# HW 01 SOLUTIONS

## Practice Problems

### 2.1

$A = \{FF\},\ B = \{MM\},\ C = \{MF, FM, MM\}$. $A \cap B = \emptyset,\ B \cap C = \{MM\},\ C \setminus B = \{MF, FM\},\ A \cup B = \{FF, MM\},\ A \cup C = S,\ B \cup C = C$.

### 2.5

a. $(A \cap B) \cup (A \cap \overline{B}) = A \cap (B \cup \overline{B}) = A \cap S = A$.

b. $B \cup (A \cap \overline{B}) = (B \cap A) \cup (B \cap \overline{B}) = (B \cap A) = A$.

c. $(A \cap B) \cap (A \cap \overline{B}) = A \cap (B \cap \overline{B}) = \emptyset$.
The result follows from part a.

d. $B \cap (A \cap \overline{B}) = A \cap (B \cap \overline{B}) = \emptyset$.
The result follows from part b.

### 2.11

### 2.11

**a.** Since $P(S) = P(E_1) + \cdots + P(E_5) = 1$,

$$1 = 0.15 + 0.15 + 0.40 + 3P(E_5).$$

Thus, $P(E_5) = 0.10$ and $P(E_4) = 0.20$.

**b.** Obviously,

$$P(E_3) + P(E_4) + P(E_5) = 0.6.$$

Thus, they are all equal to 0.2.

## 2.13

**a.** Denote the events as very likely (VL), somewhat likely (SL), unlikely (U), and other (O).

**b.** Not equally likely: $P(\text{VL}) = 0.24$, $P(\text{SL}) = 0.24$, $P(\text{U}) = 0.40$, $P(\text{O}) = 0.12$.

**c.**

$$P(\text{at least SL}) = P(\text{SL}) + P(\text{VL}) = 0.48.$$

## 2.15

a. Since the events are mutually exclusive,
   $P(S) = P(E_1) + \cdots + P(E_4) = 1$.
   So, $P(E_2) = 1 - 0.01 - 0.09 - 0.81 = 0.09$.

b. $P(\text{at least one hit}) = P(E_1) + P(E_2) + P(E_3) = 0.19$.

## 2.17

Let $B$ = bushing defect, $SH$ = shaft defect.

a. $P(B) = 0.06 + 0.02 = 0.08$

b. $P(B \text{ or } SH) = 0.06 + 0.08 + 0.02 = 0.16$

c. $P(\text{exactly one defect}) = 0.06 + 0.08 = 0.14$

d. $P(\text{neither defect}) = 1 - P(B \text{ or } SH) = 1 - 0.16 = 0.84$

## 2.19

a. $(V_1, V_1), (V_1, V_2), (V_1, V_3), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_1), (V_3, V_2), (V_3, V_3)$

b. If equally likely, all have probability 1/9.

c. $A = \{\text{same vendor gets both}\} = \{(V_1, V_1), (V_2, V_2), (V_3, V_3)\}$
   $B = \{\text{at least one } V_2\} = \{(V_1, V_2), (V_2, V_1), (V_2, V_2), (V_2, V_3), (V_3, V_2)\}$ So, $P(A) = 1/3$, $P(B) = 5/9$,
   $P(A \cup B) = 7/9$, $P(A \cap B) = 1/9$.

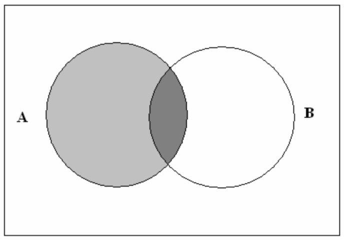
## 2.91

If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$. This value is greater than 1 if $P(A) = 0.4$ and $P(B) = 0.7$. So they cannot be mutually exclusive. It is possible if $P(A) = 0.4$ and $P(B) = 0.3$.

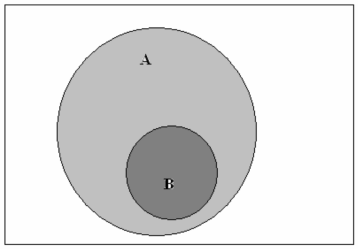
# Submission Problems

## 2.4

**2.4** **a.**




**b.**




## 2.6

$A = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(1,4),(2,4),(3,4),(4,4),(5,4),(6,4),(1,6),(2,6),(3,6),(4,6),(5,6),($

$\overline{C} = \{(2,2),(2,4),(2,6),(4,2),(4,4),(4,6),(6,2),(6,4),(6,6)\}$

$A \cap B = \{(2,2),(4,2),(6,2),(2,4),(4,4),(6,4),(2,6),(4,6),(6,6)\}$

$A \cap \overline{B} = \{(1,2),(3,2),(5,2),(1,4),(3,4),(5,4),(1,6),(3,6),(5,6)\}$

$\overline{A} \cup B =$ everything but $\{(1,2),(1,4),(1,6),(3,2),(3,4),(3,6),(5,2),(5,4),(5,6)\}$

$\overline{A} \cap C = \overline{A}$

## 2.8

a. $36 + 6 = 42$

b. 33

c. 18

## 2.14

a. $P(\text{needs glasses}) = 0.44 + 0.14 = 0.48$

b. $P(\text{needs glasses but doesn't use them}) = 0.14$

c. $P(\text{uses glasses}) = 0.44 + 0.02 = 0.46$

## 2.18

**a.** $S = \{\mathrm{HH}, \mathrm{TH}, \mathrm{HT}, \mathrm{TT}\}$

**b.** If the coin is fair, all events have probability 0.25.

**c.** $A = \{\mathrm{HT}, \mathrm{TH}\},\ B = \{\mathrm{HT}, \mathrm{TH}, \mathrm{HH}\}$

**d.** $P(A) = 0.5,\ P(B) = 0.75,\ P(A \cap B) = P(A) = 0.5,\ P(A \cup B) = P(B) = 0.75,\ P(\overline{A} \cup B) = 1.$

## 2.86

a. No. It follows from $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1$.

b. $P(A \cap B) \geq 0.5$

c. No.

d. $P(A \cap B) \leq 0.70$

## Additional Problem

Show that

a) $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$

b) $P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$

### Solution

Both parts follow from the fact that $P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$.

### Part a

$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$ because $P(A_1 \cap A_2) \geq 0$.

### Part b

$P(A_1 \cup A_2) \leq 1 \implies P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1 \implies P(A_1) + P(A_2) - 1 \leq P(A_1 \cap A_2)$