What do we Know about Standard Deviations?

- The variance (s^2) is the average squared distance of observations from the average.
- The standard deviation (s) is the square root of the variance.
- The standard deviation is a measure of the **spread** of the data, independent of the average.
- If all observations are close together, then the standard deviation will be small.

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Variance

$$s = \sqrt{s^2}$$

Standard Deviation

Using Minitab to Calculate t-critical

t-critical is usually looked up in a table. On the output from the two sample t command, the t statistic given is the calculated value, not the critical (table) value.

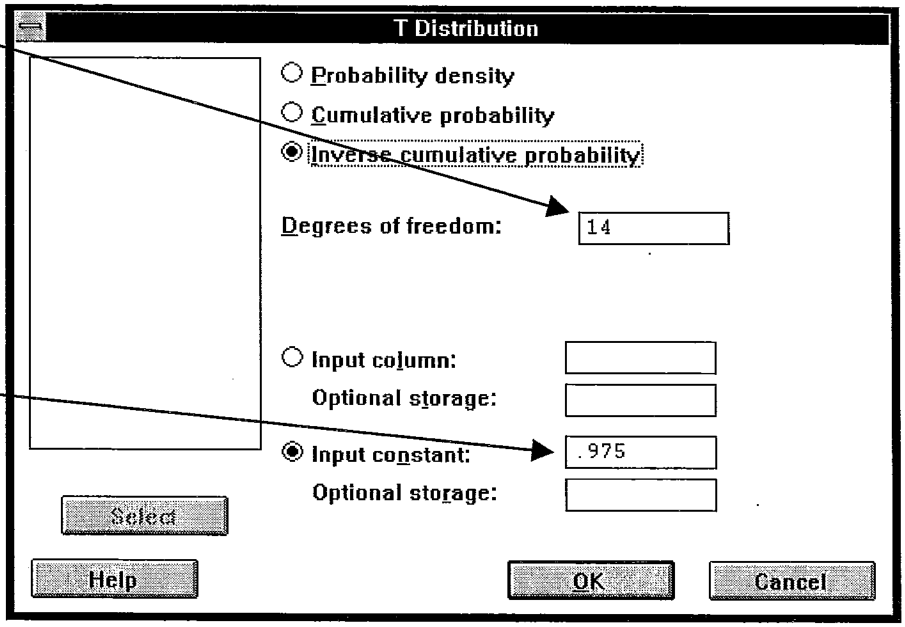
SO........How do you get t-critical in MINITAB?

Use the "inverse probability function"

Calc>Probability Distribution>T

(#groups)* (#of obs in each group -1)
= (2) * (8-1)
(IF: equal variance & balanced design)

Area under curve, 1 - $\alpha/2$ (gives a positive number)



Inverse Cumulative Distribution Function
Student's t distribution with 14 d.f.

P(X <= x)	x
0.9750	2.1448

The same value as the t value on page 18.

Class Exercise: Using Minitab to Analyze the Helicopter Flight Time

Use the data gathered on the previous page to determine whether wing width affects flight time.

1. Write H_o and H_a. H_o:
   H_a:
2. Place response data in one column, treatments ("X" levels) in a second column.
3. Use descriptive statistics with graphs to compare the two treatments.
4. Run Stat>Basic Stat>2-Sample t on your data.

WHAT ARE YOUR FINDINGS??

Using Minitab to Mathematically Compare Two Treatments (cont'd)

Session Window Output:

Two Sample T-Test and Confidence Interval

```
Two sample T for Reflect
Time          N       Mean      StDev   SE Mean
20            8      18.56       3.57       1.3
10            8      15.85       3.54       1.3

95% CI for mu (20) - mu (10): ( -1.1,  6.5)
T-Test mu (20) = mu (10) (vs not =): T= 1.53  P=0.15  DF=  14
Both use Pooled StDev = 3.56
```

'0' falls within the interval; therefore, we cannot say there is a statistical difference.

T_{calc} 1.53 < T_{table} (2.145) therefore no statistical difference can be claimed.

