$        | 0.90 | 0.02       |

 ${\bf Table~15.~No.~of~features~in~lambda-fixed~scenarios;~dataset~'dhfr'}$ 

| 1       | 41 1                                       |      |             |
|---------|--|------|-------------|
| dataset | method                                     | С    | no_features |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.10 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     | 0.30 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.50 | 0.00        |
| dhfr    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.70 | 1.05        |
| dhfr    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.90 | 3.92        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.10 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.30 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.50 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.70 | 1.26        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.90 | 4.32        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.10 | 0.00        |
| dhfr    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)      | 0.30 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.50 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.70 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.90 | 0.00        |

Table 16. No. of features in lambda-fixed scenarios; dataset 'Diabetes'

| dataset method                                       | c    | no_features |
|--|------|-------------|
| Diabetes LassoJoint_BFGS_(log(p)/n)^(1/2)            | 0.10 | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.30 | 0.99        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.50 | 1.42        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$        | 0.70 | 2.55        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.90 | 3.94        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}dtl=0$ | 0.10 | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$  | 0.30 | 0.99        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}dtl=0$ | 0.50 | 1.55        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}dtl=0$ | 0.70 | 2.65        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}dtl=0$ | 0.90 | 4.01        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.10 | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.30 | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$        | 0.50 | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.70 | 0.73        |
| Diabetes LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)       | 0.90 | 1.00        |

Table 17. No. of features in lambda-fixed scenarios; dataset 'Spambase'

| dataset method                                       | <b>c</b> 1 | no_features |
|--|------------|-------------|
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$        | 0.10       | 0.00        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.30       | 7.15        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.50       | 15.03       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.70       | 18.91       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$      | 0.90       | 22.66       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dt}$ | 0.10       | 0.00        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dt}$ | 0.30       | 7.18        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dt}$ | 0.50       | 15.29       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}_{dt}$ | 0.0.70     | 19.14       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}dt =$  | 0.90       | 22.80       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.10       | 0.00        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.30       | 0.00        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.50       | 0.01        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.70       | 3.35        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/3)}$      | 0.90       | 4.92        |

Table 18. No. of features in lambda-fixed scenarios; dataset 'Vote'

| dataset | method                                     | c    | no_features |
|---------|--|------|-------------|
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.10 | 0.00        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.30 | 2.35        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.50 | 1.41        |
| Vote    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.70 | 2.04        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.90 | 2.47        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.10 | 0.00        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.30 | 2.55        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.50 | 1.45        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.70 | 2.33        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.90 | 2.62        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.10 | 0.00        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.30 | 0.00        |
| Vote    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)      | 0.50 | 0.01        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.70 | 1.16        |
| Vote    | $LassoJoint\_BFGS\_(log(p)/n)^{}(1/3)$     | 0.90 | 1.42        |

 ${\bf Table~19.~No.~of~features~in~lambda-fixed~scenarios;~dataset~'Wdbc'}$ 

| dataset | method                                     | c    | no_features |
|---------|--|------|-------------|
| Wdbc    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.10 | 0.00        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.30 | 1.87        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | 0.50 | 2.60        |
| Wdbc    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.70 | 4.11        |
| Wdbc    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)      | 0.90 | 3.70        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.10 | 0.00        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.30 | 1.96        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.50 | 2.63        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.70 | 4.23        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)_dtl=0$ | 0.90 | 3.73        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.10 | 0.00        |
| Wdbc    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/3)      | 0.30 | 0.00        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.50 | 0.63        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.70 | 2.12        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/3)$       | 0.90 | 2.51        |

# 3 Results of the joint-wise methods for the low-dimensional datasets

Table 20. Accuracy score on 'Artif' dataset

| method                             | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|------------------------------------|------------|------------|------------|------------|------------|
| $LassoJoint_BFGS_(log(p)/n)^(1/2)$ | 0.52(0.02) | 0.78(0.05) | 0.89(0.02) | 0.88(0.04) | 0.86(0.08) |
| $LassoJoint\_BFGS\_lambda.1se$     | 0.55(0.09) | 0.88(0.01) | 0.89(0.02) | 0.88(0.04) | 0.87(0.08) |
| $LassoJoint\_BFGS\_lambda.min$     | 0.82(0.03) | 0.87(0.02) | 0.88(0.02) | 0.81(0.14) | 0.75(0.16) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$   |            |            |            |            | 0.89(0.01) |
| $LassoJoint\_MM\_lambda.1se$       |            |            |            |            | 0.89(0.01) |
| $LassoJoint\_MM\_lambda.min$       | 0.73(0.04) | 0.85(0.02) | 0.86(0.09) | 0.88(0.02) | 0.88(0.01) |
| MIF5 Joint BFGS                    | 0.75(0.06) | 0.87(0.02) | 0.89(0.01) | 0.88(0.06) | 0.61(0.12) |
| MIF5 Joint MM                      | 0.72(0.04) | 0.85(0.02) | 0.89(0.01) | 0.89(0.01) | 0.89(0.01) |

Table 21. Accuracy score on 'Banknote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.55(0.08) | 0.87(0.02) | 0.99(0.02) | 0.99(0)    | 0.98(0.05) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.57(0.11) | 0.99(0.03) | 0.99(0.01) | 0.99(0)    | 0.98(0.06) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.97(0.02) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.98(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.48(0.09) | 0.86(0.02) | 0.99(0.02) | 0.99(0.01) | 1(0)       |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.5(0.13)  | 0.95(0.05) | 0.99(0.01) | 0.99(0.01) | 1(0)       |
| LassoJoint_MM_lambda.min                          | 0.97(0.01) | 0.98(0.01) | 0.99(0.01) | 0.99(0.01) | 1(0)       |
| MIF5 Joint BFGS                                   | 0.99(0.02) | 0.99(0.01) | 0.99(0)    | 0.99(0)    | 0.95(0.09) |
| MIF5 Joint MM                                     | 0.94(0.04) | 0.98(0.01) | 0.99(0.01) | 0.99(0.01) | 1(0)       |

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.55(0.05) | 0.9(0.06)  | 0.88(0.09) | 0.95(0.02) | 0.85(0.17) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.59(0.1)  | 0.9(0.07)  | 0.88(0.1)  | 0.95(0.02) | 0.83(0.18) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.9(0.08)  | 0.91(0.04) | 0.87(0.11) | 0.94(0.03) | 0.84(0.18) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.55(0.05) | 0.92(0.03) | 0.89(0.07) | 0.95(0.02) | 0.95(0.02) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.59(0.11) | 0.93(0.03) | 0.89(0.07) | 0.95(0.02) | 0.95(0.02) |
| LassoJoint_MM_lambda.min                          | 0.91(0.04) | 0.93(0.03) | 0.88(0.08) | 0.95(0.02) | 0.95(0.02) |
| MIF5 Joint BFGS                                   | 0.86(0.05) | 0.9(0.04)  | 0.91(0.06) | 0.91(0.04) | 0.76(0.09) |
| MIF5 Joint MM                                     | 0.86(0.05) | 0.89(0.04) | 0.92(0.05) | 0.91(0.03) | 0.92(0.03) |

Table 23. Accuracy score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.45(0.04) | 0.48(0.11) | 0.83(0.11) | 0.86(0.03) | 0.86(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.45(0.04) | 0.48(0.11) | 0.83(0.11) | 0.86(0.03) | 0.74(0.13) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.53(0.14) | 0.73(0.18) | 0.82(0.1)  | 0.8(0.11)  | 0.67(0.11) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.45(0.04) | 0.87(0.03) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | 0.45(0.04) | 0.85(0.09) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.62(0.17) | 0.84(0.05) | 0.84(0.03) | 0.85(0.04) | 0.86(0.03) |
| MIF5 Joint BFGS                       | 0.66(0.19) | 0.85(0.04) | 0.84(0.05) | 0.86(0.03) | 0.86(0.03) |
| MIF5 Joint MM                         | 0.82(0.1)  | 0.85(0.03) | 0.85(0.03) | 0.85(0.03) | 0.85(0.03) |

Table 24. Accuracy score on 'Credit\_g' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.7(0.03)  | 0.64(0.16) | 0.57(0.17) | 0.64(0.13) | 0.72(0.07) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.7(0.03)  | 0.66(0.14) | 0.62(0.16) | 0.66(0.13) | 0.73(0.04) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.68(0.08) | 0.64(0.17) | 0.67(0.1)  | 0.73(0.04) | 0.73(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.3(0.03)  | 0.31(0.08) | 0.64(0.04) | 0.67(0.05) | 0.74(0.02) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.3(0.03)  | 0.29(0.03) | 0.67(0.06) | 0.7(0.04)  | 0.75(0.02) |
| $LassoJoint\_MM\_lambda.min$                      | 0.32(0.08) | 0.67(0.11) | 0.67(0.05) | 0.73(0.04) | 0.75(0.02) |
| MIF5 Joint BFGS                                   | 0.53(0.17) | 0.43(0.18) | 0.52(0.18) | 0.73(0.03) | 0.74(0.03) |
| MIF5 Joint MM                                     | 0.53(0.08) | 0.54(0.13) | 0.59(0.1)  | 0.73(0.03) | 0.74(0.03) |

Table 25. Accuracy score on 'dhfr' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.55(0.12) | 0.58(0.09) | 0.71(0.1)  | 0.78(0.02) | 0.85(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$                    |            |            |            |            | 0.88(0.05) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.49(0.08) | 0.74(0.05) | 0.82(0.04) | 0.74(0.09) | 0.83(0.1)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.39(0.05) | 0.42(0.09) | 0.71(0.1)  | 0.78(0.03) | 0.86(0.03) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.39(0.05) | 0.5(0.18)  | 0.8(0.05)  | 0.85(0.03) | 0.89(0.01) |
| $LassoJoint\_MM\_lambda.min$                      | 0.49(0.19) | 0.66(0.07) | 0.81(0.04) | 0.84(0.03) | 0.87(0.04) |
| MIF5 Joint BFGS                                   | 0.51(0.19) | 0.75(0.07) | 0.78(0.05) | 0.71(0.19) | 0.73(0.1)  |
| MIF5 Joint MM                                     | 0.51(0.19) | 0.75(0.07) | 0.74(0.05) | 0.73(0.17) | 0.76(0.09) |

Table 26. Accuracy score on 'Diabetes' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.35(0.04) | 0.74(0.03) | 0.76(0.04) | 0.64(0.18) | 0.36(0.09) |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.35(0.04) | 0.45(0.17) | 0.76(0.04) | 0.64(0.18) | 0.36(0.08) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.61(0.1)  | 0.75(0.03) | 0.76(0.04) | 0.57(0.2)  | 0.36(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.35(0.04) | 0.73(0.04) | 0.77(0.04) | 0.75(0.03) | 0.76(0.03) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.35(0.04) | 0.4(0.13)  | 0.76(0.07) | 0.75(0.03) | 0.76(0.03) |
| LassoJoint_MM_lambda.min                          | 0.61(0.07) | 0.74(0.04) | 0.76(0.03) | 0.75(0.03) | 0.77(0.03) |
| MIF5 Joint BFGS                                   | 0.73(0.03) | 0.74(0.04) | 0.75(0.03) | 0.77(0.03) | 0.36(0.04) |
| MIF5 Joint MM                                     | 0.7(0.07)  | 0.75(0.04) | 0.75(0.03) | 0.77(0.03) | 0.75(0.03) |

 ${\bf Table~27.~Accuracy~score~on~'Spambase'~dataset}$ 

| method                                 | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.61(0.01) | 0.43(0.12) | 0.69(0.25) | 0.5(0.21)  | 0.7(0.22)  |
| $LassoJoint\_BFGS\_lambda.1se$         | 0.69(0.16) | 0.86(0.09) | 0.85(0.16) | 0.41(0.07) | 0.67(0.23) |
| $LassoJoint\_BFGS\_lambda.min$         | 0.82(0.03) | 0.88(0.02) | 0.86(0.14) | 0.4(0.07)  | 0.67(0.25) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$       | 0.39(0.01) | 0.44(0.12) | 0.86(0.05) | 0.89(0.01) | 0.91(0.01) |
| $Lasso Joint\_MM\_lambda.1se$          | 0.51(0.13) | 0.84(0.05) | 0.89(0.01) | 0.9(0.01)  | 0.92(0.01) |
| $LassoJoint\_MM\_lambda.min$           | 0.62(0.05) | 0.84(0.02) | 0.89(0.01) | 0.9(0.01)  | 0.92(0.01) |
| MIF5 Joint BFGS                        | 0.43(0.05) | 0.58(0.06) | 0.51(0.07) | 0.59(0.04) | 0.52(0.04) |
| MIF5 Joint MM                          | 0.43(0.03) | 0.58(0.06) | 0.51(0.07) | 0.82(0.02) | 0.84(0.01) |

Table 28. Accuracy score on 'Vote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.68(0.07) | 0.4(0.22)  | 0.92(0.03) | 0.88(0.18) | 0.93(0.03) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0.68(0.07) | 0.51(0.29) | 0.92(0.03) | 0.93(0.04) | 0.93(0.03) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.77(0.14) | 0.66(0.27) | 0.89(0.05) | 0.93(0.03) | 0.9(0.04)  |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ |            | 0.95(0.03) |            |            |            |
| LassoJoint_MM_lambda.1se              | 0.32(0.07) | 0.93(0.09) | 0.92(0.04) | 0.93(0.03) | 0.93(0.03) |
| $Lasso Joint\_MM\_lambda.min$         | 0.64(0.19) | 0.67(0.2)  | 0.89(0.06) | 0.93(0.04) | 0.92(0.03) |

Table 29. Accuracy score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.36(0.04) | 0.92(0.02) | 0.93(0.03) | 0.94(0.02) | 0.67(0.28) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.36(0.04) | 0.93(0.03) | 0.94(0.02) | 0.91(0.12) | 0.63(0.17) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.77(0.1)  | 0.71(0.25) | 0.93(0.03) | 0.67(0.24) | 0.58(0.16) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.36(0.04) | 0.9(0.03)  | 0.92(0.03) | 0.94(0.02) | 0.96(0.02) |
| LassoJoint_MM_lambda.1se              | 0.36(0.04) | 0.9(0.04)  | 0.93(0.03) | 0.94(0.02) | 0.97(0.02) |
| LassoJoint_MM_lambda.min              | 0.64(0.08) | 0.72(0.21) | 0.88(0.06) | 0.91(0.04) | 0.97(0.02) |
| MIF5 Joint BFGS                       | 0.82(0.18) | 0.92(0.03) | 0.93(0.02) | 0.9(0.1)   | 0.52(0.24) |
| MIF5 Joint MM                         | 0.73(0.14) | 0.9(0.03)  | 0.93(0.02) | 0.92(0.03) | 0.95(0.02) |

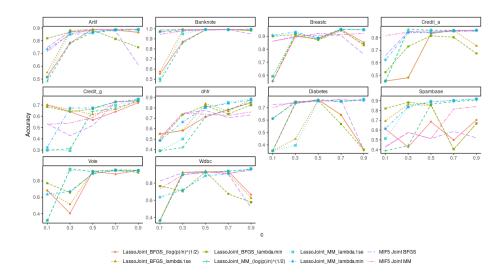


Fig. 2. The accuracy for the test datasets

### 4 Results for the low-dimensional datasets

Table 30. Accuracy score on 'Banknote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.76(0.06) | 0.71(0.1)  | 0.88(0.01) | 0.97(0.02) | 0.99(0.01) |
| $AdaS\_svm$                           | 0.82(0.05) | 0.8(0.03)  | 0.95(0)    | 0.97(0.01) | 1(0)       |
| Joint BFGS                            | 1(0)       | 1(0)       | 0.98(0)    | 1(0)       | 1(0.01)    |
| Joint MM                              | 0.96(0.01) | 0.97(0)    | 0.96(0)    | 0.97(0.01) | 0.99(0)    |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.91(0.27) | 0.87(0.05) | 1(0.02)    | 1(0)       | 0.98(0.08) |
| LassoJoint_BFGS_lambda.1se            | 0.94(0.12) | 1(0.01)    | 0.98(0.01) | 1(0)       | 0.99(0.03) |
| LassoJoint_BFGS_lambda.min            | 1(0.01)    | 1(0)       | 0.98(0.01) | 1(0)       | 0.99(0.03) |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | 0.04(0.17) | 0.81(0.04) | 0.98(0.03) | 0.98(0.01) | 0.99(0.01) |
| LassoJoint_MM_lambda.1se              | 0.39(0.31) | 0.96(0.03) | 0.97(0.01) | 0.98(0.01) | 0.99(0)    |
| LassoJoint_MM_lambda.min              | 0.95(0.01) | 0.97(0.01) | 0.97(0.01) | 0.98(0.01) | 0.99(0)    |
| Naive                                 | 0(0)       | 0(0)       | 0.4(0.01)  | 0.88(0.01) | 0.98(0)    |
| Oracle                                | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0.01)    |
| Weighted BFGS                         | 0.98(0.02) | 0.98(0.01) | 0.97(0)    | 0.99(0)    | 1(0.01)    |
|                                       |            |            |            |            |            |

 ${\bf Table~31.}~{\bf Accuracy~score~on~'Breastc'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.56(0.15) | 0.75(0.01) | 0.92(0.04) | 0.95(0.02) | 0.88(0.04) |
| AdaS_svm                              | 0.64(0.16) | 0.75(0.03) | 0.79(0.03) | 0.91(0.02) | 0.87(0.05) |
| Joint BFGS                            | 0.56(0.39) | 0.82(0.09) | 0.96(0.02) | 0.96(0.01) | 0.56(0.4)  |
| Joint MM                              | 0.73(0.06) | 0.69(0.07) | 0.94(0.03) | 0.95(0.01) | 0.87(0.05) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.89(0.14) | 0.75(0.2)  | 0.94(0.04) | 0.71(0.4)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.35(0.34) | 0.86(0.05) | 0.88(0.1)  | 0.95(0.02) | 0.59(0.35) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.84(0.09) | 0.87(0.02) | 0.9(0.11)  | 0.95(0.02) | 0.82(0.17) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.88(0.07) | 0.77(0.14) | 0.94(0.04) | 0.93(0.04) |
| LassoJoint_MM_lambda.1se              | 0.4(0.31)  | 0.82(0.03) | 0.87(0.07) | 0.95(0.02) | 0.88(0.05) |
| LassoJoint_MM_lambda.min              | 0.78(0.08) | 0.78(0.05) | 0.88(0.08) | 0.95(0.02) | 0.88(0.05) |
| Naive                                 | 0(0.01)    | 0(0)       | 0.43(0.07) | 0.88(0.03) | 0.84(0.05) |
| Oracle                                | 0.97(0.04) | 0.97(0.01) | 0.97(0.01) | 0.97(0.02) | 0.97(0.05) |
| Weighted BFGS                         | 0.48(0.34) | 0.84(0.1)  | 0.93(0.04) | 0.94(0.01) | 0.57(0.41) |

Table 32. Accuracy score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.34(0.02) | 0.66(0.01) | 0.75(0.06) | 0.71(0.07) | 0.87(0.03) |
| AdaS_svm                              | 0.16(0.02) | 0.65(0.02) | 0.76(0.05) | 0.77(0.03) | 0.83(0.03) |
| Joint BFGS                            | 0(0)       | 0.02(0.14) | 0.39(0.42) | 0.86(0.04) | 0.29(0.3)  |
| Joint MM                              | 0.22(0.02) | 0.16(0.11) | 0.8(0.05)  | 0.81(0.02) | 0.91(0.04) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.05(0.2)  | 0.75(0.2)  | 0.8(0.04)  | 0.79(0.04) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0(0)       | 0.58(0.21) | 0.28(0.3)  | 0.84(0.06) | 0.73(0.16) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.73(0.23) | 0.6(0.13)  | 0.56(0.33) | 0.83(0.1)  | 0.55(0.09) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.8(0.04)  | 0.8(0.04)  | 0.8(0.04)  | 0.79(0.04) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.64(0.08) | 0.82(0.04) | 0.8(0.03)  | 0.83(0.04) |
| LassoJoint_MM_lambda.min              | 0.33(0.12) | 0.64(0.05) | 0.82(0.05) | 0.81(0.03) | 0.87(0.05) |
| Naive                                 | 0(0)       | 0.11(0.01) | 0.36(0.07) | 0.52(0.07) | 0.89(0.04) |
| Oracle                                | 0.92(0)    | 0.92(0.01) | 0.92(0.04) | 0.92(0.07) | 0.92(0.03) |
| Weighted BFGS                         | 0(0)       | 0.02(0.14) | 0.36(0.39) | 0.86(0.04) | 0.46(0.11) |

Table 33. Accuracy score on 'Credit\_g' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.56(0.1)  | 0.45(0.05) | 0.53(0.04) | 0.58(0.01) | 0.64(0.03) |
| AdaS_svm                              | 0.34(0.15) | 0.38(0.09) | 0.47(0.05) | 0.61(0.01) | 0.65(0.05) |
| Joint BFGS                            | 0.65(0.05) | 0.76(0.05) | 0.82(0.24) | 0.77(0.02) | 0.75(0.06) |
| Joint MM                              | 0.45(0.06) | 0.66(0.05) | 0.75(0.13) | 0.66(0.03) | 0.8(0.04)  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.84(0.37) | 0.53(0.35) | 0.65(0.26) | 0.81(0.13) |
| $LassoJoint\_BFGS\_lambda.1se$        | 1(0)       | 0.61(0.43) | 0.43(0.33) | 0.23(0.18) | 0.78(0.03) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.36(0.25) | 0.42(0.3)  | 0.67(0.22) | 0.6(0.08)  | 0.75(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.03(0.12) | 0.59(0.09) | 0.69(0.12) | 0.83(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0(0)       | 0.61(0.09) | 0.6(0.06)  | 0.81(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.15(0.14) | 0.56(0.2)  | 0.7(0.12)  | 0.63(0.07) | 0.81(0.04) |
| Naive                                 | 0(0)       | 0.01(0.01) | 0.2(0.04)  | 0.6(0.02)  | 0.8(0.04)  |
| Oracle                                | 0.91(0.01) | 0.91(0.04) | 0.91(0.02) | 0.91(0)    | 0.91(0.03) |
| Weighted BFGS                         | 0.68(0.06) | 0.79(0.06) | 0.79(0.23) | 0.75(0.01) | 0.79(0.04) |

Table 34. Accuracy score on 'dhfr' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.72(0.13) | 0.54(0.09) | 0.77(0.11) | 0.83(0.04) | 0.88(0.01) |
| AdaS_svm                              | 0.04(0.04) | 0.19(0.31) | 0.62(0.16) | 0.79(0.06) | 0.74(0.02) |
| Joint BFGS                            | 0.21(0.1)  | 0.25(0.11) | 0.46(0.22) | 0.67(0.06) | 0.74(0.02) |
| Joint MM                              | 0.3(0.04)  | 0.41(0.06) | 0.53(0.08) | 0.64(0.08) | 0.62(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.6(0.55)  | 1(0)       | 0.69(0.15) | 0.89(0.04) | 0.9(0.03)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.74(0.43) | 0.8(0.14)  | 0.74(0.17) | 0.93(0.03) | 0.88(0.01) |
| LassoJoint_BFGS_lambda.min            | 0.28(0.18) | 0.37(0.31) | 0.65(0.22) | 0.71(0.13) | 0.76(0.02) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0(0)       | 0.69(0.14) | 0.88(0.03) | 0.91(0.04) |
| $LassoJoint\_MM\_lambda.1se$          | 0(0)       | 0.48(0.29) | 0.79(0.03) | 0.93(0.03) | 0.83(0.01) |
| LassoJoint_MM_lambda.min              | 0.13(0.1)  | 0.36(0.18) | 0.69(0.09) | 0.82(0.11) | 0.72(0.03) |
| Naive                                 | 0.17(0.03) | 0.26(0.04) | 0.49(0.06) | 0.65(0.03) | 0.72(0.01) |
| Oracle                                | 0.94(0.04) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0)    |
| Weighted BFGS                         | 0.21(0.06) | 0.33(0.11) | 0.53(0.12) | 0.62(0.04) | 0.74(0.03) |

