HW 04 SOLUTIONS

Practice Problems

3.1

$P(Y = 0) = P(\text{no impurities}) = 0.2$, $P(Y = 1) = P(\text{exactly one impurity}) = 0.7$, $P(Y = 2) = 0.1$.

3.3

$p(2) = P(DD) = 1/6$, $p(3) = P(DGD) + P(GDD) = 2(2/4)(2/3)(1/2) = 2/6$, $p(4) = P(GGDD) + P(DGGD) + P(GDGD) = 3(2/4)(1/3)(1) = 1/2$.

3.5

There are $3! = 6$ possible ways to assign the words to the pictures. Of these, one is a perfect match, three have one match, and two have zero matches. Thus, $p(0) = 2/6$, $p(1) = 3/6$, $p(3) = 1/6$.

3.15

a. $p(0) = P(Y = 0) = (.48)^3 = .1106$, $p(1) = P(Y = 1) = 3(.48)^2(.52) = .3594$, $p(2) = P(Y = 2) = 3(.48)(.52)^2 = .3894$, $p(3) = P(Y = 3) = (.52)^3 = .1406$.

b. The graph is omitted.

c. $P(Y = 1) = .3594$.

3.21

Note that $E(N) = E(8\pi R^2) = 8\pi E(R^2)$. So, $E(R^2) = 21^2(.05) + 22^2(.20) + \cdots + 26^2(.05) = 549.1$. Therefore $E(N) = 8\pi(549.1) = 13{,}800.388$.

3.31

a. The mean of W will be larger than the mean of Y if $\mu > 0$. If $\mu < 0$, the mean of W will be smaller than μ. If $\mu = 0$, the mean of W will equal μ.

b. $E(W) = E(2Y) = 2E(Y) = 2\mu$.

c. The variance of W will be larger than σ^2, since the spread of values of W has increased.

d. $V(X) = E[(X - E(X))^2] = E[(2Y - 2\mu)^2] = 4E[(Y - \mu)^2] = 4\sigma^2$.

3.37

a. Not a binomial random variable.

b. Not a binomial random variable.

c. Binomial with $n = 100$, $p =$ proportion of high school students who scored above 1026.

d. Not a binomial random variable (not discrete).

e. Not binomial, since the sample was not selected among all female HS grads.

3.39

Let $Y =$ number of components failing in less than 1000 hours. Then $Y \sim Binomial(n = 4, p = .2)$.
a. $P(Y = 2) = \binom{4}{2}(.2)^2(.8)^2 = 0.1536$.
b. The system operates if 0, 1, or 2 components fail. $P(\text{system operates}) = .4096 + .4096 + .1536 = .9728$.

3.51

a. $P(\text{at least one 6 in four rolls}) = 1–P(\text{no 6's in four rolls}) = 1–(5/6)^4 = 0.51775$.

b. In a single toss of two dice, $P(\text{double 6}) = 1/36$. Then $P(\text{at least one double 6 in 24 rolls}) = 1–P(\text{no double 6's in 24 rolls}) = 1–(35/36)^{24} = 0.4914$.

3.55

We use the identity

$$\begin{aligned} E[Y(Y-1)(Y-2)] &= \sum_{y=0}^{n} \frac{y(y-1)(y-2)n!}{y!(n-y)!} p^y (1-p)^{n-y} \\ &= \sum_{y=3}^{n} \frac{n(n-1)(n-2)(n-3)!}{(y-3)!(n-3-(y-3))!} p^y (1-p)^{n-y} \\ &= n(n-1)(n-2)p^3 \sum_{z=0}^{n-3} \binom{n-3}{z} p^z (1-p)^{n-3-z} \\ &= n(n-1)(n-2)p^3 \end{aligned}$$

Equating this to $E(Y^3) - 3E(Y^2) + 2E(Y)$, it is found that $E(Y^3) = 3n(n-1)p^2 - n(n-1)(n-2)p^3 + np$.

3.59

If Y = number of defective motors, then $Y \sim \mathit{Binomial}(n = 10, p = .08)$. Then $E(Y) = .8$. The seller's expected net gain is $1000–200E(Y) = 840$.