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__Sol 1:__

$$f(x_1, x_2, x_3) = \overline{x_1}\, x_2 x_3 + x_1 \overline{x_2} \;\;\cancel{+ x_1 x_3}$$

Input $x_i \in \{-1, 1\}$ where $1 = TRUE$
$-1 = FALSE$.

$$f(x_1, x_2, x_3) = OR(\; AND(NOT(x_1), AND(x_2, x_3)), \; AND(x_1, NOT(x_2)))$$

__NOT.__

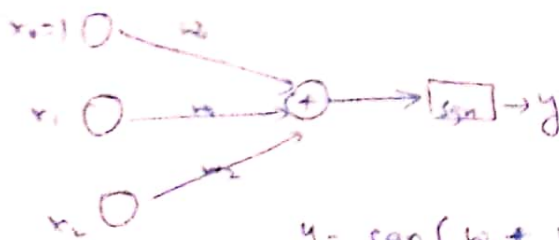$$y = \begin{cases} 1, & x_1 = -1 \\ -1, & x_1 = 1 \end{cases}.$$

$$y = sgn(w_0 + w x_1)$$

$x_1 = 1$ $\qquad y = sgn(w_0 + w_1) = -1$ $\qquad \Rightarrow w_0 + w_1 < 0$

$x_1 = -1$ $\qquad y = sgn(w_0 - w_1) = +1$ $\qquad \Rightarrow w_0 - w_1 > 0$

__Solving; weight:__ $\quad w_1 < 0,$

$$\boxed{w_1 = -1, \; w_0 = 0.5} \;\; \text{satisfies the equations}$$

__AND.__

x_1	x_2	y
1	1	1
-1	+1	-1
1	-1	-1
-1	-1	-1

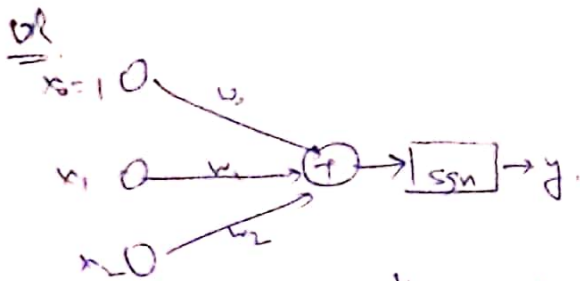$$y = sgn(w_0 + w_1 x_1 + w_2 x_2)$$

Using truth table:

$$y = sgn(w_0 + w_1 + w_2) = 1 \qquad \Rightarrow \quad w_0 + w_1 + w_2 > 0$$

$$y = sgn(w_0 - w_1 + w_2) = -1 \qquad \Rightarrow \quad w_0 - w_1 + w_2 < 0$$

$$y = sgn(w_0 + w_1 - w_2) = -1 \qquad \Rightarrow \quad w_0 + w_1 - w_2 < 0$$

$$y = sgn(w_0 - w_1 - w_2) = -1 \qquad \Rightarrow \quad w_0 - w_1 - w_2 < 0$$

$$\boxed{w_0 = -1; \; w_1 = 1, \; w_2 = 1} \; \text{satisfies all equations.}$$

$x_0 = 1$ w_0

x_1 w_1 → ⊕ → $\boxed{sgn}$ → y.

x_2 w_2

r_1	x_2	y
1	1	1
1	-1	1
-1	1	1
-1	-1	-1

$$y = sgn(w_0 + w_1 x_1 + w_2 x_2)$$

Using truth table:-

$sgn(w_0 + w_1 + w_2) = 1$ ⟹ $w_0 + w_1 + w_2 > 0$

$sgn(w_0 + w_1 - w_2) = 1$ ⟹ $w_0 + w_1 - w_2 > 0$

$sgn(w_0 - w_1 + w_2) = 1$ ⟹ $w_0 - w_1 + w_2 > 0$

$sgn(w_0 - w_1 - w_2) = -1$ ⟹ $w_0 - w_1 - w_2 \not> 0$

$\boxed{w_0 = 1, w_1 = 1; w_2 = 1}$ satisfies all equations.

$$f(x_1, x_2, x_3) = OR\Big(AND\big(NOT(x_1), AND(x_2, x_3)\big), AND(x_1, NOT(x_1))\Big)$$