 ${\bf Table~35.}~{\rm Accuracy~score~on~'Diabetes'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.49(0.05) | 0.59(0.01) | 0.61(0.01) | 0.69(0.02) | 0.76(0.05) |
| $AdaS_sym$                            | 0.2(0.18)  | 0.51(0.02) | 0.55(0)    | 0.67(0.02) | 0.7(0.05)  |
| Joint BFGS                            | 0.89(0.04) | 0.72(0.02) | 0.91(0.04) | 0.8(0.14)  | 0.44(0.42) |
| Joint MM                              | 0.78(0.01) | 0.69(0.02) | 0.89(0.02) | 0.81(0.05) | 0.9(0.03)  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.88(0.05) | 0.87(0.05) | 0.59(0.35) | 0.04(0.17) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.06(0.17) | 0.91(0.02) | 0.8(0.17)  | 0.68(0.23) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.82(0.15) | 0.73(0.05) | 0.89(0.03) | 0.71(0.18) | 0.18(0.3)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.82(0.05) | 0.86(0.05) | 0.8(0.04)  | 0.87(0.04) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.57(0.2)  | 0.9(0.05)  | 0.87(0.05) | 0.89(0.03) |
| LassoJoint_MM_lambda.min              | 0.71(0.09) | 0.69(0.04) | 0.86(0.02) | 0.81(0.03) | 0.9(0.03)  |
| Naive                                 | 0(0)       | 0(0)       | 0.08(0.01) | 0.58(0.01) | 0.84(0.05) |
| Oracle                                | 0.91(0)    | 0.91(0.01) | 0.91(0)    | 0.91(0.01) | 0.91(0.03) |
| Weighted BFGS                         | 0.92(0.01) | 0.72(0.02) | 0.91(0.04) | 0.83(0.14) | 0.51(0.38) |

Table 36. Accuracy score on 'Spambase' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.45(0.03) | 0.71(0.03) | 0.79(0.03) | 0.84(0.02) | 0.9(0.01)  |
| $AdaS\_svm$                           | 0.4(0.06)  | 0.68(0.03) | 0.78(0.02) | 0.86(0.02) | 0.93(0)    |
| Joint BFGS                            | 0.84(0.01) | 0.91(0.02) | 0.75(0.34) | 0.01(0.01) | 0.02(0.13) |
| Joint MM                              | 0.42(0.05) | 0.79(0.03) | 0.89(0.01) | 0.93(0.01) | 0.94(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.07(0.2)  | 0.55(0.44) | 0.2(0.39)  | 0.56(0.4)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.16(0.19) | 0.91(0.06) | 0.83(0.23) | 0.03(0.09) | 0.75(0.23) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.75(0.06) | 0.91(0.02) | 0.91(0.09) | 0.02(0.05) | 0.48(0.41) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.09(0.2)  | 0.85(0.1)  | 0.92(0.02) | 0.94(0.01) |
| LassoJoint_MM_lambda.1se              | 0.23(0.19) | 0.79(0.06) | 0.89(0.01) | 0.93(0.01) | 0.95(0.01) |
| LassoJoint_MM_lambda.min              | 0.37(0.07) | 0.77(0.04) | 0.88(0.01) | 0.93(0.01) | 0.94(0.01) |
| Naive                                 | 0(0)       | 0.02(0.01) | 0.28(0.03) | 0.85(0.01) | 0.92(0.01) |
| Oracle                                | 0.95(0.01) | 0.95(0.01) | 0.95(0.01) | 0.95(0)    | 0.95(0)    |
| Weighted BFGS                         | 0.81(0.01) | 0.9(0.02)  | 0.74(0.31) | 0.08(0.02) | 0.12(0.11) |

Table 37. Accuracy score on 'Vote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.65(0.17) | 0.8(0.05)  | 0.76(0.05) | 0.9(0.01)  | 0.85(0.01) |
| AdaS_svm                              | 0.44(0.26) | 0.66(0.11) | 0.72(0.07) | 0.9(0.01)  | 0.89(0.01) |
| Joint BFGS                            | 0.51(0.05) | 0.36(0.16) | 0.73(0.07) | 0.88(0.02) | 0.87(0.02) |
| Joint MM                              | 0.23(0.08) | 0.33(0.09) | 0.73(0.07) | 0.86(0.01) | 0.85(0.03) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.13(0.32) | 0.91(0.05) | 0.85(0.26) | 0.92(0.04) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0.79(0.33) | 0.45(0.35) | 0.85(0.05) | 0.91(0.02) | 0.89(0.02) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.66(0.11) | 0.44(0.27) | 0.83(0.05) | 0.94(0.02) | 0.9(0.04)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.95(0.03) | 0.9(0.06)  | 0.93(0.04) | 0.92(0.04) |
| LassoJoint_MM_lambda.1se              | 0.24(0.34) | 0.76(0.14) | 0.84(0.05) | 0.94(0.02) | 0.9(0.02)  |
| LassoJoint_MM_lambda.min              | 0.43(0.13) | 0.34(0.15) | 0.8(0.04)  | 0.89(0.02) | 0.9(0.02)  |
| Naive                                 | 0.03(0.02) | 0.17(0.05) | 0.43(0.08) | 0.64(0.01) | 0.82(0.03) |
| Oracle                                | 0.95(0.05) | 0.95(0.04) | 0.95(0.05) | 0.95(0.01) | 0.95(0.03) |
| Weighted BFGS                         | 0.48(0.04) | 0.38(0.18) | 0.72(0.07) | 0.83(0)    | 0.81(0.03) |

Table 38. Accuracy score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.59(0.03) | 0.78(0.03) | 0.77(0.09) | 0.94(0.02) | 0.97(0.02) |
| $AdaS\_svm$                           | 0.73(0.02) | 0.78(0.03) | 0.72(0.06) | 0.9(0.04)  | 0.99(0.01) |
| Joint BFGS                            | 0.03(0.07) | 0.83(0.01) | 0.53(0.38) | 0.02(0.04) | 0.24(0.13) |
| Joint MM                              | 0.02(0.03) | 0.7(0.02)  | 0.73(0.05) | 0.91(0.04) | 0.96(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.91(0.03) | 0.95(0.05) | 0.96(0.02) | 0.49(0.46) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.96(0.03) | 0.89(0.07) | 0.75(0.31) | 0.23(0.26) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.66(0.06) | 0.85(0.18) | 0.84(0.05) | 0.53(0.36) | 0.18(0.14) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.86(0.05) | 0.92(0.05) | 0.94(0.03) | 0.97(0.02) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.89(0.03) | 0.82(0.07) | 0.95(0.02) | 0.98(0.01) |
| $LassoJoint\_MM\_lambda.min$          | 0.15(0.11) | 0.81(0.15) | 0.78(0.05) | 0.92(0.04) | 0.98(0.01) |
| Naive                                 | 0.04(0)    | 0.07(0.01) | 0.3(0.04)  | 0.83(0.02) | 0.96(0.01) |
| Oracle                                | 1(0)       | 1(0.01)    | 1(0.01)    | 1(0.02)    | 1(0.01)    |
| Weighted BFGS                         | 0.02(0.06) | 0.75(0.02) | 0.51(0.36) | 0.15(0.06) | 0.63(0.07) |

Table 39. Recall score on 'Banknote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.76(0.06) | 0.71(0.1)  | 0.88(0.01) | 0.97(0.02) | 0.99(0.01) |
| $AdaS\_svm$                           | 0.82(0.05) | 0.8(0.03)  | 0.95(0)    | 0.97(0.01) | 1(0)       |
| Joint BFGS                            | 1(0)       | 1(0)       | 0.98(0)    | 1(0)       | 1(0.01)    |
| Joint MM                              | 0.96(0.01) | 0.97(0)    | 0.96(0)    | 0.97(0.01) | 0.99(0)    |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.91(0.27) | 0.87(0.05) | 1(0.02)    | 1(0)       | 0.98(0.08) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.94(0.12) | 1(0.01)    | 0.98(0.01) | 1(0)       | 0.99(0.03) |
| $LassoJoint\_BFGS\_lambda.min$        | 1(0.01)    | 1(0)       | 0.98(0.01) | 1(0)       | 0.99(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.04(0.17) | 0.81(0.04) | 0.98(0.03) | 0.98(0.01) | 0.99(0.01) |
| LassoJoint_MM_lambda.1se              | 0.39(0.31) | 0.96(0.03) | 0.97(0.01) | 0.98(0.01) | 0.99(0)    |
| LassoJoint_MM_lambda.min              | 0.95(0.01) | 0.97(0.01) | 0.97(0.01) | 0.98(0.01) | 0.99(0)    |
| Naive                                 | 0(0)       | 0(0)       | 0.4(0.01)  | 0.88(0.01) | 0.98(0)    |
| Oracle                                | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0.01)    |
| Weighted BFGS                         | 0.98(0.02) | 0.98(0.01) | 0.97(0)    | 0.99(0)    | 1(0.01)    |

Table 40. Recall score on 'Breastc' dataset

| .1 1                                  | 0.1        | 0.0        | 0.5        | 0.7        | 0.0        |
|---------------------------------------|------------|------------|------------|------------|------------|
| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
| AdaS_knn                              | 0.56(0.15) | 0.75(0.01) | 0.92(0.04) | 0.95(0.02) | 0.88(0.04) |
| AdaS_svm                              | 0.64(0.16) | 0.75(0.03) | 0.79(0.03) | 0.91(0.02) | 0.87(0.05) |
| Joint BFGS                            | 0.56(0.39) | 0.82(0.09) | 0.96(0.02) | 0.96(0.01) | 0.56(0.4)  |
| Joint MM                              | 0.73(0.06) | 0.69(0.07) | 0.94(0.03) | 0.95(0.01) | 0.87(0.05) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.89(0.14) | 0.75(0.2)  | 0.94(0.04) | 0.71(0.4)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.35(0.34) | 0.86(0.05) | 0.88(0.1)  | 0.95(0.02) | 0.59(0.35) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.84(0.09) | 0.87(0.02) | 0.9(0.11)  | 0.95(0.02) | 0.82(0.17) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.88(0.07) | 0.77(0.14) | 0.94(0.04) | 0.93(0.04) |
| LassoJoint_MM_lambda.1se              | 0.4(0.31)  | 0.82(0.03) | 0.87(0.07) | 0.95(0.02) | 0.88(0.05) |
| LassoJoint_MM_lambda.min              | 0.78(0.08) | 0.78(0.05) | 0.88(0.08) | 0.95(0.02) | 0.88(0.05) |
| Naive                                 | 0(0.01)    | 0(0)       | 0.43(0.07) | 0.88(0.03) | 0.84(0.05) |
| Oracle                                | 0.97(0.04) | 0.97(0.01) | 0.97(0.01) | 0.97(0.02) | 0.97(0.05) |
| Weighted BFGS                         | 0.48(0.34) | 0.84(0.1)  | 0.93(0.04) | 0.94(0.01) | 0.57(0.41) |

Table 41. Recall score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.34(0.02) | 0.66(0.01) | 0.75(0.06) | 0.71(0.07) | 0.87(0.03) |
| AdaS_svm                              | 0.16(0.02) | 0.65(0.02) | 0.76(0.05) | 0.77(0.03) | 0.83(0.03) |
| Joint BFGS                            | 0(0)       | 0.02(0.14) | 0.39(0.42) | 0.86(0.04) | 0.29(0.3)  |
| Joint MM                              | 0.22(0.02) | 0.16(0.11) | 0.8(0.05)  | 0.81(0.02) | 0.91(0.04) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.05(0.2)  | 0.75(0.2)  | 0.8(0.04)  | 0.79(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.58(0.21) | 0.28(0.3)  | 0.84(0.06) | 0.73(0.16) |
| LassoJoint_BFGS_lambda.min            | 0.73(0.23) | 0.6(0.13)  | 0.56(0.33) | 0.83(0.1)  | 0.55(0.09) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.8(0.04)  | 0.8(0.04)  | 0.8(0.04)  | 0.79(0.04) |
| LassoJoint_MM_lambda.1se              | 0(0)       | 0.64(0.08) | 0.82(0.04) | 0.8(0.03)  | 0.83(0.04) |
| LassoJoint_MM_lambda.min              | 0.33(0.12) | 0.64(0.05) | 0.82(0.05) | 0.81(0.03) | 0.87(0.05) |
| Naive                                 | 0(0)       | 0.11(0.01) | 0.36(0.07) | 0.52(0.07) | 0.89(0.04) |
| Oracle                                | 0.92(0)    | 0.92(0.01) | 0.92(0.04) | 0.92(0.07) | 0.92(0.03) |
| Weighted BFGS                         | 0(0)       | 0.02(0.14) | 0.36(0.39) | 0.86(0.04) | 0.46(0.11) |

Table 42. Recall score on 'Credit\_g' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.56(0.1)  | 0.45(0.05) | 0.53(0.04) | 0.58(0.01) | 0.64(0.03) |
| $AdaS_sym$                            | 0.34(0.15) | 0.38(0.09) | 0.47(0.05) | 0.61(0.01) | 0.65(0.05) |
| Joint BFGS                            | 0.65(0.05) | 0.76(0.05) | 0.82(0.24) | 0.77(0.02) | 0.75(0.06) |
| Joint MM                              | 0.45(0.06) | 0.66(0.05) | 0.75(0.13) | 0.66(0.03) | 0.8(0.04)  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.84(0.37) | 0.53(0.35) | 0.65(0.26) | 0.81(0.13) |
| $LassoJoint\_BFGS\_lambda.1se$        | 1(0)       | 0.61(0.43) | 0.43(0.33) | 0.23(0.18) | 0.78(0.03) |
| LassoJoint_BFGS_lambda.min            | 0.36(0.25) | 0.42(0.3)  | 0.67(0.22) | 0.6(0.08)  | 0.75(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.03(0.12) | 0.59(0.09) | 0.69(0.12) | 0.83(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0(0)       | 0.61(0.09) | 0.6(0.06)  | 0.81(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.15(0.14) | 0.56(0.2)  | 0.7(0.12)  | 0.63(0.07) | 0.81(0.04) |
| Naive                                 | 0(0)       | 0.01(0.01) | 0.2(0.04)  | 0.6(0.02)  | 0.8(0.04)  |
| Oracle                                | 0.91(0.01) | 0.91(0.04) | 0.91(0.02) | 0.91(0)    | 0.91(0.03) |
| Weighted BFGS                         | 0.68(0.06) | 0.79(0.06) | 0.79(0.23) | 0.75(0.01) | 0.79(0.04) |

Table 43. Recall score on 'dhfr' dataset

|                                       | 0.1        | 0.2        | 0.5        | 0.7        | 0.0        |
|---------------------------------------|------------|------------|------------|------------|------------|
| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
| AdaS_knn                              | 0.72(0.13) | 0.54(0.09) | 0.77(0.11) | 0.83(0.04) | 0.88(0.01) |
| AdaS_svm                              | 0.04(0.04) | 0.19(0.31) | 0.62(0.16) | 0.79(0.06) | 0.74(0.02) |
| Joint BFGS                            | 0.21(0.1)  | 0.25(0.11) | 0.46(0.22) | 0.67(0.06) | 0.74(0.02) |
| Joint MM                              | 0.3(0.04)  | 0.41(0.06) | 0.53(0.08) | 0.64(0.08) | 0.62(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.6(0.55)  | 1(0)       | 0.69(0.15) | 0.89(0.04) | 0.9(0.03)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.74(0.43) | 0.8(0.14)  | 0.74(0.17) | 0.93(0.03) | 0.88(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.28(0.18) | 0.37(0.31) | 0.65(0.22) | 0.71(0.13) | 0.76(0.02) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0(0)       | 0.69(0.14) | 0.88(0.03) | 0.91(0.04) |
| LassoJoint_MM_lambda.1se              | 0(0)       | 0.48(0.29) | 0.79(0.03) | 0.93(0.03) | 0.83(0.01) |
| LassoJoint_MM_lambda.min              | 0.13(0.1)  | 0.36(0.18) | 0.69(0.09) | 0.82(0.11) | 0.72(0.03) |
| Naive                                 | 0.17(0.03) | 0.26(0.04) | 0.49(0.06) | 0.65(0.03) | 0.72(0.01) |
| Oracle                                | 0.94(0.04) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0)    |
| Weighted BFGS                         | 0.21(0.06) | 0.33(0.11) | 0.53(0.12) | 0.62(0.04) | 0.74(0.03) |

Table 44. Recall score on 'Diabetes' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.49(0.05) | 0.59(0.01) | 0.61(0.01) | 0.69(0.02) | 0.76(0.05) |
| AdaS_svm                              | 0.2(0.18)  | 0.51(0.02) | 0.55(0)    | 0.67(0.02) | 0.7(0.05)  |
| Joint BFGS                            | 0.89(0.04) | 0.72(0.02) | 0.91(0.04) | 0.8(0.14)  | 0.44(0.42) |
| Joint MM                              | 0.78(0.01) | 0.69(0.02) | 0.89(0.02) | 0.81(0.05) | 0.9(0.03)  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.88(0.05) | 0.87(0.05) | 0.59(0.35) | 0.04(0.17) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.06(0.17) | 0.91(0.02) | 0.8(0.17)  | 0.68(0.23) |
| LassoJoint_BFGS_lambda.min            | 0.82(0.15) | 0.73(0.05) | 0.89(0.03) | 0.71(0.18) | 0.18(0.3)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.82(0.05) | 0.86(0.05) | 0.8(0.04)  | 0.87(0.04) |
| $LassoJoint\_MM\_lambda.1se$          | 0(0)       | 0.57(0.2)  | 0.9(0.05)  | 0.87(0.05) | 0.89(0.03) |
| LassoJoint_MM_lambda.min              | 0.71(0.09) | 0.69(0.04) | 0.86(0.02) | 0.81(0.03) | 0.9(0.03)  |
| Naive                                 | 0(0)       | 0(0)       | 0.08(0.01) | 0.58(0.01) | 0.84(0.05) |
| Oracle                                | 0.91(0)    | 0.91(0.01) | 0.91(0)    | 0.91(0.01) | 0.91(0.03) |
| Weighted BFGS                         | 0.92(0.01) | 0.72(0.02) | 0.91(0.04) | 0.83(0.14) | 0.51(0.38) |

 ${\bf Table~45.}~{\bf Recall~score~on~'Spambase'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.45(0.03) | 0.71(0.03) | 0.79(0.03) | 0.84(0.02) | 0.9(0.01)  |
| $AdaS\_svm$                           | 0.4(0.06)  | 0.68(0.03) | 0.78(0.02) | 0.86(0.02) | 0.93(0)    |
| Joint BFGS                            | 0.84(0.01) | 0.91(0.02) | 0.75(0.34) | 0.01(0.01) | 0.02(0.13) |
| Joint MM                              | 0.42(0.05) | 0.79(0.03) | 0.89(0.01) | 0.93(0.01) | 0.94(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.07(0.2)  | 0.55(0.44) | 0.2(0.39)  | 0.56(0.4)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.16(0.19) | 0.91(0.06) | 0.83(0.23) | 0.03(0.09) | 0.75(0.23) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.75(0.06) | 0.91(0.02) | 0.91(0.09) | 0.02(0.05) | 0.48(0.41) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.09(0.2)  | 0.85(0.1)  | 0.92(0.02) | 0.94(0.01) |
| LassoJoint_MM_lambda.1se              | 0.23(0.19) | 0.79(0.06) | 0.89(0.01) | 0.93(0.01) | 0.95(0.01) |
| LassoJoint_MM_lambda.min              | 0.37(0.07) | 0.77(0.04) | 0.88(0.01) | 0.93(0.01) | 0.94(0.01) |
| Naive                                 | 0(0)       | 0.02(0.01) | 0.28(0.03) | 0.85(0.01) | 0.92(0.01) |
| Oracle                                | 0.95(0.01) | 0.95(0.01) | 0.95(0.01) | 0.95(0)    | 0.95(0)    |
| Weighted BFGS                         | 0.81(0.01) | 0.9(0.02)  | 0.74(0.31) | 0.08(0.02) | 0.12(0.11) |

Table 46. Recall score on 'Vote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.65(0.17) | 0.8(0.05)  | 0.76(0.05) | 0.9(0.01)  | 0.85(0.01) |
| $AdaS\_svm$                           | 0.44(0.26) | 0.66(0.11) | 0.72(0.07) | 0.9(0.01)  | 0.89(0.01) |
| Joint BFGS                            | 0.51(0.05) | 0.36(0.16) | 0.73(0.07) | 0.88(0.02) | 0.87(0.02) |
| Joint MM                              | 0.23(0.08) | 0.33(0.09) | 0.73(0.07) | 0.86(0.01) | 0.85(0.03) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.13(0.32) | 0.91(0.05) | 0.85(0.26) | 0.92(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.79(0.33) | 0.45(0.35) | 0.85(0.05) | 0.91(0.02) | 0.89(0.02) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.66(0.11) | 0.44(0.27) | 0.83(0.05) | 0.94(0.02) | 0.9(0.04)  |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | 0(0)       | 0.95(0.03) | 0.9(0.06)  | 0.93(0.04) | 0.92(0.04) |
| LassoJoint_MM_lambda.1se              | 0.24(0.34) | 0.76(0.14) | 0.84(0.05) | 0.94(0.02) | 0.9(0.02)  |
| LassoJoint_MM_lambda.min              | 0.43(0.13) | 0.34(0.15) | 0.8(0.04)  | 0.89(0.02) | 0.9(0.02)  |
| Naive                                 | 0.03(0.02) | 0.17(0.05) | 0.43(0.08) | 0.64(0.01) | 0.82(0.03) |
| Oracle                                | 0.95(0.05) | 0.95(0.04) | 0.95(0.05) | 0.95(0.01) | 0.95(0.03) |
| Weighted BFGS                         | 0.48(0.04) | 0.38(0.18) | 0.72(0.07) | 0.83(0)    | 0.81(0.03) |

Table 47. Recall score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.59(0.03) | 0.78(0.03) | 0.77(0.09) | 0.94(0.02) | 0.97(0.02) |
| AdaS_svm                              | 0.73(0.02) | 0.78(0.03) | 0.72(0.06) | 0.9(0.04)  | 0.99(0.01) |
| Joint BFGS                            | 0.03(0.07) | 0.83(0.01) | 0.53(0.38) | 0.02(0.04) | 0.24(0.13) |
| Joint MM                              | 0.02(0.03) | 0.7(0.02)  | 0.73(0.05) | 0.91(0.04) | 0.96(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.91(0.03) | 0.95(0.05) | 0.96(0.02) | 0.49(0.46) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.96(0.03) | 0.89(0.07) | 0.75(0.31) | 0.23(0.26) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.66(0.06) | 0.85(0.18) | 0.84(0.05) | 0.53(0.36) | 0.18(0.14) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.86(0.05) | 0.92(0.05) | 0.94(0.03) | 0.97(0.02) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.89(0.03) | 0.82(0.07) | 0.95(0.02) | 0.98(0.01) |
| $LassoJoint\_MM\_lambda.min$          | 0.15(0.11) | 0.81(0.15) | 0.78(0.05) | 0.92(0.04) | 0.98(0.01) |
| Naive                                 | 0.04(0)    | 0.07(0.01) | 0.3(0.04)  | 0.83(0.02) | 0.96(0.01) |
| Oracle                                | 1(0)       | 1(0.01)    | 1(0.01)    | 1(0.02)    | 1(0.01)    |
| Weighted BFGS                         | 0.02(0.06) | 0.75(0.02) | 0.51(0.36) | 0.15(0.06) | 0.63(0.07) |

