Drying-gas flow →

(a) Constant-rate period

(b) First falling-rate period

(c) Second falling-rate period

Figure 18.29 Drying stages for granular solids.

linearly with decreasing average moisture content. This is the *first falling-rate drying period*. It is not always observed with wet solids of the second category.

During the period from C to D, the liquid in the pores of wet solids of the first category begins to recede from the exposed surface. In the final period from D to E, as shown in Figure 18.29c, evaporation occurs from liquid surfaces in the pores, where the wet-bulb temperature prevails. However, the temperature of the exposed surface of the solid rises to approach the dry-bulb temperature of the gas. During this period, the *second falling-rate drying period*, the rate of drying may be controlled by vapor diffusion for wet solids of the first category and by liquid diffusion for wet solids of the second category. The rate falls exponentially with decreasing moisture content.

§18.4.1 Constant-Rate Drying Period

In direct-heat equipment, drying involves transfer of heat from the gas to the surface and interior of the wet solid, and mass transfer of moisture from the interior and the solid to the gas. During the constant-rate rate of mass transfer is determined by gas-pha ary-layer or film resistance at the wet surface o The wet solid is assumed to be at a uniform te so the only resistance to convective heat transfe gas phase. The rate of moisture evaporation c based on convective heat transfer or mass according to conventional, but simplified, tran tionships in which thermal radiation, the bulk- for mass transfer, and the sensible-heat effe evaporated moisture are often ignored:

$$\frac{dm_v}{dt} = \frac{h(T_g - T_i)A}{\Delta H_i^{vap}} = M_A k_y (y_i - y_g)A$$

where subscript i refers to the gas–solid interface

As discussed previously, the interface at these is at the wet-bulb temperature, T_w. Although dryi culations could be based on mass transfer using (more common to use the heat-transfer relation when air is the gas and water is the moisture bec wide availability of the psychrometric chart for tl the equality of wet-bulb and adiabatic-saturatio tures, and a wider availability of correlations for heat transfer than for mass transfer, although an be used to derive one from the other. Combining ((18-33), the drying-rate flux for constant-rate dry R_c, in terms of heat transfer, becomes

$$R_c = \frac{h(T_g - T_w)}{\Delta H_w^{vap}}$$

while an equivalent, but less-useful, mass-transf obtained by combining (18-16), (18-32), and (18-3

$$R_c = M_B k_y (\mathcal{H}_w - \mathcal{H}_d)$$

where subscripts d and w refer to gas dry-bulb an conditions.

For some dryers, it is preferable to use a volun transfer coefficient, (ha), defined by

$$\frac{dm_v}{dt} = \frac{(ha)(T_g - T_i)V}{\Delta H_w^{vap}}$$

where a = external surface area of wet solids per u of dryer, and V = volume of dryer. Then, the dryi unit dryer volume during the constant-rate drying $\;$

$$(R_c)_V = \frac{(ha)(T_d - T_w)}{\Delta H_w^{vap}}$$

Interphase heat-transfer coefficients were dis several geometries in §3.5. Empirical equations drying-rate calculations are summarized in Muj and representative equations, when the gas is air, a Table 18.6, where in (1), G is the mass velocity o flow channel that passes over the wet surface. In (mass velocity of the air impinging on the wet surf to (8), d_p is the particle diameter and G is the super velocity.

trihydrate forms; at 0.8–5.6 torr, the monohydrate is favored. Below 0.8 torr, $CuSO_4$ crystals are free of water.

EXAMPLE 18.6 Effect of Equilibrium-Moisture Content.

One-kg blocks of wet Borax laundry soap with an initial H_2O content of 20.2 wt% on a dry basis are dried with air in a tunnel dryer at 1 atm. In the limit, if the soap were brought to equilibrium with the air at 25°C and a relative humidity of 20%, determine the kg of moisture evaporated from each block.

