Exam II Review

Wednesday, October 14, 2020 1:04 AM

MA 125-6C, CALCULUS I
October 14, 2015

Name (Print last name first): ....

Show all your work and justify your answer!

No partial credit will be given for the answer only!

PART I

You must simplify your answer when possible.
All problems in Part I are 10 points each.

1. Find the derivative of the function $y = f(x) = \cos(x^3)$.

$f(x)$

$g(x) = 3x^2$

Chain Rule

$f'(g(x)) \cdot g'(x)$

$f'(g(x)) = -\sin(x^3)$

$g'(x) = 3x^2$

$-\sin(x^3) \cdot 3x^2 = -3x^2 \sin(x^3)$

2. Find the derivative of $f(x) = (x^2 + x)^8$.

$f(x)$, $g(x)$

$f'(g(x)) \cdot g'(x)$

$f'(g(x)) = 8(x^2+x)^7$

$g'(x) = 2x + 1$

$\rightarrow 8(x^2+x)^7 \cdot 2x+1$

3. Find the absolute maximum and minimum of the function $y = f(x) = (x-2)^2(x+1)^3$ on the interval $[0, 1]$.

$f(x)$ $g(x)$

Product Rule: $f'(x)g(x) + f(x)g'(x)$

$f'(x) = 2(x-2)$

$g'(x) = 3(x+1)^2$

$2(x-2)(x+1)^3 + (x-2)^2 \cdot 3(x+1)^2$

① ② ③

$(x-2)(x+1)^2 \left(2(x+1) + 3(x-2)\right)$

$x-2=0$ $x=2$ ✗

$(x+1)^2 = 0$ $x=-1$ ✗

$x = \frac{4}{5}$ ✓

$(2(x+1) + 3(x-2)) = 0 \rightarrow 2x+2+3x-6=0$

$\rightarrow 5x-4=0$ $x = 4/5$

x	0	4/5	1
y	-4	8.4	-6

Abs max = $(4/5, 8.4)$

Abs min = $(1, -6)$

$(1-2)^2(1+1)^3$

$(1^2-2^2)(1^3+1^3) = (1-4)(1+1) = -6$

4. Find the linearization of the function $f(x) = x\tan(x)$ at the point $a = \pi/4$ and use it to estimate the value $f(\pi/4 + 0.1)$.

$L(x) = f(a) + f'(a)(x-a)$

$f(a) = \left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right) = \pi/4$

$f'(x) = \tan(x) + x\sec^2(x)$

$f'(a) = \tan\left(\frac{\pi}{4}\right) + \left(\frac{\pi}{4}\right)\sec^2(\pi/4)$

$1 + \pi/4 \sec^2(\pi/4)$

$= 1 + \frac{\pi}{2\cancel{4}}(\cancel{2}) \rightarrow 1 + \frac{\pi}{2}$

$L(x) = \frac{\pi}{4} + \left(1+\frac{\pi}{2}\right)\left(x - \frac{\pi}{4}\right)$

$= \frac{\pi}{4} + \left(1+\frac{\pi}{2}\right)\left(\cancel{\frac{\pi}{4}} + 0.1 - \cancel{\frac{\pi}{4}}\right)$

$= \frac{\pi}{4} + \left(1+\frac{\pi}{2}\right)(0.1)$

5. Find two positive numbers so that their sum is 200 and their product is maximal. [As always you must justify your answer!]

$x + y = 200$ $\quad$ $xy \to max$

$y = 200 - x$ $\quad$ $y(200 - x) \to max$

$(200 - x)(200 - x) = -x^2 + 200x = f(x)$

$f'(x) = -2x + 200 \to -2(x - 100)$

$-2 = 0$ x

$x - 100 = 0 \to x = 100$

$x = 100$

$x + y = 200$

$100 + y = 200$

$y = 100$

6. Suppose that the **derivative** of a function $y = f(x)$ is given:

$f'(x) = (x + 2)(3 - x)$.

(a) Find the x-coordinates of all local max/min of the function $y = f(x)$.

$x + 2 = 0 \to x = -2 \to$ min

$3 - x = 0 \to x = 3 \to$ max

(b) At which x value is the function $y = f(x)$ most rapidly increasing?

$f''(x) = f'(x)g(x) + f(x)g'(x)$

$f(x) = x + 2$

$g(x) = 3 - x$

$(1)(3 - x) + (x + 2)(-1)$

$x = \frac{1}{2}$

$3 - x + (-x - 2) = 0 \to 3 - x - x - 2 = 0$

$1 - 2x = 0$

$-2x = -1$

$x = 1/2$

$\boxed{\text{PART II}}$

7. [**15 points**] You work for a soup company. Your boss asks you to design a soup can of volume $1\,dm^3$ and **minimal** surface area. Either specify the radius of top/bottom of such a can or show that such a can does not exist. Then your boss asks you to design a soup can of volume $1\,dm^3$ and **maximal** surface area. Either specify the radius of top/bottom of such a can or show that such a can does not exist.

You may use that the volume of a can of radius r and height h is $V = \pi r^2 h$ while the surface are of the side is $2\pi rh$ and of the top (and bottom) is πr^2.

① Volume

$1 = \pi r^2 h$

② Height

$h = 1/\pi r^2$

③ Surface Area

$SA = 2\pi rh + 2\pi r^2$

$= 2\pi r^2 + 2\pi r\left(\frac{1}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{2\cancel{\pi}\cancel{r}}{\cancel{\pi} r^{\cancel{2}}}$

$= 2\pi r^2 + \frac{2}{r}$

④ Minimal

(Take derivative)

$S(r) = 2\pi r^2 + \frac{2}{r}$

$S'(r) = 2\pi(2r) + 2\left(\frac{-1}{r^2}\right)$

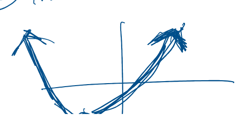
$= 4\pi r - \frac{2}{r^2}$

$4\pi r - \frac{2}{r^2} = 0 \to \frac{4\pi r^3 - 2}{r^2} = 0$

$4\pi r^3 - 2 = 0 \to 4\pi r^3 = 2 \to r^3 = \frac{2}{4\pi} \to r = \sqrt[3]{\frac{1}{2\pi}}$

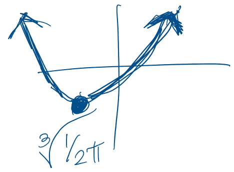
$r^2 = 0 \to 0$

⑤ Maximal



There is no maximum

$S(r) \to \infty$



maximum

$s(r) \to \infty$