Table 48. Precision score on 'Banknote' dataset

| .1 1                                  | 0.1        | 0.0        |            | 0 =        | 0.0        |
|---------------------------------------|------------|------------|------------|------------|------------|
| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
| AdaS_knn                              | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| AdaS_svm                              | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| Joint BFGS                            | 0.99(0.01) | 0.99(0)    | 1(0)       | 0.99(0)    | 0.99(0.01) |
| Joint MM                              | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.56(0.09) | 0.9(0.03)  | 0.99(0.02) | 0.99(0.01) | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.65(0.15) | 0.99(0.01) | 1(0)       | 0.99(0)    | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.99(0.01) | 0.99(0)    | 1(0)       | 0.99(0)    | 0.99(0.01) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.96(0.03) | 0.92(0.02) | 1(0.02)    | 1(0)       | 1(0)       |
| LassoJoint_MM_lambda.1se              | 0.97(0.02) | 1(0.01)    | 1(0)       | 1(0)       | 1(0)       |
| LassoJoint_MM_lambda.min              | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| Naive                                 | NaN(NA)    | 1(NA)      | 1(0)       | 1(0)       | 1(0)       |
| Oracle                                | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0.01)    |
| Weighted BFGS                         | 0.96(0.03) | 0.95(0.01) | 0.98(0)    | 0.99(0)    | 0.99(0.01) |

Table 49. Precision score on 'Breastc' dataset

| method                               | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                             | 0.88(0.05) | 0.94(0)    | 1(0.01)    | 0.95(0.04) | 0.9(0.02)  |
| $AdaS_{-svm}$                        | 1(0)       | 1(0)       | 1(0)       | 0.98(0.03) | 0.92(0.02) |
| Joint BFGS                           | 0.96(0.05) | 1(0.01)    | 1(0.01)    | 0.92(0.02) | 0.91(0)    |
| Joint MM                             | 0.96(0.02) | 1(0)       | 1(0.01)    | 0.93(0.03) | 0.91(0)    |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | NaN(NA)    | 0.89(0.06) | 0.99(0.01) | 0.95(0.04) | 0.94(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.91(0.03) | 0.97(0.04) | 0.99(0.01) | 0.93(0.03) | 0.92(0.03) |
| $LassoJoint\_BFGS\_lambda.min$       | 0.93(0.04) | 0.97(0.04) | 1(0.01)    | 0.92(0.03) | 0.92(0.01) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | NaN(NA)    | 0.94(0.04) | 1(0.01)    | 0.96(0.04) | 0.95(0.03) |
| $LassoJoint\_MM\_lambda.1se$         | 0.95(0.03) | 0.99(0.02) | 1(0)       | 0.94(0.03) | 0.93(0.02) |
| $LassoJoint\_MM\_lambda.min$         | 0.98(0.02) | 0.99(0.02) | 1(0)       | 0.94(0.03) | 0.92(0.02) |
| Naive                                | 1(0)       | NaN(NA)    | 1(0)       | 1(0)       | 0.96(0.03) |
| Oracle                               | 0.97(0.04) | 0.97(0.01) | 0.97(0.02) | 0.97(0.03) | 0.97(0.01) |
| Weighted BFGS                        | 0.84(0.02) | 0.95(0)    | 0.99(0.02) | 0.93(0.02) | 0.92(0.03) |

Table 50. Precision score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.89(0)    | 0.83(0.01) | 0.82(0.02) | 0.83(0.04) | 0.84(0.03) |
| AdaS_svm                              | 0.73(0.02) | 0.89(0.01) | 0.94(0.01) | 0.96(0.01) | 0.93(0.01) |
| Joint BFGS                            | NaN(NA)    | 0.02(0.13) | 0.84(0.1)  | 0.82(0.03) | 0.99(0.01) |
| Joint MM                              | 0.84(0)    | 0.84(0.02) | 0.85(0.04) | 0.86(0.03) | 0.91(0.03) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.92(0.01) | 0.94(0.02) | 0.88(0.03) | 0.91(0.06) |
| LassoJoint_BFGS_lambda.min            | 0.78(0.02) | 0.91(0.03) | 0.9(0.07)  | 0.86(0.04) | 0.97(0.02) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) |
| LassoJoint_MM_lambda.1se              | NaN(NA)    | 0.91(0.02) | 0.93(0.02) | 0.9(0.02)  | 0.95(0.02) |
| LassoJoint_MM_lambda.min              | 0.92(0.01) | 0.92(0.01) | 0.91(0.04) | 0.91(0.02) | 0.92(0.03) |
| Naive                                 | 1(NA)      | 0.89(0.02) | 0.91(0.04) | 0.95(0.03) | 0.92(0.03) |
| Oracle                                | 0.94(0)    | 0.94(0.01) | 0.94(0.03) | 0.94(0.02) | 0.94(0.02) |
| Weighted BFGS                         | NaN(NA)    | 0.02(0.13) | 0.88(0.11) | 0.83(0.02) | 0.97(0.02) |

 ${\bf Table~51.~Precision~score~on~'Credit\_g'~dataset}$ 

| method                                 | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--|------------|------------|------------|------------|------------|
| AdaS_knn                               | 0.78(0.03) | 0.77(0.03) | 0.77(0.05) | 0.78(0)    | 0.82(0.03) |
| AdaS_svm                               | 0.85(0.03) | 0.86(0.03) | 0.88(0.04) | 0.85(0.01) | 0.84(0.02) |
| Joint BFGS                             | 0.76(0.02) | 0.76(0.07) | 0.75(0.05) | 0.81(0.01) | 0.83(0.02) |
| Joint MM                               | 0.9(0.02)  | 0.82(0.04) | 0.82(0.05) | 0.83(0.01) | 0.83(0.01) |
| LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.7(0.03)  | 0.71(0.03) | 0.79(0.06) | 0.81(0.05) | 0.8(0.03)  |
| $LassoJoint\_BFGS\_lambda.1se$         | 0.69(0.02) | 0.7(0.02)  | 0.79(0.05) | 0.92(0.09) | 0.83(0.01) |
| $LassoJoint\_BFGS\_lambda.min$         | 0.79(0.09) | 0.8(0.03)  | 0.82(0.05) | 0.84(0.02) | 0.83(0.01) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$       | NaN(NA)    | 0.86(0.05) | 0.86(0.05) | 0.82(0.04) | 0.8(0.03)  |
| $Lasso Joint\_MM\_lambda.1se$          | NaN(NA)    | NaN(NA)    | 0.84(0.04) | 0.82(0.02) | 0.82(0.02) |
| LassoJoint_MM_lambda.min               | 0.91(0.05) | 0.83(0.05) | 0.82(0.04) | 0.82(0.02) | 0.83(0.01) |
| Naive                                  | NaN(NA)    | 0.96(0.16) | 0.92(0.04) | 0.87(0.01) | 0.83(0.01) |
| Oracle                                 | 0.82(0.03) | 0.82(0.02) | 0.82(0.03) | 0.82(0)    | 0.82(0.01) |
| Weighted BFGS                          | 0.82(0.03) | 0.8(0.03)  | 0.78(0.04) | 0.8(0)     | 0.83(0.01) |

Table 52. Precision score on 'dhfr' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.63(0.04) | 0.74(0.05) | 0.75(0.04) | 0.79(0.03) | 0.79(0.01) |
| $AdaS\_svm$                           | 0.3(0.11)  | 0.72(0.08) | 0.9(0.06)  | 0.94(0.02) | 0.89(0.01) |
| Joint BFGS                            | 0.85(0.1)  | 0.98(0.03) | 0.96(0.04) | 0.96(0.02) | 0.91(0.01) |
| Joint MM                              | 0.64(0.07) | 0.77(0.09) | 0.83(0.06) | 0.79(0.03) | 0.87(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.64(0.05) | 0.58(0.09) | 0.82(0.06) | 0.78(0.05) | 0.87(0.07) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.59(0.01) | 0.8(0.12)  | 0.91(0.02) | 0.94(0.01) | 0.97(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.83(0.03) | 0.95(0.05) | 0.92(0.05) | 0.95(0.04) | 0.92(0.01) |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | NaN(NA)    | NaN(NA)    | 0.83(0.06) | 0.78(0.05) | 0.87(0.06) |
| LassoJoint_MM_lambda.1se              | NaN(NA)    | 0.86(0.08) | 0.91(0.02) | 0.93(0.01) | 0.95(0)    |
| LassoJoint_MM_lambda.min              | 0.99(0.06) | 0.94(0.05) | 0.93(0.03) | 0.93(0.02) | 0.94(0)    |
| Naive                                 | 0.85(0.09) | 0.98(0.03) | 0.96(0.05) | 0.92(0.01) | 0.91(0)    |
| Oracle                                | 0.95(0.02) | 0.95(0.04) | 0.95(0.03) | 0.95(0.02) | 0.95(0.01) |
| Weighted BFGS                         | 0.88(0.08) | 0.98(0.03) | 0.93(0.05) | 0.91(0.02) | 0.89(0.01) |

Table 53. Precision score on 'Diabetes' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.66(0.01) | 0.87(0.01) | 0.91(0.02) | 0.82(0.01) | 0.79(0.02) |
| $AdaS_sym$                            | 0.63(0.34) | 0.92(0.01) | 0.92(0.01) | 0.82(0.02) | 0.81(0.02) |
| Joint BFGS                            | 0.77(0.03) | 0.84(0.01) | 0.8(0.01)  | 0.79(0.03) | 0.9(0.11)  |
| Joint MM                              | 0.8(0.01)  | 0.84(0.01) | 0.82(0)    | 0.79(0.03) | 0.76(0.03) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.76(0.04) | 0.79(0.05) | 0.81(0.1)  | 0.97(0.07) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.79(0.03) | 0.77(0.02) | 0.74(0.03) | 0.74(0.05) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.75(0.03) | 0.81(0.02) | 0.82(0.02) | 0.79(0.03) | 0.94(0.09) |
| $LassoJoint_MM_{(log(p)/n)^{(1/2)}$   | NaN(NA)    | 0.78(0.04) | 0.8(0.05)  | 0.82(0.04) | 0.79(0.04) |
| $LassoJoint\_MM\_lambda.1se$          | NaN(NA)    | 0.81(0.01) | 0.79(0.02) | 0.75(0.04) | 0.75(0.04) |
| LassoJoint_MM_lambda.min              | 0.81(0.03) | 0.83(0.02) | 0.83(0.02) | 0.8(0.02)  | 0.77(0.03) |
| Naive                                 | NaN(NA)    | 1(0)       | 1(0.01)    | 0.86(0.01) | 0.8(0.02)  |
| Oracle                                | 0.82(0.01) | 0.82(0.01) | 0.82(0.01) | 0.82(0.01) | 0.82(0.03) |
| Weighted BFGS                         | 0.73(0.01) | 0.83(0.01) | 0.81(0.01) | 0.79(0.04) | 0.87(0.1)  |

Table 54. Precision score on 'Spambase' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.84(0.03) | 0.88(0.02) | 0.9(0.01)  | 0.92(0.01) | 0.89(0.01) |
| $AdaS\_svm$                           | 0.92(0.04) | 0.95(0.01) | 0.95(0.01) | 0.97(0)    | 0.95(0)    |
| Joint BFGS                            | 0.88(0.02) | 0.92(0.01) | 0.94(0.04) | 1(0)       | 1(0.01)    |
| Joint MM                              | 0.93(0.01) | 0.95(0.01) | 0.93(0.01) | 0.96(0)    | 0.92(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.61(0.01) | 0.96(0.03) | 0.92(0.04) | 0.95(0.05) | 0.94(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.91(0.11) | 0.91(0.01) | 0.92(0.02) | 1(0.02)    | 0.94(0.02) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.91(0.02) | 0.91(0.01) | 0.92(0.01) | 0.99(0.02) | 0.96(0.03) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | NaN(NA)    | 0.97(0.03) | 0.92(0.02) | 0.9(0.01)  | 0.91(0.01) |
| $LassoJoint\_MM\_lambda.1se$          | 0.9(0.06)  | 0.95(0.01) | 0.93(0.01) | 0.94(0.01) | 0.92(0.01) |
| $LassoJoint\_MM\_lambda.min$          | 0.93(0.01) | 0.95(0.01) | 0.94(0.01) | 0.95(0.01) | 0.92(0.01) |
| Naive                                 | 0.76(0.43) | 1(0)       | 0.99(0.01) | 0.97(0.01) | 0.92(0)    |
| Oracle                                | 0.94(0.01) | 0.94(0.01) | 0.94(0.01) | 0.94(0)    | 0.94(0.01) |
| Weighted BFGS                         | 0.85(0.02) | 0.9(0.01)  | 0.92(0.04) | 0.99(0.01) | 0.98(0.01) |

Table 55. Precision score on 'Vote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.96(0.02) | 0.99(0.01) | 1(0)       | 0.97(0)    | 0.97(0.01) |
| $AdaS\_svm$                           | 0.87(0.32) | 1(0)       | 1(0)       | 1(0)       | 0.98(0.01) |
| Joint BFGS                            | 0.95(0.04) | 0.96(0.04) | 0.91(0.03) | 0.88(0.01) | 0.98(0.01) |
| Joint MM                              | 0.93(0.09) | 0.96(0.03) | 0.95(0.02) | 0.97(0)    | 0.98(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.68(0.07) | 0.99(0.02) | 0.97(0.03) | 0.98(0.02) | 0.98(0.02) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.72(0.14) | 0.99(0.01) | 0.97(0.05) | 0.99(0.01) | 0.98(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.97(0.04) | 0.98(0.02) | 0.93(0.03) | 0.97(0.01) | 0.97(0.02) |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | NaN(NA)    | 0.98(0.02) | 0.98(0.02) | 0.98(0.02) | 0.98(0.02) |
| LassoJoint_MM_lambda.1se              | 0.99(0.01) | 0.99(0.01) | 0.98(0.02) | 0.98(0.01) | 0.97(0.01) |
| LassoJoint_MM_lambda.min              | 0.99(0.02) | 0.99(0.02) | 0.97(0.02) | 0.97(0.01) | 0.99(0.01) |
| Naive                                 | 0.74(0.35) | 1(0.01)    | 0.98(0.03) | 1(0)       | 0.98(0.02) |
| Oracle                                | 1(0.03)    | 1(0.02)    | 1(0.02)    | 1(0)       | 1(0.01)    |
| Weighted BFGS                         | 0.94(0.04) | 0.92(0.07) | 0.86(0.03) | 0.97(0)    | 0.99(0.01) |

Table 56. Precision score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.93(0.01) | 0.96(0.01) | 0.98(0.03) | 0.97(0.01) | 0.99(0.01) |
| $AdaS\_svm$                           | 0.96(0)    | 1(0.01)    | 1(0)       | 0.99(0.01) | 0.99(0.01) |
| Joint BFGS                            | 0.67(0.03) | 0.95(0)    | 0.98(0.03) | 1(0.01)    | 1(0)       |
| Joint MM                              | 0.5(0.05)  | 0.91(0.01) | 0.99(0.02) | 0.99(0.01) | 0.97(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.96(0.03) | 0.94(0.03) | 0.94(0.03) | 0.97(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.92(0.02) | 0.98(0.01) | 0.98(0.02) | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.91(0.02) | 0.96(0.01) | 0.96(0.03) | 0.99(0.02) | 1(0.01)    |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | NaN(NA)    | 0.98(0.02) | 0.96(0.02) | 0.97(0.02) | 0.96(0.02) |
| $LassoJoint\_MM\_lambda.1se$          | NaN(NA)    | 0.98(0.01) | 0.99(0.01) | 0.98(0.02) | 0.98(0.01) |
| LassoJoint_MM_lambda.min              | 0.88(0.08) | 0.97(0.01) | 0.98(0.02) | 0.99(0.01) | 0.97(0.01) |
| Naive                                 | 0.75(0.03) | 0.85(0.04) | 0.98(0.02) | 0.99(0.01) | 0.98(0.02) |
| Oracle                                | 0.99(0.01) | 0.99(0.01) | 0.99(0.02) | 0.99(0.02) | 0.99(0.02) |
| Weighted BFGS                         | 0.5(0.04)  | 0.93(0.01) | 0.97(0.01) | 0.99(0.02) | 1(0)       |

Table 57. F1 score on 'Banknote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.86(0.04) | 0.82(0.07) | 0.94(0.01) | 0.98(0.01) | 0.99(0)    |
| AdaS_svm                              | 0.9(0.03)  | 0.89(0.02) | 0.98(0)    | 0.99(0.01) | 1(0)       |
| Joint BFGS                            | 1(0)       | 1(0)       | 0.99(0)    | 1(0)       | 0.99(0)    |
| Joint MM                              | 0.98(0)    | 0.98(0)    | 0.98(0)    | 0.99(0)    | 0.99(0)    |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.71(0.05) | 0.88(0.03) | 0.99(0.02) | 0.99(0)    | 0.98(0.05) |
| LassoJoint_BFGS_lambda.1se            | 0.75(0.08) | 0.99(0.01) | 0.99(0)    | 1(0)       | 0.99(0.02) |
| LassoJoint_BFGS_lambda.min            | 0.99(0.01) | 1(0)       | 0.99(0)    | 1(0)       | 0.99(0.02) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.81(0.02) | 0.86(0.02) | 0.99(0.02) | 0.99(0.01) | 1(0)       |
| LassoJoint_MM_lambda.1se              | 0.7(0.15)  | 0.98(0.02) | 0.98(0)    | 0.99(0)    | 1(0)       |
| LassoJoint_MM_lambda.min              | 0.98(0.01) | 0.98(0)    | 0.98(0)    | 0.99(0)    | 1(0)       |
| Naive                                 | NaN(NA)    | 0.01(NA)   | 0.57(0.01) | 0.94(0.01) | 0.99(0)    |
| Oracle                                | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| Weighted BFGS                         | 0.97(0.03) | 0.96(0.01) | 0.97(0)    | 0.99(0)    | 0.99(0)    |

Table 58. F1 score on 'Breastc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.67(0.13) | 0.84(0.01) | 0.96(0.02) | 0.95(0.02) | 0.89(0.03) |
| $AdaS_sym$                            | 0.77(0.12) | 0.86(0.02) | 0.88(0.02) | 0.94(0.02) | 0.89(0.04) |
| Joint BFGS                            | 0.89(0.03) | 0.9(0.03)  | 0.98(0.01) | 0.94(0.01) | 0.88(0.02) |
| Joint MM                              | 0.83(0.05) | 0.82(0.06) | 0.97(0.02) | 0.94(0.01) | 0.89(0.03) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.9(0.04)  | 0.87(0.08) | 0.95(0.03) | 0.94(0.02) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.71(0.15) | 0.91(0.03) | 0.93(0.07) | 0.94(0.01) | 0.8(0.15)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.88(0.06) | 0.92(0.02) | 0.94(0.07) | 0.94(0.02) | 0.86(0.1)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.91(0.04) | 0.86(0.11) | 0.95(0.02) | 0.94(0.02) |
| $LassoJoint\_MM\_lambda.1se$          | 0.69(0.15) | 0.9(0.02)  | 0.93(0.04) | 0.94(0.01) | 0.9(0.02)  |
| LassoJoint_MM_lambda.min              | 0.86(0.05) | 0.87(0.04) | 0.93(0.05) | 0.94(0.01) | 0.9(0.03)  |
| Naive                                 | 0.05(0)    | NaN(NA)    | 0.59(0.07) | 0.94(0.02) | 0.9(0.02)  |
| Oracle                                | 0.96(0.03) | 0.96(0)    | 0.96(0.01) | 0.96(0.01) | 0.96(0.03) |
| Weighted BFGS                         | 0.77(0.03) | 0.89(0.04) | 0.96(0.03) | 0.94(0.01) | 0.89(0.01) |

Table 59. F1 score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.49(0.02) | 0.73(0.01) | 0.78(0.04) | 0.76(0.03) | 0.86(0.03) |
| AdaS_svm                              | 0.26(0.02) | 0.75(0.01) | 0.84(0.04) | 0.86(0.02) | 0.88(0.02) |
| Joint BFGS                            | NaN(NA)    | 0.8(0.04)  | 0.67(0.26) | 0.84(0.01) | 0.36(0.34) |
| Joint MM                              | 0.36(0)    | 0.26(0.11) | 0.83(0.04) | 0.84(0.01) | 0.91(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.88(0.04) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_BFGS\_lambda.1se$       | NaN(NA)    | 0.7(0.15)  | 0.65(0.15) | 0.86(0.02) | 0.8(0.09)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.74(0.13) | 0.72(0.11) | 0.71(0.25) | 0.84(0.05) | 0.7(0.08)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.87(0.03) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | NaN(NA)    | 0.75(0.07) | 0.87(0.02) | 0.85(0.02) | 0.89(0.03) |
| $LassoJoint\_MM\_lambda.min$          |            |            |            |            | 0.89(0.02) |
| Naive                                 | 0.03(NA)   | 0.2(0.02)  | 0.51(0.07) | 0.67(0.07) | 0.9(0.02)  |
| Oracle                                | 0.93(0)    | 0.93(0.01) | 0.93(0.03) | 0.93(0.04) | 0.93(0.02) |
| Weighted BFGS                         | NaN(NA)    | 0.78(0.07) | 0.52(0.36) | 0.84(0.01) | 0.61(0.1)  |

Table 60. F1 score on 'Credit\_g' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.65(0.07) | 0.56(0.04) | 0.63(0.04) | 0.66(0)    | 0.72(0.02) |
| AdaS_svm                              | 0.48(0.16) | 0.52(0.08) | 0.61(0.04) | 0.71(0)    | 0.73(0.04) |
| Joint BFGS                            | 0.7(0.03)  | 0.76(0.03) | 0.81(0.03) | 0.79(0)    | 0.79(0.03) |
| Joint MM                              | 0.6(0.06)  | 0.73(0.04) | 0.77(0.07) | 0.73(0.01) | 0.81(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.82(0.02) | 0.83(0.02) | 0.75(0.06) | 0.73(0.12) | 0.81(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.82(0.01) | 0.75(0.11) | 0.66(0.13) | 0.33(0.21) | 0.8(0.02)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.58(0.1)  | 0.67(0.12) | 0.73(0.15) | 0.7(0.05)  | 0.79(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.64(0.04) | 0.69(0.05) | 0.74(0.06) | 0.82(0.02) |
| LassoJoint_MM_lambda.1se              | NaN(NA)    | NaN(NA)    | 0.7(0.05)  | 0.69(0.04) | 0.82(0.02) |
| LassoJoint_MM_lambda.min              | 0.23(0.2)  | 0.64(0.18) | 0.75(0.06) | 0.71(0.04) | 0.82(0.02) |
| Naive                                 | NaN(NA)    | 0.03(0.02) | 0.33(0.06) | 0.71(0.02) | 0.81(0.02) |
| Oracle                                | 0.86(0.02) | 0.86(0.02) | 0.86(0.02) | 0.86(0)    | 0.86(0.01) |
| Weighted BFGS                         | 0.74(0.04) | 0.79(0.03) | 0.81(0.03) | 0.77(0.01) | 0.81(0.02) |

