STAT 5700 — Quiz 5
Date: October 23, 2025

Name: _____

Problem 1 (3pts) Let $m(t) = \frac{1}{4}e^t + \frac{1}{4}e^{3t} + \frac{1}{2}e^{5t}$. Find the following:

- $E(Y)$
- $V(Y)$
- The probability distribution of Y

Solution

$$m'(t) = \frac{d}{dt}\left(\frac{1}{4}e^t + \frac{1}{4}e^{3t} + \frac{1}{2}e^{5t}\right) = \frac{1}{4}e^t + \frac{3}{4}e^{3t} + \frac{5}{2}e^{5t}.$$

$$E(Y) = m'(0) = \frac{1}{4}(1) + \frac{1}{4}(3) + \frac{1}{2}(5) = \frac{1+3+10}{4} = \frac{14}{4} = 3.5$$

$$m''(t) = \frac{d}{dt}\left(\frac{1}{4}e^t + \frac{3}{4}e^{3t} + \frac{5}{2}e^{5t}\right) = \frac{1}{4}e^t + \frac{9}{4}e^{3t} + \frac{25}{2}e^{5t}.$$

$$E(Y^2) = m''(0) = \frac{1}{4}(1^2) + \frac{1}{4}(3^2) + \frac{1}{2}(5^2) = \frac{1+9+50}{4} = \frac{60}{4} = 15$$

$$V(Y) = E(Y^2) - [E(Y)]^2 = 15 - (3.5)^2 = 15 - 12.25 = 2.75$$

From the MGF, the probabilities correspond to the coefficients of e^{yt}:

Y	1	3	5
$P(Y = y)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$

Problem 2 (1pt) Find an expression for the 100th moment of the Bernoulli distribution (support $y = 0, 1$). You answer should be a function of p only.

Solution

$p(y) = p^y(1-p)^{1-y}$ for $y = 0, 1$

The n-th moment is:

$$E(Y^n) = \sum_{y=0}^{1} y^n p^y (1-p)^{1-y} = 0^n p^0 (1-p)^1 + 1^n p^1 (1-p)^0 = p$$

So the **100th moment** is:

$$E(Y^{100}) = p$$

TRUE/FALSE (0.5pt each)

3. FALSE The first central moment, $E(Y - \mu)$, is equal to σ^2 for all random variables Y, where μ is the mean of Y and σ^2 is the variance.
4. TRUE If two random variables have the same moment-generating function for all real values of t then they must have the same probability distribution.