# HW 04 SOLUTIONS

## Practice Problems

### 3.15

a. $p(0) = P(Y = 0) = (.48)^3 = .1106$, $p(1) = P(Y = 1) = 3(.48)^2(.52) = .3594$, $p(2) = P(Y = 2) = 3(.48)(.52)^2 = .3894$, $p(3) = P(Y = 3) = (.52)^3 = .1406$.

b. The graph is omitted.

c. $P(Y = 1) = .3594$.

### 3.21

Note that $E(N) = E(8\pi R^2) = 8\pi E(R^2)$. So, $E(R^2) = 21^2(.05) + 22^2(.20) + \cdots + 26^2(.05) = 549.1$. Therefore $E(N) = 8\pi(549.1) = 13{,}800.388$.

### 3.31

a. The mean of $W$ will be larger than the mean of $Y$ if $\mu > 0$. If $\mu < 0$, the mean of $W$ will be smaller than $\mu$. If $\mu = 0$, the mean of $W$ will equal $\mu$.

b. $E(W) = E(2Y) = 2E(Y) = 2\mu$.

c. The variance of $W$ will be larger than $\sigma^2$, since the spread of values of $W$ has increased.

d. $V(X) = E[(X - E(X))^2] = E[(2Y - 2\mu)^2] = 4E[(Y - \mu)^2] = 4\sigma^2$.

## 3.37

a. Not a binomial random variable.

b. Not a binomial random variable.

c. Binomial with $n = 100$, $p =$ proportion of high school students who scored above 1026.

d. Not a binomial random variable (not discrete).

e. Not binomial, since the sample was not selected among all female HS grads.

## 3.39

Let $Y$ = number of components failing in less than 1000 hours. Then $Y \sim Binomial(n = 4, p = .2)$.
a. $P(Y = 2) = \binom{4}{2}(.2)^2(.8)^2 = 0.1536$.
b. The system operates if 0, 1, or 2 components fail. $P(\text{system operates}) = .4096 + .4096 + .1536 = .9728$.

## 3.51

a. $P(\text{at least one 6 in four rolls}) = 1 - P(\text{no 6's in four rolls}) = 1-(5/6)^4 = 0.51775$.

b. In a single toss of two dice, $P(\text{double 6}) = 1/36$. Then $P(\text{at least one double 6 in 24 rolls}) = 1-P(\text{no double 6's in 24 rolls}) = 1-(35/36)^{24} = 0.4914$.

## 3.55

We use the identity

$$\begin{aligned}
E[Y(Y-1)(Y-2)] &= \sum_{y=0}^{n} \frac{y(y-1)(y-2)n!}{y!(n-y)!} p^y(1-p)^{n-y} \\
&= \sum_{y=3}^{n} \frac{n(n-1)(n-2)(n-3)!}{(y-3)!(n-3-(y-3))!} p^y(1-p)^{n-y} \\
&= n(n-1)(n-2)p^3 \sum_{z=0}^{n-3} \binom{n-3}{z} p^z(1-p)^{n-3-z} \\
&= n(n-1)(n-2)p^3
\end{aligned}$$

Equating this to $E(Y^3) - 3E(Y^2) + 2E(Y)$, it is found that $E(Y^3) = 3n(n-1)p^2 - n(n-1)(n-2)p^3 + np$.

## 3.59

If $Y$ = number of defective motors, then $Y \sim Binomial(n = 10, p = .08)$. Then $E(Y) = .8$. The seller's expected net gain is $1000–200E(Y) = 840$.