1 2 3 4 本来是 $\dfrac{e^{1-4}\ e^{2}\ e^{3-4}\ e^{4-4}}{\sum_{x=0}^{4} e^{x-4}}$

现在分块

一块 $\dfrac{e^{1-2}, e^{2-2}}{e^{1-2}+e^{2-2}}$ 二块 $\dfrac{e^{3-4}, e^{4-4}}{e^{3-4}+e^{4-4}}$

1 2 0, 1

先找到一块最大值

$m_0 = max(-inf, x_0, x_1) = 2$

$l_0 = e^{1-m_0} + e^{2-m_0} \left\{ e^{x-m_0} \right.$

$O_0 = \sum e^{x-m_0} \cdot V = e^{1-m_0} V_0 + e^{1-m_0} V_1$

$O_f = \dfrac{O_0}{l_0}$

假如第二块小

$m_1 = max(0, 1)$

$m_1 = max(m_0 m_1) = max(m_{old}, m_1) = 2$

理论上：$l_1 = \underbrace{e^{1-2}+e^{2-2}}_{l_0} + e^{0-2} + e^{1-2}$

$O_1 = (e^{1-mg} V_0 + e^{2-mg} V_1 + e^{0-mg} V_2 + e^{1-mg} V_3)$

$= (e^{1-m_0} V_0 + e^{2-m_0}) e^{m_0-m_1} + e^{0-mg} V_2 + e^{1-mg} V_3)$

$= O_0 e^{m_0-m_1} + O_b$

$m_1 = max(m_0, x_2, x_3)$

$l_1 = e^{3-m_1} + e^{4-m_1} + \boxed{e^{1-m_1} + e^{2-m_1}}$

$= (e^{1-m_0} + e^{2-m_0}) \cdot e^{m_0-m_1}$

$= e^{3-m_1} + e^{4-m_1} + e^{m_0-m_1} \cdot \boxed{l_0}$

$O_1 = \dfrac{1}{l_1} (e^{1-m_1} V_0 + e^{2-m_1} V_1 + e^{3-m_1} V_2 + e^{4-m_1} V_3$

$= \dfrac{1}{l_1} (e^{3-m_1} V_2 + e^{4-m_1} V_3 + (e^{1-m_0} V_0 + e^{2-m_0} V_1) \cdot e^{m_0-m_1}$

$= \dfrac{1}{l_1} (e^{3-m_1} V_2 + {}^{4-m_1} V_3 + l_0 \cdot O_0 \cdot e^{m_0-m_1})$

$m_2 = max(m_1, x)$ 令 $S_n = e$

$l_2 = e^{m_2-m_1} \cdot l_1 + \sum_{i=2}^{3} S_n^m$

$P_2 = S @ V + P_1 e^{m_1-m_2}$

$O_2 = \dfrac{P_2}{l_2}$

最终只要保存上一个的

m_1 最大值

l_1

P_1

公式：局部子块

m_b 子块的最大值，

y_b 子块每个元素求 e^{x-m_i}，

l_b y_b 子块中求和 $\sum_{i=0}^{n} e^{x-m_i} = \sum y_{bi}$

O_b 子块输出，每一个 y_b 元素乘以对应 V，$= y_b \cdot V$

这不是最后输出，最后为 $O_{final} = \dfrac{O_b}{l_b}$

全局更新

$m_g = max(m_b, m_{b0})$

$l_g = l_{b0} e^{m_{b0}-m_g} + l_b e^{m_b-m_g}$

$O_g = O_{b0} e^{m_{b0}-m_g} + O_b e^{m_b-m_g}$

$O_f = \dfrac{O_g}{l_g}$

大概思想，先计算当前块的 m_b. l_b. O_b，再结合 $e^{\cdots-m_g}$ 更新之前和当下的值，计算全局值

计算 $m_g = max(m_g, m_b)$，

$y_b = e^{x-m_g}$

$l_b = \sum e^{x_i-m_g}$，

$O_b = y_b \cdot V$，

$l_g = l_{b0} e^{m_{b0}-m_g} + l_b$

$O_g = O_{b0} e^{m_{b0}-m_g} + O_b$

每次循环进来，① 首先计算子块最大值 m_b，与上一轮 m_g 比较更新新的 $m_g = max(m_g, m_b)$，然而后面要用到未更新的上一轮最大值，所以 m_g 更新前保存一下，$m_0 = m_g$

② 用新的 m_g 构造 $y_b = e^{x-m_g}$ 以及 $l_b = \sum e^{x_i-m_g}$，这个也要保存更新前的值 $l_{b0} = l_b$，同理先保存 $O_{b0} = O_b$ 再更新 $O_b = y_b \cdot V$.

③ 最后更新，$l_g = l_{b0} e^{m_{b0}-m_g} + l_b$，$O_g = O_{b0} e^{m_{b0}-m_g} + O_b$