STAT 5700 Practice Final Exam

1. **(6pts)** Consider the function $f(x) = \frac{x}{12}$, $-1 < x < 5$. Determine whether $f(x)$ is a valid probability density function or not.

2. **(8pts)** The random variables X and Y have joint probability density function

$$f(x, y) = 1, \quad 0 < y < x < 2, \quad x + y < 2.$$

   Find the marginal probability density function $f_X(x)$.

3. **(10 points)** The random variables X and Y have joint probability density function

$$f(x, y) = x + y, \quad 0 < x < 1, \quad 0 < y < 1.$$

   Find the covariance between X and Y.

4. **(12pts)** A casino patron will continue to make \$5 bets on red in roulette until she has won 4 of these bets. Note that on each bet she will either win \$5 with probability 18/38 or lose \$5 with probability 20/38.

   a. What is the probability she places a total of 9 bets?
   b. What are her expected winnings when she stops?
   c. What is the variance of her winnings when she stops?

5. **(8pts)** A wildlife reserve tracks bird sightings during migration season. Based on historical data, 12% of birds are tagged with one tracking band, 6% are tagged with multiple bands, and the remainder are untagged. A researcher observes 15 birds at random. Let W_1 be the number of birds with one band and W_2 be the number with multiple bands. If the data processing cost is given by $5W_1 + 12W_2$ dollars, find the mean and variance of the total processing cost.

6. **(6pts)** Suppose that X is Poisson with mean 2. Find $P(X \geq 4 | X > 2)$.

7. **(8pts)** A cabinet has three drawers, labeled A, B, and C; drawer A has two gold balls, drawer B has two silver balls, and drawer C has one gold ball and one silver ball. A drawer is chosen at random (i.e., so that each of the three drawers is equally likely to be chosen) and a ball is chosen at random from the drawer. Suppose the ball is gold. What is the probability that the second ball drawn from the same drawer will also be gold? That is, what is the probability that drawer A was chosen?

8. **(6pts)** An urn contains five balls labeled with the numbers $1, 2, 3, 4$, and 6. Suppose that we select two balls at random without replacement. Find the probability mass function for the product X of the two numbers. For example, if the numbers are 2 and 3, then $X = 2 \cdot 3 = 6$.

9. **(8pts)** A class has 28 students. Of the 28 students, 16 are statistics majors, and 12 are not. You plan to select k students at random from the class without replacement. What value of k maximizes the probability that there are exactly five statistics majors among the k students?

10. **(6pts)** Suppose that $P(A) = 0.8$, $P(B) = 0.6$, and $P(B \mid A) = 0.7$. Find $P(A \cup B)$.

11. **(10pts)** WNBA star Caitlin Clark makes 90 of her free throw attempts. Suppose that she attempts a sequence of free throws, with each attempt independent of all other attempts.

    a. What is the probability that she misses at least two of her first ten attempts?
    b. What is the probability that her first missed attempt is on an even-numbered attempt like the second attempt, the fourth attempt, and so on?

12. **(12pts)** The random variable X has probability density function

$$f(x) = \begin{cases} \frac{x}{3}, & 0 < x < 2, \\ \frac{1}{9}, & 2 < x < 5, \\ 0, & otherwise \end{cases}$$

    a. Find the cdf for X.
    b. Find the median of this distribution.