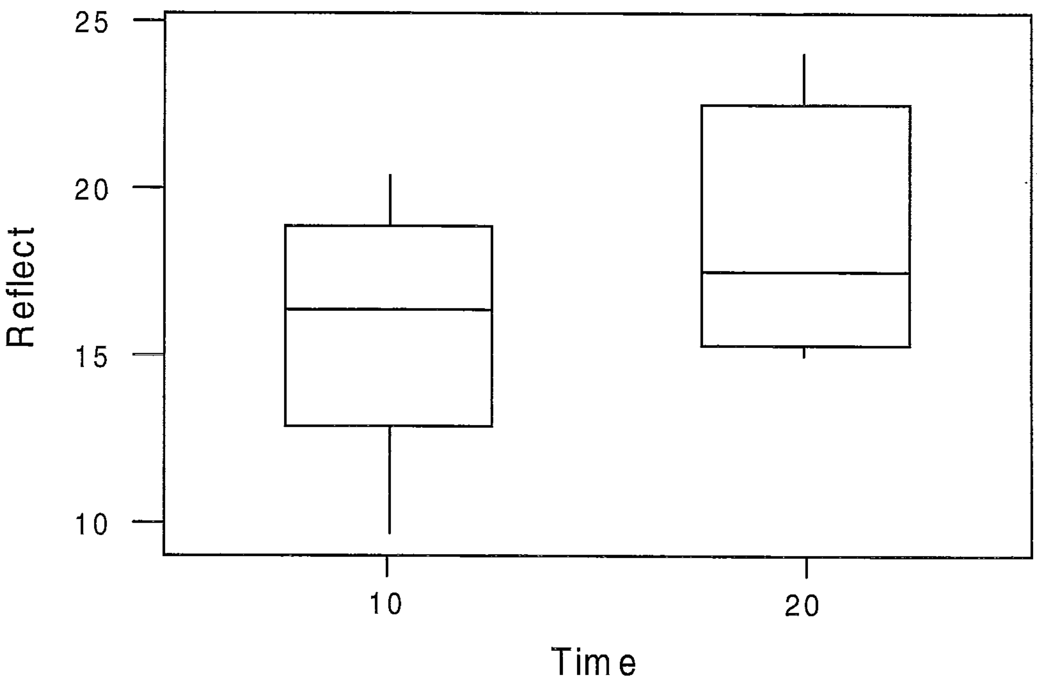
p>0.05; another way to determine no statistical difference can be claimed.

H_o: the means at 20 and 10 minute wash time are the same.
H_a: the means are different

NOTE: The confidence interval is a **Range of Plausible Values** for the difference between the averages.

If you want to reduce the width of the interval, reduce the variation or increase the sample size.

Graphically Compare Two Treatments (cont'd)



- There appears to be a possible difference in reflectance between the 10 minute and 20 minute wash! Is this difference due to chance, or does a 20 minute wash significantly brighten clothes?

- Remember that we can subtract the averages and get a "point estimate" of the difference, but we would also like an idea of what variation to expect for this point estimate (a "confidence interval").

Using Minitab to Graphically Compare Two Treatments

File > Open >

L:\minitab\training\minitab\Session 3\reflect.mtw

C1 Twenty	C2 Ten	C3 Reflect	C4 Time
17.4	20.4	17.4	20
17.7	19.3	17.7	20
23.2	17.6	23.2	20
20.4	16.3	20.4	20
15.0	9.7	15.0	20
24.0	16.4	24.0	20
15.6	14.8	15.6	20
15.2	12.3	15.2	20
		20.4	10
		19.3	10
		17.6	10
		16.3	10
		9.7	10
		16.4	10
		14.8	10
		12.3	10

Reflectance for 20 minute wash.

Reflectance for 10 minute wash.

Stacked data.

Reference data.

The data in columns C3 and C4 will be used to generate all of the graphs and numerical output.

The Effect of Sample Size on the Distribution of an Average (cont'd)

Daily average	1 per day (30 samples)	Avg of 4 per day (120 total samples)
24	X	
23	XX	
22	XX	
21	XXX	XX
20	XX	XXXXXX
19	XXXXXX	XXXXX
18	XXXXX	XXXXXXXX
17	X	XXXXXX
16	XXXX	XXX
15	XX	
14		
13	XX	
average:	**18.6**	**18.4**
standard deviation:	**2.84**	**1.45**

The standard deviation for the average of the 4 samples per day is calculated using:

The Standard Error of the Mean: $$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}}$$

The standard deviation (variation) is smaller when an average of 4 measurements per day is used. In fact, it is smaller by about a factor of $\sqrt{4} = 2$.

