1.$U \sim \mathbb{U}([0,1])$

$$\mathbb{P}(U \leq t) = F_U(t) = \begin{cases} 0 & \text{si } t < 0 \\ \frac{t-0}{1-0} = t & \text{si } t \in [0,1] \\ 1 & \text{si } t > 1 \end{cases}$$

$\mathbb{P}(G(U) < t) = \mathbb{P}(\frac{-1}{\lambda}\ln(1-U) < t) = \mathbb{P}(\ln(1-U) > -\lambda t) = \mathbb{P}(1-U > e^{-\lambda t}) = \mathbb{P}(U < 1-e^{-\lambda t}) = F_U(1-e^{-\lambda t}) = \begin{cases} 0 & \text{si } 1-e^{-\lambda t} < 0 \\ 1-e^{-\lambda t} & \text{si } 1-e^{-\lambda t} \in [0,1] \\ 1 & \text{si } 1-e^{-\lambda t} > 1 \end{cases}$

$$\begin{aligned} 1-e^{-\lambda t} < 0 &\iff e^{-\lambda t} > 1 \\ &\iff \underbrace{-\lambda}_{<0} t > 0 \\ &\iff t < 0 \end{aligned}$$

$$\begin{aligned} 1-e^{-\lambda t} \in [0,1] &\iff 0 \leq 1-e^{-\lambda t} \leq 1 \\ &\iff e^{-\lambda t} \leq 1 \\ &\iff \underbrace{-\lambda}_{<0} t \leq 0 \\ &\iff t \geq 0 \end{aligned}$$

$1-e^{-\lambda t} > 1 \iff e^{-\lambda t} < 0$ aucun réel t ne satisfait l'inéquation

D'où $\mathbb{P}(G(U) < t) = \begin{cases} 0 & \text{si } t < 0 \\ 1-e^{-\lambda t} & \text{si } t \geq 0 \end{cases}$

Ainsi, G(U) suit bien une loi exponentielle de paramètre λ.

2. $\mathbb{P}(G(1-U) < t) = \mathbb{P}(\frac{-1}{\lambda}\ln(U) < t) = \mathbb{P}(\ln(U) > -\lambda t) = \mathbb{P}(U > e^{-\lambda t}) = 1-\mathbb{P}(U < e^{-\lambda t}) = 1-F_U(e^{-\lambda t})$

$$F_U(e^{-\lambda t}) = \begin{cases} 0 & \text{si } e^{-\lambda t} < 0 \text{ Aucun t ne satisfait l'inéquation} \\ e^{-\lambda t} & \text{si } e^{-\lambda t} \in [0,1] \\ 1 & \text{si } e^{-\lambda t} > 1 \end{cases}$$

$$-F_U(e^{-\lambda t}) = \begin{cases} -e^{-\lambda t} & \text{si } -e^{-\lambda t} \in [-1,0] \\ -1 & \text{si } -e^{-\lambda t} < -1 \end{cases}$$

$$\mathbb{P}(G(1-U) < t) = 1-F_U(e^{-\lambda t}) = \begin{cases} 1-e^{-\lambda t} & \text{si } 1-e^{-\lambda t} \in [0,1] \iff t \geq 0 \\ 0 & \text{si } 1-e^{-\lambda t} < 0 \iff t < 0 \end{cases} = F_U(1-e^{-\lambda t})$$

On retombe bien sur la même loi exponentielle que G(U). Si $U \sim \mathbb{U}([0,1])$ on aura G(1-U)~G(U)~$exp(\lambda)$