Solution

The initial moisture content of the soap on a wet basis is obtained from a rearrangement of (18-30):

$$W = \frac{100X}{100 + X} = \frac{100(20.2)}{100 + 20.2} = 16.8 \text{ wt\%}$$

Initial weight of moisture $= 0.168(1.0) = 0.168$ kg H_2O

Initial weight of dry soap $= 1 - 0.168 = 0.832$ kg dry soap

From Figure 18.24, for soap at $\mathcal{H}_R = 0.20$, $X^* = 0.037$

Final weight of moisture $= 0.037(0.832) = 0.031$ kg

Moisture evaporated $= 0.168 - 0.031 = 0.137$ kg H_2O/kg soap

§18.4 DRYING PERIODS

The decrease in average moisture content, X, as a function of time, t, for drying either category of solids in a direct-heat dryer was observed experimentally by Sherwood [11,12] to exhibit the type of relationship shown in Figure 18.28a, provided the exposed surface of the solid is initially covered with observable moisture. If that curve is differentiated with respect to time and multiplied by the ratio of the mass of dry solid to the interfacial area between the mass of wet solid and the gas, a plot can be made of drying-rate flux, R,

$$R = \frac{dm_v}{A\,dt} = -\frac{m_s}{A}\frac{dX}{dt} \qquad (18\text{-}32)$$

where $m_v =$ mass of moisture evaporated and $m_s =$ mass of bone-dry solid as a function of moisture content, as shown in Figure 18.28b. In Figure 18.28a, the final equilibrium-moisture content is X^*. Although both plots exhibit four drying periods, the periods are more distinct in the drying-rate curve. For some wet materials and/or some hot-gas conditions, fewer than four drying periods are observed. From A to B, the wet solid is being preheated to an exposed-surface temperature equal to the wet-bulb gas temperature, while moisture is evaporated at an increasing rate. At the end of the preheat period, if the wet solid is of the granular, first category, a cross section has the appearance of Figure 18.29a, where the exposed surface is still covered by a film of moisture. A wet solid of the second category is covered on the exposed surface by free moisture. The drying rate now
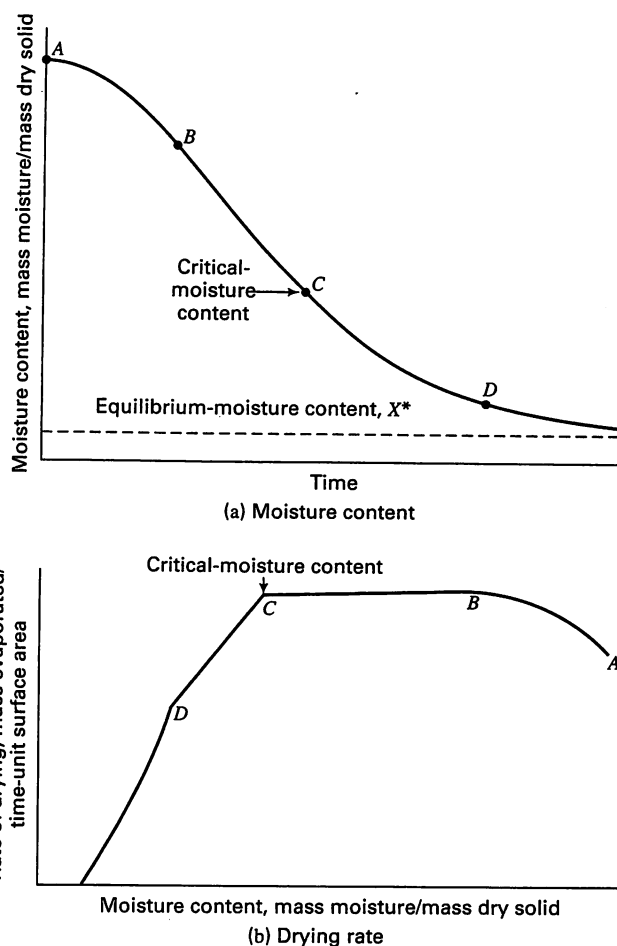

Figure 18.28 Drying curves for constant drying conditions.

becomes constant during the period from B to C, which prevails as long as free moisture covers the exposed surface.

This surface moisture may be part of the original moisture that covered the surface, or it may be moisture brought to the surface by *capillary action* in the case of wet solids of the first category or by *liquid diffusion* in the case of wet solids of the second category. In either case, the rate of drying is controlled by external mass and heat transfer between the exposed surface of the wet solid and the bulk gas. Migration of moisture from the interior of the wet solid to the exposed surface is not a rate-affecting factor. This period, the *constant-rate drying period*, terminates at point C, the *critical moisture content*. When drying wet solids of the first category under agitated conditions—as in a direct-heat rotary dryer, fluidized-bed dryer, flash dryer, or agitated batch dryer—such that all particle surfaces are in direct contact with the gas, the constant-rate drying period may extend all the way to X^*.