Table 61. F1 score on 'dhfr' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.67(0.07) | 0.62(0.07) | 0.76(0.07) | 0.81(0.03) | 0.83(0.01) |
| $AdaS\_svm$                           | 0.09(0.05) | 0.65(0.15) | 0.72(0.11) | 0.85(0.04) | 0.81(0.01) |
| Joint BFGS                            | 0.32(0.13) | 0.38(0.14) | 0.58(0.23) | 0.79(0.04) | 0.82(0.01) |
| Joint MM                              | 0.41(0.04) | 0.53(0.08) | 0.65(0.07) | 0.71(0.06) | 0.72(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.78(0.04) | 0.73(0.07) | 0.75(0.1)  | 0.83(0.02) | 0.88(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.73(0.04) | 0.78(0.07) | 0.8(0.13)  | 0.93(0.02) | 0.92(0.01) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.5(0.11)  | 0.45(0.33) | 0.73(0.18) | 0.8(0.07)  | 0.83(0.01) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | NaN(NA)    | 0.74(0.1)  | 0.83(0.03) | 0.89(0.03) |
| LassoJoint_MM_lambda.1se              | NaN(NA)    | 0.73(0.07) | 0.85(0.02) | 0.93(0.01) | 0.89(0)    |
| LassoJoint_MM_lambda.min              | 0.28(0.11) | 0.49(0.21) | 0.79(0.06) | 0.87(0.07) | 0.81(0.02) |
| Naive                                 | 0.29(0.05) | 0.41(0.05) | 0.65(0.06) | 0.76(0.02) | 0.8(0.01)  |
| Oracle                                | 0.95(0.02) | 0.95(0.03) | 0.95(0.02) | 0.95(0.03) | 0.95(0.01) |
| Weighted BFGS                         | 0.34(0.08) | 0.49(0.12) | 0.67(0.1)  | 0.73(0.02) | 0.81(0.02) |

Table 62. F1 score on 'Diabetes' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.56(0.03) | 0.71(0.01) | 0.73(0.01) | 0.75(0.01) | 0.77(0.03) |
| $AdaS\_svm$                           | 0.29(0.26) | 0.66(0.02) | 0.69(0)    | 0.74(0.02) | 0.75(0.03) |
| Joint BFGS                            | 0.82(0.01) | 0.77(0.01) | 0.85(0.02) | 0.78(0.11) | 0.44(0.37) |
| Joint MM                              | 0.79(0.01) | 0.76(0.01) | 0.85(0.01) | 0.8(0.02)  | 0.83(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.81(0.03) | 0.83(0.03) | 0.6(0.34)  | 0.12(0.28) |
| $Lasso Joint\_BFGS\_lambda.1se$       | NaN(NA)    | 0.58(0.1)  | 0.84(0.01) | 0.76(0.09) | 0.69(0.13) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.77(0.08) | 0.77(0.02) | 0.85(0.02) | 0.74(0.12) | 0.22(0.31) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.8(0.03)  | 0.83(0.03) | 0.81(0.03) | 0.83(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | NaN(NA)    | 0.66(0.13) | 0.84(0.03) | 0.8(0.01)  | 0.82(0.02) |
| LassoJoint_MM_lambda.min              | 0.75(0.05) | 0.75(0.02) | 0.84(0.01) | 0.8(0.01)  | 0.83(0.02) |
| Naive                                 | NaN(NA)    | 0.02(0.01) | 0.15(0.02) | 0.69(0.01) | 0.82(0.03) |
| Oracle                                | 0.86(0.01) | 0.86(0.01) | 0.86(0.01) | 0.86(0.01) | 0.86(0.01) |
| Weighted BFGS                         | 0.81(0.01) | 0.77(0.01) | 0.86(0.02) | 0.79(0.11) | 0.53(0.31) |

Table 63. F1 score on 'Spambase' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.59(0.03) | 0.79(0.02) | 0.84(0.02) | 0.88(0.01) | 0.9(0)     |
| $AdaS_sym$                            | 0.55(0.05) | 0.79(0.02) | 0.86(0.01) | 0.91(0.01) | 0.94(0)    |
| Joint BFGS                            | 0.86(0.01) | 0.91(0.01) | 0.76(0.31) | 0.03(0.01) | 0.04(0.18) |
| Joint MM                              | 0.58(0.05) | 0.86(0.02) | 0.91(0.01) | 0.94(0)    | 0.93(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.76(0.01) | 0.53(0.35) | 0.79(0.3)  | 0.51(0.46) | 0.73(0.31) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.38(0.24) | 0.91(0.04) | 0.85(0.19) | 0.06(0.13) | 0.81(0.18) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.82(0.03) | 0.91(0.01) | 0.91(0.06) | 0.03(0.07) | 0.53(0.41) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.51(0.27) | 0.88(0.07) | 0.91(0.01) | 0.92(0.01) |
| $LassoJoint\_MM\_lambda.1se$          | 0.4(0.21)  | 0.86(0.04) | 0.91(0.01) | 0.94(0.01) | 0.93(0.01) |
| $LassoJoint\_MM\_lambda.min$          | 0.53(0.07) | 0.85(0.02) | 0.91(0.01) | 0.94(0.01) | 0.93(0.01) |
| Naive                                 | 0.01(0)    | 0.03(0.01) | 0.44(0.03) | 0.91(0)    | 0.92(0)    |
| Oracle                                | 0.95(0.01) | 0.95(0)    | 0.95(0)    | 0.95(0)    | 0.95(0.01) |
| Weighted BFGS                         | 0.83(0.01) | 0.9(0.01)  | 0.76(0.26) | 0.14(0.03) | 0.21(0.1)  |

Table 64. F1 score on 'Vote' dataset

|   | 0.4        | 0.0        |            | ~ =        |            |
|---|------------|------------|------------|------------|------------|
| method                                  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
| AdaS_knn                                | 0.76(0.15) | 0.89(0.03) | 0.86(0.03) | 0.94(0.01) | 0.9(0.01)  |
| AdaS_svm                                | 0.63(0.23) | 0.79(0.08) | 0.83(0.05) | 0.95(0.01) | 0.93(0.01) |
| Joint BFGS                              | 0.66(0.05) | 0.5(0.18)  | 0.81(0.04) | 0.88(0.01) | 0.92(0.01) |
| Joint MM                                | 0.36(0.12) | 0.48(0.1)  | 0.83(0.05) | 0.91(0.01) | 0.91(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)   | 0.81(0.05) | 0.96(0.02) | 0.94(0.03) | 0.95(0.03) | 0.95(0.02) |
| $LassoJoint\_BFGS\_lambda.1se$          | 0.78(0.1)  | 0.75(0.19) | 0.9(0.03)  | 0.95(0.01) | 0.93(0.01) |
| $LassoJoint\_BFGS\_lambda.min$          | 0.77(0.08) | 0.55(0.29) | 0.87(0.03) | 0.96(0.01) | 0.93(0.02) |
| LassoJoint_ $MM_{-}(\log(p)/n)^{(1/2)}$ | NaN(NA)    | 0.96(0.02) | 0.93(0.03) | 0.95(0.03) | 0.95(0.02) |
| LassoJoint_MM_lambda.1se                | 0.74(0.17) | 0.85(0.09) | 0.9(0.03)  | 0.96(0.01) | 0.94(0.01) |
| $LassoJoint\_MM\_lambda.min$            | 0.59(0.13) | 0.48(0.16) | 0.88(0.02) | 0.93(0.01) | 0.94(0.01) |
| Naive                                   | 0.07(0.02) | 0.28(0.07) | 0.6(0.08)  | 0.78(0)    | 0.89(0.02) |
| Oracle                                  | 0.97(0.02) | 0.97(0.03) | 0.97(0.02) | 0.97(0)    | 0.97(0.02) |
| Weighted BFGS                           | 0.63(0.03) | 0.5(0.19)  | 0.78(0.05) | 0.9(0)     | 0.89(0.01) |

Table 65. F1 score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| AdaS_knn                              | 0.72(0.03) | 0.86(0.02) | 0.86(0.06) | 0.96(0.01) | 0.98(0.01) |
| $AdaS\_svm$                           | 0.83(0.02) | 0.88(0.02) | 0.83(0.05) | 0.94(0.02) | 0.99(0.01) |
| Joint BFGS                            | 0.06(0.07) | 0.89(0.01) | 0.86(0.11) | 0.11(0.07) | 0.37(0.16) |
| Joint MM                              | 0.03(0.04) | 0.79(0.01) | 0.84(0.04) | 0.95(0.02) | 0.96(0.01) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.94(0.02) | 0.94(0.02) | 0.95(0.02) | 0.77(0.35) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.94(0.01) | 0.93(0.04) | 0.8(0.26)  | 0.54(0.2)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.76(0.04) | 0.89(0.12) | 0.89(0.04) | 0.62(0.32) | 0.4(0.1)   |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | NaN(NA)    | 0.92(0.03) | 0.94(0.02) | 0.95(0.02) | 0.96(0.02) |
| LassoJoint_MM_lambda.1se              | NaN(NA)    | 0.93(0.02) | 0.9(0.04)  | 0.97(0.01) | 0.98(0.01) |
| LassoJoint_MM_lambda.min              | 0.25(0.16) | 0.88(0.1)  | 0.87(0.03) | 0.95(0.02) | 0.97(0.01) |
| Naive                                 | 0.08(0)    | 0.13(0.02) | 0.46(0.04) | 0.91(0.02) | 0.97(0.01) |
| Oracle                                | 0.97(0)    | 0.97(0)    | 0.97(0)    | 0.97(0.01) | 0.97(0.01) |
| Weighted BFGS                         | 0.03(0.07) | 0.83(0.01) | 0.83(0.13) | 0.26(0.08) | 0.77(0.05) |

Table 66. Method avg.rank based on Accuracy metrics

| method                               | 0.1   | 0.3   | 0.5   | 0.7   | 0.9   |
|--------------------------------------|-------|-------|-------|-------|-------|
| Oracle                               | 1.11  | 1.22  | 1.44  | 1.44  | 1.67  |
| $LassoJoint\_MM\_lambda.min$         | 6.30  | 6.17  | 5.54  | 5.77  | 5.31  |
| $LassoJoint\_BFGS\_lambda.min$       | 3.60  | 5.08  | 5.15  | 7.54  | 8.23  |
| $LassoJoint\_BFGS\_lambda.1se$       | 7.50  | 4.42  | 5.46  | 6.31  | 6.62  |
| $LassoJoint\_MM\_lambda.1se$         | 9.60  | 6.08  | 5.46  | 5.23  | 4.31  |
| Joint MM                             | 6.22  | 8.11  | 6.33  | 6.67  | 6.11  |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 7.90  | 6.17  | 5.69  | 6.15  | 7.92  |
| AdaS_svm                             | 7.11  | 8.67  | 8.58  | 5.00  | 5.00  |
| AdaS_knn                             | 5.00  | 8.44  | 7.17  | 6.67  | 7.25  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 12.00 | 6.58  | 6.15  | 6.00  | 4.23  |
| Joint BFGS                           | 4.67  | 5.89  | 7.33  | 8.44  | 10.11 |
| Weighted BFGS                        | 6.56  | 7.67  | 8.44  | 9.56  | 11.00 |
| Naive                                | 10.56 | 12.00 | 12.44 | 10.22 | 7.22  |

Table 67. Method avg.rank based on Recall metrics

| method                                | 0.1   | 0.3   | 0.5   | 0.7   | 0.9   |
|---------------------------------------|-------|-------|-------|-------|-------|
| Oracle                                | 1.78  | 1.56  | 1.33  | 1.44  | 1.33  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 5.40  | 5.17  | 5.08  | 5.31  | 6.69  |
| LassoJoint_BFGS_lambda.1se            | 6.70  | 4.33  | 5.00  | 5.92  | 7.31  |
| LassoJoint_BFGS_lambda.min            | 3.90  | 5.08  | 5.23  | 7.00  | 8.31  |
| $LassoJoint\_MM\_lambda.min$          | 6.60  | 6.75  | 6.38  | 5.85  | 6.00  |
| AdaS_knn                              | 5.56  | 7.44  | 6.58  | 6.92  | 5.75  |
| $LassoJoint\_MM\_lambda.1se$          | 10.60 | 6.50  | 5.62  | 5.23  | 4.77  |
| Joint MM                              | 6.33  | 8.22  | 6.56  | 6.89  | 6.33  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 11.10 | 6.33  | 6.54  | 6.46  | 4.31  |
| Joint BFGS                            | 5.11  | 6.44  | 6.78  | 6.44  | 10.11 |
| AdaS_svm                              | 7.33  | 9.33  | 8.75  | 7.58  | 5.58  |
| Weighted BFGS                         | 6.44  | 7.33  | 8.67  | 8.11  | 10.00 |
| Naive                                 | 11.56 | 12.11 | 12.67 | 11.44 | 8.44  |

Table 68. Method avg.rank based on Precision metrics

| method                                 | $0.1 \ \ 0.3$ | 0.5   | 0.7 - 0.9     |
|--|---------------|-------|---------------|
| AdaS_svm                               | 5.22 3.56     | 3.25  | $2.92\ 4.67$  |
| Oracle                                 | $2.33 \ 5.56$ | 6.56  | $6.00\ 5.56$  |
| $LassoJoint\_MM\_lambda.min$           | $2.60\ 4.83$  | 5.62  | $6.62\ 7.00$  |
| $Lasso Joint\_BFGS\_lambda.min$        | $5.50\ 6.58$  | 6.62  | $6.08\ 4.77$  |
| Naive                                  | $8.22\ 5.33$  | 5.22  | $3.89\ 7.78$  |
| Joint BFGS                             | $6.22\ 7.33$  | 6.89  | $7.56\ 4.33$  |
| $LassoJoint\_MM\_lambda.1se$           | $8.80\ 5.83$  | 5.38  | $6.77\ 6.46$  |
| Joint MM                               | 4.897.11      | 6.33  | $7.22\ 8.22$  |
| AdaS_knn                               | $6.22\ 7.44$  | 5.83  | $6.83\ 8.33$  |
| $LassoJoint\_BFGS\_lambda.1se$         | $9.10\ 7.17$  | 7.08  | $5.85\ 6.15$  |
| $LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)$ | $9.80\ 7.67$  | 8.15  | $7.23\ 6.38$  |
| $LassoJoint\_MM\_(log(p)/n)^(1/2)$     | $11.10\ 7.00$ | 7.15  | $7.23\ 8.00$  |
| Weighted BFGS                          | 8.11 9.56     | 10.44 | $10.22\ 6.11$ |

 ${\bf Table~69.~Method~avg.rank~based~on~F1~metrics}$ 

| method                                 | 0.1   | 0.3   | 0.5   | 0.7   | 0.9   |
|--|-------|-------|-------|-------|-------|
| Oracle                                 | 1.11  | 1.22  | 1.33  | 1.44  | 2.00  |
| $LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)$ | 6.40  | 3.92  | 4.38  | 5.31  | 6.46  |
| $Lasso Joint\_BFGS\_lambda.1se$        | 7.20  | 4.58  | 5.15  | 6.23  | 7.00  |
| $LassoJoint\_MM\_lambda.1se$           | 10.40 | 6.58  | 4.85  | 5.15  | 3.69  |
| $LassoJoint\_BFGS\_lambda.min$         | 3.80  | 5.33  | 5.85  | 7.77  | 8.69  |
| $LassoJoint\_MM\_lambda.min$           | 6.70  | 7.25  | 6.54  | 6.31  | 5.08  |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$  | 11.10 | 6.58  | 6.77  | 5.69  | 4.38  |
| Joint MM                               | 6.33  | 8.67  | 6.44  | 6.56  | 6.56  |
| AdaS_svm                               | 7.22  | 8.44  | 8.42  | 4.92  | 5.58  |
| Joint BFGS                             | 4.67  | 5.22  | 6.89  | 7.89  | 10.56 |
| AdaS_knn                               | 5.89  | 8.89  | 7.00  | 7.00  | 6.92  |
| Weighted BFGS                          | 6.78  | 7.22  | 8.78  | 9.89  | 10.89 |
| Naive                                  | 10.67 | 12.67 | 12.89 | 11.00 | 7.67  |

# 5 Results of the joint-wise methods for the high-dimensional datasets

 ${\bf Table~70.~Accuracy~score~on~'Alon\_DS'~dataset}$ 

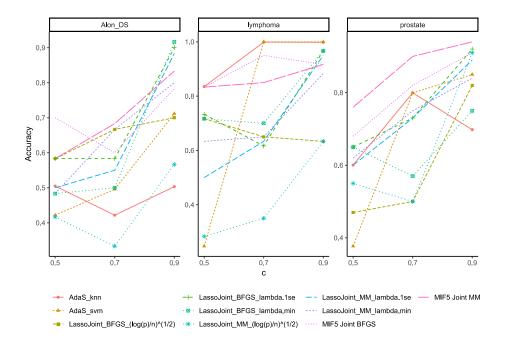
| method  | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.58(0.1)  | 0.67(0.1)  | 0.7(0.17)  |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.58(0.1)  | 0.58(0.1)  | 0.9(0.11)  |
| LassoJoint_BFGS_lambda.min                        | 0.48(0.11) | 0.5(0.16)  | 0.92(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.42(0.1)  | 0.33(0.1)  | 0.57(0.27) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.5(0.1)   | 0.55(0.17) | 0.88(0.11) |
| $LassoJoint\_MM\_lambda.min$                      | 0.48(0.11) | 0.67(0.16) | 0.8(0.17)  |
| MIF5 Joint BFGS                                   | 0.7(0.13)  | 0.6(0.22)  | 0.78(0.17) |
| MIF5 Joint MM                                     | 0.58(0.2)  | 0.68(0.25) | 0.83(0.16) |

Table 71. Accuracy score on 'lymphoma' dataset

| method   | 0.5        | 0.7        | 0.9        |
|--|------------|------------|------------|
| $\overline{\text{LassoJoint\_BFGS\_(log(p)/n)^(1/2)}}$ | 0.72(0.1)  | 0.65(0.09) | 0.63(0.16) |
| $LassoJoint\_BFGS\_lambda.1se$                         | 0.73(0.09) | 0.62(0.25) | 0.97(0.05) |
| LassoJoint_BFGS_lambda.min                             | 0.72(0.29) | 0.7(0.15)  | 0.97(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                       | 0.28(0.1)  | 0.35(0.09) | 0.63(0.16) |
| $LassoJoint\_MM\_lambda.1se$                           | 0.5(0.32)  | 0.63(0.19) | 0.95(0.05) |
| $LassoJoint\_MM\_lambda.min$                           | 0.63(0.27) | 0.65(0.19) | 0.88(0.07) |
| MIF5 Joint BFGS  | 0.83(0.08) | 0.95(0.05) | 0.92(0.08) |
| MIF5 Joint MM  | 0.83(0.1)  | 0.85(0.11) | 0.92(0.08) |

Table 72. Accuracy score on 'prostate' dataset

| method  | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.47(0.1)  | 0.5(0.09)  | 0.82(0.24) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.65(0.17) | 0.73(0.06) | 0.92(0.08) |
| $Lasso Joint\_BFGS\_lambda.min$                   | 0.65(0.05) | 0.57(0.15) | 0.75(0.12) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.55(0.09) | 0.5(0.09)  | 0.91(0.1)  |
| $LassoJoint\_MM\_lambda.1se$                      | 0.6(0.12)  | 0.73(0.08) | 0.89(0.05) |
| LassoJoint_MM_lambda.min                          | 0.62(0.08) | 0.75(0.05) | 0.84(0.07) |
| MIF5 Joint BFGS                                   | 0.68(0.1)  | 0.82(0.06) | 0.91(0.08) |
| MIF5 Joint MM                                     | 0.76(0.08) | 0.9(0.04)  | 0.94(0.08) |



 ${\bf Fig.\,3.}$  The accuracy for the high-dimensional datasets

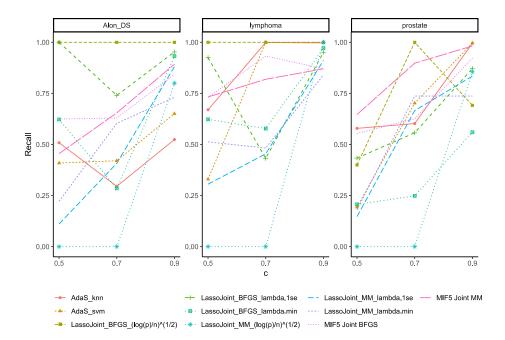


Fig. 4. The recall for the high-dimensional datasets

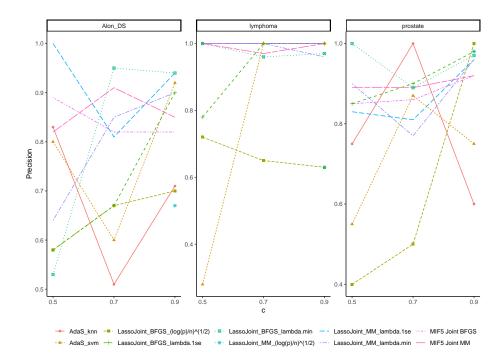


Fig. 5. The precision for the high-dimensional datasets

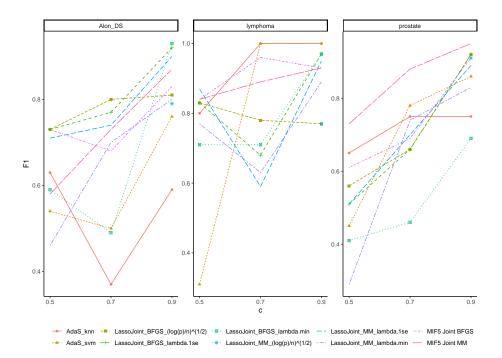


Fig. 6. The F1 score for the high-dimensional datasets

 ${\bf Table~73.~Accuracy~score~on~'Alon\_DS'~dataset}$ 

| method                               | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|
| AdaS_knn                             | 0.5(0.03)  | 0.42(0.03) | 0.5(0.03)  |
| AdaS_svm                             | 0.42(0.03) | 0.5(0.02)  | 0.71(0.05) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.58(0.1)  | 0.67(0.1)  | 0.7(0.17)  |
| $Lasso Joint\_BFGS\_lambda.1se$      | 0.58(0.1)  | 0.58(0.1)  | 0.9(0.11)  |
| LassoJoint_BFGS_lambda.min           | 0.48(0.11) | 0.5(0.16)  | 0.92(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 0.42(0.1)  | 0.33(0.1)  | 0.57(0.27) |
| $Lasso Joint\_MM\_lambda.1se$        | 0.5(0.1)   | 0.55(0.17) | 0.88(0.11) |
| $LassoJoint\_MM\_lambda.min$         | 0.48(0.11) | 0.67(0.16) | 0.8(0.17)  |
| MIF5 Joint BFGS                      | 0.7(0.13)  | 0.6(0.22)  | 0.78(0.17) |
| MIF5 Joint MM                        | 0.58(0.2)  | 0.68(0.25) | 0.83(0.16) |

Table 74. Accuracy score on 'lymphoma' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.84(0.02) | 1(0.01)    | 1(0.01)    |
| AdaS_svm                              | 0.25(0.03) | 1(0)       | 1(0)       |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.72(0.1)  | 0.65(0.09) | 0.63(0.16) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0.73(0.09) | 0.62(0.25) | 0.97(0.05) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.72(0.29) | 0.7(0.15)  | 0.97(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.28(0.1)  | 0.35(0.09) | 0.63(0.16) |
| $LassoJoint\_MM\_lambda.1se$          | 0.5(0.32)  | 0.63(0.19) | 0.95(0.05) |
| LassoJoint_MM_lambda.min              | 0.63(0.27) | 0.65(0.19) | 0.88(0.07) |
| MIF5 Joint BFGS                       | 0.83(0.08) | 0.95(0.05) | 0.92(0.08) |
| MIF5 Joint MM                         | 0.83(0.1)  | 0.85(0.11) | 0.92(0.08) |

