

Lista de ejercicios de la lección 2.1

Cocientes indeterminados

Instrucciones. Calcular el límite si existe.

1.
$$\lim_{x \to 2} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$$
 10. $\lim_{x \to \frac{\pi}{2}^+} \frac{\tan x}{\cot 2x}$

10.
$$\lim_{x \to \frac{\pi}{2}^+} \frac{\tan x}{\cot 2x}$$

19.
$$\lim_{x\to 0} \frac{\arcsin{(2x)}}{\arcsin{x}}$$

2.
$$\lim_{x \to \infty} \frac{2x^2 - 5x + 2}{5x^2 - 7x - 6}$$
 11. $\lim_{t \to \frac{\pi}{2}} \frac{\tan t}{\tan 3t}$

11.
$$\lim_{t \to \frac{\pi}{2}} \frac{\tan t}{\tan 3t}$$

20.
$$\lim_{x\to 0} \frac{x - \arctan x}{x \sin x}$$

3.
$$\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x^2-25}$$

3.
$$\lim_{x \to 5} \frac{\sqrt{x-1}-2}{x^2-25}$$
 12. $\lim_{x \to 0} \frac{\tan x - \sin x}{x^3 \tan x}$ 21. $\lim_{x \to -\infty} \frac{3-3^x}{5-5^x}$

21.
$$\lim_{x \to -\infty} \frac{3-3^x}{5-5^x}$$

4.
$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2}$$
 13. $\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$ 22. $\lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$

13.
$$\lim_{x \to 0} \frac{\sin x - x}{\tan x - x}$$

$$22. \lim_{x \to \infty} \frac{\ln x}{\sqrt{x}}$$

$$5. \lim_{x \to \infty} \frac{x}{\sqrt{x^2 + 1}}$$

14.
$$\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos(2\pi - x)}$$

5.
$$\lim_{x \to \infty} \frac{x}{\sqrt{x^2 + 1}}$$
 14.
$$\lim_{x \to \frac{\pi}{2}} \frac{2x - \pi}{\cos(2\pi - x)}$$
 23.
$$\lim_{x \to \infty} \left[\frac{x \ln x}{x + \ln x} \right]$$

6.
$$\lim_{x \to \infty} \frac{x^{\frac{3}{2}} + 5x - 4}{x \ln x}$$
 15. $\lim_{x \to 0} \frac{x(1 + \cos x)}{x - \sin x}$ 24. $\lim_{x \to 0^+} \frac{\ln(\sin x)}{\ln(\sin 2x)}$

15.
$$\lim_{x \to 0} \frac{x(1 + \cos x)}{x - \sin x}$$

24.
$$\lim_{x\to 0^+} \frac{\ln(\sin x)}{\ln(\sin 2x)}$$

7.
$$\lim_{x \to \frac{\pi}{2}^{-}} \left[\frac{2 + \sec x}{3 \tan x} \right]$$
 16. $\lim_{x \to \infty} \frac{2e^{3x} + \ln x}{e^{3x} + x^2}$ 25. $\lim_{x \to \infty} \frac{x + \cosh x}{x^2 + 1}$

16.
$$\lim_{x \to \infty} \frac{2e^{3x} + \ln x}{e^{3x} + x^2}$$

$$25. \lim_{x \to \infty} \frac{x + \cosh x}{x^2 + 1}$$

8.
$$\lim_{x \to \frac{3\pi}{2}} \frac{1 + \sin x}{\cos^2 x}$$

8.
$$\lim_{x \to \frac{3\pi}{2}} \frac{1 + \sin x}{\cos^2 x}$$
 17. $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\sin x}{x \sin x}$

9.
$$\lim_{\theta \to \frac{\pi}{2}} \frac{1 - \sin \theta}{1 + \cos 2\theta}$$

9.
$$\lim_{\theta \to \frac{\pi}{2}} \frac{1 - \sin \theta}{1 + \cos 2\theta}$$
 18. $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x}$

Productos indeterminados

Instrucciones. Calcular el límite si existe.

