Appendix B

Georeferencing

B.1 Introduction

Image data can be obtained from various sources, like satellite, aerial cameras, and maps. However, the spatial reference information in such image data is not typically available. So, we need to convert image data into real-world data means we need to georeference this image data to a map coordinate system. As we know that a map coordinate system is a method by which the earth's curved surface is portrayed on a flat surface. When our image data is georeferenced, we describe its position using map coordinates and assign the map frame's coordinate system. In this, we are going to learn how to georeference an image data using ground control points. The focal length of a nearly vertical photograph is given. Photo coordinate in image coordinate system and ground coordinate is given for five points. We are interested in knowing the extrinsic parameter using the collinearity condition.

B.2 Methodology:

B.2.1 Problem Statement

A near-vertical aerial photograph taken with a 151.876-mm-focal-length camera contains images of five ground control points A through E. Refined photo coordinates and ground control coordinates in a local vertical system of the five points are listed in the following

Point	x(mm)	y(mm)	X(m)	Y(m)	Z(m)
A	-53.845	65.230	6934.954	23961.10	160.136
B	104.50	68.324	7860.202	23941.56	152.653
C	4.701	-12.153	7261.078	23491.497	142.208
D	-61.372	-79.559	6836.650	23087.47	137.719
E	93.825	-62.060	7791.556	23166.680	138.827

TABLE B.1: Table showing image coordinates (x, y) and ground control points coordinates (X, Y, Z)

table B.1. Calculate the exterior orientation parameters for this photograph using space resection.

B.2.2 Solution-

The translation parameter (X_0, Y_0, Z_0), orientation parameter (ω, ϕ, κ) and principal point coordinates (x_0, y_0) are called extrinc paramters. Principal of collinerity equation is use to find extrinsic parameter. It says principal point, image point and ground point lies in a single line. Here X,Y,Z are coordinate of ground point in local coordinate system and x,y are coordinate at image in image coordinate system.

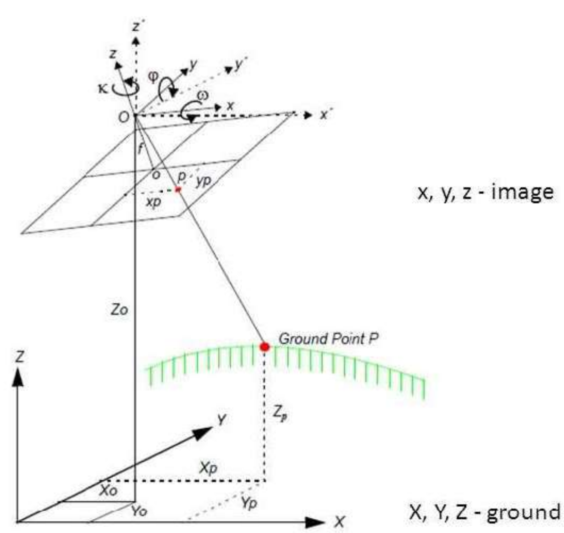

FIGURE B.1: Collinearity condition.

From figure B.1, using collinearity condition, we can say that,

$$S.\vec{op} = \vec{OP} \tag{B.1}$$

$$S.\begin{bmatrix} x_p \\ y_p \\ -f \end{bmatrix} = R.\begin{bmatrix} X - X_0 \\ Y - Y_0 \\ Z - Z_0 \end{bmatrix}$$

Here, value of R is:

$$\begin{aligned} R &= \begin{bmatrix} R_x(\omega) & R_y(\phi) & R_z(\kappa) \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\omega) & sin(\omega) \\ 0 & -sin(\omega) & cos(\omega) \end{bmatrix} \begin{bmatrix} cos(\phi) & 0 & -sin(\phi) \\ 0 & 1 & 0 \\ sin(\phi) & 0 & cos(\phi) \end{bmatrix} \begin{bmatrix} cos(\kappa) & sin(\kappa) & 0 \\ -sin(\kappa) & cos(\kappa) & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{B.2}$$

$$R = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}$$

Using element of rotation matrix and collinearity equation can be written as-

$$S.x_p = m_{11}(X - X_0) + m_{12}(Y - Y_0) + m_{13}(Z - Z_0) \tag{B.3}$$

$$S.y_p = m_{21}(X - X_0) + m_{22}(Y - Y_0) + m_{23}(Z - Z_0) \tag{B.4}$$

$$-S.f = m_{31}(X - X_0) + m_{32}(Y - Y_0) + m_{33}(Z - Z_0) \tag{B.5}$$

Using equation B.3 and B.5

$$x_p = -f\frac{m_{11}(X - X_0) + m_{12}(Y - Y_0) + m_{13}(Z - Z_0)}{m_{31}(X - X_0) + m_{32}(Y - Y_0) + m_{33}(Z - Z_0)} \tag{B.6}$$