1 0.5

x_1 -1 → ⊕ → $\boxed{sgn.}$ 1

1 → -1 → ⊕ → $\boxed{sgn}$

1 -1

x_2 1 → ⊕ → $\boxed{ssn}$

x_3 1

1 1 → ⊕ → $\boxed{sgn}$ → y

-1 → ⊕ → $\boxed{sgn}$ 1 → ⊕ → $\boxed{ssn}$
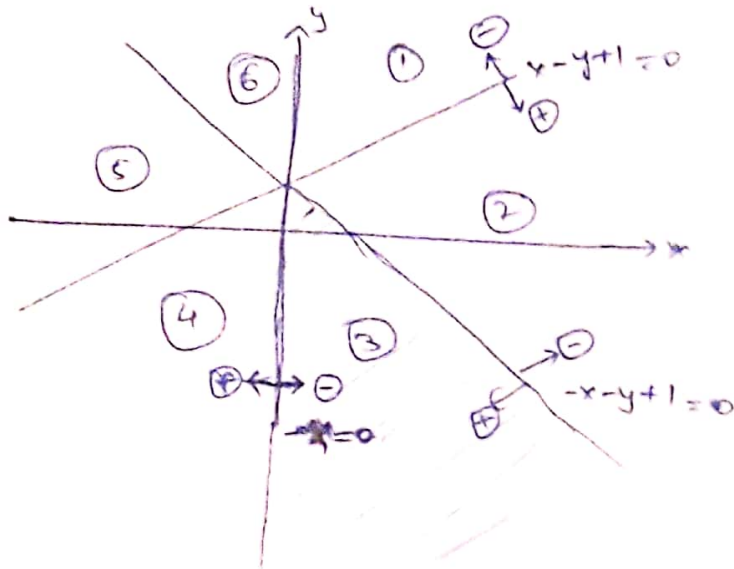
0.5 1 1

1 -1

1
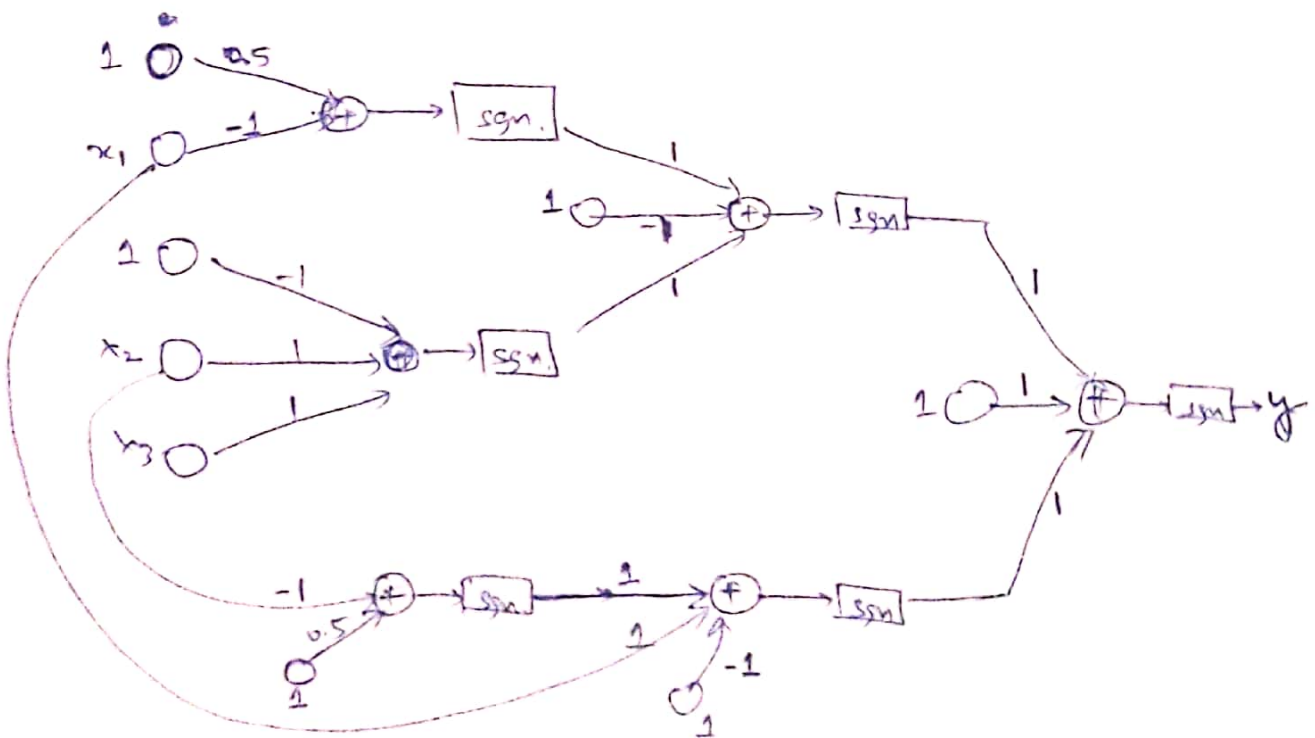
② $u(x) = \begin{cases} 1, & x \geq 0 \\ 0, & x < 0 \end{cases}$

1st Hidden layer eq^{ns}.

$$f_1 = u(x - y + 1)$$

$$f_2 = u(-x - y + 1)$$

$$f_3 = u(-x).$$

2nd weights:

$$z = u\left(f_1 + f_2^* - 1.5\right) \bar{e} f_3)$$

6 regime

Region	f_1	f_2	f_3	$f_1 + f_2 + f_3 + 1.5$	z (n0)
①	0	0	0	-1.5	0
②	1	0	0	1-1.5	0
③	1	1	0	2-1.5	1
④	1	1	1	1-1.5	0
⑤	0	1	1	-1.5	0
⑥	0	0	1	-1-1.5	0

In Region③, z=1 (marked above)