STAT 5700 — Quiz 9 SOLUTIONS
Date: December 4, 2025

(2pts) Problem 1 Let X and Y be two independent, continuous random variables with pdfs $f_X(x) = 2x$, $0 < x < 1$, and $f_Y(y) = 3y^2$, $0 < y < 1$. Find the following:

  a. $P(X < Y)$
  b. ρ_{XY}

SOLUTION part a

Because X and Y are independent, $\rho_{XY} = 0$

SOLUTION part b

Due to independence, the covariance is 0, which means the correlation is 0.

(2pts) Problem 2 Let $\mu_X = 2$, $\sigma_X^2 = 9$, $\mu_Y = 1$, $\sigma_Y^2 = 16$, and $\sigma_{XY} = 3$. Find the following:

  a. $V(2X + 4Y)$
  b. $Cov(X, 2Y)$
  c. $Corr(X, 2Y)$

SOLUTION part a

$$V(2X + 4Y) = 2^2\sigma_X^2 + 4^2\sigma_Y^2 + 2(2)(4)\sigma_{XY} = 4 \cdot 9 + 16 \cdot 16 + 16 \cdot 3 = 36 + 256 + 48 = 340.$$

SOLUTION part b

$$\text{Cov}(X, 2Y) = E(X2Y) - E(X)E(2Y) = 2[E(XY) - E(X)E(Y)] = 2Cov(X, Y) = 2(3) = 6.$$

SOLUTION part c

$$\text{Corr}(X, 2Y) = \frac{\text{Cov}(X, 2Y)}{\sqrt{V(X)}\sqrt{V(2Y)}} = \frac{6}{(3)(2)(4)} = \frac{1}{4}.$$

MULTIPLE CHOICE (0.5 each)

3. Consider two continuous random variables X and Y. Which of the following can be negative? Select all that apply.

  A) μ_X
  B) σ_X
  C) $Cov(X, Y)$
  D) ρ_{XY}

4. The correlation between W and Z is calculated to be exactly 1. Z is plotted against W. A line is drawn through all of the data points. The slope of the line _____ equal to 0.5. The slope of the line _____ equal to -2.

   A) could be, could be

   B) could be, could NOT be

   C) could NOT be, could be

   D) could NOT be, could NOT be