Table 75. Accuracy score on 'prostate' dataset

| method                               | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|
| AdaS_knn                             | 0.6(0.01)  | 0.8(0.01)  | 0.7(0.02)  |
| AdaS_svm                             |            | 0.8(0.01)  |            |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.47(0.1)  | 0.5(0.09)  | 0.82(0.24) |
| $Lasso Joint\_BFGS\_lambda.1se$      | 0.65(0.17) | 0.73(0.06) | 0.92(0.08) |
| LassoJoint_BFGS_lambda.min           | 0.65(0.05) | 0.57(0.15) | 0.75(0.12) |
| $LassoJoint\_MM\_(log(p)/n)^(1/2)$   | 0.55(0.09) | 0.5(0.09)  | 0.91(0.1)  |
| $Lasso Joint\_MM\_lambda.1se$        | 0.6(0.12)  | 0.73(0.08) | 0.89(0.05) |
| $LassoJoint\_MM\_lambda.min$         |            | 0.75(0.05) |            |
| MIF5 Joint BFGS                      | 0.68(0.1)  | 0.82(0.06) | 0.91(0.08) |
| MIF5 Joint MM                        | 0.76(0.08) | 0.9(0.04)  | 0.94(0.08) |

Table 76. Recall score on 'Alon\_DS' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.51(0.05) | 0.3(0.07)  | 0.52(0.15) |
| $AdaS\_svm$                           | 0.41(0.06) | 0.42(0.05) | 0.65(0.04) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 1(0)       | 1(0)       |
| $LassoJoint\_BFGS\_lambda.1se$        | 1(0)       | 0.74(0.43) | 0.95(0.06) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.62(0.52) | 0.28(0.23) | 0.93(0.06) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | 0(0)       | 0(0)       | 0.8(0.45)  |
| $LassoJoint\_MM\_lambda.1se$          | 0.11(0.25) | 0.41(0.37) | 0.88(0.17) |
| $LassoJoint\_MM\_lambda.min$          | 0.22(0.24) | 0.6(0.19)  | 0.73(0.24) |
| MIF5 Joint BFGS                       | 0.62(0.13) | 0.63(0.27) | 0.84(0.14) |
| MIF5 Joint MM                         | 0.46(0.2)  | 0.65(0.28) | 0.89(0.15) |

Table 77. Recall score on 'lymphoma' dataset

| method                               | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|
| AdaS_knn                             | 0.67(0.03) | 1(0.01)    | 1(0.02)    |
| AdaS_svm                             | 0.33(0.03) | 1(0)       | 1(0)       |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 1(0)       | 1(0)       | 1(0)       |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.92(0.17) | 0.43(0.32) | 0.95(0.07) |
| $LassoJoint\_BFGS\_lambda.min$       | 0.62(0.37) | 0.58(0.19) | 0.97(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 0(0)       | 0(0)       | 1(0)       |
| $Lasso Joint\_MM\_lambda.1se$        | 0.31(0.42) | 0.45(0.24) | 0.91(0.09) |
| LassoJoint_MM_lambda.min             | 0.51(0.34) | 0.48(0.22) | 0.84(0.11) |
| MIF5 Joint BFGS                      | 0.73(0.11) | 0.93(0.06) | 0.87(0.13) |
| MIF5 Joint MM                        | 0.73(0.12) | 0.82(0.11) | 0.87(0.13) |

Table 78. Recall score on 'prostate' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.58(0.04) | 0.6(0.02)  | 1(0.04)    |
| AdaS_svm                              | 0.19(0.19) | 0.7(0.02)  | 1(0.04)    |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$  | 0.4(0.55)  | 1(0)       | 0.69(0.4)  |
| $Lasso Joint\_BFGS\_lambda.1se$       |            | 0.56(0.14) |            |
| $LassoJoint\_BFGS\_lambda.min$        | 0.21(0.13) | 0.25(0.36) | 0.56(0.19) |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | 0(0)       | 0(0)       | 0.86(0.13) |
| $Lasso Joint\_MM\_lambda.1se$         | 0.15(0.2)  | 0.67(0.2)  | 0.83(0.07) |
| LassoJoint_MM_lambda.min              | 0.18(0.1)  | 0.74(0.17) | 0.74(0.14) |
| MIF5 Joint BFGS                       | 0.55(0.3)  | 0.63(0.32) | 0.92(0.13) |
| MIF5 Joint MM                         | 0.65(0.19) | 0.9(0.12)  | 0.98(0.04) |

Table 79. F1 score on 'Alon\_DS' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.63(0.02) | 0.37(0.05) | 0.59(0.09) |
| $AdaS\_svm$                           | 0.54(0.02) | 0.5(0.03)  | 0.76(0.05) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.73(0.08) | 0.8(0.07)  | 0.81(0.13) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.73(0.08) | 0.77(0.04) | 0.92(0.1)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.59(0.26) | 0.49(0.2)  | 0.93(0.06) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | NaN(NA)    | NaN(NA)    | 0.79(0.14) |
| $LassoJoint\_MM\_lambda.1se$          | 0.71(NA)   | 0.74(0.03) | 0.9(0.1)   |
| $LassoJoint\_MM\_lambda.min$          | 0.46(0.19) | 0.7(0.15)  | 0.8(0.2)   |
| MIF5 Joint BFGS                       | 0.73(0.14) | 0.68(0.2)  | 0.83(0.14) |
| MIF5 Joint MM                         | 0.58(0.24) | 0.73(0.22) | 0.87(0.13) |

Table 80. F1 score on 'lymphoma' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.8(0.02)  | 1(0.01)    | 1(0.01)    |
| AdaS_svm                              | 0.31(0)    | 1(0)       | 1(0)       |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.83(0.07) | 0.78(0.07) | 0.77(0.12) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.83(0.07) | 0.68(0.2)  | 0.97(0.04) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.71(0.33) | 0.71(0.15) | 0.97(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | NaN(NA)    | 0.77(0.12) |
| $LassoJoint\_MM\_lambda.1se$          | 0.87(0.01) | 0.59(0.22) | 0.95(0.05) |
| LassoJoint_MM_lambda.min              | 0.77(0.16) | 0.63(0.21) | 0.89(0.08) |
| MIF5 Joint BFGS                       | 0.84(0.07) | 0.96(0.03) | 0.93(0.08) |
| MIF5 Joint MM                         | 0.84(0.09) | 0.89(0.09) | 0.93(0.08) |

Table 81. F1 score on 'prostate' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.65(0.02) | 0.75(0.01) | 0.75(0.02) |
| $AdaS_svm$                            | 0.45(0)    | 0.78(0)    | 0.86(0.02) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.56(0.15) | 0.66(0.09) | 0.92(0.06) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.51(0.33) | 0.66(0.07) | 0.92(0.07) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.41(0.09) | 0.46(0.3)  | 0.69(0.18) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | NaN(NA)    | 0.91(0.09) |
| LassoJoint_MM_lambda.1se              | 0.51(0.09) | 0.7(0.08)  | 0.89(0.05) |
| LassoJoint_MM_lambda.min              | 0.29(0.13) | 0.74(0.06) | 0.83(0.09) |
| MIF5 Joint BFGS                       | 0.61(0.2)  | 0.69(0.26) | 0.91(0.08) |
| MIF5 Joint MM                         | 0.73(0.11) | 0.88(0.08) | 0.95(0.07) |

Table 82. Precision score on 'Alon\_DS' dataset

| method                                | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|
| AdaS_knn                              | 0.83(0.02) | 0.51(0.04) | 0.71(0.05) |
| AdaS_svm                              | 0.8(0.03)  | 0.6(0.01)  | 0.92(0.08) |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.58(0.1)  | 0.67(0.1)  | 0.7(0.17)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.58(0.1)  | 0.67(0.08) | 0.9(0.16)  |
| $LassoJoint\_BFGS\_lambda.min$        | 0.53(0.36) | 0.95(0.1)  | 0.94(0.13) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | NaN(NA)    | NaN(NA)    | 0.67(0.18) |
| $LassoJoint\_MM\_lambda.1se$          | 1(NA)      | 0.81(0.03) | 0.94(0.13) |
| LassoJoint_MM_lambda.min              | 0.64(0.13) | 0.85(0.09) | 0.9(0.15)  |
| MIF5 Joint BFGS                       | 0.89(0.18) | 0.82(0.18) | 0.82(0.14) |
| MIF5 Joint MM                         | 0.82(0.29) | 0.91(0.12) | 0.85(0.12) |

Table 83. Precision score on 'lymphoma' dataset

| method                               | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|
| AdaS_knn                             | 1(0)       | 1(0)       | 1(0)       |
| AdaS_svm                             | 0.28(0.03) | 1(0)       | 1(0)       |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.72(0.1)  | 0.65(0.09) | 0.63(0.16) |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.78(0.15) | 1(0)       | 1(0)       |
| $LassoJoint\_BFGS\_lambda.min$       | 1(0)       | 0.96(0.09) | 0.97(0.07) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | NaN(NA)    | NaN(NA)    | 0.63(0.16) |
| $LassoJoint\_MM\_lambda.1se$         | 1(0)       | 1(0)       | 1(0)       |
| LassoJoint_MM_lambda.min             | 1(0)       | 1(0)       | 0.96(0.09) |
| MIF5 Joint BFGS                      | 1(0)       | 1(0)       | 1(0)       |
| MIF5 Joint MM                        | 1(0)       | 0.97(0.06) | 1(0)       |

Table 84. Precision score on 'prostate' dataset

| method                               | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|
| AdaS_knn                             | 0.75(0.03) | 1(0.02)    | 0.6(0)     |
| AdaS_svm                             | 0.55(0.01) | 0.87(0.01) | 0.75(0.02) |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.4(0.14)  | 0.5(0.09)  | 1(0)       |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.85(0.24) | 0.9(0.16)  | 0.98(0.04) |
| $LassoJoint\_BFGS\_lambda.min$       | 1(0)       | 0.89(0.19) | 0.97(0.06) |
| $LassoJoint_MM_{(log(p)/n)^{(1/2)}}$ | NaN(NA)    | NaN(NA)    | 0.98(0.05) |
| $LassoJoint\_MM\_lambda.1se$         | 0.83(0.24) | 0.81(0.14) | 0.96(0.06) |
| $LassoJoint\_MM\_lambda.min$         | 0.9(0.22)  | 0.77(0.09) | 0.96(0.05) |
| MIF5 Joint BFGS                      | 0.85(0.17) | 0.86(0.22) | 0.92(0.12) |
| MIF5 Joint MM                        | 0.89(0.12) | 0.89(0.16) | 0.92(0.12) |

 ${\bf Table~85.~Method~avg.rank~based~on~Accuracy~metrics}$ 

| method                                | 0.5  | 0.7 - 0.9     |
|---------------------------------------|------|---------------|
| MIF5 Joint MM                         | 2.67 | 2.00 4.00     |
| MIF5 Joint BFGS                       | 1.67 | $3.00\ 5.33$  |
| $LassoJoint\_BFGS\_lambda.1se$        | 3.33 | $6.67\ 2.33$  |
| AdaS_knn                              | 4.00 | 4.337.00      |
| $LassoJoint\_BFGS\_lambda.min$        | 5.67 | $7.00\ 4.67$  |
| $LassoJoint\_MM\_lambda.1se$          | 7.00 | $7.00\ 4.33$  |
| $LassoJoint\_MM\_lambda.min$          | 6.67 | $5.00\ 6.67$  |
| AdaS_svm                              | 9.67 | $4.33\ 5.00$  |
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 5.33 | $5.67\ 8.33$  |
| LassoJoint_MM_ $(\log(p)/n)^(1/2)$    |      | $10.00\ 7.33$ |

 ${\bf Table~86.~Method~avg.rank~based~on~Recall~metrics}$ 

| method  | 0.5   | 0.7 0.9         |
|---|-------|-----------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 2.33  | 1.67 4.33       |
| MIF5 Joint MM                                     | 3.67  | $3.33 \ 5.33$   |
| AdaS_knn  | 4.00  | $5.33\ 4.00$    |
| $LassoJoint\_BFGS\_lambda.1se$                    | 2.67  | $6.33\ 4.33$    |
| MIF5 Joint BFGS                                   | 3.33  | $4.67\ 6.00$    |
| AdaS_svm  | 7.33  | $4.00\ 4.33$    |
| LassoJoint_BFGS_lambda.min                        | 5.00  | $8.00\ 6.00$    |
| LassoJoint_MM_lambda.min                          | 7.67  | $5.00\ 8.67$    |
| LassoJoint_MM_lambda.1se                          | 9.00  | $6.67\ 6.33$    |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 10.00 | $10.00 \; 5.67$ |

Table 87. Method avg.rank based on Precision metrics

| method                               | 0.5   | 0.7 0.9       |
|--------------------------------------|-------|---------------|
| LassoJoint_BFGS_lambda.min           | 4.00  | 4.00 4.00     |
| $Lasso Joint\_MM\_lambda.1se$        | 3.33  | $5.33\ 3.67$  |
| $LassoJoint\_BFGS\_lambda.1se$       | 6.33  | $4.00\ 3.00$  |
| AdaS_knn                             | 3.67  | 3.67  6.33    |
| MIF5 Joint MM                        | 4.33  | $4.33\ 6.67$  |
| LassoJoint_MM_lambda.min             | 4.00  | $5.33 \ 6.33$ |
| MIF5 Joint BFGS                      | 4.00  | $5.33 \ 6.33$ |
| AdaS_svm                             | 7.33  | $5.00\ 4.67$  |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 8.00  | 8.00  6.33    |
| $LassoJoint\_MM\_(log(p)/n)^{}(1/2)$ | 10.00 | $10.00\ 7.67$ |

Table 88. Method avg.rank based on F1 metrics

| method                               | 0.5   | 0.7 0.9        |
|--------------------------------------|-------|----------------|
| MIF5 Joint MM                        | 3.67  | 3.00 4.00      |
| $LassoJoint\_BFGS\_lambda.1se$       | 4.00  | $5.67\ 2.67$   |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 3.00  | $4.33\ 5.67$   |
| MIF5 Joint BFGS                      | 2.67  | $5.00\ 5.33$   |
| $Lasso Joint\_MM\_lambda.1se$        | 3.67  | $5.67\ 4.67$   |
| AdaS_knn                             | 4.33  | $4.33\ 6.67$   |
| AdaS_svm                             | 8.00  | $3.67 \; 6.00$ |
| LassoJoint_BFGS_lambda.min           | 7.33  | $7.67\ 5.00$   |
| LassoJoint_MM_lambda.min             | 8.33  | $5.67\ 7.67$   |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 10.00 | $10.00\ 7.33$  |

## 6 Selection methods: the LassoJoint and the MIF5

Table 89. No. of features; dataset 'Alon\_DS'

| dataset method                                 |      | no_features |
|--|------|-------------|
| Alon_DS LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.30 | 0.00        |
| Alon_DS LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.50 | 0.00        |
| Alon_DS LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.70 | 0.00        |
| Alon_DS LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.90 | 0.00        |
| Alon_DS LassoJoint_BFGS_lambda.1se             | 0.30 | 1.00        |
| Alon_DS LassoJoint_BFGS_lambda.1se             | 0.50 | 0.80        |
| Alon_DS LassoJoint_BFGS_lambda.1se             | 0.70 | 0.40        |
| Alon_DS LassoJoint_BFGS_lambda.1se             | 0.90 | 5.60        |
| Alon_DS LassoJoint_BFGS_lambda.min             | 0.30 | 10.60       |
| Alon_DS LassoJoint_BFGS_lambda.min             | 0.50 | 2.80        |
| Alon_DS LassoJoint_BFGS_lambda.min             | 0.70 | 9.40        |
| Alon_DS LassoJoint_BFGS_lambda.min             | 0.90 | 13.40       |
| Alon_DS LassoMM_ $(\log(p)/n)^(1/2)$           | 0.30 | 0.00        |
| Alon_DS LassoMM_ $(\log(p)/n)^(1/2)$           | 0.50 | 0.00        |
| Alon_DS LassoMM_ $(\log(p)/n)^(1/2)$           | 0.70 | 0.00        |
| Alon_DS LassoMM_ $(\log(p)/n)^(1/2)$           | 0.90 | 0.00        |
| Alon_DS LassoMM_lambda.1se                     | 0.30 | 2.60        |
| Alon_DS LassoMM_lambda.1se                     | 0.50 | 0.60        |
| Alon_DS LassoMM_lambda.1se                     | 0.70 | 1.60        |
| Alon_DS LassoMM_lambda.1se                     | 0.90 | 6.60        |
| Alon_DS LassoMM_lambda.min                     | 0.30 | 9.20        |
| Alon_DS LassoMM_lambda.min                     | 0.50 | 4.00        |
| Alon_DS LassoMM_lambda.min                     | 0.70 | 12.80       |
| Alon_DS LassoMM_lambda.min                     | 0.90 | 13.80       |

Table 90. No. of features; dataset 'Artif'

| dataset | method                                 | c    | no_features |
|---------|--|------|-------------|
| Artif   | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.10 | 0.00        |
| Artif   | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.30 | 3.64        |
| Artif   | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.50 | 5.00        |
| Artif   | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.70 | 5.00        |
| Artif   | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.90 | 5.00        |
| Artif   | LassoJoint_BFGS_lambda.1se             | 0.10 | 0.50        |
| Artif   | $LassoJoint\_BFGS\_lambda.1se$         | 0.30 | 5.04        |
| Artif   | $LassoJoint\_BFGS\_lambda.1se$         | 0.50 | 5.00        |
| Artif   | $LassoJoint\_BFGS\_lambda.1se$         | 0.70 | 5.02        |
| Artif   | $LassoJoint\_BFGS\_lambda.1se$         | 0.90 | 5.20        |
| Artif   | $LassoJoint\_BFGS\_lambda.min$         | 0.10 | 12.66       |
| Artif   | $LassoJoint\_BFGS\_lambda.min$         | 0.30 | 12.64       |
| Artif   | $LassoJoint\_BFGS\_lambda.min$         | 0.50 | 9.62        |
| Artif   | $LassoJoint\_BFGS\_lambda.min$         | 0.70 | 9.22        |
| Artif   | $LassoJoint\_BFGS\_lambda.min$         | 0.90 | 13.94       |
| Artif   | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.10 | 0.00        |
| Artif   | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.30 | 3.64        |
| Artif   | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.50 | 5.00        |
| Artif   | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.70 | 5.00        |
| Artif   | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.90 | 5.00        |
| Artif   | $LassoMM\_lambda.1se$                  | 0.10 | 0.74        |
| Artif   | $Lasso MM\_lamb da.1se$                | 0.30 | 5.02        |
| Artif   | LassoMM_lambda.1se                     | 0.50 | 5.00        |
| Artif   | LassoMM_lambda.1se                     | 0.70 | 5.02        |
| Artif   | LassoMM_lambda.1se                     | 0.90 | 5.14        |
| Artif   | LassoMM_lambda.min                     | 0.10 | 11.96       |
| Artif   | LassoMM_lambda.min                     | 0.30 | 12.40       |
| Artif   | LassoMM_lambda.min                     | 0.50 | 9.76        |
| Artif   | LassoMM_lambda.min                     | 0.70 | 8.84        |
| Artif   | LassoMM_lambda.min                     | 0.90 | 13.32       |

Table 91. No. of features; dataset 'Banknote'

| dataset  | method                                 | С    | no_features |
|----------|--|------|-------------|
|          | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 0.06        |
|          | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)  |      | 1.68        |
|          | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 2.98        |
|          | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 3.00        |
|          | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.90 | 3.00        |
|          | LassoJoint_BFGS_lambda.1se             | 0.10 | 0.06        |
| Banknote | LassoJoint_BFGS_lambda.1se             | 0.30 | 2.80        |
| Banknote | LassoJoint_BFGS_lambda.1se             | 0.50 | 3.02        |
| Banknote | LassoJoint_BFGS_lambda.1se             | 0.70 | 3.00        |
| Banknote | LassoJoint_BFGS_lambda.1se             | 0.90 | 3.22        |
| Banknote | LassoJoint_BFGS_lambda.min             | 0.10 | 3.90        |
| Banknote | LassoJoint_BFGS_lambda.min             | 0.30 | 3.70        |
| Banknote | LassoJoint_BFGS_lambda.min             | 0.50 | 3.88        |
| Banknote | LassoJoint_BFGS_lambda.min             | 0.70 | 3.74        |
| Banknote | LassoJoint_BFGS_lambda.min             | 0.90 | 4.00        |
| Banknote | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.10 | 0.06        |
| Banknote | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.30 | 1.68        |
| Banknote | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.50 | 2.98        |
| Banknote | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.70 | 3.00        |
| Banknote | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.90 | 3.00        |
| Banknote | LassoMM_lambda.1se                     | 0.10 | 0.08        |
| Banknote | LassoMM_lambda.1se                     | 0.30 | 2.80        |
| Banknote | LassoMM_lambda.1se                     | 0.50 | 3.04        |
| Banknote | LassoMM_lambda.1se                     | 0.70 | 3.00        |
| Banknote | LassoMM_lambda.1se                     | 0.90 | 3.26        |
| Banknote | LassoMM_lambda.min                     | 0.10 | 3.88        |
| Banknote | LassoMM_lambda.min                     | 0.30 | 3.72        |
| Banknote | LassoMM_lambda.min                     | 0.50 | 3.86        |
| Banknote | LassoMM_lambda.min                     | 0.70 | 3.72        |
| Banknote | LassoMM_lambda.min                     | 0.90 | 4.00        |

Table 92. No. of features; dataset 'Breastc'

| dataset method                                     | C    | no_features |
|--|------|-------------|
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       |      | 0.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 3.00        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 4.38        |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      |             |
| Breastc LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 6.92        |
| Breastc LassoJoint_BFGS_lambda.1se                 | 0.10 | 0.22        |
| Breastc LassoJoint_BFGS_lambda.1se                 | 0.30 | -           |
| Breastc LassoJoint_BFGS_lambda.1se                 | 0.50 | 5.26        |
| Breastc LassoJoint_BFGS_lambda.1se                 | 0.70 |             |
| Breastc LassoJoint_BFGS_lambda.1se                 | 0.90 | 7.34        |
| Breastc LassoJoint_BFGS_lambda.min                 | 0.10 | 5.58        |
| Breastc LassoJoint_BFGS_lambda.min                 | 0.30 |             |
| Breastc LassoJoint_BFGS_lambda.min                 | 0.50 | 6.64        |
| Breastc LassoJoint_BFGS_lambda.min                 | 0.70 | 8.24        |
| Breastc LassoJoint_BFGS_lambda.min                 | 0.90 | 8.22        |
| Breastc LassoMM_ $(\log(p)/n)^{(1/2)}$             | 0.10 | 0.00        |
| Breastc LassoMM <sub>-</sub> $(\log(p)/n)^{(1/2)}$ | 0.30 | 3.00        |
| Breastc LassoMM <sub>-</sub> $(\log(p)/n)^{(1/2)}$ | 0.50 | 4.38        |
| Breastc LassoMM_ $(\log(p)/n)^{(1/2)}$             | 0.70 | 5.18        |
| Breastc LassoMM_ $(\log(p)/n)^{(1/2)}$             | 0.90 | 6.92        |
| Breastc LassoMM_lambda.1se                         | 0.10 | 0.14        |
| Breastc LassoMM_lambda.1se                         | 0.30 | 3.92        |
| Breastc LassoMM_lambda.1se                         | 0.50 | 5.42        |
| Breastc LassoMM_lambda.1se                         | 0.70 | 6.10        |
| Breastc LassoMM_lambda.1se                         | 0.90 | 7.34        |
| Breastc LassoMM_lambda.min                         | 0.10 | 5.58        |
| Breastc LassoMM_lambda.min                         | 0.30 | 5.28        |
| Breastc LassoMM_lambda.min                         | 0.50 | 6.70        |
| Breastc LassoMM_lambda.min                         | 0.70 | 8.20        |
| $Breastc\ LassoMM\_lambda.min$                     | 0.90 | 8.28        |

