AMS310 Homework 5

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Chapter 7

1. The correct answer is $\boxed{d}$. This is because we should reject H_0 when $p \leq \alpha$ and do not reject when H_0 if $p > \alpha$. Both values of α are greater than our p-value.

2a. Our testing hypothesis is such: The mean warm up time of a certain brand of printer is greater than 15 seconds $\boxed{(H_0:\ p = 15, H_1:\ p > 15)}$.

b. It would be appropriate to reject $\boxed{\mathrm{H}_0}$.

c. The engineer did not make the correct decision, as $14 < 15$, which makes the null hypothesis true. This is a type I error.

3a. $\mathrm{H}_0:\ p \geq 8$, $\mathrm{H}_1:\ p < 8$

b.

$$
\begin{aligned}
\alpha &= P(\text{type II error}) \\
&= P(\text{failure to reject } \mathrm{H}_0 \text{ when } \mu = 5) \\
&= P\left(\overline{X} \leq 6 \text{ when } \overline{X} \sim N\left(5, \left(\frac{3}{\sqrt{20}}\right)^2\right)\right) \\
&= P\left(Z > \frac{6-5}{\frac{3}{\sqrt{20}}}\right) \\
&= 1 - \Phi(1.49) \\
&= 1 - 0.9319 \\
&= \boxed{0.0681}
\end{aligned}
$$

Chapter 8

1a. For $n = 120,\ s = 3.7,\ \overline{x} = 36.1,\ z_{0.025} = 1.96$

$\left(\overline{x} - z_{\alpha/2}\frac{s}{\sqrt{n}},\ \overline{x} + z_{\alpha/2}\frac{s}{\sqrt{n}}\right)$
$\left(36.1 - 1.96\frac{3.7}{\sqrt{120}},\ 36.1 + 1.96\frac{3.7}{\sqrt{120}}\right)$
$\boxed{(35.44, 36.76)}$

b.

$$\begin{aligned} n &= \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 \\ &= \left(\frac{2.57 \cdot 4}{0.5}\right)^2 \\ &= \boxed{423} \end{aligned}$$

2a. $\mathrm{H}_0:\ \mu = 28$, $\mathrm{H}_1:\ \mu \neq 28$

$$\begin{aligned} z &= \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} \\ &= \frac{27.32 - 28}{\frac{12}{\sqrt{16}}} \\ &= -2.27 \end{aligned}$$

Thus the rejection region $\rightarrow |z| \geq z_{0.01}$
$|z| = 2.27, z_{0.01} = 2.33,$ $\boxed{\text{thus we reject } \mathrm{H}_0}$

$$\begin{aligned} P(|Z| > 2.27) &= 2P(Z < -2.27) \\ &= 2(0.0116) \\ &= \boxed{0.0232}\ \text{(p value)} \end{aligned}$$

b.

$$\begin{aligned}
\text{Probability of type II error } &= \Phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{\frac{\sigma}{\sqrt{n}}}\right) - \Phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{\frac{\sigma}{\sqrt{n}}}\right) \\
&= \Phi\left(2.57 + \frac{28 - 27}{\frac{1.2}{\sqrt{16}}}\right) - \Phi\left(-2.57 + \frac{28 - 27}{\frac{1.2}{\sqrt{16}}}\right) \\
&= \Phi(5.90) - \Phi(0.763) \\
&= 1 - 0.7764 \\
&= \boxed{0.2236}
\end{aligned}$$

c. $\beta(27) = 0.1,\ \alpha = 0.01$

$$\begin{aligned}
n &= \left(\frac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu'}\right)^2 \\
&= \left(\frac{1.2(2.57 + 1.28)}{28 - 27}\right)^2 \\
&= \boxed{22}
\end{aligned}$$

3a. $\mathrm{H}_0 :\ \mu \geq 240$, $\mathrm{H}_1 :\ \mu < 240$

$$\begin{aligned}
z &= \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} \\
&= \frac{230 - 240}{20/\sqrt{16}} \\
z &= -2
\end{aligned}$$

$z = -2,\ z_{0.05} = -1.64\ z \leq -z_{0.05}? \rightarrow \boxed{-2 \leq 1.64 \checkmark}$(We do not reject the hypothesis)

$$\begin{aligned}
p\text{-value } &= P(Z \leq z) \\
&= P(Z \leq -2) \\
&= \Phi(-2) \\
&= \boxed{0.0228}
\end{aligned}$$

b. $\overline{x} = 230,\ \sigma = 20,\ n = 16$

$$\left(\overline{x} - 1.96\left(\frac{\sigma}{\sqrt{n}}\right),\ \overline{x} + 1.96\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$
$$(230 - 1.96 \cdot 5,\ 230 + 1.96 \cdot 5)$$
$$\boxed{(220.2, 239.8)}$$