At C, the moisture just barely covers the exposed surface; and then until point D is reached, as shown in Figure 18.29b, the surface tends to a dry state because the rate of liquid travel by diffusion or capillary action to the exposed surface is not sufficiently fast. In this period, the exposed-surface temperature remains at the wet-bulb temperature if heat conduction is adequate, but the wetted exposed area for mass transfer decreases. Consequently, the rate of drying decreases

plant, vegetable, and animal materials such as wood, leather, soap, eggs, glues, cereals, starch, cotton, and wool. These materials are affected by moisture removal, often shrinking when dried and swelling when wetted. With these materials, drying in the later stages can be slow. If the surface is dried too rapidly, moisture and temperature gradients can cause checking, warping, case hardening, and/or cracking. Therefore, selection of drying conditions is a critical factor. Drying to low moisture contents is possible only when using a gas of low humidity.

In a direct-heat drying process, the extent to which moisture can be removed from a solid is limited, particularly for the second category, by the *equilibrium-moisture content* of the solid, which depends on factors that include temperature, pressure, and moisture content of the gas. Even if the drying conditions produce a completely dry solid, subsequent exposure of the solid to a different humidity can result in an increase in moisture content.

Terms used to describe equilibrium-moisture content are shown in Figure 18.23 with reference to a hypothetical equilibrium isotherm. Moisture content, X, is expressed as mass of moisture per 100 mass units of bone-dry solid. This is the most common way to express moisture content and is equivalent to wt% moisture on a dry-solid basis. This is analogous

to expressions for humidity and is most convenient in dryi calculations where the mass of bone-dry solid and dry g remain constant while moisture is transferred from solid gas. Less common is wt% moisture on a wet-solid basis, The two moisture contents are related by the expression

$$X = \frac{100W}{100 - W} \qquad (18\text{-}3$$

or

$$W = \frac{100X}{100 + X} \qquad (18\text{-}3$$

Rarely used is moisture content on a volume basis becau wet solids of the second category shrink during drying. Als moisture content is never expressed on a mole basis becau the molecular weight of the dry solid may not be known.

In Figure 18.23, equilibrium-moisture content, X^*, is plo ted for a second-category solid for a given temperature ar pressure, against relative humidity, $\mathcal{H}_R$. In some cases, humi ity, $\mathcal{H}$, is used with a limit of the saturation humidity, $\mathcal{H}_s$. $\mathcal{H}_R = 100\%$, equilibrium-moisture content is called *bour moisture*, X_B. If the wet solid has a total moisture conten $X_T > X_B$, the excess, $X_T - X_B$, is *unbound moisture*. At a rel: tive humidity $< 100\%$, the excess of X_T over the equilibriun moisture content, i.e., $X_T - X^*$, is the *free-moisture* content.

In the presence of a saturated gas, only unbound moistu can be removed during drying. For a partially saturated ga only free moisture can be removed. But if $\mathcal{H}_R = 0$, all solid given enough time, may be dried to a bone-dry state. Soli materials that can contain bound moisture are *hygroscopi* Bound moisture exhibits a vapor pressure less than the nor mal vapor pressure. The bound-moisture content of cellula materials such as wood is referred to as the fiber-saturatio point.

Experimental equilibrium-moisture isotherms at 25°C an 1 atm are shown in Figure 18.24 for second-category materi als. At low values of $\mathcal{H}_R$, e.g., $<10\%$, moisture is bound to th solid on its surfaces as an adsorbed monomolecular laye Such bound moisture can also be present on solids of the fir: category. At intermediate values of $\mathcal{H}_R$, e.g., 20–60%, multi molecular layers may build up on the monolayer. At larg values of $\mathcal{H}_R$, e.g., $>60\%$, moisture is held in micropores s small (e.g., <1 μm in radius) that vapor-pressure lowerin; occurs, as predicted by the Kelvin equation (15-14). In cellu lar materials such as plant and tree matter, some moisture i held osmotically in fibers behind semipermeable membrane of cell walls.

