Practice Assignment

Monday, October 19, 2020 12:56 AM

MA 125-6B FALL 2020 **Written Assignment #3 (2.5, 2.8, 3.1)** **NAME**__________________________

25 points

Show all work/steps in the problems below.

1-3. Find the derivative of each of the following. In each case, simplify your answers as much as possible (eliminate negative exponents, collect like terms, combine fractions, factor final answer completely).

[3] 1. $f(x) = \sec^2(5x)$ $f'(g(x)) \cdot g'(x)$

$\sec^2 \tan(5x) \cdot 5$

$5\sec(5x)\tan(5x)$

$f(x)$

[3] 2. $g(x) = x\sqrt{4-x^2}$ $\leftarrow g(x)$

$f'(x)g(x) + f(x)g'(x)$

$f'(x) = 1$

$g'(x) = \frac{1}{2}(4-x^2)^{-1/2} \Rightarrow \frac{1}{2} \frac{1}{\sqrt{4-x^2}} \Rightarrow \frac{1}{2\sqrt{4-x^2}}$

$\sqrt{4-x^2} + x\left(\frac{1}{2\sqrt{4-x^2}}\right) \Rightarrow \sqrt{4-x^2} + \frac{x}{2\sqrt{4-x^2}}$

[3] 3. $h(x) = \dfrac{\cos(\frac{1}{x})}{x}$

$\dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)} = \dfrac{\sin(1/x)}{x^3} - \dfrac{\cos(1/x)}{x^2}$

4-5. Find the critical number(s) x for the given functions.

$f(x)$

[3] 4. $f(x) = \sqrt[3]{x^2+x}$ $\leftarrow g(x)$ $\leftarrow$ Chain Rule

$f'(g(x)) \cdot g'(x)$

$\frac{1}{3}(x^2+x)^{-2/3} = \frac{1}{3} \frac{1}{\sqrt[3]{(x^2+x)^2}} = \frac{1}{3\sqrt[3]{(x^2+x)^2}} = f'(g(x))$

$g'(x) = 2x+1$

$\frac{1}{3\sqrt[3]{(x^2+x)^2}} \cdot 2x+1 = \frac{2x+1}{3\sqrt[3]{(x^2+x)^2}} = 0$

$2x+1 = 0 = x = -1/2$

$\frac{1}{3\sqrt[3]{(x^2+x)^2}} = 0 \Rightarrow x = 0$

$fx \rightarrow$

[3] 5. $f(x) = \dfrac{x}{x^2+4}$ on the interval [-1, 4]

$gx \rightarrow$

$\dfrac{f'(x)g(x) - f(x)g'(x)}{g^2(x)}$

$\dfrac{(x^2+4) - 3x}{(x^2+4)^2} = \dfrac{(x^2+4)-3x}{x^4+16} = 0$ $\quad \frac{1}{x^4+16} = 0$

No solution

$(x^2+4) - 3x = 0 \Rightarrow x^2 - 3x + 4 = 0$

$(x-4)(x+1)$

$x-4 = 0 \Rightarrow \boxed{x = 4}$

$x+1 = 0 \Rightarrow \boxed{x = -1}$

$f(x)$ $g(x)$

[3] 6. Use calculus to determine the x-values in the domain of the following function where the tangent is horizontal.

$f(x) = (1-x)(2x+1)^3 / 2$ $\leftarrow$ Product Rule

$f'(x)g(x) + f(x)g'(x)$

horizontal.

$f(x) = \boxed{(1-x)^2}\boxed{(2x+1)^3}$ / 2 ← Product Rule

$f'(x)g(x) + f(x)g'(x)$

$f'(x) = 2(1-x) = 2-2x$

$g'(x) = 3(2x+1)^2 = 3(2x^2+1) = 6x^2+3$

$(1-x^2) \times \frac{d}{dx}(2x+1)^3 + (2x+1)^3 \frac{d}{dx}(1-x)^2$

$(1-x^2)(6x+3) + (2x+1)^3(-2x+2)$

$1-x^2=0 \rightarrow x=\pm 1$

$6x+3=0 \rightarrow x=-1/2$

$8x^3+1=0 \rightarrow 8x^3=-1 \rightarrow x=\sqrt{\frac{-1}{8}} \rightarrow x=i$

$-2x+2=0 \rightarrow -2x=-2 \rightarrow x=1$

$x = 1, -1, -1/2$

[3] 7. $\sqrt[3]{8.2}$ is the value of a function $y = f(x)$. Identify the function. Then approximate this value by using a linearization of the function at an appropriate value $\boxed{x=a}$ in the domain of f to give a good estimate. Ideally, *a* is such that *f (a)* and *f '(a)* can be computed exactly and are not approximations themselves.

$L(x) = f(a) + f'(a)(x-a)$

1) Find $f(x)$

2) $f(a)$, $f'(x)$, $f'(a)$

3) Plug into $L(x)$

[4] 8. Let $f(x) = x^3 - 12x - 1$ on $[-5, 4]$. Find the critical numbers for $y = f(x)$, and show how you use the Closed Interval Method to find the absolute maximum and minimum values of the function.