26.
$$\lim_{x \to 0^+} (x^2 \ln x)$$

26.
$$\lim_{x \to 0^+} (x^2 \ln x)$$
 31. $\lim_{x \to \infty} \left[x \left(\frac{\pi}{2} - \arctan x \right) \right]$ 36. $\lim_{x \to \infty} \left[x \left(e^{-\frac{1}{x}} - 1 \right) \right]$

$$36. \lim_{x \to \infty} \left[x \left(e^{-\frac{1}{x}} - 1 \right) \right]$$

$$27. \lim_{x \to \infty} \left(x \sin \frac{1}{x} \right)$$

27.
$$\lim_{x \to \infty} \left(x \sin \frac{1}{x} \right)$$
 32. $\lim_{x \to 0^{+}} [\sin x \ln (\sin x)]$ 37. $\lim_{x \to \infty} \left[\frac{1}{x} \left(e^{\frac{1}{x}} - x \right) \right]$

$$37. \lim_{x \to \infty} \left[\frac{1}{x} \left(e^{\frac{1}{x}} - x \right) \right]$$



28.
$$\lim_{x \to 0^{+}} (x \cot x)$$

28.
$$\lim_{x \to 0^+} (x \cot x)$$
 33. $\lim_{x \to \frac{\pi}{2}^-} [\tan x \ln (\sin x)]$ 38. $\lim_{x \to 0^+} (e^{-x} \ln x)$

38.
$$\lim_{x \to 0^+} (e^{-x} \ln x)$$

$$29. \lim_{x \to \frac{\pi}{2}} (\sec x \cos 3x)$$

34.
$$\lim_{x \to \infty} (x^3 2^{-x})$$

29.
$$\lim_{x \to \frac{\pi}{2}} (\sec x \cos 3x)$$
 34. $\lim_{x \to \infty} (x^3 2^{-x})$ 39. $\lim_{x \to \infty} [(x^2 - 1)e^{-x^2}]$

30.
$$\lim_{x \to 0^+} x \ln(\sin x)$$
 35. $\lim_{x \to -\infty} (x^2 e^x)$

35.
$$\lim_{x \to -\infty} (x^2 e^x)$$

Diferencias Indeterminadas

Instrucciones. Calcular el límite si existe.

40.
$$\lim_{x \to \infty} \left[\frac{x^2}{x-1} - \frac{x^2}{x+1} \right]$$
 44. $\lim_{x \to \frac{\pi}{2}} [\sec x - \tan x]$

44.
$$\lim_{x \to \frac{\pi}{2}} [\sec x - \tan x]$$

$$48. \lim_{x \to \infty} [\sinh x - x]$$

41.
$$\lim_{x \to -3} \left[\frac{4}{x^2 + 2x - 3} - \frac{4}{x + 3} \right]$$
 45. $\lim_{x \to \infty} [\ln 2x - \ln (x + 1)]$ 49. $\lim_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$

45.
$$\lim_{x \to \infty} [\ln 2x - \ln (x+1)]$$

49.
$$\lim_{x \to \infty} \left(x - \sqrt{x^2 + x} \right)$$

42.
$$\lim_{x \to 1} \left[\frac{1}{x - 1} - \frac{1}{\ln x} \right]$$

46.
$$\lim_{x \to \infty} [\ln(4x+3) - \ln(3x+4)]$$

$$43. \lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{\sin x} \right]$$

47.
$$\lim_{x \to \infty} \left(\sqrt{x^4 + 5x^2 + 3} - x^2 \right)$$

Potencias Indeterminadas

Instrucciones. Calcular el límite si existe.

50.
$$\lim_{x \to \infty} (1+x^3)^{x^{-2}}$$

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 58. $\lim_{x \to 0} (x + \cos 2x)^{\csc 3x}$ 66. $\lim_{x \to \infty} \left[(\ln x)^{\frac{1}{x}} \right]$

66.
$$\lim_{x \to \infty} \left[(\ln x)^{\frac{1}{x}} \right]$$

51.
$$\lim_{x \to \infty} \left[\left(1 + \frac{1}{x} \right)^{5x} \right]$$
 59. $\lim_{x \to 0} \left[(1 + \sin 4x)^{\cot x} \right]$ 67. $\lim_{x \to \infty} \left[(1 + 2x)^{\frac{1}{2\ln x}} \right]$

59.
$$\lim_{x \to 0} \left[(1 + \sin 4x)^{\cot x} \right]$$

67.
$$\lim_{x \to \infty} \left[(1+2x)^{\frac{1}{2\ln x}} \right]$$

52.
$$\lim_{x \to 0^+} \left[\left(1 + \frac{1}{x} \right)^x \right]$$
 60. $\lim_{x \to \frac{\pi}{2}^-} \left[(\tan x)^{\cos x} \right]$ 68. $\lim_{x \to e} \left[(\ln x)^{\frac{1}{x - e}} \right]$

