# Practice Exam 2

**1. (20 points)** The daily sales at a local bookstore that focuses on books in mathematics, statistics, and finance are normally distributed with mean $1200 and standard deviation $200. You may find the `qnorm()` and/or `pnorm()` output below to be useful in completing this problem.

(a) Find the probability that the daily sales are between $900 and $1500.

(b) Find the interval that contains the middle 40% of daily sales numbers.

(c) Daily sales are independent from one day to the next. What is the probability that the daily sales are outside the interval ($900,$1500) at least twice in a five-day work week?

| z | pnorm(z) |
|---|---|
| -2 | 0.023 |
| -1.5 | 0.067 |
| -1 | 0.159 |
| -0.5 | 0.309 |
| 0.5 | 0.691 |
| 1 | 0.841 |
| 1.5 | 0.933 |
| 2 | 0.977 |

| p | qnorm(p) |
|---|---|
| 0.1 | -1.282 |
| 0.2 | -0.842 |
| 0.3 | -0.524 |
| 0.4 | -0.253 |
| 0.6 | 0.253 |
| 0.7 | 0.524 |
| 0.8 | 0.842 |
| 0.9 | 1.282 |

**2. (10 points)** An urn contains four balls labeled with the integers 1 to 4. Suppose that two balls are selected at random from the urn without replacement. Let $X$ be the smaller of the two integers on the selected balls, and let $Y$ be the larger of the two. For example, if the integers are 4 (first draw) and 1 (second), then $X = 1$ and $Y = 4$.

(a) Find the joint distribution for $X$ and $Y$.

(b) Find the marginal distribution of $Y$

(c) Find $p(x|y = 3)$

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**3. (15 points)** The random variable $X$ has probability density function

$$f(x) = \begin{cases} x^2, & -1 < x < 0, \\ cx, & 0 < x < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of $c$ that makes $f(x)$ a valid probability density function.

(b) Find the cumulative distribution function for $X$.

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**4. (15 points)** The side length $S$ of a square is uniformly distributed on the interval $[2, 4]$.

(a) Write down the probability density function for $S$.

(b) Find the expected value for the area $A = S^2$ of the square.

(c) Find the variance for the area $A = S^2$ of the square.

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**5. (10 points)** Find a formula for $E[X^3]$ when $X$ follows the Gamma$(\alpha, \beta)$ distribution. Put your answer in terms of $\alpha$ and $\beta$, with no instances of the gamma function showing up.

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**6. (10 points)** The random variable $X$ has moment generating function

$$m(t) = (0.5 + 0.5e^{2t})^2.$$

(a) Find $E[X]$.

(b) Find $V(X)$.

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**7. (15 points)** The total medical claims each year for employees in a company is modeled with pdf $f(y) = 2(1 - y), \ \ 0 \le y \le 1$, where $y$ is in \$100,000s.

  (a) Find the mean of the total claims
  (b) Find the 90th percentile of the distribution
  (c) If the company set aside \$75,000, what is the probability they cover all the claims for a year.

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**8. (5 points)** A compter randomly generates a number $Y$ that ranges from 100 to 150. Consider the following function:

$$h(y) = \begin{cases} 0, & y < 100 \\ 0.02y - 2, & 100 \le y \le 150 \\ 0, & y > 150 \end{cases}$$

The function $h(y)$ is _____.

  A) the cumulative distribution function (cdf) of $Y$
  B) the probability density function (pdf) of $Y$
  C) neither the cdf nor the pdf of $Y$

Justify your answer.