**Table 93.** No. of features; dataset 'Credit\_a'

| 1.4.4.4.1.1                                     |      | <u> </u>    |
|---|------|-------------|
| dataset method                                  |      | no_features |
| Credit_a LassoJoint_BFGS_(log(p)/n)^(1/2)       |      | 0.00        |
| Credit_a LassoJoint_BFGS_(log(p)/n)^(1/2)       |      | 1.00        |
| Credit_a LassoJoint_BFGS_(log(p)/n)^(1/2)       | 0.50 | 1.00        |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.70 | 1.96        |
| Credit_a LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 1.82        |
| Credit_a LassoJoint_BFGS_lambda.1se             | 0.10 | 0.04        |
| Credit_a LassoJoint_BFGS_lambda.1se             | 0.30 | 0.90        |
| Credit_a LassoJoint_BFGS_lambda.1se             | 0.50 | 1.06        |
| Credit_a LassoJoint_BFGS_lambda.1se             | 0.70 | 2.16        |
| Credit_a LassoJoint_BFGS_lambda.1se             | 0.90 | 5.90        |
| $Credit_a LassoJoint_BFGS_lambda.min$           | 0.10 | 3.76        |
| Credit_a LassoJoint_BFGS_lambda.min             | 0.30 | 5.20        |
| Credit_a LassoJoint_BFGS_lambda.min             | 0.50 | 9.32        |
| Credit_a LassoJoint_BFGS_lambda.min             | 0.70 | 9.62        |
| Credit_a LassoJoint_BFGS_lambda.min             | 0.90 | 17.30       |
| Credit_a LassoMM_ $(\log(p)/n)^(1/2)$           | 0.10 | 0.00        |
| Credit_a LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.30 | 1.00        |
| Credit_a LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.50 | 1.00        |
| Credit_a LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.70 | 1.96        |
| Credit_a LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.90 | 1.82        |
| Credit_a LassoMM_lambda.1se                     | 0.10 | 0.00        |
| Credit_a LassoMM_lambda.1se                     | 0.30 | 0.96        |
| Credit_a LassoMM_lambda.1se                     | 0.50 | 1.20        |
| Credit_a LassoMM_lambda.1se                     | 0.70 | 2.38        |
| Credit_a LassoMM_lambda.1se                     | 0.90 | 5.88        |
| Credit_a LassoMM_lambda.min                     | 0.10 | 3.68        |
| Credit_a LassoMM_lambda.min                     | 0.30 | 6.44        |
| Credit_a LassoMM_lambda.min                     | 0.50 | 9.34        |
| Credit_a LassoMM_lambda.min                     | 0.70 | 9.64        |
| Credit_a LassoMM_lambda.min                     | 0.90 | 17.60       |

 $\textbf{Table 94.} \ \ \text{No. of features; dataset 'Credit\_g'}$ 

| dataset method                                  |      | no_features |
|---|------|-------------|
| Credit_g LassoJoint_BFGS_(log(p)/n)^(1/2)       |      | 0.00        |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 0.08        |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 1.08        |
| Credit_g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 2.50        |
| Credit-g LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 3.70        |
| Credit_g LassoJoint_BFGS_lambda.1se             | 0.10 | 0.00        |
| Credit_g LassoJoint_BFGS_lambda.1se             | 0.30 | 0.00        |
| Credit_g LassoJoint_BFGS_lambda.1se             | 0.50 | 2.92        |
| Credit_g LassoJoint_BFGS_lambda.1se             | 0.70 | 7.26        |
| Credit_g LassoJoint_BFGS_lambda.1se             | 0.90 | 11.42       |
| Credit_g LassoJoint_BFGS_lambda.min             | 0.10 | 0.36        |
| Credit_g LassoJoint_BFGS_lambda.min             | 0.30 | 4.84        |
| Credit_g LassoJoint_BFGS_lambda.min             | 0.50 | 11.38       |
| Credit_g LassoJoint_BFGS_lambda.min             | 0.70 | 15.02       |
| Credit_g LassoJoint_BFGS_lambda.min             | 0.90 | 18.40       |
| Credit_g LassoMM_ $(\log(p)/n)^(1/2)$           | 0.10 | 0.00        |
| Credit_g LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.30 | 0.08        |
| Credit_g LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.50 | 1.08        |
| Credit_g LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.70 | 2.50        |
| Credit_g LassoMM_ $(\log(p)/n)^(1/2)$           | 0.90 | 3.70        |
| Credit_g LassoMM_lambda.1se                     | 0.10 | 0.00        |
| Credit_g LassoMM_lambda.1se                     | 0.30 | 0.00        |
| Credit_g LassoMM_lambda.1se                     | 0.50 | 3.06        |
| Credit_g LassoMM_lambda.1se                     | 0.70 | 7.08        |
| $Credit_g LassoMM_lambda.1se$                   | 0.90 | 11.38       |
| Credit_g LassoMM_lambda.min                     | 0.10 | 0.32        |
| Credit_g LassoMM_lambda.min                     | 0.30 | 4.86        |
| Credit_g LassoMM_lambda.min                     | 0.50 | 11.44       |
| Credit_g LassoMM_lambda.min                     | 0.70 | 15.08       |
| $Credit_g LassoMM_lambda.min$                   | 0.90 | 18.16       |

Table 95. No. of features; dataset 'dhfr'

| dataset | method                                 | C    | no_features |
|---------|--|------|-------------|
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.30 | 0.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 1.00        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ |      | 1.20        |
| dhfr    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.90 | 4.60        |
| dhfr    | LassoJoint_BFGS_lambda.1se             | 0.10 | 0.00        |
| dhfr    | LassoJoint_BFGS_lambda.1se             | 0.30 |             |
| dhfr    | LassoJoint_BFGS_lambda.1se             | 0.50 | 6.80        |
| dhfr    | LassoJoint_BFGS_lambda.1se             | 0.70 | 12.00       |
| dhfr    | LassoJoint_BFGS_lambda.1se             | 0.90 |             |
| dhfr    | LassoJoint_BFGS_lambda.min             | 0.10 | 3.00        |
| dhfr    | LassoJoint_BFGS_lambda.min             | 0.30 | 15.60       |
| dhfr    | LassoJoint_BFGS_lambda.min             | 0.50 | 11.80       |
| dhfr    | LassoJoint_BFGS_lambda.min             | 0.70 | 26.60       |
| dhfr    | LassoJoint_BFGS_lambda.min             | 0.90 | 23.20       |
| dhfr    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.10 | 0.00        |
| dhfr    | LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.30 | 0.00        |
| dhfr    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.50 | 1.00        |
| dhfr    | LassoMM_ $(\log(p)/n)^{(1/2)}$         | 0.70 | 1.20        |
| dhfr    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.90 | 4.60        |
| dhfr    | LassoMM_lambda.1se                     | 0.10 | 0.00        |
| dhfr    | $LassoMM\_lambda.1se$                  | 0.30 | 0.20        |
| dhfr    | LassoMM_lambda.1se                     | 0.50 | 5.80        |
| dhfr    | $LassoMM\_lambda.1se$                  | 0.70 | 13.40       |
| dhfr    | $LassoMM\_lambda.1se$                  | 0.90 | 11.80       |
| dhfr    | LassoMM_lambda.min                     | 0.10 | 3.20        |
| dhfr    | LassoMM_lambda.min                     | 0.30 | 15.00       |
| dhfr    | LassoMM_lambda.min                     | 0.50 | 13.40       |
| dhfr    | LassoMM_lambda.min                     | 0.70 | 23.20       |
| dhfr    | LassoMM_lambda.min                     | 0.90 | 21.40       |

Table 96. No. of features; dataset 'Diabetes'

| dataset method                                      | C    | no_features |
|---|------|-------------|
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$       | -    | 0.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 1.00        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 1.84        |
| Diabetes LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$     |      | 2.76        |
| Diabetes Lasso Joint_BFGS_ $(\log(p)/n)^{(1/2)}$    |      | 3.52        |
| Diabetes LassoJoint_BFGS_lambda.1se                 | 0.10 | 0.00        |
| Diabetes LassoJoint_BFGS_lambda.1se                 | 0.30 | 0.26        |
| Diabetes LassoJoint_BFGS_lambda.1se                 | 0.50 | 1.98        |
| Diabetes LassoJoint_BFGS_lambda.1se                 | 0.70 | 2.86        |
| Diabetes LassoJoint_BFGS_lambda.1se                 | 0.90 | 4.14        |
| Diabetes LassoJoint_BFGS_lambda.min                 | 0.10 | 2.76        |
| Diabetes LassoJoint_BFGS_lambda.min                 | 0.30 | 3.98        |
| Diabetes LassoJoint_BFGS_lambda.min                 | 0.50 | 6.36        |
| Diabetes LassoJoint_BFGS_lambda.min                 | 0.70 | 4.94        |
| Diabetes LassoJoint_BFGS_lambda.min                 | 0.90 | 7.60        |
| Diabetes LassoMM_ $(\log(p)/n)^{(1/2)}$             | 0.10 | 0.00        |
| Diabetes LassoMM <sub>-</sub> ( $\log(p)/n$ )^(1/2) | 0.30 | 1.00        |
| Diabetes LassoMM <sub>-</sub> $(\log(p)/n)^{(1/2)}$ | 0.50 | 1.84        |
| Diabetes LassoMM <sub>-</sub> $(\log(p)/n)^{(1/2)}$ | 0.70 | 2.76        |
| Diabetes LassoMM_ $(\log(p)/n)^{(1/2)}$             | 0.90 | 3.52        |
| Diabetes LassoMM_lambda.1se                         | 0.10 | 0.00        |
| Diabetes LassoMM_lambda.1se                         | 0.30 | 0.14        |
| Diabetes LassoMM_lambda.1se                         | 0.50 | 2.00        |
| Diabetes LassoMM_lambda.1se                         | 0.70 | 2.88        |
| Diabetes LassoMM_lambda.1se                         | 0.90 | 4.12        |
| Diabetes LassoMM_lambda.min                         | 0.10 | 2.84        |
| Diabetes LassoMM_lambda.min                         | 0.30 | 4.04        |
| Diabetes LassoMM_lambda.min                         | 0.50 | 6.14        |
| Diabetes LassoMM_lambda.min                         | 0.70 | 5.08        |
| Diabetes LassoMM_lambda.min                         | 0.90 | 7.50        |

Table 97. No. of features; dataset 'lymphoma'

| dataset  | method                                 | С    | no_features |
|----------|--|------|-------------|
| lymphoma | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)  | 0.50 | 0.00        |
| lymphoma | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.70 | 0.00        |
| lymphoma | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.90 | 0.00        |
| lymphoma | $LassoJoint\_BFGS\_lambda.1se$         | 0.50 | 2.60        |
| lymphoma | $LassoJoint\_BFGS\_lambda.1se$         | 0.70 | 2.00        |
| lymphoma | $LassoJoint\_BFGS\_lambda.1se$         | 0.90 | 4.80        |
| lymphoma | $LassoJoint\_BFGS\_lambda.min$         | 0.50 | 16.80       |
| lymphoma | $LassoJoint\_BFGS\_lambda.min$         | 0.70 | 10.20       |
| lymphoma | $LassoJoint\_BFGS\_lambda.min$         | 0.90 | 12.80       |
| lymphoma | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.50 | 0.00        |
| lymphoma | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.70 | 0.00        |
| lymphoma | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.90 | 0.00        |
| lymphoma | LassoMM_lambda.1se                     | 0.50 | 4.60        |
| lymphoma | $Lasso MM\_lamb da.1se$                | 0.70 | 4.00        |
| lymphoma | $Lasso MM\_lamb da.1se$                | 0.90 | 3.80        |
| lymphoma | LassoMM_lambda.min                     | 0.50 | 15.00       |
| lymphoma | LassoMM_lambda.min                     | 0.70 | 9.40        |
| lymphoma | $Lasso MM\_lamb da.min$                | 0.90 | 15.20       |

Table 98. No. of features; dataset 'prostate'

| dataset method  | c    | no_features |
|---|------|-------------|
| $\overline{\text{prostate LassoJoint\_BFGS\_(log(p)/n)^(1/2)}}$ | 0.30 | 0.00        |
| prostate LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                 | 0.50 | 0.00        |
| prostate LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                 | 0.70 | 0.00        |
| prostate LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$                 | 0.90 | 1.00        |
| $prostate\ LassoJoint\_BFGS\_lambda.1se$                        | 0.30 | 1.80        |
| $prostate\ LassoJoint\_BFGS\_lambda.1se$                        | 0.50 | 1.00        |
| $prostate\ LassoJoint\_BFGS\_lambda.1se$                        | 0.70 | 4.80        |
| $prostate\ LassoJoint\_BFGS\_lambda.1se$                        | 0.90 | 6.20        |
| $prostate\ LassoJoint\_BFGS\_lambda.min$                        | 0.30 | 10.80       |
| $prostate\ LassoJoint\_BFGS\_lambda.min$                        | 0.50 | 7.80        |
| $prostate\ LassoJoint\_BFGS\_lambda.min$                        | 0.70 | 25.40       |
| $prostate\ LassoJoint\_BFGS\_lambda.min$                        | 0.90 | 23.40       |
| prostate LassoMM_ $(\log(p)/n)^(1/2)$                           | 0.30 | 0.00        |
| prostate LassoMM_ $(\log(p)/n)^(1/2)$                           | 0.50 | 0.00        |
| prostate LassoMM_ $(\log(p)/n)^(1/2)$                           | 0.70 | 0.00        |
| prostate LassoMM_ $(\log(p)/n)^(1/2)$                           | 0.90 | 1.00        |
| $prostate\ LassoMM\_lambda.1se$                                 | 0.30 | 0.20        |
| $prostate\ LassoMM\_lambda.1se$                                 | 0.50 | 0.60        |
| prostate LassoMM_lambda.1se                                     | 0.70 | 5.80        |
| prostate LassoMM_lambda.1se                                     | 0.90 | 8.00        |
| prostate LassoMM_lambda.min                                     | 0.30 | 12.00       |
| prostate LassoMM_lambda.min                                     | 0.50 | 6.40        |
| $prostate\ Lasso MM\_lambda.min$                                | 0.70 | 21.00       |
| $prostate\ Lasso MM\_lambda.min$                                | 0.90 | 21.80       |

Table 99. No. of features; dataset 'Spambase'

| dataset method  | c    | no_features |
|---|------|-------------|
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$   | 0.10 | 0.00        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$   | 0.30 | 7.16        |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$   | 0.50 | 16.30       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$   | 0.70 | 18.88       |
| Spambase LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$   | 0.90 | 23.12       |
| Spambase LassoJoint_BFGS_lambda.1se   | 0.10 | 5.12        |
| Spambase LassoJoint_BFGS_lambda.1se   | 0.30 | 20.84       |
| Spambase LassoJoint_BFGS_lambda.1se   | 0.50 | 32.10       |
| Spambase LassoJoint_BFGS_lambda.1se   | 0.70 | 30.56       |
| Spambase LassoJoint_BFGS_lambda.1se   | 0.90 | 36.06       |
| Spambase LassoJoint_BFGS_lambda.min   | 0.10 | 33.08       |
| Spambase LassoJoint_BFGS_lambda.min   | 0.30 | 39.20       |
| Spambase LassoJoint_BFGS_lambda.min   | 0.50 | 45.90       |
| Spambase LassoJoint_BFGS_lambda.min   | 0.70 | 44.54       |
| Spambase LassoJoint_BFGS_lambda.min   | 0.90 | 46.38       |
| Spambase LassoMM <sub>-</sub> ( $log(p)/n$ ) <sup><math>^{^{^{^{^{^{^{^{^{^{^{^{^{^{^{^{^{^{^{</math></sup> | 0.10 | 0.00        |
| Spambase LassoMM_ $(\log(p)/n)^{(1/2)}$   | 0.30 | 7.16        |
| Spambase LassoMM <sub>-</sub> $(log(p)/n)^{(1/2)}$  | 0.50 | 16.30       |
| Spambase LassoMM <sub>-</sub> $(log(p)/n)^{(1/2)}$  | 0.70 | 18.88       |
| Spambase LassoMM <sub>-</sub> $(log(p)/n)^{(1/2)}$  | 0.90 | 23.12       |
| Spambase LassoMM_lambda.1se   | 0.10 | 5.20        |
| Spambase LassoMM_lambda.1se   | 0.30 | 20.78       |
| Spambase LassoMM_lambda.1se   | 0.50 | 32.70       |
| Spambase LassoMM_lambda.1se   | 0.70 | 30.26       |
| Spambase LassoMM_lambda.1se   | 0.90 | 35.90       |
| Spambase LassoMM_lambda.min   | 0.10 | 32.16       |
| Spambase LassoMM_lambda.min   | 0.30 | 39.76       |
| Spambase LassoMM_lambda.min   | 0.50 | 46.18       |
| Spambase LassoMM_lambda.min   | 0.70 | 44.58       |
| Spambase LassoMM_lambda.min   | 0.90 | 46.52       |

Table 100. No. of features; dataset 'Vote'

| dataset | method                               | c    | no_features |
|---------|--------------------------------------|------|-------------|
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.10 | 0.00        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.30 | 1.12        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.50 | 3.56        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.70 | 1.92        |
| Vote    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.90 | 3.10        |
| Vote    | $LassoJoint\_BFGS\_lambda.1se$       | 0.10 | 0.00        |
| Vote    | $LassoJoint\_BFGS\_lambda.1se$       | 0.30 | 1.48        |
| Vote    | $LassoJoint\_BFGS\_lambda.1se$       | 0.50 | 4.72        |
| Vote    | $LassoJoint\_BFGS\_lambda.1se$       | 0.70 | 2.50        |
| Vote    | $LassoJoint\_BFGS\_lambda.1se$       | 0.90 | 4.16        |
| Vote    | $LassoJoint\_BFGS\_lambda.min$       | 0.10 | 3.86        |
| Vote    | $LassoJoint\_BFGS\_lambda.min$       | 0.30 | 6.58        |
| Vote    | LassoJoint_BFGS_lambda.min           | 0.50 | 8.86        |
| Vote    | LassoJoint_BFGS_lambda.min           | 0.70 | 6.42        |
| Vote    | $LassoJoint\_BFGS\_lambda.min$       | 0.90 | 12.56       |
| Vote    | $LassoMM_{-}(log(p)/n)^{(1/2)}$      | 0.10 | 0.00        |
| Vote    | $LassoMM_{-}(log(p)/n)^{(1/2)}$      | 0.30 | 1.12        |
| Vote    | $LassoMM_{-}(log(p)/n)^{1/2}$        | 0.50 | 3.56        |
| Vote    | $LassoMM_{-}(log(p)/n)^{1/2}$        | 0.70 | 1.92        |
| Vote    | $LassoMM_{-}(log(p)/n)^{1/2}$        | 0.90 | 3.10        |
| Vote    | $Lasso MM\_lamb da.1se$              | 0.10 | 0.00        |
| Vote    | $Lasso MM\_lamb da.1se$              | 0.30 | 1.34        |
| Vote    | $Lasso MM\_lamb da.1se$              | 0.50 | 4.68        |
| Vote    | LassoMM_lambda.1se                   | 0.70 | 2.52        |
| Vote    | LassoMM_lambda.1se                   | 0.90 | 4.62        |
| Vote    | LassoMM_lambda.min                   | 0.10 | 4.22        |
| Vote    | LassoMM_lambda.min                   | 0.30 | 6.26        |
| Vote    | LassoMM_lambda.min                   | 0.50 | 8.60        |
| Vote    | LassoMM_lambda.min                   | 0.70 | 6.66        |
| Vote    | $LassoMM\_lambda.min$                | 0.90 | 12.62       |

Table 101. No. of features; dataset 'Wdbc'

| dataset | method                                 | c    | no_features |
|---------|--|------|-------------|
| Wdbc    | LassoJoint_BFGS_( $\log(p)/n$ )^(1/2)  | 0.10 | 0.00        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.30 | 1.70        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^{(1/2)}$ | 0.50 | 3.60        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.70 | 3.12        |
| Wdbc    | LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$   | 0.90 | 3.96        |
| Wdbc    | LassoJoint_BFGS_lambda.1se             | 0.10 | 0.14        |
| Wdbc    | LassoJoint_BFGS_lambda.1se             | 0.30 | 3.52        |
| Wdbc    | $LassoJoint\_BFGS\_lambda.1se$         | 0.50 | 6.80        |
| Wdbc    | $LassoJoint\_BFGS\_lambda.1se$         | 0.70 | 5.24        |
| Wdbc    | $LassoJoint\_BFGS\_lambda.1se$         | 0.90 | 7.44        |
| Wdbc    | $LassoJoint\_BFGS\_lambda.min$         | 0.10 | 5.86        |
| Wdbc    | $LassoJoint\_BFGS\_lambda.min$         | 0.30 | 11.50       |
| Wdbc    | $LassoJoint\_BFGS\_lambda.min$         | 0.50 | 13.20       |
| Wdbc    | $LassoJoint\_BFGS\_lambda.min$         | 0.70 | 15.62       |
| Wdbc    | $LassoJoint\_BFGS\_lambda.min$         | 0.90 | 11.10       |
| Wdbc    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.10 | 0.00        |
| Wdbc    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.30 | 1.70        |
| Wdbc    | $LassoMM_{-}(log(p)/n)^{1/2}$          | 0.50 | 3.60        |
| Wdbc    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.70 | 3.12        |
| Wdbc    | $LassoMM_{-}(log(p)/n)^{(1/2)}$        | 0.90 | 3.96        |
| Wdbc    | LassoMM_lambda.1se                     | 0.10 | 0.14        |
| Wdbc    | LassoMM_lambda.1se                     | 0.30 | 3.36        |
| Wdbc    | $LassoMM\_lambda.1se$                  | 0.50 | 6.88        |
| Wdbc    | LassoMM_lambda.1se                     | 0.70 | 5.64        |
| Wdbc    | LassoMM_lambda.1se                     | 0.90 | 7.26        |
| Wdbc    | LassoMM_lambda.min                     | 0.10 | 6.06        |
| Wdbc    | LassoMM_lambda.min                     | 0.30 | 11.38       |
| Wdbc    | LassoMM_lambda.min                     | 0.50 | 12.74       |
| Wdbc    | LassoMM_lambda.min                     | 0.70 | 15.84       |
| Wdbc    | LassoMM_lambda.min                     | 0.90 | 10.90       |

Table 102. Accuracy score on 'Banknote' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.55(0.08) | 0.87(0.02) | 0.99(0.02) | 0.99(0)    | 0.98(0.05) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.57(0.11) | 0.99(0.03) | 0.99(0.01) | 0.99(0)    | 0.98(0.06) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.97(0.02) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.98(0.05) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.48(0.09) | 0.86(0.02) | 0.99(0.02) | 0.99(0.01) | 1(0)       |
| $Lasso Joint\_MM\_lambda.1se$         | 0.5(0.13)  | 0.95(0.05) | 0.99(0.01) | 0.99(0.01) | 1(0)       |
| LassoJoint_MM_lambda.min              | 0.97(0.01) | 0.98(0.01) | 0.99(0.01) | 0.99(0.01) | 1(0)       |
| MIF5 Joint BFGS                       | 0.99(0.02) | 0.99(0.01) | 0.99(0)    | 0.99(0)    | 0.95(0.09) |
| MIF5 Joint MM                         | 0.94(0.04) | 0.98(0.01) | 0.99(0.01) | 0.99(0.01) | 1(0)       |

