HW 02 SOLUTIONS

Practice Problems

2.25

Unless exactly 1/2 of all cars in the lot are Volkswagens, the claim is not true.

2.31

a. There are four "good" systems and two "defective" systems. If two out of the six systems are chosen randomly, there are 15 possible unique pairs. Denoting the systems as $g_1, g_2, g_3, g_4, d_1, d_2$, the sample space is $S = \{g_1g_2, g_1g_3, g_1g_4, g_1d_1, g_1d_2, g_2g_3, g_2g_4, g_2d_1, g_2d_2, g_3g_4, g_3d_1, g_3$
Thus: $P(\text{at least one defective}) = 9/15$, $P(\text{both defective}) = P(d_1d_2) = 1/15$.

b. If four are defective: $P(\text{at least one defective}) = 14/15$, $P(\text{both defective}) = 6/15$.

2.37

a. There are $6! = 720$ possible itineraries.

b. In the 720 orderings, exactly 360 have Denver before San Francisco and 360 have San Francisco before Denver. So, the probability is 0.5.

2.41

If the first digit cannot be zero, there are 9 possible values. For the remaining six digits, there are 10 possible values each. Thus, the total number is $9 \cdot 10^6$.

2.43

The number of ways to choose 3 objects from 9, 6, 5, and 1 is
$\binom{9}{3}\binom{6}{5}\binom{1}{1} = 504$ ways.

2.51

There are $\binom{50}{3} = 19,600$ ways to choose 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3} = 4$ ways for the organizers to win all of the prizes. The probability is $\frac{4}{19600}$.

b. There are $\binom{4}{2}\binom{46}{1} = 276$
ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $\frac{276}{19600}$.

c. There are $\binom{4}{1}\binom{46}{2} = 4140$ ways. The probability is $\frac{4140}{19600}$.

d. There are $\binom{46}{3} = 15,180$ ways. The probability is $\frac{15180}{19600}$.

2.57

There are $\binom{52}{2} = 1326$ ways to draw two cards from the deck. The probability is
$\frac{4 \cdot 12}{1326} = 0.0362$.

2.59

There are $\binom{52}{5} = 2,598,960$ ways to draw five cards from the deck.

a. $\binom{4}{1}^5 = 1024$ ways. So, the probability is $\frac{1024}{2{,}598{,}960} = 0.000394$.

b. There are 9 different types of "straight" hands. So, the probability is $\frac{9 \cdot 45}{2{,}598{,}960} = 0.00355$. Note that this includes "straight flush" and "royal straight flush" hands.

2.61

a. $\frac{364(364)(364)\cdots(364)}{365^n} = \frac{364^n}{365^n}$

b. With $n = 253$,
$1 - \left(\frac{364}{365}\right)^{253} \approx 0.5005$.

Submitted Problems

2.26

a. Let N_1, N_2 denote the empty cans and W_1, W_2 denote the cans filled with water. Thus, $S = \{N_1N_2, N_1W_2, N_2W_2, N_1W_1, N_2W_1, W_1W_2\}$.

b. If this is merely a guess, the events are equally likely. So, $P(W_1W_2) = 1/6$.

2.28

a. Denote the four candidates as A_1, A_2, A_3, and M. Since order is not important, the outcomes are $\{A_1A_2, A_1A_3, A_1M, A_2A_3, A_2M, A_3M\}$.

b. Assuming equally likely outcomes, all have probability 1/6.

c. $P(\text{minority hired}) = P(A_1M) + P(A_2M) + P(A_3M) = 0.5$

2.30

a. Let w_1 denote the first wine, w_2 the second, and w_3 the third. Each sample point is an ordered triple indicating the ranking.

b. Triples: $(w_1, w_2, w_3), (w_1, w_3, w_2), (w_2, w_1, w_3), (w_2, w_3, w_1), (w_3, w_1, w_2), (w_3, w_2, w_1)$

c. For each wine, there are 4 ordered triples where it is not last. So, the probability is 2/3.

2.38

By the mn rule, $4 \cdot 3 \cdot 4 \cdot 5 = 240$.

2.42

There are three different positions to fill using ten engineers. Then, there are

${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$ different ways to fill the positions.

2.46

There are $\binom{10}{2}$ ways to choose two teams for the first game, $\binom{8}{2}$ for the second, etc. So,

$\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{10!}{(2!)^5} = 113,400$ ways to assign the ten teams to five games.

2.48

Same answer: $\binom{8}{5} = \binom{8}{3} = 56$.

2.50

Two numbers, 4 and 6, are possible for each of the three digits. So, there are $2 \cdot 2 \cdot 2 = 8$ potential winning three-digit numbers.

2.58

There are $\binom{52}{5} = 2{,}598{,}960$ ways to draw five cards from the deck.

a. To draw three Aces and two Kings: $\binom{4}{3}\binom{4}{2} = 24$ So the probability is $\frac{24}{2{,}598{,}960}$.

b. There are $13 \cdot 12 = 156$ types of full house hands. From part (a), each type can be made in 24 different ways.
   So the probability is $\frac{156 \cdot 24}{2{,}598{,}960} \approx 0.00144$

2.64

$6!\left(\frac{1}{6}\right)^5 = \frac{5}{324}$

2.68a–c

a. $\binom{n}{n} = \dfrac{n!}{n!(n-n)!} = 1$. There is only one way to choose all of the items.

b. $\binom{n}{0} = \dfrac{n!}{0!(n-0)!} = 1$. There is only one way to choose none of the items.

c. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!(n-(n-r)!)} = \binom{n}{n-r}$. There are the same number of ways to choose r out of n objects as there are to choose $n-r$ out of n objects.