LA - Prac Prob

Monday, November 2, 2020 10:03 PM

1. **[14 points]** Evaluate $\int \frac{\sin(\frac{1}{x})}{x^2}\,dx$

2. Evaluate $\int (1+\tan(x))^5 \sec^2(x)\,dx.$

3. Evaluate $\int_0^1 x^4\sqrt{2x^5+1}\,dx$

7. Find the average value of the function $f(x) = \sin(x)$ on $[0, \pi]$.

Choosing u in u-substitution

1. Parantheses (x)
2. Exponents (xe^x)
3. Trigonometry $(x^2\cos(x))$
4. Square root $(\sqrt{x+1})$

① $\int \frac{\sin(1/x)}{x^2}\,dx$

$u = 1/x$

$du = -\frac{1}{x^2}\,dx$

$dx = -x^2\,du$

$\int \frac{\sin u}{x^2}\,(-x^2\,du)$

$-\int \sin u\,du$

$-(-\cos u)$

$= \cos(1/x) + C$

② $\int (1+\tan x)^5 \sec^2 x\,dx$

$u = \tan x$

$du = \sec^2 x\,dx$

$dx = \frac{du}{\sec^2 x}$

$\int (1+u)^5 \sec^2 x \left(\frac{du}{\sec^2 x}\right)$

$\int (1+u)^5\,du \rightarrow \int 1^5 + u^5\,du$

$\int 1 + u^5\,du \rightarrow \int 1\,du + \int u^5\,du$

$\int 1\,du \rightarrow \frac{1u^{0+1}}{0+1} = \frac{1u}{1} = u$

$\int u^5\,du \rightarrow \frac{u^6}{6}$

$\frac{u^6}{6} + u \rightarrow \frac{\tan(x)^6}{6} + \tan(x) + C$

③ $\int_0^1 x^4\sqrt{2x^5+1}\,dx$

(3) $\int_0^1 x^4\sqrt{2x^5+1}\,dx$

$u = 2x^5+1$

$du = 10x^4\,dx$

$dx = \frac{1}{10x^4}$

$\int_0^1 x^4\sqrt{u}\left(\frac{1}{10x^4}\right)$

$\frac{1}{10}\int_0^1 \sqrt{u} = \frac{1}{10}\int_0^1 u^{1/2+\frac{2}{2}} =$

$\frac{1}{10}\int_0^1 \frac{u^{3/2}}{3/2} \rightarrow \frac{1}{10}\left(\frac{2\sqrt{u^3}}{3}\right)\Big|_0^1$

$\frac{2}{3}u^{3/2} \rightarrow \frac{2}{3}\sqrt{u^3}$

$\rightarrow \frac{\sqrt{u^3}}{15}\Big|_0^1 \rightarrow \frac{\sqrt{(2x^5+1)^3}}{15}\Big|_0^1 = b - a$

$\left(\frac{\sqrt{(2(1)^5+1)^3}}{15} - \frac{\sqrt{(2(0)^5+1)^3}}{15}\right)$

$\left(\frac{\sqrt{27}}{15} - \frac{1}{15}\right)$