MA 125 - Limits & Derivatives

Tuesday, September 8, 2020 7:25 PM

Functions

- Equation that says if I give A, I will receive B
- Example: Find y if x=5

$\frac{3x}{5} = y \rightarrow \frac{3(5)}{5} = \frac{15}{5} = 3 = y$

In function notation: **f(x) =3** in $\frac{3x}{5} = y$

Limits

- Says that all numbers up to a certain value can be used in a function
- Example: We can use all numbers under curve for the function except 3 and over

"The limit of..." "...the function f..."

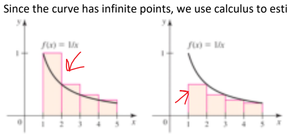
$$\lim_{x \to 3} f(x)$$

"...as x approaches 3."

- If lim has (+), limit goes towards right
- If lim has (-), limit goes towards left

Derivatives

- Definition
- Identifies the area under a curve
- Since the curve has infinite points, we use calculus to estimate the area under the curve



- Basic Properties

1) Power Rule
- Put exponent value in front & subtract one from exponent

$\frac{d}{dx}(x^n) = nx^{n-1}$ $2x^{2-1} \rightarrow 4x$

2) Sum Rule
- Two functions being added can be separated
- Separate, then add the answer to the functions together to get final answer

$(f(x) \pm g(x))' = f'(x) \pm g'(x)$ $df(x) + dg(x)$

3) Product Rule
- Two functions being multiplied can be separated
- First, take derivative of f and multiply it by g
- Second, take derivative of g and multiply it by f
- Add two answers together

$(fg)' = f'g + fg'$ $(df \cdot g) + (f\, dg)$

4) Quotient Rule
- Two functions being multiplied can be separated
- Do the same as the product rule, but subtraction instead of addition
- Since the denominator was split, it is multiplied by itself (AKA it's squared)

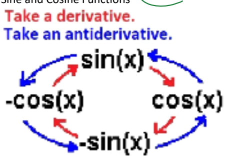
$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ $\frac{(df \cdot g) - (f \cdot dg)}{g^2}$

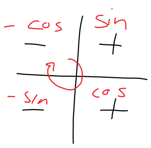
5) Chain Rule
- Asks to find the derivative of a function within a function
- First, take derivative of f and multiply it by g
- Second, take derivative of g
- Multiple the two answers together

$\boxed{\frac{d}{dx}(f(g(x)))} = f'(g(x))g'(x)$ $(df(gx) \cdot dg)$

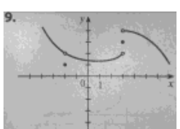
6) Sine and Cosine Functions

Take a derivative.
Take an antiderivative.





Example Answers



51. 3

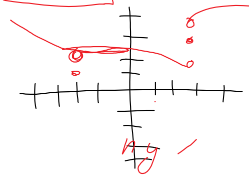
25. $6a - 4$

29. $-\frac{1}{\sqrt{1-2a}}$

Examples

7–10 ■ Sketch the graph of an example of a function f that satisfies all of the given conditions.

9. $\lim_{x \to 3^+} f(x) = 4$, $\lim_{x \to 3^-} f(x) = 2$, $\lim_{x \to -2} f(x) = 2$, $f(3) = 3$, $\boxed{f(-2) = 1}$



O - Many x val gives us y

● - 1 x val gives us y

51. $\lim_{t \to 0} \frac{\tan 6t}{\sin 2t}$

$\lim_{t \to 0} \frac{\tan 6t}{\sin 2t} \rightarrow \lim_{t \to 0} \frac{\sin 6t}{(\cos 6t)(\sin 2t)} \rightarrow \lim_{t \to 0} \frac{3(\sin 6t)}{(\cos 6t)\, 3(\sin 2t)}$

$\lim_{t \to 0} \frac{3(\sin 6t)}{(\cos 6t)(\sin 6t)} \rightarrow \lim_{t \to 0} \frac{3}{\cos 6t} = \lim_{t \to 0} \frac{3}{\cos(6 \cdot 0)} \Rightarrow \frac{3}{1} = \boxed{3}$

$* \lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1 \rightarrow \cos 0 = 1$

25. $f(x) = \boxed{3x^2} \boxed{- 4x} \boxed{+ 1}$

$3x^{2-1} = 6x$

$-4x^{1-1} = -4$

$\cancel{1}$

$6x - 4 = f'(x)$

29. $f(x) = \sqrt{1 - 2x}$

$\sqrt{1-2x} = fx$ $(x)^{\frac{1}{2}} = \sqrt{x}$

$1 - 2x = gx$ $(x)^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$

$(1-2x)^{1/2 - 1} = \frac{1}{2}(1-2x)^{\frac{1}{2} - \frac{2}{2}}$

$\frac{1}{2}(1-2x)^{-\frac{1}{2}} = \frac{1}{2}\left(\frac{1}{\sqrt{1-2x}}\right) = \boxed{\frac{1}{2\sqrt{1-2x}}}$

$1 - 2x = gx \rightarrow g'(x) = -2x^{1-1} + \cancel{X}$

$= \boxed{-2}$

$\left(\frac{1}{2\sqrt{1-2x}}\right)(-2) = \frac{-2}{2\sqrt{1-2x}} = \boxed{\frac{-1}{\sqrt{1-2x}}}$