# Exam IV Review

Tuesday, November 17, 2020 10:41 PM



**MA 125-CW, CALCULUS I**

Test 4, November 29, 2018

Name (Print last name first): ........................................

**Show all your work and justify your answer!**

**No partial credit will be given for the answer only!**

**PART I**

**You must simplify your answer when possible.**
**All problems in Part I are 8 points each.**

1. If $f(x) = \sin(x)e^{\cos(x)}$, find the derivative $f'(x)$.

Chain Rule: $f' \cdot g + f \cdot g'$

$-\cos(x) e^{\cos x} + \sin(x) \cdot e^{\cos x} \cdot \cos x$

$f'(x) = -\cos(x) e^{\cos x} + \sin(x) e^{\cos x} \cos(x)$

2. Find the derivative of $f(x) = \ln(\cos(x))$.

f g

$\ln(\cos x) \cdot (-\sin x)$

f g'

$f' = \frac{1}{\cos x}$

$\rightarrow \frac{1}{\cos x} \cdot (-\sin x)$



3. Evaluate $\int (x-2) \cdot e^{x^2-4x}\, dx$ u-substitution

$u = x^2 - 4x$

$du = 2x - 4\, dx$

$\frac{4 + du = 2x\, dx}{2x}$

$\frac{4}{2x} + du = dx$

$\int (x-2) \cdot e^u\, dx$

$\int (x-2) \cdot e^u \cdot \frac{4}{2x} + du$

$\int \cancel{x} - 2 \cdot e^u \cdot \frac{2}{\cancel{x}} + du$

$\int -2 \cdot e^u \cdot 2 + du$

$\int -4e^u + du$

$-4 \int e^u\, du$

$\frac{e^{u+1}}{u+1} \rightarrow e^u \cdot 1$

$-4e^u \rightarrow -4e^{x^2+4x} + C$

4. Evaluate $\int \frac{\cos(x)}{2+\sin(x)}\, dx$ u-substitution

$u = 2 + \sin(x)$

$du = -\cos x\, dx$

$dx = -\frac{1}{\cos x} du$

$\int \frac{\cancel{\cos x}}{u} \cdot -\frac{1}{\cancel{\cos x}} du$

$\int -\frac{1}{u} du \rightarrow -\int \frac{1}{u} du$

$\int \frac{1}{u} = \ln|u|$

$-\ln|u| + C \rightarrow -\ln|2 + \sin x| + C$



5. Solve $12e^{10x} = 7$.

6. Solve $\ln(5x+3) = 2$.



7. Use Newton's method to approximate the solution of the equation $f(x) = ln(x+2) - 0.1 = 0$. Hint: choose $x_1$ for which the value of $f(x_1)$ is close to 0 and then find $x_2$. Give only the expression for $x_2$ and not its decimal value.

8. Consider $f(x) = e^x + \cos(x)$ on the set of all non-negative reals. Show that $f$ is one-to-one on $x > 0$, and find $f^{-1}(2)$. [Recall that $e \approx 2.7 > 1$.]



**PART II**

1. **[10 points]** Evaluate the indefinite integral $\int x^3 e^{x^4} + \frac{2}{x+2}\, dx$. [Hint: treat it as two separate integrals.]



2. **[12 points]** Consider the function $s(x) = 3x^3 + x^2 + x + 12$. Verify that this function is one-to-one. (1) Make a table of values of $s(x)$ for $x = 0, 1, 2$. (2) Find $(s^{-1})'(17)$. (3)

(1) $s(x) = 3x^3 + x^2 + x + 12$

Passes horiz. line test

(2) $s(0) = 3(0)^3 + (0)^2 + (0) + 12 = 12$

$s(1) = 3(1)^3 + 1^2 + 1 + 12 = 17$

$s(2) = 3(2)^3 + 2^2 + 2 + 12 = 42$

(3) $(s^{-1})'(17)$

$\frac{1}{s'(s^{-1}(17))} = \frac{1}{s'(1)} = \frac{1}{12}$

$s = 3x^3 + x^2 + x + 12$

$s' = 9x^2 + 2x + 1$

$s'(1) = 9(1)^2 + 2(1) + 1 = 12$



3. **[14 points]**

Find absolute maximum of the function $f(x) = e^x(x^2 - 5x + 7)$ on the interval $[1.2,\ 2.9]$. Note that we want the Absolute Maximum on the interval $[1.2,\ 2.9]$.

(1) Product Rule

(2) **x = 0** to find y

(3) x = 1.2 and 2.9

(4) Highest value of 1.2 or 2.9 is abs max

$f(x) = e^x(x^2 - 5x + 7)$

$f'(x) = e^x \cdot 2x - 5 + e^x(x^2 + 5x - 7)$

$= e^0 \cdot 2(0) - 5 + e^0(0^2 + 5(0) - 7)$

$= 1 \cdot 0 - 5 + 1 \cdot (0 + 0 - 7)$

$= 1 - 5 - 7 = -11$

$f'(1.2) = e^{1.2} \cdot 2(1.2) - 5 + e^{1.2}(1.2^2 + 5(1.2) - 7) = 4.43$

$f'(2.9) = e^{2.9} \cdot 2(2.9) - 5 + e^{2.9}(2.9^2 + 5(2.9) - 7) = 389$

| x | 0 | 1.2 | 2.9 |
|---|---|---|---|
| y | -11 | 4.43 | 389 |

Abs Max = (2.9, 389)