Averages are less variable than individuals. Lower variability provides a greater ability to detect differences.

Interpretation of the Confidence Interval

1. ($\bar{X}_1$ - $\bar{X}_2$) is the point estimate, or the most likely estimate, of the true difference between the population averages.

2. But all observations are subject to error, so the true difference between population averages may be a bit above or below this value. A confidence interval gives a **range of plausible values** for the true difference between population averages.

3. 95% of the time, the true difference in population averages will fall within the range described by the Confidence Interval.

A confidence interval gives a range of plausible values

Confidence Intervals

A 95 % Confidence Interval for the Average Difference:

Test the Hypothesis

H_o: sameness (0 is in the interval)

H_a: difference (0 is NOT in the interval) using:

$$(\bar{x}_1 - \bar{x}_2) +/- t_{\left(n_1+n_2-2,\frac{\alpha}{2}\right)} * s_p * \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

where:

- $t_{(n_1+n_2-2,.025)}$ is a value from the t-Table with $n_1 + n_2 - 2$ degrees of freedom, and .025 in each tail (95% in the middle).
- n_1 and n_2 are the sample sizes for the two samples.
- s_p is the pooled standard deviation - the square root of the weighted average of the two individual variances.

$$s_p = \sqrt{\frac{\left\{(n_1 - 1) * s_1^2 + (n_2 - 1) * s_2^2\right\}}{(n_1 + n_2 - 2)}}$$

NOTE: The interval will be narrower if n_1 and n_2 are larger.

If the confidence interval range includes "0", we cannot conclude that the two means are statistically different.

Example: Horizontal Axis Washer

<u>Objective</u>: Estimate the effect of wash time on the ability to remove soot.

- The dependent variable (Y) is the change in "reflectance" - a measure of the brightness of the clothes.
- The independent variable (X) is the wash time.
- A "treatment" is a <u>level</u> of an X variable. The treatments in this example are 20-minute and 10-minute wash times.

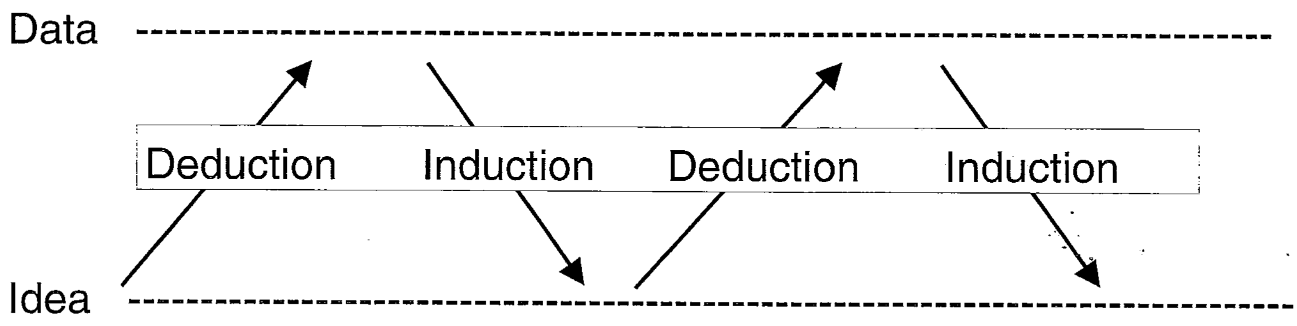
The change in reflectance for 16 wash loads is recorded below:

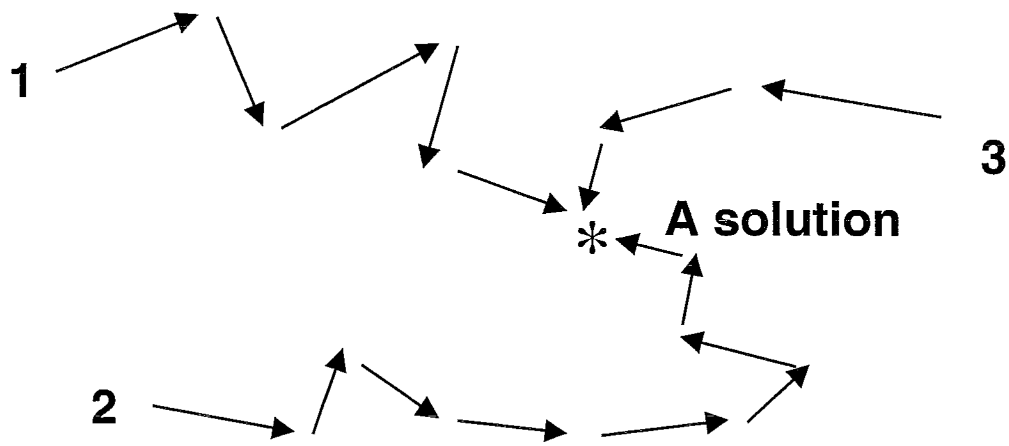
20-minute wash: 17.4, 17.7, 23.2, 20.4, 15.0, 24.0, 15.6, 15.2
10-minute wash: 20.4, 19.3, 17.6, 16.3, 9.7, 16.4, 14.8, 12.3

20-minute wash	Change In Reflectance	**10-minute wash**
x	24	
x	23	
	22	
	21	
x	20	x
	19	x
x	18	x
x	17	
x	16	xx
xx	15	x
	14	
	13	
	12	x
	11	
	10	x
18.56	average	15.85
3.57	standard deviation	3.54

Are they Statistically Different?

The Adaptive Nature of Investigation



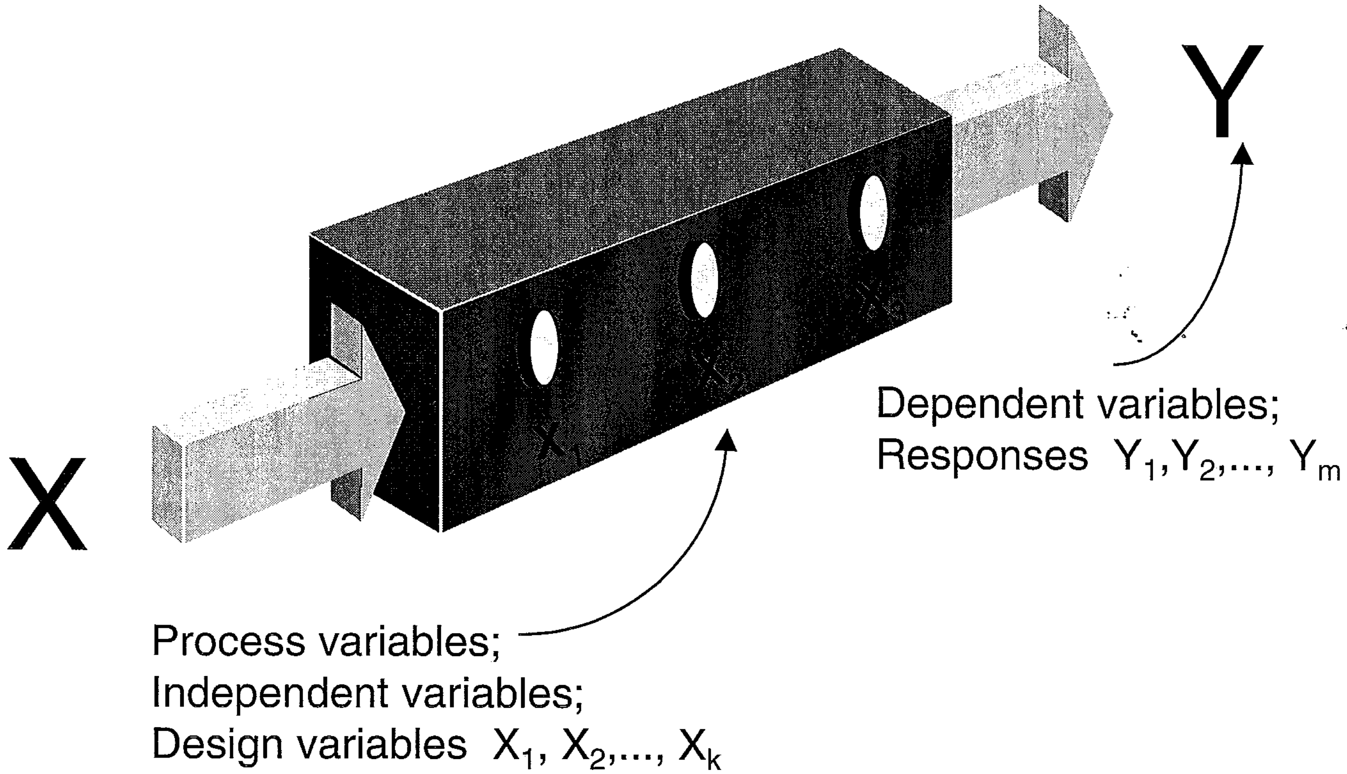


Science is usually an **<u>iterative</u>** process; perhaps a series of experiments which lead to a solution.