Temperature has a significant effect on equilibrium moisture content, an example of which is shown in Figur 18.25 for cotton at 96–302°F. At an $\mathcal{H}_R$ of 20%, equilibrium moisture content decreases from 0.037 to 0.012 lb H_2O/lb dr cotton. Experimental determination of equilibrium-moistur isotherms is complicated by a hysteresis effect, shown ii Figure 18.26 for sulfite pulp. Sorption and desorption curve: were obtained, respectively, by wetting and drying the solid and it is seen that equilibrium-moisture content in drying i: always somewhat higher, particularly in the relative-humidit
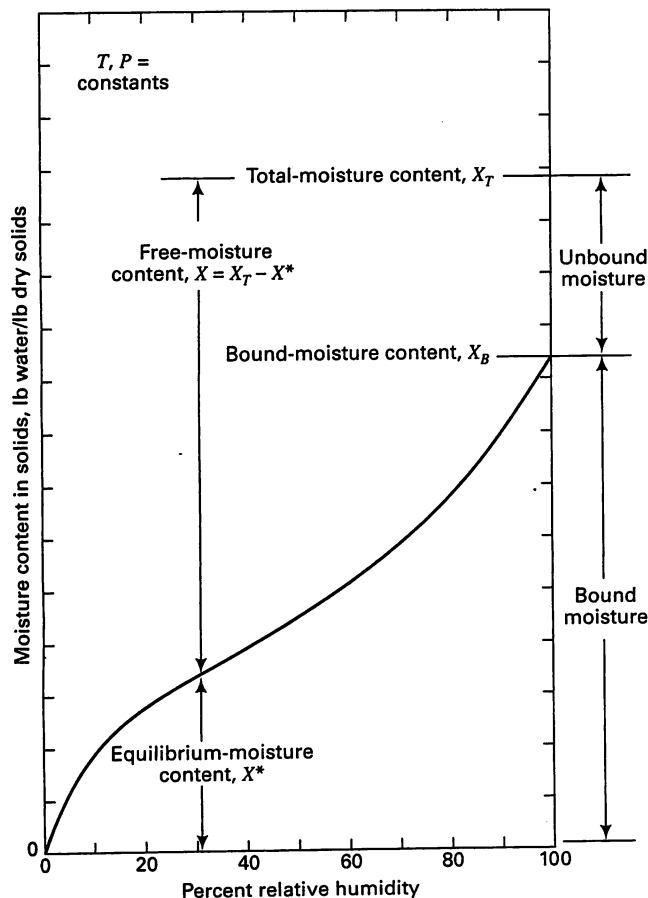

Figure 18.23 Typical isotherm for equilibrium-moisture content of a solid.

50°F = 45.6 torr, (b) humidity of the nitrogen–benzene mixture, (c) saturation humidity of the mixture, and (d) percentage humidity of the mixture.

Solution

$$\mathcal{H}_R = 35\%, \ P = 1.2 \text{ atm} = 912 \text{ torr},$$

$$M_A = 78.1, \ M_B = 28, \ P^s_{\text{benzene}} = 45.6 \text{ torr}$$

(a) From (18-8),

$$p_{\text{benzene}} = \frac{P^s_{\text{benzene}} \mathcal{H}_R}{100} = \frac{(45.6)(35)}{100} = 16 \text{ torr}$$

(b) From (18-5),

$$\mathcal{H} = \left(\frac{78.1}{28}\right)\left(\frac{16}{912 - 16}\right) = 0.050 \frac{\text{lb benzene}}{\text{lb dry nitrogen}}$$

(c) From (18-7),

$$\mathcal{H}_s = \left(\frac{78.1}{28}\right)\left(\frac{45.6}{912 - 45.6}\right) = 0.147 \frac{\text{lb benzene}}{\text{lb dry nitrogen}}$$

(d) From (99), $\quad \mathcal{H}_P = 100\left(\dfrac{0.050}{0.147}\right) = 34\%$

§18.2.2 Wet-Bulb Temperature

The temperature at which moisture evaporates in a direct-heat dryer is difficult to determine and varies from the dryer inlet to the dryer outlet. When the dryer operates isobarically and adiabatically, with all energy for moisture evaporation supplied from the hot gas by convective heat transfer, with no energy required for heating the wet solid to the evaporation temperature, use of simplified heat- and mass-transfer equations leads to an expression for the temperature of evaporation at a particular location in a dryer operating under continuous, steady-state conditions, or at a particular time in a batch dryer cycle.