Using equation B.4 and B.5

$$y_p = -f\frac{m_{21}(X - X_0) + m_{22}(Y - Y_0) + m_{23}(Z - Z_0)}{m_{31}(X - X_0) + m_{32}(Y - Y_0) + m_{33}(Z - Z_0)} \tag{B.7}$$

If origin and principal point and origin are not same-

$$x_p - x_0 = -f\frac{m_{11}(X - X_0) + m_{12}(Y - Y_0) + m_{13}(Z - Z_0)}{m_{31}(X - X_0) + m_{32}(Y - Y_0) + m_{33}(Z - Z_0)} \tag{B.8}$$

$$y_p - y_0 = -f\frac{m_{21}(X - X_0) + m_{22}(Y - Y_0) + m_{23}(Z - Z_0)}{m_{31}(X - X_0) + m_{32}(Y - Y_0) + m_{33}(Z - Z_0)} \tag{B.9}$$

Solution of this matrix is given by least square method and solution is given below:

For observation equation, we have:

$$V = AX - L \tag{B.10}$$

Type of this equation is-

$$L_0 = f(X_0) \tag{B.11}$$

Here, value of L is given by-

$$L = L_b - L_0 \tag{B.12}$$

Solution of the observation equation is given as -

$$X = (A^T PA)^{-1}(A^T PL) \tag{B.13}$$

$$Here(A^T PA) = N \tag{B.14}$$

To find co-variance matrix for given parameters, following equations are used-

$$\Sigma_X = N^{-1} \tag{B.15}$$

$$\Sigma V = P^{-1} - AN^{-1}A^T \tag{B.16}$$

$$\Sigma_{L_a} = AN^{-1}A^T \tag{B.17}$$

$$\Sigma_{L_b} = \Sigma_V + \Sigma_{L_a} = P^{-1} \tag{B.18}$$

$$\hat{\sigma_o^2} = \frac{V^T PV}{n - u} \tag{B.19}$$

B.2.2.1 Proof Covariance Matrix

The detailed proof the the above results is given below. Let's we have simple eqation in general given below. We are interested in knowing the covarainace of Y (i.e. Σ_Y) when covariance of X (i.e. Σ_X) is known.

$$
\begin{aligned}
&Y = AX + B \\
&E(Y) = E(AX + B) = AE(X) + B \\
&\Sigma_X = E[X - E(X)(X - E(X))^T] \\
&\Sigma_Y = E((Y - E(Y))(Y - E(Y))^T) \\
&\quad = E(((AX + B) - (AE(X) + B))((AX + B) - (AE(X) + B))^T) \\
&\quad = E(A(x - E(X))(A(X - E(X)))^T) \\
&\quad = AE((X - E(X))(X - E(X))^T)A^T \\
&\quad = A\Sigma_X A^T \qquad \text{where,} \quad \Sigma_X = E((X - E(X))(X - E(X))^T)
\end{aligned}
$$

We can use the above result to get the equation B.13.

Now, in order to obtain the equation B.15, again we use the above result. It is given that, $\Sigma_L = P^{-1}$. So we have,

$$
\begin{aligned}
&\Sigma_X = (A^TPA)^{-1}A^TP\Sigma_L((A^TPA)^{-1}A^TP)^T \\
&\quad = (A^TPA)^{-1}A^TPP^{-1}((A^TPA)^{-1}A^TP)^T && using \quad \Sigma_L = P^{-1} \\
&\quad = N^{-1}A^TPP^{-1}(N^{-1}A^TP)^T && let \quad (A^TPA) = N \\
&\quad = N^{-1}A^T(N^{-1}A^TP)^T \\
&\quad = N^{-1}A^T(A^TP)^T(N^{-1})^T && using \quad (N^{-1})^T = N^{-1} \\
&\quad = N^{-1}(A^TPA)^T(N^{-1}) && using \quad (N^{-1})^T = N^{-1} \\
&\quad = N^{-1}(N)^T(N^{-1}) && A^TPA = N \\
&\quad = N^{-1}N(N^{-1}) && N^T = N \\
&\quad = N^{-1} \\
&\quad = (A^TPA)^{-1}
\end{aligned}
$$

B.2.2.2 Result

The task in this problem is to find extrinsic parameter using some a priori information about the camera say focal length and coordinate of some points on the image and ground.

In the given image B.1, principal point and center of image does not coincide. We have successfully implemented this space resection in python OpenCV. We have get satisfactory results here as follows:

- Focal length = 151.876mm
- Translation parameter=

  X_0=7247.6841m

  Y_0=23595.371m

  Z_0=1058.0708m
- Orientation parameter=

  ω=0.0307675

  ϕ=-0.01283911

  κ=0.03845134
- Principal point coordinate=

  x_0= -0.13390760mm

  y_0= 0.26860202mm