 ${\bf Table~103.~Accuracy~score~on~'Breastc'~dataset}$ 

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.55(0.05) | 0.9(0.06)  | 0.88(0.09) | 0.95(0.02) | 0.85(0.17) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.59(0.1)  | 0.9(0.07)  | 0.88(0.1)  | 0.95(0.02) | 0.83(0.18) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.9(0.08)  | 0.91(0.04) | 0.87(0.11) | 0.94(0.03) | 0.84(0.18) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$                | 0.55(0.05) | 0.92(0.03) | 0.89(0.07) | 0.95(0.02) | 0.95(0.02) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.59(0.11) | 0.93(0.03) | 0.89(0.07) | 0.95(0.02) | 0.95(0.02) |
| $LassoJoint\_MM\_lambda.min$                      | 0.91(0.04) | 0.93(0.03) | 0.88(0.08) | 0.95(0.02) | 0.95(0.02) |
| MIF5 Joint BFGS                                   | 0.86(0.05) | 0.9(0.04)  | 0.91(0.06) | 0.91(0.04) | 0.76(0.09) |
| MIF5 Joint MM                                     | 0.86(0.05) | 0.89(0.04) | 0.92(0.05) | 0.91(0.03) | 0.92(0.03) |

 ${\bf Table~104.~Accuracy~score~on~'Credit\_a'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.45(0.04) | 0.48(0.11) | 0.83(0.11) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0.45(0.04) | 0.48(0.11) | 0.83(0.11) | 0.86(0.03) | 0.74(0.13) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.53(0.14) | 0.73(0.18) | 0.82(0.1)  | 0.8(0.11)  | 0.67(0.11) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0.45(0.04) | 0.87(0.03) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | 0.45(0.04) | 0.85(0.09) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.62(0.17) | 0.84(0.05) | 0.84(0.03) | 0.85(0.04) | 0.86(0.03) |
| MIF5 Joint BFGS                       | 0.66(0.19) | 0.85(0.04) | 0.84(0.05) | 0.86(0.03) | 0.86(0.03) |
| MIF5 Joint MM                         | 0.82(0.1)  | 0.85(0.03) | 0.85(0.03) | 0.85(0.03) | 0.85(0.03) |

 ${\bf Table~105.~Accuracy~score~on~'Credit\_g'~dataset}$ 

| method   | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{*}(1/2)}$ | 0.7(0.03)  | 0.64(0.16) | 0.57(0.17) | 0.64(0.13) | 0.72(0.07) |
| $Lasso Joint\_BFGS\_lambda.1se$                    | 0.7(0.03)  | 0.66(0.14) | 0.62(0.16) | 0.66(0.13) | 0.73(0.04) |
| $LassoJoint\_BFGS\_lambda.min$                     | 0.68(0.08) | 0.64(0.17) | 0.67(0.1)  | 0.73(0.04) | 0.73(0.03) |
| $LassoJoint\_MM\_(log(p)/n)^(1/2)$                 | 0.3(0.03)  | 0.31(0.08) | 0.64(0.04) | 0.67(0.05) | 0.74(0.02) |
| $Lasso Joint\_MM\_lambda.1se$                      | 0.3(0.03)  | 0.29(0.03) | 0.67(0.06) | 0.7(0.04)  | 0.75(0.02) |
| $LassoJoint\_MM\_lambda.min$                       | 0.32(0.08) | 0.67(0.11) | 0.67(0.05) | 0.73(0.04) | 0.75(0.02) |
| MIF5 Joint BFGS                                    | 0.53(0.17) | 0.43(0.18) | 0.52(0.18) | 0.73(0.03) | 0.74(0.03) |
| MIF5 Joint MM                                      | 0.53(0.08) | 0.54(0.13) | 0.59(0.1)  | 0.73(0.03) | 0.74(0.03) |

Table 106. Accuracy score on 'dhfr' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.55(0.12) | 0.58(0.09) | 0.71(0.1)  | 0.78(0.02) | 0.85(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.55(0.12) | 0.58(0.1)  | 0.84(0.02) | 0.77(0.08) | 0.88(0.05) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.49(0.08) | 0.74(0.05) | 0.82(0.04) | 0.74(0.09) | 0.83(0.1)  |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$                | 0.39(0.05) | 0.42(0.09) | 0.71(0.1)  | 0.78(0.03) | 0.86(0.03) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.39(0.05) | 0.5(0.18)  | 0.8(0.05)  | 0.85(0.03) | 0.89(0.01) |
| $LassoJoint\_MM\_lambda.min$                      | 0.49(0.19) | 0.66(0.07) | 0.81(0.04) | 0.84(0.03) | 0.87(0.04) |
| MIF5 Joint BFGS                                   | 0.51(0.19) | 0.75(0.07) | 0.78(0.05) | 0.71(0.19) | 0.73(0.1)  |
| MIF5 Joint MM                                     | 0.51(0.19) | 0.75(0.07) | 0.74(0.05) | 0.73(0.17) | 0.76(0.09) |

Table 107. Accuracy score on 'Diabetes' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.35(0.04) | 0.74(0.03) | 0.76(0.04) | 0.64(0.18) | 0.36(0.09) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.35(0.04) | 0.45(0.17) | 0.76(0.04) | 0.64(0.18) | 0.36(0.08) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.61(0.1)  | 0.75(0.03) | 0.76(0.04) | 0.57(0.2)  | 0.36(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.35(0.04) | 0.73(0.04) | 0.77(0.04) | 0.75(0.03) | 0.76(0.03) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.35(0.04) | 0.4(0.13)  | 0.76(0.07) | 0.75(0.03) | 0.76(0.03) |
| $LassoJoint\_MM\_lambda.min$                      | 0.61(0.07) | 0.74(0.04) | 0.76(0.03) | 0.75(0.03) | 0.77(0.03) |
| MIF5 Joint BFGS                                   | 0.73(0.03) | 0.74(0.04) | 0.75(0.03) | 0.77(0.03) | 0.36(0.04) |
| MIF5 Joint MM                                     | 0.7(0.07)  | 0.75(0.04) | 0.75(0.03) | 0.77(0.03) | 0.75(0.03) |

 ${\bf Table\ 108.\ Accuracy\ score\ on\ 'Spambase'\ dataset}$ 

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ |            |            |            |            |            |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.69(0.16) | 0.86(0.09) | 0.85(0.16) | 0.41(0.07) | 0.67(0.23) |
| $Lasso Joint\_BFGS\_lambda.min$                   | 0.82(0.03) | 0.88(0.02) | 0.86(0.14) | 0.4(0.07)  | 0.67(0.25) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.39(0.01) | 0.44(0.12) | 0.86(0.05) | 0.89(0.01) | 0.91(0.01) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.51(0.13) | 0.84(0.05) | 0.89(0.01) | 0.9(0.01)  | 0.92(0.01) |
| $LassoJoint\_MM\_lambda.min$                      | 0.62(0.05) | 0.84(0.02) | 0.89(0.01) | 0.9(0.01)  | 0.92(0.01) |
| MIF5 Joint BFGS                                   | 0.43(0.05) | 0.58(0.06) | 0.51(0.07) | 0.59(0.04) | 0.52(0.04) |
| MIF5 Joint MM                                     | 0.43(0.03) | 0.58(0.06) | 0.51(0.07) | 0.82(0.02) | 0.84(0.01) |

Table 109. Accuracy score on 'Vote' dataset

| method                               | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|------------|------------|
| $LassoJoint\_BFGS\_(log(p)/n)^(1/2)$ | 0.68(0.07) | 0.4(0.22)  | 0.92(0.03) | 0.88(0.18) | 0.93(0.03) |
| $Lasso Joint\_BFGS\_lambda.1se$      | 0.68(0.07) | 0.51(0.29) | 0.92(0.03) | 0.93(0.04) | 0.93(0.03) |
| $LassoJoint\_BFGS\_lambda.min$       | 0.77(0.14) | 0.66(0.27) | 0.89(0.05) | 0.93(0.03) | 0.9(0.04)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 0.32(0.07) | 0.95(0.03) | 0.91(0.04) | 0.93(0.03) | 0.93(0.03) |
| $Lasso Joint\_MM\_lambda.1se$        | 0.32(0.07) | 0.93(0.09) | 0.92(0.04) | 0.93(0.03) | 0.93(0.03) |
| $Lasso Joint\_MM\_lambda.min$        | 0.64(0.19) | 0.67(0.2)  | 0.89(0.06) | 0.93(0.04) | 0.92(0.03) |

Table 110. Accuracy score on 'Wdbc' dataset

| method                               | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 0.36(0.04) | 0.92(0.02) | 0.93(0.03) | 0.94(0.02) | 0.67(0.28) |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.36(0.04) | 0.93(0.03) | 0.94(0.02) | 0.91(0.12) | 0.63(0.17) |
| $LassoJoint\_BFGS\_lambda.min$       | 0.77(0.1)  | 0.71(0.25) | 0.93(0.03) | 0.67(0.24) | 0.58(0.16) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 0.36(0.04) | 0.9(0.03)  | 0.92(0.03) | 0.94(0.02) | 0.96(0.02) |
| $Lasso Joint\_MM\_lambda.1se$        | 0.36(0.04) | 0.9(0.04)  | 0.93(0.03) | 0.94(0.02) | 0.97(0.02) |
| $LassoJoint\_MM\_lambda.min$         | 0.64(0.08) | 0.72(0.21) | 0.88(0.06) | 0.91(0.04) | 0.97(0.02) |
| MIF5 Joint BFGS                      | 0.82(0.18) | 0.92(0.03) | 0.93(0.02) | 0.9(0.1)   | 0.52(0.24) |
| MIF5 Joint MM                        | 0.73(0.14) | 0.9(0.03)  | 0.93(0.02) | 0.92(0.03) | 0.95(0.02) |

Table 111. Recall score on 'Banknote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.91(0.27) | 0.87(0.05) | 1(0.02)    | 1(0)       | 0.98(0.08) |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.9(0.27)  | 0.99(0.03) | 1(0)       | 1(0)       | 0.98(0.09) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.99(0.01) | 1(0.01)    | 1(0)       | 1(0)       | 0.98(0.08) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.04(0.17) | 0.81(0.04) | 0.98(0.03) | 0.98(0.01) | 0.99(0.01) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.09(0.24) | 0.93(0.07) | 0.99(0.01) | 0.98(0.01) | 0.99(0.01) |
| LassoJoint_MM_lambda.min                          | 0.94(0.03) | 0.96(0.02) | 0.99(0.01) | 0.98(0.01) | 0.99(0.01) |
| MIF5 Joint BFGS                                   | 0.99(0.02) | 0.99(0.01) | 1(0)       | 1(0)       | 0.92(0.16) |
| MIF5 Joint MM                                     | 0.9(0.06)  | 0.96(0.01) | 0.97(0.01) | 0.99(0.01) | 0.99(0.01) |

Table 112. Recall score on 'Breastc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) |            |            |            | 0.94(0.04) |            |
| $LassoJoint\_BFGS\_lambda.1se$        |            |            |            | 0.94(0.04) |            |
| $LassoJoint\_BFGS\_lambda.min$        | 0.85(0.16) | 0.93(0.05) | 0.73(0.23) | 0.92(0.05) | 0.67(0.42) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.88(0.07) | 0.77(0.14) | 0.94(0.04) | 0.93(0.04) |
| $Lasso Joint\_MM\_lambda.1se$         | 0.09(0.24) | 0.89(0.06) | 0.76(0.15) | 0.93(0.04) | 0.93(0.04) |
| $LassoJoint\_MM\_lambda.min$          | 0.8(0.08)  | 0.89(0.06) | 0.76(0.16) | 0.93(0.04) | 0.93(0.04) |
| MIF5 Joint BFGS                       | 0.54(0.15) | 0.82(0.1)  | 0.82(0.2)  | 0.85(0.09) | 0.13(0.3)  |
| MIF5 Joint MM                         | 0.53(0.16) | 0.74(0.11) | 0.82(0.15) | 0.84(0.09) | 0.81(0.1)  |

Table 113. Recall score on 'Credit\_a' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0(0)       | 0.05(0.2)  | 0.75(0.2)  | 0.8(0.04)  | 0.79(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0(0)       | 0.05(0.2)  | 0.75(0.19) | 0.8(0.04)  | 0.56(0.25) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.15(0.27) | 0.59(0.37) | 0.78(0.19) | 0.72(0.23) | 0.43(0.21) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0(0)       | 0.8(0.04)  | 0.8(0.04)  | 0.8(0.04)  | 0.79(0.04) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0(0)       | 0.77(0.16) | 0.8(0.04)  | 0.8(0.04)  | 0.8(0.04)  |
| $LassoJoint\_MM\_lambda.min$                      | 0.33(0.34) | 0.76(0.09) | 0.8(0.05)  | 0.8(0.07)  | 0.83(0.04) |
| MIF5 Joint BFGS                                   | 0.44(0.41) | 0.84(0.07) | 0.75(0.1)  | 0.8(0.07)  | 0.78(0.07) |
| MIF5 Joint MM                                     | 0.71(0.19) | 0.75(0.06) | 0.75(0.06) | 0.76(0.06) | 0.76(0.06) |

 ${\bf Table~114.~Recall~score~on~'Credit\_g'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 1(0)       | 0.84(0.37) | 0.53(0.35) | 0.65(0.26) | 0.81(0.13) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 1(0)       | 0.88(0.33) | 0.61(0.33) | 0.68(0.26) | 0.77(0.06) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.95(0.19) | 0.66(0.35) | 0.65(0.18) | 0.81(0.1)  | 0.76(0.07) |
| $LassoJoint_MM_{-}(log(p)/n)^{(1/2)}$ | 0(0)       | 0.03(0.12) | 0.59(0.09) | 0.69(0.12) | 0.83(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0(0)       | 0.68(0.14) | 0.73(0.12) | 0.82(0.03) |
| LassoJoint_MM_lambda.min              | 0.05(0.16) | 0.69(0.2)  | 0.66(0.1)  | 0.79(0.09) | 0.82(0.04) |
| MIF5 Joint BFGS                       | 0.54(0.4)  | 0.24(0.35) | 0.4(0.34)  | 0.89(0.07) | 0.9(0.04)  |
| MIF5 Joint MM                         | 0.49(0.17) | 0.4(0.23)  | 0.51(0.19) | 0.87(0.07) | 0.89(0.04) |

Table 115. Recall score on 'dhfr' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.6(0.55)  | 1(0)       | 0.69(0.15) | 0.89(0.04) | 0.9(0.03)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.6(0.55)  | 0.93(0.16) | 0.85(0.06) | 0.67(0.18) | 0.87(0.07) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.3(0.41)  | 0.67(0.07) | 0.84(0.09) | 0.6(0.15)  | 0.79(0.16) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0(0)       | 0.69(0.14) | 0.88(0.03) | 0.91(0.04) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.24(0.33) | 0.78(0.06) | 0.83(0.05) | 0.87(0.01) |
| $Lasso Joint\_MM\_lambda.min$         | 0.2(0.33)  | 0.48(0.09) | 0.78(0.07) | 0.81(0.07) | 0.84(0.06) |
| MIF5 Joint BFGS                       | 0.35(0.37) | 0.88(0.06) | 0.91(0.13) | 0.9(0.16)  | 0.97(0.03) |
| MIF5 Joint MM                         | 0.32(0.35) | 0.84(0.05) | 0.82(0.1)  | 0.91(0.16) | 0.97(0.03) |

Table 116. Recall score on 'Diabetes' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.88(0.05) | 0.87(0.05) | 0.59(0.35) | 0.04(0.17) |
| $LassoJoint\_BFGS\_lambda.1se$        | 0(0)       | 0.2(0.37)  | 0.88(0.06) | 0.59(0.35) | 0.04(0.17) |
| LassoJoint_BFGS_lambda.min            | 0.5(0.22)  | 0.86(0.05) | 0.84(0.07) | 0.45(0.39) | 0.02(0.08) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.82(0.05) | 0.86(0.05) | 0.8(0.04)  | 0.87(0.04) |
| $LassoJoint\_MM\_lambda.1se$          | 0(0)       | 0.1(0.26)  | 0.84(0.14) | 0.8(0.04)  | 0.87(0.04) |
| LassoJoint_MM_lambda.min              | 0.47(0.12) | 0.8(0.05)  | 0.83(0.06) | 0.79(0.05) | 0.88(0.04) |
| MIF5 Joint BFGS                       | 0.91(0.05) | 0.9(0.11)  | 0.88(0.04) | 0.9(0.03)  | 0.01(0.04) |
| MIF5 Joint MM                         | 0.69(0.12) | 0.82(0.08) | 0.87(0.04) | 0.89(0.03) | 0.85(0.04) |

Table 117. Recall score on 'Spambase' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 1(0)       | 0.07(0.2)  | 0.55(0.44) | 0.2(0.39)  | 0.56(0.4)  |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.62(0.32) | 0.86(0.16) | 0.82(0.29) | 0.03(0.13) | 0.48(0.42) |
| $Lasso Joint\_BFGS\_lambda.min$                   | 0.79(0.06) | 0.89(0.03) | 0.84(0.25) | 0.03(0.13) | 0.49(0.44) |
| $LassoJoint\_MM\_(log(p)/n)^(1/2)$                | 0(0)       | 0.09(0.2)  | 0.85(0.1)  | 0.92(0.02) | 0.94(0.01) |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.21(0.23) | 0.78(0.09) | 0.89(0.02) | 0.93(0.01) | 0.95(0.01) |
| $LassoJoint\_MM\_lambda.min$                      | 0.39(0.09) | 0.78(0.04) | 0.87(0.02) | 0.93(0.01) | 0.94(0.01) |
| MIF5 Joint BFGS                                   | 0.02(0.09) | 0(0)       | 0(0)       | 0(0)       | 0(0)       |
| MIF5 Joint MM                                     | 0.01(0.04) | 0(0)       | 0(0)       | 0.67(0.05) | 0.88(0.03) |

Table 118. Recall score on 'Vote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 1(0)       | 0.13(0.32) | 0.91(0.05) | 0.85(0.26) | 0.92(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 1(0)       | 0.28(0.44) | 0.91(0.05) | 0.92(0.04) | 0.92(0.04) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.73(0.22) | 0.5(0.41)  | 0.89(0.07) | 0.92(0.04) | 0.89(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0(0)       | 0.95(0.03) | 0.9(0.06)  | 0.93(0.04) | 0.92(0.04) |
| $LassoJoint\_MM\_lambda.1se$                      | 0(0)       | 0.92(0.12) | 0.89(0.05) | 0.93(0.04) | 0.92(0.04) |
| $LassoJoint\_MM\_lambda.min$                      | 0.48(0.28) | 0.54(0.28) | 0.86(0.08) | 0.91(0.04) | 0.92(0.04) |

Table 119. Recall score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0(0)       | 0.91(0.03) | 0.95(0.05) | 0.96(0.02) | 0.49(0.46) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0(0)       | 0.91(0.04) | 0.95(0.04) | 0.91(0.2)  | 0.41(0.26) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.67(0.15) | 0.54(0.41) | 0.93(0.05) | 0.5(0.42)  | 0.33(0.24) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | 0(0)       | 0.86(0.05) | 0.92(0.05) | 0.94(0.03) | 0.97(0.02) |
| $Lasso Joint\_MM\_lambda.1se$         | 0(0)       | 0.85(0.06) | 0.91(0.05) | 0.94(0.04) | 0.98(0.02) |
| LassoJoint_MM_lambda.min              | 0.44(0.12) | 0.56(0.33) | 0.84(0.1)  | 0.88(0.07) | 0.98(0.02) |
| MIF5 Joint BFGS                       | 0.77(0.31) | 0.93(0.04) | 0.95(0.03) | 0.87(0.16) | 0.23(0.39) |
| MIF5 Joint MM                         | 0.59(0.23) | 0.87(0.04) | 0.93(0.03) | 0.88(0.04) | 0.95(0.02) |

Table 120. Precision score on 'Banknote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.56(0.09) | 0.9(0.03)  | 0.99(0.02) | 0.99(0.01) | 0.99(0.01) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.59(0.12) | 0.98(0.03) | 0.99(0.01) | 0.99(0.01) | 0.99(0.02) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.96(0.02) | 0.99(0.01) | 0.99(0.01) | 0.99(0.01) | 0.99(0.02) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.96(0.03) | 0.92(0.02) | 1(0.02)    | 1(0)       | 1(0)       |
| $LassoJoint\_MM\_lambda.1se$                      | 0.95(0.03) | 0.98(0.04) | 1(0)       | 1(0)       | 1(0)       |
| $LassoJoint\_MM\_lambda.min$                      | 1(0)       | 1(0)       | 1(0)       | 1(0)       | 1(0)       |
| MIF5 Joint BFGS                                   | 0.99(0.02) | 0.99(0.01) | 0.99(0.01) | 0.99(0.01) | 0.99(0.01) |
| MIF5 Joint MM                                     | 1(0.01)    | 1(0)       | 1(0)       | 1(0)       | 1(0)       |

Table 121. Precision score on 'Breastc' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | NaN(NA)    | 0.89(0.06) | 0.99(0.01) | 0.95(0.04) | 0.94(0.04) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.9(0.09)  | 0.89(0.07) | 1(0.01)    | 0.95(0.04) | 0.94(0.04) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.93(0.06) | 0.89(0.07) | 1(0.01)    | 0.95(0.05) | 0.95(0.04) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | NaN(NA)    | 0.94(0.04) | 1(0.01)    | 0.96(0.04) | 0.95(0.03) |
| $LassoJoint\_MM\_lambda.1se$                      | 0.94(0.04) | 0.95(0.04) | 1(0.01)    | 0.96(0.03) | 0.96(0.03) |
| $LassoJoint\_MM\_lambda.min$                      | 0.98(0.03) | 0.96(0.04) | 1(0.01)    | 0.97(0.03) | 0.95(0.04) |
| MIF5 Joint BFGS                                   | 0.94(0.08) | 0.91(0.08) | 0.9(0.07)  | 0.83(0.09) | 0.89(0.1)  |
| MIF5 Joint MM                                     | 0.97(0.07) | 0.97(0.04) | 0.95(0.05) | 0.85(0.09) | 0.88(0.09) |

Table 122. Precision score on 'Credit\_a' dataset

| method                               | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$       | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0.04) |
| $LassoJoint\_BFGS\_lambda.min$       | 0.92(0.06) | 0.89(0.07) | 0.87(0.09) | 0.91(0.05) | 0.94(0.04) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$   | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.94(0.03) |
| $LassoJoint\_MM\_lambda.1se$         | NaN(NA)    | 0.95(0.03) | 0.94(0.03) | 0.94(0.03) | 0.93(0.03) |
| $LassoJoint\_MM\_lambda.min$         | 0.93(0.07) | 0.94(0.03) | 0.91(0.04) | 0.93(0.03) | 0.91(0.04) |
| MIF5 Joint BFGS                      | 0.85(0.09) | 0.85(0.06) | 0.9(0.04)  | 0.89(0.04) | 0.91(0.04) |
| MIF5 Joint MM                        | 0.93(0.04) | 0.91(0.04) | 0.91(0.05) | 0.91(0.04) | 0.91(0.04) |

