

$$\int \frac{1}{u} du = \ln |u| + C$$

$$\textcircled{1} \int \frac{x}{x^2+4} dx \quad \underline{u\text{-substitution}}$$

$$u = x^2 + 4$$

$$du = 2x dx$$

$$dx = \frac{du}{2x}$$

$$\int \frac{x}{u} \cdot \frac{du}{2x} \rightarrow \int \frac{du}{2u}$$

$$\frac{1}{2} \int \frac{1}{u} du \rightarrow \frac{1}{2} \ln |u| + C$$

$$\frac{1}{2} \ln |x^2 + 4| + C$$

$$\textcircled{2} \int \frac{x^2 - 4}{x} dx \quad \underline{\text{Separate}}$$

$$\int \frac{x^2}{x} dx - \int \frac{4}{x} dx$$

↓

$$\int x dx$$

$$\frac{x^2}{2}$$

↓

$$4 \ln |x|$$

↓

$$\frac{x^2}{2} - 4 \ln |x| + C$$

$$\int e^x dx = \underline{e^x} + C \quad \leftarrow$$

$$\int a e^{ax} dx = e^{ax} + C$$

$$\int a e^x dx = e^{ax} + C$$

$$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$$

$$\textcircled{3} \int e^{-5x} dx \quad \underline{u\text{-substitution}}$$

$$\begin{aligned} u &= -5x \\ du &= -5 dx \\ dx &= \frac{du}{-5} \end{aligned}$$

$$\int e^u dx \rightarrow \int e^u \frac{du}{-5}$$

$$-\frac{1}{5} \int e^u du \rightarrow \frac{1}{-5} e^u + C$$

$$-\frac{1}{5} e^{-5x} + C$$

$$\int e^{\textcircled{-5x}} dx \rightarrow \frac{e^{-5x}}{-5} + C$$

$$\textcircled{4} \int (x+2) \cdot e^{x^2+4x} dx \quad \underline{u\text{-substitution}}$$

$$\begin{aligned} u &= x^2 + 4x \\ du &= 2x + 4 dx \\ dx &= \frac{du}{2x+4} \end{aligned}$$

$$\int (x+2) \cdot e^u dx$$

$$\int (x+2) \cdot e^u \cdot \frac{1}{2x+4} du$$

$$\cancel{\int (x+2) \cdot e^u \cdot \frac{1}{2(x+2)} du}$$

$$\int e^u du$$

$$\frac{1}{2} \int e^u du$$

$$\frac{1}{2} e^u + C \rightarrow \frac{1}{2} e^{x^2+4x} + C$$

$$\textcircled{5} \int \left(x^2 e^{x^3} + \frac{1}{x-2} \right) dx$$

$$\int x^2 e^{x^3} dx + \int \frac{1}{x-2} dx$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$dx = \frac{1}{3x^2} du$$

$$\int \cancel{x^2} e^u \cdot \frac{1}{3\cancel{x^2}} du$$

$$\frac{1}{3} \int e^u du$$

$$\frac{1}{3} e^{x^3}$$

$$u = x-2$$

$$du = dx$$

$$\int \frac{1}{u} dx$$

$$\ln|u|$$

$$\ln|x-2| + C$$

$$= \frac{e^{x^3}}{3} + \ln|x-2| + C$$