STAT 5700 – Practice Exam #1a

Instructions

You must **SHOW YOUR WORK** to receive full credit.

Problem 1

The random variable X has the following probability distribution:

x	2	4	10
$p(x)$	0.2	0.5	0.3

(a) Find the expected value of X.

(b) Find the variance and standard deviation of X.

(c) Find $E[X^3]$.

(d) Define the cdf of X

a) $E(X) = \sum x\,p(x) = 2(.2) + 4(.5) + 10(.3) = \boxed{5.4}$

b) $V(X) = E(X^2) - (E(X))^2$

$E(X^2) = \sum x^2 p(x) = 2^2(.2) + 4^2(.5) + 10^2(.3) = 38.8$

$V(X) = 38.8 - 5.4^2 = \boxed{9.64}$

$SD(X) = \sqrt{V(X)} = \boxed{3.1}$

c) $E(X^3) = \sum x^3 p(x) = 2^3(.2) + 4^3(.5) + 10^3(.3) = \boxed{333.6}$

d) $F(x) = P(X \le x) = \begin{cases} 0 & x < 2 \\ 0.2 & 2 \le x < 4 \\ .7 & 4 \le x < 10 \\ 1 & x \ge 10 \end{cases}$

Problem 2

An urn contains five balls labeled with the numbers 1, 2, 3, 4, and 6. Suppose that we select two balls at random without replacement.

(a) Find the probability mass function (pmf) for the product X of the two numbers. For example, if the numbers are 2 and 3, then $X = 2 \cdot 3 = 6$.

(b) Find $P(X$ is divisible by 3)

a)

	1	2	3	4	6
1	·	2	3	4	6
2	2	·	6	8	12
3	3	6	·	12	18
4	4	8	12	·	24
6	6	12	18	24	·

X	P(X = x)
2	2/20
3	2/20
4	2/20
6	4/20
8	2/20
12	4/20
18	2/20
24	2/20
	20/20 ✓

b) P(X divisible by 3)
= P(X = 3, 6, 12, 18, 24)
= 14/20

Problem 3

A process is repeated independently, where each trial results in a "success" with probability 0.04.

(a) Write an expression (in terms of n) for the probability of getting at least one success in n trials.

(b) Find the smallest integer n such that the probability of getting at least one success is at least 0.50.

a) $P(\text{at least one}) = 1 - P(0) = 1 - (.96)^n$

b) $1 - .96^n \geq .5$

$0.5 \geq .96^n$

$\ln(0.5) \geq n \ln(.96)$

$\frac{\ln(0.5)}{\ln(.96)} \leq n$ ($\ln(.96)$ is negative, so have to flip the inequality)

$16.98 \leq n \Rightarrow \boxed{n = 17}$

Problem 4

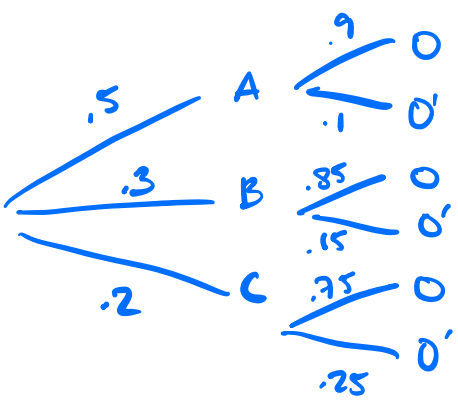
A medical clinic refers patients to one of three doctors for checkups:

- Dr. A (50% of patients),
- Dr. B (30% of patients),
- Dr. C (20% of patients).

The doctors complete the checkups on time with probabilities 0.9 (Dr. A), 0.85 (Dr. B), and 0.75 (Dr. C).

(a) What percentage of all checkups are completed on time?

(b) If a checkup was **not** completed on time, what is the probability that Dr. C was the doctor?



a) $P(O) = .5(.9) + .3(.85) + .2(.75) = \boxed{0.855}$

b) $P(C|O') = \dfrac{P(C \cap O')}{P(O')}$

$= \dfrac{(.2)(.25)}{1 - .855} = \boxed{0.345}$

Problem 5

A student organization has 20 members: 12 undergraduates and 8 graduate students. A committee of 4 members is chosen at random (without replacement).

(a) What is the probability that all 4 committee members are undergraduates?

(b) What is the probability that the committee has at least one undergraduate and at least one graduate student?

a) $\frac{\binom{12}{4}\binom{8}{0}}{\binom{20}{4}} = \frac{\frac{12!}{8!4!}\cdot\frac{8!}{0!8!}}{\frac{20!}{16!4!}} = \frac{495}{4845} = \boxed{0.1022}$

b) $1 - [P(\text{all undergrad}) + P(\text{all grad})]$

$P(\text{all grad}) = \frac{\binom{12}{0}\binom{8}{4}}{\binom{20}{4}} = \frac{\frac{8!}{4!4!}}{4845} = \frac{70}{4845} = 0.0144$

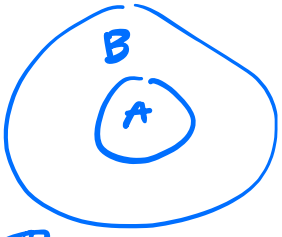
$1 - .1022 - .0144 = \boxed{.8834}$

Problem 6

Prove the following statement. If $A \subset B$, then $P(A|B) = \frac{P(A)}{P(B)}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$

$A \cap B = A$ b/c $A \subset B$



Problem 7

Two fair coins are flipped. $S = \{HH, HT, TH, TT\}$

- Let A be the event that exactly one head is observed. $A = \{HT, TH\}$
- Let B be the event that the first coin shows heads. $B = \{HH, HT\}$

By checking an appropriate probability condition, determine whether A and B are independent.

$P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$A \cap B = \{HT\}$

$P(A \cap B) = \frac{1}{4}$

$P(A) = \frac{2}{4} = P(B)$

$P(A)P(B) = \frac{1}{2}\frac{1}{2} = \frac{1}{4} = P(A \cap B)$ ✓ A, B are indep.

Multiple Choice

Problem 8

Suppose that a fair coin is flipped 10 times. Which is more likely – that the flips result in 5 heads and 5 tails, or that the flips result in 6 of one outcome and 4 of the other?

A. 5 of each
B. 6–4 split
C. These are equally likely
D. Not enough information to decide which of (A) or (B) is greater

Problem 9

(S) and (T) are events with P(S) = 0.7 and P(T) = 0.6. Which of the following – A or B – is greater? Or are they equal? Or is there not enough information to decide?

A. 0.42
B. $P(S \cap T)$ C. 0.42 and $P(S \cap T)$ are exactly the same
D. There is not enough information to determine which of (A) or (B) is greater

Need to know whether S,T are independent