Table 123. Precision score on 'Credit\_g' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.7(0.03)  | 0.71(0.03) | 0.79(0.06) | 0.81(0.05) | 0.8(0.03)  |
| $LassoJoint\_BFGS\_lambda.1se$        | 0.7(0.03)  | 0.71(0.03) | 0.8(0.05)  | 0.83(0.06) | 0.83(0.03) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.7(0.03)  | 0.8(0.05)  | 0.85(0.05) | 0.82(0.05) | 0.85(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.86(0.05) | 0.86(0.05) | 0.82(0.04) | 0.8(0.03)  |
| $Lasso Joint\_MM\_lambda.1se$         | NaN(NA)    | NaN(NA)    | 0.83(0.05) | 0.83(0.04) | 0.83(0.03) |
| LassoJoint_MM_lambda.min              | 0.76(0.03) | 0.83(0.04) | 0.85(0.04) | 0.82(0.04) | 0.83(0.03) |
| MIF5 Joint BFGS                       | 0.73(0.06) | 0.83(0.07) | 0.85(0.07) | 0.76(0.04) | 0.77(0.03) |
| MIF5 Joint MM                         | 0.75(0.07) | 0.89(0.06) | 0.86(0.05) | 0.77(0.04) | 0.77(0.03) |

 ${\bf Table~124.~Precision~score~on~'dhfr'~dataset}$ 

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | 0.64(0.05) | 0.58(0.09) | 0.82(0.06) | 0.78(0.05) | 0.87(0.07) |
| $Lasso Joint\_BFGS\_lambda.1se$       | 0.64(0.05) | 0.61(0.14) | 0.89(0.06) | 0.92(0.03) | 0.93(0.07) |
| LassoJoint_BFGS_lambda.min            | 0.73(0.1)  | 0.86(0.06) | 0.87(0.06) | 0.93(0.04) | 0.95(0.1)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | NaN(NA)    | 0.83(0.06) | 0.78(0.05) | 0.87(0.06) |
| $LassoJoint\_MM\_lambda.1se$          | NaN(NA)    | 0.74(0.02) | 0.89(0.07) | 0.92(0.06) | 0.94(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.71(0.17) | 0.92(0.09) | 0.9(0.05)  | 0.92(0.05) | 0.96(0.02) |
| MIF5 Joint BFGS                       | 0.66(0.18) | 0.73(0.07) | 0.77(0.05) | 0.73(0.16) | 0.7(0.11)  |
| MIF5 Joint MM                         | 0.67(0.29) | 0.74(0.08) | 0.77(0.04) | 0.75(0.16) | 0.73(0.1)  |

Table 125. Precision score on 'Diabetes' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.76(0.04) | 0.79(0.05) | 0.81(0.1)  | 0.97(0.07) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.78(0.03) | 0.79(0.05) | 0.81(0.1)  | 0.96(0.09) |
| $Lasso Joint\_BFGS\_lambda.min$       | 0.85(0.08) | 0.78(0.04) | 0.8(0.05)  | 0.84(0.11) | 0.97(0.04) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.78(0.04) | 0.8(0.05)  | 0.82(0.04) | 0.79(0.04) |
| $Lasso Joint\_MM\_lamb da.1se$        | NaN(NA)    | 0.77(0.05) | 0.8(0.05)  | 0.82(0.05) | 0.79(0.04) |
| $LassoJoint\_MM\_lambda.min$          | 0.87(0.05) | 0.8(0.04)  | 0.81(0.05) | 0.82(0.04) | 0.8(0.04)  |
| MIF5 Joint BFGS                       | 0.73(0.04) | 0.76(0.06) | 0.77(0.04) | 0.78(0.04) | 1(0.01)    |
| MIF5 Joint MM                         | 0.82(0.05) | 0.8(0.04)  | 0.78(0.04) | 0.78(0.04) | 0.79(0.03) |

Table 126. Precision score on 'Spambase' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.61(0.01) | 0.96(0.03) | 0.92(0.04) | 0.95(0.05) | 0.94(0.04) |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.88(0.11) | 0.92(0.02) | 0.93(0.03) | 1(0.02)    | 0.96(0.03) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.91(0.02) | 0.91(0.01) | 0.94(0.03) | 0.98(0.07) | 0.96(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | NaN(NA)    | 0.97(0.03) | 0.92(0.02) | 0.9(0.01)  | 0.91(0.01) |
| $Lasso Joint\_MM\_lamb da.1se$                    | 0.97(0.03) | 0.94(0.01) | 0.94(0.01) | 0.91(0.01) | 0.92(0.01) |
| $LassoJoint\_MM\_lambda.min$                      | 0.95(0.02) | 0.95(0.01) | 0.95(0.01) | 0.92(0.01) | 0.92(0.01) |
| MIF5 Joint BFGS                                   | 0.94(0.04) | NaN(NA)    | NaN(NA)    | NaN(NA)    | NaN(NA)    |
| MIF5 Joint MM                                     | 0.94(0.07) | NaN(NA)    | NaN(NA)    | 0.85(0.04) | 0.8(0.03)  |

Table 127. Precision score on 'Vote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ |            |            |            |            |            |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.68(0.07) | 0.98(0.02) | 0.98(0.02) | 0.98(0.02) | 0.97(0.02) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.92(0.07) | 0.98(0.02) | 0.96(0.04) | 0.97(0.03) | 0.97(0.03) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$                | NaN(NA)    | 0.98(0.02) | 0.98(0.02) | 0.98(0.02) | 0.98(0.02) |
| $LassoJoint\_MM\_lambda.1se$                      | NaN(NA)    | 0.98(0.02) | 0.99(0.02) | 0.98(0.02) | 0.97(0.02) |
| $Lasso Joint\_MM\_lambda.min$                     | 0.96(0.07) | 0.98(0.03) | 0.98(0.02) | 0.98(0.02) | 0.97(0.03) |

Table 128. Precision score on 'Wdbc' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | NaN(NA)    | 0.96(0.03) | 0.94(0.03) | 0.94(0.03) | 0.97(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$                    | NaN(NA)    | 0.97(0.03) | 0.96(0.02) | 0.95(0.03) | 0.99(0.02) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.95(0.05) | 0.99(0.02) | 0.96(0.02) | 0.98(0.03) | 0.99(0.01) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | NaN(NA)    | 0.98(0.02) | 0.96(0.02) | 0.97(0.02) | 0.96(0.02) |
| $LassoJoint\_MM\_lambda.1se$                      | NaN(NA)    | 0.99(0.02) | 0.97(0.02) | 0.97(0.02) | 0.97(0.02) |
| $LassoJoint\_MM\_lambda.min$                      | 0.98(0.03) | 0.99(0.01) | 0.97(0.02) | 0.97(0.02) | 0.97(0.02) |
| MIF5 Joint BFGS                                   | 0.94(0.04) | 0.95(0.03) | 0.95(0.02) | 0.98(0.02) | 0.99(0.02) |
| MIF5 Joint MM                                     | 0.98(0.03) | 0.96(0.03) | 0.97(0.02) | 0.98(0.02) | 0.96(0.02) |

Table 129. F1 score on 'Banknote' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.71(0.05) | 0.88(0.03) | 0.99(0.02) | 0.99(0)    | 0.98(0.05) |
| $LassoJoint\_BFGS\_lambda.1se$                    | 0.72(0.06) | 0.99(0.03) | 0.99(0.01) | 0.99(0)    | 0.98(0.06) |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.98(0.02) | 0.99(0)    | 0.99(0.01) | 0.99(0)    | 0.98(0.06) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | 0.81(0.02) | 0.86(0.02) | 0.99(0.02) | 0.99(0.01) | 1(0)       |
| $Lasso Joint\_MM\_lambda.1se$                     | 0.83(0.03) | 0.95(0.05) | 0.99(0.01) | 0.99(0.01) | 1(0)       |
| $LassoJoint\_MM\_lambda.min$                      | 0.97(0.01) | 0.98(0.01) | 0.99(0)    | 0.99(0.01) | 1(0)       |
| MIF5 Joint BFGS                                   | 0.99(0.02) | 0.99(0)    | 0.99(0)    | 0.99(0)    | 0.94(0.1)  |
| MIF5 Joint MM                                     | 0.95(0.04) | 0.98(0.01) | 0.99(0.01) | 0.99(0)    | 1(0)       |

Table 130. F1 score on 'Breastc' dataset

| method                               | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | NaN(NA)    | 0.9(0.04)  | 0.87(0.08) | 0.95(0.03) | 0.94(0.02) |
| $LassoJoint\_BFGS\_lambda.1se$       | 0.76(0.06) | 0.9(0.04)  | 0.86(0.14) | 0.94(0.03) | 0.92(0.1)  |
| $LassoJoint\_BFGS\_lambda.min$       | 0.89(0.06) | 0.91(0.04) | 0.87(0.1)  | 0.94(0.03) | 0.94(0.04) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | NaN(NA)    | 0.91(0.04) | 0.86(0.11) | 0.95(0.02) | 0.94(0.02) |
| $Lasso Joint\_MM\_lambda.1se$        | 0.81(0.05) | 0.92(0.03) | 0.85(0.12) | 0.95(0.02) | 0.94(0.02) |
| $LassoJoint\_MM\_lambda.min$         | 0.88(0.05) | 0.92(0.03) | 0.85(0.13) | 0.95(0.02) | 0.94(0.02) |
| MIF5 Joint BFGS                      | 0.68(0.13) | 0.86(0.06) | 0.87(0.09) | 0.84(0.06) | 0.69(0.26) |
| MIF5 Joint MM                        | 0.67(0.15) | 0.83(0.08) | 0.87(0.12) | 0.84(0.06) | 0.84(0.07) |

Table 131. F1 score on 'Credit\_a' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.88(0.04) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.88(0.04) | 0.86(0.02) | 0.86(0.03) | 0.67(0.23) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.66(0.22) | 0.85(0.03) | 0.81(0.18) | 0.78(0.18) | 0.56(0.22) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$      | NaN(NA)    | 0.87(0.03) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $Lasso Joint\_MM\_lambda.1se$         | NaN(NA)    | 0.87(0.03) | 0.86(0.02) | 0.86(0.03) | 0.86(0.03) |
| $LassoJoint\_MM\_lambda.min$          | 0.6(0.29)  | 0.84(0.07) | 0.85(0.03) | 0.86(0.04) | 0.87(0.03) |
| MIF5 Joint BFGS                       | 0.76(0.22) | 0.84(0.04) | 0.82(0.04) | 0.84(0.04) | 0.83(0.05) |
| MIF5 Joint MM                         | 0.79(0.17) | 0.82(0.04) | 0.82(0.05) | 0.83(0.04) | 0.83(0.04) |

Table 132. F1 score on 'Credit\_g' dataset

| method                             | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|------------------------------------|------------|------------|------------|------------|------------|
| $LassoJoint_BFGS_(log(p)/n)^(1/2)$ | 0.82(0.02) | 0.83(0.02) | 0.75(0.06) | 0.73(0.12) | 0.81(0.04) |
| $Lasso Joint\_BFGS\_lambda.1se$    | 0.82(0.02) | 0.83(0.02) | 0.77(0.05) | 0.73(0.17) | 0.8(0.03)  |
| $LassoJoint\_BFGS\_lambda.min$     | 0.81(0.08) | 0.81(0.04) | 0.72(0.14) | 0.8(0.05)  | 0.8(0.03)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$   | NaN(NA)    | 0.64(0.04) | 0.69(0.05) | 0.74(0.06) | 0.82(0.02) |
| $LassoJoint\_MM\_lambda.1se$       | NaN(NA)    | NaN(NA)    | 0.74(0.09) | 0.77(0.06) | 0.82(0.02) |
| $Lasso Joint\_MM\_lambda.min$      | 0.51(0.21) | 0.75(0.11) | 0.73(0.07) | 0.8(0.04)  | 0.82(0.02) |
| MIF5 Joint BFGS                    | 0.72(0.17) | 0.66(0.23) | 0.63(0.22) | 0.82(0.03) | 0.83(0.02) |
| MIF5 Joint MM                      | 0.59(0.14) | 0.52(0.23) | 0.61(0.16) | 0.81(0.03) | 0.83(0.02) |

Table 133. F1 score on 'dhfr' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 0.78(0.04) | 0.73(0.07) | 0.75(0.1)  | 0.83(0.02) | 0.88(0.03) |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 0.78(0.04) | 0.72(0.07) | 0.87(0.01) | 0.76(0.11) | 0.9(0.05)  |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.38(0.31) | 0.74(0.03) | 0.85(0.05) | 0.72(0.11) | 0.85(0.1)  |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                  | NaN(NA)    | NaN(NA)    | 0.74(0.1)  | 0.83(0.03) | 0.89(0.03) |
| $Lasso Joint\_MM\_lambda.1se$                     | NaN(NA)    | 0.66(0.08) | 0.83(0.05) | 0.87(0.03) | 0.91(0.01) |
| LassoJoint_MM_lambda.min                          | 0.31(0.36) | 0.62(0.07) | 0.83(0.04) | 0.86(0.04) | 0.89(0.04) |
| MIF5 Joint BFGS                                   | 0.49(0.33) | 0.8(0.07)  | 0.83(0.05) | 0.8(0.14)  | 0.81(0.07) |
| MIF5 Joint MM                                     | 0.47(0.35) | 0.79(0.06) | 0.79(0.04) | 0.81(0.13) | 0.83(0.07) |

Table 134. F1 score on 'Diabetes' dataset

| method  | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---|------------|------------|------------|------------|------------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | NaN(NA)    | 0.81(0.03) | 0.83(0.03) | 0.6(0.34)  | 0.12(0.28) |
| $LassoJoint\_BFGS\_lambda.1se$                    | NaN(NA)    | 0.81(0.03) | 0.83(0.03) | 0.6(0.34)  | 0.15(0.3)  |
| $LassoJoint\_BFGS\_lambda.min$                    | 0.59(0.18) | 0.82(0.03) | 0.82(0.03) | 0.47(0.37) | 0.19(0.22) |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$                | NaN(NA)    | 0.8(0.03)  | 0.83(0.03) | 0.81(0.03) | 0.83(0.03) |
| $LassoJoint\_MM\_lambda.1se$                      | NaN(NA)    | 0.79(0.03) | 0.83(0.03) | 0.81(0.03) | 0.83(0.03) |
| $LassoJoint\_MM\_lambda.min$                      | 0.6(0.12)  | 0.8(0.03)  | 0.82(0.03) | 0.8(0.03)  | 0.83(0.03) |
| MIF5 Joint BFGS                                   | 0.81(0.03) | 0.81(0.06) | 0.82(0.02) | 0.83(0.02) | 0.03(0.07) |
| MIF5 Joint MM                                     | 0.74(0.1)  | 0.81(0.04) | 0.82(0.02) | 0.83(0.02) | 0.82(0.03) |

 ${\bf Table~135.}~{\rm F1~score~on~'Spambase'~dataset}$ 

| method                             | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|------------------------------------|------------|------------|------------|------------|------------|
| $LassoJoint_BFGS_(log(p)/n)^(1/2)$ | 0.76(0.01) | 0.53(0.35) | 0.79(0.3)  | 0.51(0.46) | 0.73(0.31) |
| $LassoJoint\_BFGS\_lambda.1se$     | 0.76(0.15) | 0.87(0.15) | 0.84(0.25) | 0.04(0.13) | 0.58(0.4)  |
| $LassoJoint\_BFGS\_lambda.min$     | 0.84(0.03) | 0.9(0.01)  | 0.85(0.23) | 0.04(0.13) | 0.62(0.41) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$   | NaN(NA)    | 0.51(0.27) | 0.88(0.07) | 0.91(0.01) | 0.92(0.01) |
| $Lasso Joint\_MM\_lambda.1se$      | 0.4(0.25)  | 0.85(0.07) | 0.91(0.01) | 0.92(0.01) | 0.93(0.01) |
| $LassoJoint\_MM\_lambda.min$       | 0.55(0.09) | 0.86(0.03) | 0.9(0.01)  | 0.92(0.01) | 0.93(0.01) |
| MIF5 Joint BFGS                    | 0.42(0.28) | NaN(NA)    | NaN(NA)    | NaN(NA)    | NaN(NA)    |
| MIF5 Joint MM                      | 0.23(0.11) | NaN(NA)    | NaN(NA)    | 0.75(0.04) | 0.84(0.02) |

Table 136. F1 score on 'Vote' dataset

| method   | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|--|------------|------------|------------|------------|------------|
| $\overline{\text{LassoJoint\_BFGS\_(log(p)/n)^(1/2)}}$ | 0.81(0.05) | 0.96(0.02) | 0.94(0.03) | 0.95(0.03) | 0.95(0.02) |
| $Lasso Joint\_BFGS\_lambda.1se$                        | 0.81(0.05) | 0.96(0.02) | 0.94(0.03) | 0.95(0.03) | 0.95(0.02) |
| $LassoJoint\_BFGS\_lambda.min$                         | 0.81(0.15) | 0.88(0.08) | 0.92(0.04) | 0.94(0.02) | 0.93(0.03) |
| $LassoJoint_MM_(log(p)/n)^(1/2)$                       | NaN(NA)    | 0.96(0.02) | 0.93(0.03) | 0.95(0.03) | 0.95(0.02) |
| $Lasso Joint\_MM\_lambda.1se$                          | NaN(NA)    | 0.94(0.09) | 0.94(0.03) | 0.95(0.03) | 0.95(0.02) |
| $LassoJoint\_MM\_lambda.min$                           | 0.6(0.27)  | 0.65(0.25) | 0.91(0.05) | 0.94(0.03) | 0.94(0.03) |

Table 137. F1 score on 'Wdbc' dataset

| method                                | 0.1        | 0.3        | 0.5        | 0.7        | 0.9        |
|---------------------------------------|------------|------------|------------|------------|------------|
| LassoJoint_BFGS_( $\log(p)/n$ )^(1/2) | NaN(NA)    | 0.94(0.02) | 0.94(0.02) | 0.95(0.02) | 0.77(0.35) |
| $LassoJoint\_BFGS\_lambda.1se$        | NaN(NA)    | 0.94(0.02) | 0.95(0.02) | 0.91(0.17) | 0.68(0.12) |
| $LassoJoint\_BFGS\_lambda.min$        | 0.78(0.12) | 0.6(0.41)  | 0.94(0.02) | 0.55(0.38) | 0.6(0.17)  |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$    | NaN(NA)    | 0.92(0.03) | 0.94(0.02) | 0.95(0.02) | 0.96(0.02) |
| $LassoJoint\_MM\_lambda.1se$          | NaN(NA)    | 0.91(0.04) | 0.94(0.02) | 0.95(0.02) | 0.97(0.01) |
| $LassoJoint\_MM\_lambda.min$          | 0.59(0.13) | 0.65(0.34) | 0.9(0.06)  | 0.92(0.04) | 0.97(0.01) |
| MIF5 Joint BFGS                       | 0.86(0.2)  | 0.94(0.02) | 0.95(0.02) | 0.91(0.13) | 0.45(0.44) |
| MIF5 Joint MM                         | 0.71(0.21) | 0.91(0.03) | 0.95(0.02) | 0.93(0.02) | 0.96(0.02) |

 ${\bf Table~138.~Method~avg.rank~based~on~Accuracy~metrics}$ 

| method                               | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|--------------------------------------|------|------|------|------|------|
| LassoJoint_BFGS_lambda.1se           | 4.20 | 3.83 | 3.31 | 4.54 | 4.46 |
| $LassoJoint\_MM\_lambda.min$         | 3.70 | 3.67 | 5.46 | 4.31 | 3.54 |
| MIF5 Joint MM                        | 3.78 | 4.36 | 4.75 | 4.25 | 4.25 |
| MIF5 Joint BFGS                      | 2.56 | 3.64 | 4.42 | 4.75 | 6.58 |
| LassoJoint_BFGS_ $(\log(p)/n)^(1/2)$ | 4.20 | 4.83 | 4.00 | 3.85 | 5.15 |
| $LassoJoint\_BFGS\_lambda.min$       | 2.40 | 3.75 | 4.38 | 5.69 | 6.08 |
| LassoJoint_MM_lambda.1se             | 7.10 | 4.83 | 4.23 | 3.85 | 2.38 |
| $LassoJoint_MM_(log(p)/n)^(1/2)$     | 7.20 | 5.50 | 5.00 | 4.31 | 3.23 |
|                                      |      |      |      |      |      |

Table 139. Method avg.rank based on Recall metrics

| method  | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|---|------|------|------|------|------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 3.10 | 3.75 | 3.31 | 2.85 | 4.15 |
| $Lasso Joint\_BFGS\_lambda.1se$                   | 3.90 | 3.08 | 3.08 | 4.46 | 5.15 |
| MIF5 Joint BFGS                                   | 2.89 | 3.36 | 3.83 | 4.42 | 5.75 |
| MIF5 Joint MM                                     | 4.11 | 5.00 | 5.08 | 3.67 | 3.75 |
| $LassoJoint\_BFGS\_lambda.min$                    | 2.70 | 3.75 | 4.23 | 6.23 | 6.46 |
| LassoJoint_MM_lambda.min                          | 4.20 | 4.58 | 5.54 | 5.00 | 4.31 |
| LassoJoint_MM_lambda.1se                          | 7.10 | 5.33 | 5.00 | 4.31 | 3.23 |
| LassoJoint_MM_ $(\log(p)/n)^{(1/2)}$              | 7.20 | 5.58 | 5.46 | 4.54 | 2.77 |

Table 140. Method avg.rank based on Precision metrics

| method                                 | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|--|------|------|------|------|------|
| LassoJoint_MM_lambda.min               | 1.20 | 2.67 | 3.00 | 4.23 | 4.92 |
| $LassoJoint\_BFGS\_lambda.min$         | 3.50 | 4.17 | 4.31 | 3.31 | 2.92 |
| $LassoJoint\_MM\_lambda.1se$           | 5.80 | 4.08 | 3.00 | 3.69 | 3.85 |
| $LassoJoint\_BFGS\_lambda.1se$         | 5.80 | 4.58 | 4.31 | 3.38 | 3.00 |
| MIF5 Joint MM                          | 2.56 | 3.55 | 4.92 | 5.67 | 6.83 |
| $LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)$ | 5.60 | 5.25 | 5.62 | 4.62 | 3.77 |
| LassoJoint_ $MM_(\log(p)/n)^(1/2)$     | 6.90 | 4.83 | 4.31 | 4.54 | 5.00 |
| MIF5 Joint BFGS                        | 3.78 | 5.36 | 6.25 | 6.33 | 5.50 |

Table 141. Method avg.rank based on F1 metrics

| method  | 0.1  | 0.3  | 0.5  | 0.7  | 0.9  |
|---|------|------|------|------|------|
| $\overline{LassoJoint\_BFGS\_(log(p)/n)^{}(1/2)}$ | 3.80 | 3.00 | 2.54 | 3.00 | 4.38 |
| $LassoJoint\_BFGS\_lambda.1se$                    | 4.20 | 2.50 | 2.62 | 4.85 | 4.69 |
| $LassoJoint\_MM\_lambda.1se$                      | 6.70 | 5.25 | 4.08 | 3.46 | 2.85 |
| LassoJoint_BFGS_lambda.min                        | 2.50 | 3.42 | 4.69 | 6.31 | 5.62 |
| MIF5 Joint BFGS                                   | 2.67 | 4.09 | 4.50 | 5.08 | 6.33 |
| LassoJoint_MM_lambda.min                          | 4.10 | 4.92 | 5.85 | 4.46 | 4.08 |
| MIF5 Joint MM                                     | 4.22 | 5.64 | 5.83 | 4.17 | 4.33 |
| $LassoJoint\_MM\_(log(p)/n)^{}(1/2)$              | 7.00 | 5.75 | 5.54 | 4.23 | 3.38 |
|   |      |      |      |      |      |