You may be able to start at different places and still eventually get to a solution.

There may be more than one path to the solution; it will more than likely take multiple experiments to find it.

Directed Experimentation (cont'd)



- Assume we have direct **control of the process variables** ($X_1, X_2, \ldots, X_k$).
  - These might be temperatures and pressures in the process, or the trucking company we use to ship product and the shift on which the trailer is loaded.
- We would like to **find settings that improve the responses** ($Y_1, Y_2, \ldots, Y_m$).

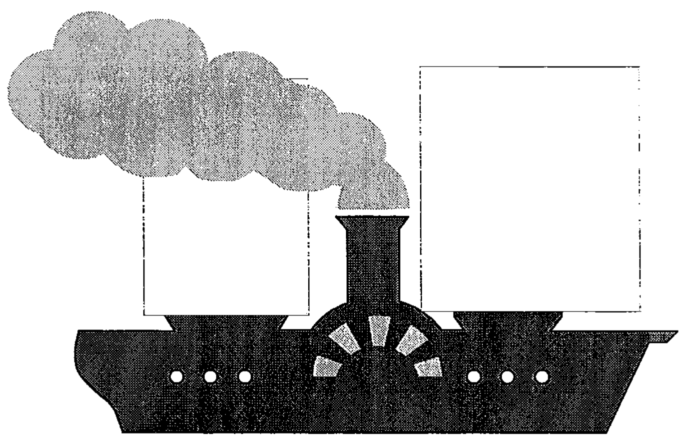
The Objective of an Experiment is to Estimate the Effects of the Independent Variables on the Responses.

Passive Observation

Passive observation means watching something happen as opposed to making something happen.

EXAMPLE: Champagne was first created naturally. It is likely that many people observed this without noting anything special. But eventually a critical observer noted the promise of champagne.

EXAMPLE: James Watt started the industrial revolution with the simple observation of the power of steam escaping from his tea kettle. This led to the invention of the steam engine.



Directed Experimentation Increases the Probability of Informative Events.

(Adapted from a lecture by George Box during the course: "An Explanation of Taguchi's Contributions to Quality Improvement," University of Wisconsin, April 27-30, 1987.)

The Role of Statistics

The field of statistics deals with variation in the following ways:

- **Descriptive statistics** -- describing a set of data with graphs and a few summary numbers (mean, variance, standard deviation).
- **Statistical inference**-- determining when differences in results might be due to random variation, and when differences in results cannot be attributed to random variation.

  (**Confidence intervals** and **hypothesis testing**).
- **Design of Experiments** (DOE) -- collecting and analyzing data to:

  1. Move the average of the distribution of the results.
  2. Reduce the variation in the results.
  3. Give results that apply to a wide range of conditions to provide a more robust process.
  4. Determine if the potential Vital Few “X’s” do or do not influence the “Y” response.

Observations Vary

When measurements are repeated, one usually gets different answers...this is variation!

1. **Systematic Variation** (Signal)

   Differences in the measurements which are **expected** and **predictable**.

   Example:

   The volume of electric range sales is different from the summer season to the Christmas holidays.

2. **Random Variation** (Noise)

   Differences in the measurements which are **NOT predictable.**

   Example:

   Two refrigerators of the same design, tested for energy usage on **two different days**, with the same technician, in the same test stand, with the same air temperature, with the same measuring instruments, etc... will probably give two different results.

Comparing Two Treatments

PURPOSE:

Introduce the Design of Experiments methodology for one "X" at two levels.

OBJECTIVES:

1. Understand the objectives of an experiment -- to estimate the effect of a treatment ("X") on the response ("Y").

2. Understand the rationale for collecting data for experiments to help control the variation.

3. Be able to interpret the results of two-treatment experiments.

4. Provide examples of two-treatment experiments -- some that we will conduct in class.