HW 08 SOLUTIONS

Practice Problems

4.89

(a) Note that $\int_2^\infty \frac{1}{\beta}e^{-y/\beta}dy = e^{-2/\beta} = 0.0821 \Rightarrow \beta = 0.8$

(b) $P(Y \leq 1.7) = 1 - e^{-1.7/0.8} = 0.5075$

4.93

Let Y be the time between fatal airplane accidents. So $Y \sim$ Exponential($\beta = 44$).
(a) $P(Y \leq 31) = \int_0^{31} \frac{1}{44}e^{-y/44} = 1 - e^{-31/44} = 0.5057$
(b) $V(Y) = \beta^2 = 44^2 = 1936$

4.109

$Y \sim$ Gamma($\alpha = 3,\ \beta = 2$), $L = 30Y + 2Y^2$
$E(L) = 30E(Y) + 2E(Y^2) = 30(6) + 2(12 + 6^2) = 276$
$V(L) = E(L^2) - [E(L)]^2 = E(900Y^2 + 120Y^3 + 4Y^4) - 276^2$. $E(Y^3) = \int +0\infty \frac{y^5}{16}e^{-y/2} = 480$ and $E(Y^4) = \int +0\infty \frac{y^6}{16}e^{-y/2} = 5760$ Thus, $V(Y) = 900(48) + 120(480) + 4(5760) - 276^2 = 47664$

4.123a

$Y \sim$ Beta($\alpha = 4,\ \beta = 3$), $k = \frac{\Gamma(4+3)}{\Gamma(4)\Gamma(3)} = 60$
95th percentile: $\phi_{.95} = 0.84684$

4.127

For $\alpha = \beta = 1,\ f(y) = \frac{\Gamma(2)}{\Gamma(1)\Gamma(1)} y^{1-1}(1-y)^{1-1} = 1 \Rightarrow Y \sim \text{Uniform}(0,1)$

4.165

The pdf of Y is in the form of a gamma density with $Y \sim \text{Gamma}(\alpha = 2,\ \beta = 0.5)$
(a) $c = \frac{1}{\Gamma(2)0.5^2} = 4$
(b) $E(Y) = \alpha\beta = 2(0.5) = 1$, $V(Y) = \alpha\beta^2 = 2(.5)^2 = 0.5$
(c) MGF: $M(t) = (1-\beta t)^{-\alpha} = (1-0.5t)^{-2},\ t < 2$

Submitted Problems

4.96abc

$Y \sim \text{Gamma}(\alpha = 4,\ \beta = 2)$ (a) $k = 1/(\Gamma(4)2^4) = 1/96$ (b) $Y \sim \chi^2$ with $\nu = 2\alpha = 8$ degrees of freedom (c) $E(Y) = \alpha\beta = 4(2) = 8$, $V(Y) = \alpha\beta^2 = 4(2^2) = 16$

4.104

$Y \sim \text{Exponential}(\beta = 100),\ P(Y > 200) = e^{-2}$ Let the random variable X be the number of components that operate in the equipment for more than 200 hours. Then $X \sim \text{Binomial}(n = 3, p = e^{-2}),\ P(X \geq 2) = P(X = 2) + P(X = 3) = 3(e^{-2})^2(1 - e^{-2}) + (e^{-2})^3 = 0.05$

4.110

$Y \sim \text{Gamma}(\alpha = 3,\ \beta = 0.5),\ E(Y) = 1.5,\ V(Y) = 0.75$

4.126

(a) $\int_0^y (6t - 6t^2)dt = 3y^2 - 2y^3$, so $F(y) = \begin{cases} 0, & y < 0 \\ 3y^2 - 2y^3, & 0 \leq y \leq 1 \\ 1, & y > 1 \end{cases}$

(b) not shown

(c) $P(0.5 \leq Y \leq 0.8) = F(0.8) - F(0.5) = 1.92 - 1.092 - 0.75 + 0.25 = 0.396$

4.130

$\sigma^2 = E(Y^2) - \mu^2,$

$$E(Y^2) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \int_0^1 y^{\alpha+1}(1-y)^{\beta-1} dy = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \frac{\Gamma(\alpha+2)\Gamma(\beta)}{\Gamma(\alpha+2+\beta)} = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)}$$

$V(Y) = \frac{(\alpha+1)\alpha}{(\alpha+\beta)(\alpha+\beta+1)} - \left(\frac{\alpha}{\alpha+\beta}\right)^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$