60.
$$\lim_{x \to \frac{\pi}{2}^{-}} \left[(\tan x)^{\cos x} \right]$$

68.
$$\lim_{x \to e} \left[(\ln x)^{\frac{1}{x-e}} \right]$$

53.
$$\lim_{x \to \infty} \left(1 + \frac{2}{x} \right)^{x^2}$$
 61. $\lim_{x \to 0^+} (e^x - 1)^x$ 69. $\lim_{x \to 0} (\sinh x)^{\tan x}$

61.
$$\lim_{x\to 0^+} (e^x - 1)^x$$

69.
$$\lim_{x\to 0} (\sinh x)^{\tan x}$$



54.
$$\lim_{x \to \infty} \left(\frac{3x}{3x+1} \right)^x$$
 62. $\lim_{x \to \infty} (x+e^x)^{\frac{2}{x}}$ 70. $\lim_{x \to 0} \left((\cos x)e^{x^2} \right)^{\frac{4}{x^4}}$

62.
$$\lim_{x \to \infty} (x + e^x)^{\frac{2}{x}}$$

70.
$$\lim_{x \to 0} \left((\cos x) e^{x^2} \right)^{\frac{4}{x^4}}$$

55.
$$\lim_{x\to 0} \left[(\cos x)^{x+1} \right]$$

63.
$$\lim_{x \to 0^+} \left[(e^x + 3x)^{\frac{1}{x}} \right]$$

55.
$$\lim_{x \to 0} \left[(\cos x)^{x+1} \right]$$
 63. $\lim_{x \to 0^+} \left[(e^x + 3x)^{\frac{1}{x}} \right]$ 71. $\lim_{x \to \infty} \ln \left(\frac{x^2 - 1}{x^2 + 1} \right)^{3x}$

56.
$$\lim_{x \to 0^+} \left[(2x+1)^{\cot x} \right]$$
 64. $\lim_{x \to \infty} \left[(1+e^x)^{e^{-x}} \right]$

64.
$$\lim_{x \to \infty} \left[(1 + e^x)^{e^{-x}} \right]$$

57.
$$\lim_{x \to \frac{\pi}{2}^-} \left[(1 + \cos x)^{\tan x} \right]$$
 65. $\lim_{x \to 1^-} (1 - x)^{\ln x}$

65.
$$\lim_{x \to 1^{-}} (1-x)^{\ln x}$$

72. Demostrar que si
$$a>0$$
, entonces $\lim_{n\to\infty}n\left(a^{\frac{1}{n}}-1\right)=\ln a$

73. Hallar los valores de
$$a$$
 y b tales que $\lim_{x\to 0} \frac{\cos(ax)-b}{2x^2} = -4$

74. Hallar los valores de
$$a$$
 y b tales que $\lim_{x\to 0} \frac{\sin{(2x)} + ax + bx^3}{x^3} = 0$

75. Hallar los valores de
$$a$$
 y b tales que $\lim_{x\to 0} \frac{\sin{(3x)} + ax + bx^2}{x^3} = 0$

76. Si
$$\lim_{x\to\infty} \ln\left(\frac{nx+1}{nx-1}\right)^x = 9$$
, determinar n .

77. Demostrar que
$$\lim_{x\to\infty}\frac{e^x}{x^n}=\infty$$
 para cualquier entero $n.$

78. Demostrar que
$$\lim_{x\to\infty} \frac{\ln x}{x^p} = 0$$
 para cualquier $p > 0$.

79. Determinar los valores de
$$a$$
, b y c para los cuales $\lim_{x\to 1} \frac{ax^4 + bx^3 + 1}{(x-1)\sin(\pi x)} = c$

80. Calcular
$$\lim_{x\to 0} \frac{e^x - 1 - x - \frac{x^2}{2} - \frac{x^3}{6}}{x^4}$$

81. Calcular
$$\lim_{x\to a} \frac{\sqrt{2a^3x - x^4} - a\sqrt[3]{a^2x}}{a - \sqrt[4]{ax^3}}$$
, donde $a > 0$.