If it is further assumed that the moisture being evaporated is free liquid exerting its full vapor pressure at the surface of the solid, this temperature of evaporation is called the *wet bulb temperature*, T_w, because it can be measured by cover ing a thermometer bulb with a wick saturated with the liqu being evaporated and passing a partially saturated gas pa the wick, as indicated in Figure 18.18a.

In Figure 18.18b, where the wetted wick is replaced by incremental amount of wet solid, assume the heat-transf area = mass-transfer area = A. At steady state, the rate convective heat transfer from the gas to the wet solid is giv by Newton's law of cooling:

$$Q = h(T - T_w)A \qquad (18\text{-}1$$

The molar rate of mass transfer of evaporated moisture fro the wet surface of the solid, A, is

$$n_A = \frac{k_y(y_{A_w} - y_A)A}{(1 - y_A)_{LM}} \qquad (18\text{-}1$$

An enthalpy balance on the moisture evaporated and heat to the gas temperature couples the heat- and mass-transs equations to give

$$Q = n_A M_A \left[\Delta H_w^{\text{vap}} + (C_P)_A(T - T_w)\right] \qquad (18\text{-}1$$

To obtain a simplified relationship for the coupling in terms T and $\mathcal{H}$, assume that the mole fraction of moisture in the b gas and at the wet solid–gas interface is small. Then, the b flow effect in (18-14) becomes $(1 - y_A)_{LM} \approx 1.0$. Also, fr (18-5), replacing p_A with $y_A P$,
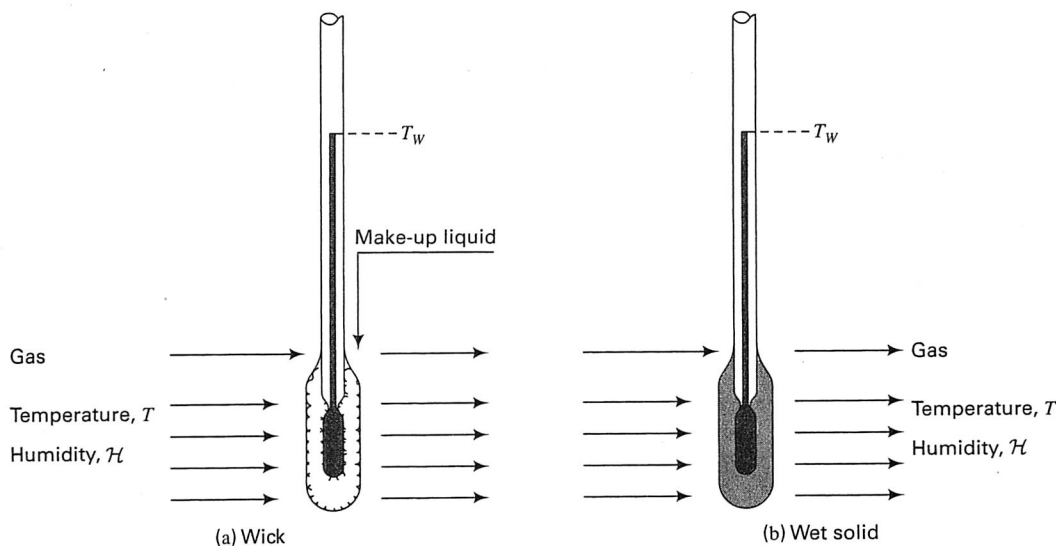
$$y_A = \frac{\mathcal{H} M_A}{\dfrac{1}{M_B} + \dfrac{\mathcal{H}}{M_A}} \approx \frac{\mathcal{H} M_B}{M_A} \qquad (18\text{-}$$

If the latent heat in (18-15) is much greater than the ser ble heat,

$$\Delta H_w^{\text{vap}} + (C_P)_A(T - T_w) \approx \Delta H_w^{\text{vap}} \qquad (18\text{-}$$

Simplifying, and combining (18-13) to (18-16),

$$T_w = T - \frac{k_y M_B \Delta H_w^{\text{vap}}}{h}(\mathcal{H}_w - \mathcal{H}) \qquad (18\text{-}$$

Figure 18.18 Wet-bulb temperature.

[From W.L. McCabe, J.C. Sn and P. Harriott, *Unit Operati of Chemical Engineering*, 5t McGraw-Hill, New York (19 with permission.]