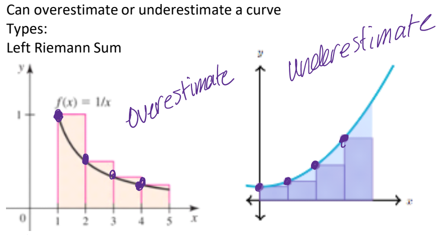
Integrals & Riemann

Wednesday, October 21, 2020 12:49 AM

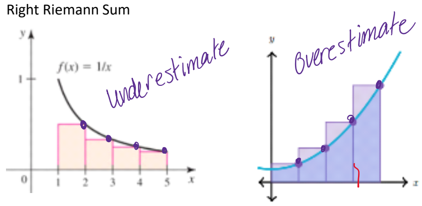
Riemann Sum

- Definition
  - Approximates the area under a curve
  - Can overestimate or underestimate a curve
  - Types:

1) Left Riemann Sum



2) Right Riemann Sum



- Formulas

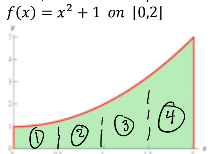
$$L_n = f(x_0)\Delta x + f(x_1)\Delta x + \cdots + f(x_{n-1})\Delta x = \sum_{i=0}^{n-1} f(x_i)\Delta x,$$

$$R_n = f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_n)\Delta x = \sum_{i=1}^{n} f(x_i)\Delta x,$$

$$M_n = f(\overline{x}_1)\Delta x + f(\overline{x}_2)\Delta x + \cdots + f(\overline{x}_n)\Delta x = \sum_{i=1}^{n} f(\overline{x}_i)\Delta x,$$

- Examples

$f(x) = x^2 + 1$ on $[0,2]$ a, b



1) Find Delta x

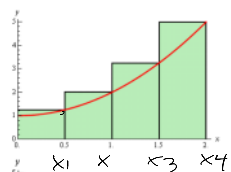
$\Delta x = \frac{b-a}{n}$ $[a,b] = [0,2]$

n = # of sections

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

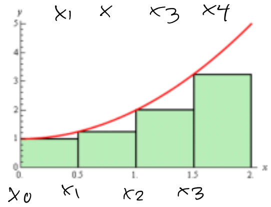
2) Take right riemann sum

$$A_r = \frac{1}{2}f\left(\frac{1}{2}\right) + \frac{1}{2}f(1) + \frac{1}{2}f\left(\frac{3}{2}\right) + \frac{1}{2}f(2)$$
$$= \frac{1}{2}\left(\frac{5}{4}\right) + \frac{1}{2}(2) + \frac{1}{2}\left(\frac{13}{4}\right) + \frac{1}{2}(5)$$
$$= 5.75$$



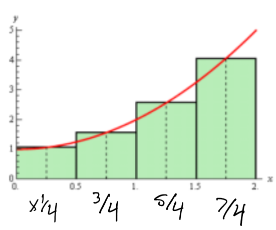
3) Take left riemann sum

$$A_l = \frac{1}{2}f(0) + \frac{1}{2}f\left(\frac{1}{2}\right) + \frac{1}{2}f(1) + \frac{1}{2}f\left(\frac{3}{2}\right)$$
$$= \frac{1}{2}(1) + \frac{1}{2}\left(\frac{5}{4}\right) + \frac{1}{2}(2) + \frac{1}{2}\left(\frac{13}{4}\right)$$
$$= 3.75$$



4) Find median sum

$$A_m = \frac{1}{2}f\left(\frac{1}{4}\right) + \frac{1}{2}f\left(\frac{3}{4}\right) + \frac{1}{2}f\left(\frac{5}{4}\right) + \frac{1}{2}f\left(\frac{7}{4}\right)$$
$$= \frac{1}{2}\left(\frac{17}{16}\right) + \frac{1}{2}\left(\frac{25}{16}\right) + \frac{1}{2}\left(\frac{41}{16}\right) + \frac{1}{2}\left(\frac{65}{16}\right)$$
$$= 4.625$$



$$\frac{R+L}{2} = \frac{5.75 + 3.75}{2} = 4.75$$

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Integrals (Antiderivative)

- Description
  - Finding the "original" of the function
  - Inverse of derivative, so we go backwards from the derivative function given
  - Add "c" as a constant since the derivative takes away coefficients without an x value
- Steps

1) Add one to each coefficient (lets say that number equals A)
2) Divide each x by the value of A
3) Add c (constant) to end of antiderivative

- Examples

$\int 6x^5 - 18x^2 + 7\,dx$

$6x^5 \to 6x^{5+1} \to \frac{6x^6}{5+1} \to \frac{6x^6}{6} \to x^6$

$-18x^2 \to \frac{-18x^3}{3} \to -6x^3$

$7 \to 7x$

$+C$

$= \boxed{x^6 - 6x^3 + 7x + C}$

$\int 40x^3 + 12x^2\,dx - 9x + 14$

$40x^3 \to \frac{40x^4}{4} \to 10x^4$

$12x^2 \to \frac{12x^3}{3} \to 4x^3$

$-9x + 14$

$= 10x^4 + 4x^3 - 9x + 14$

$\int 12x^3 + x^2 + 5t^2\,dt$

$12t^{0+1} = \frac{12t^1}{0+1} = 12t$

$1t^0 = \frac{t}{0+1} = t$

$12x^3t + x^2t + \frac{5t^3}{3} + C$