# HW 05 SOLUTIONS

## Practice Problems

### 3.67

$$(0.7)^4(0.3) = 0.07203$$

### 3.71

(a)

$$P(Y > a) = \sum_{y=a+1}^{\infty} q^{y-1}p = pq^a \sum_{y=1}^{\infty} q^{y-1} = \frac{pq^a}{1-q} = q^a$$

(b) From part (a),

$$P(Y > a + b \mid Y > a) = \frac{P(Y > a + b)}{P(Y > a)} = \frac{q^{a+b}}{q^a} = q^b$$

(c)

$$P(Y > a + b \mid Y > a) = P(Y > b)$$

(d) The results in the past are not relevant to a future outcome (independent trials).

### 3.73

Let $Y$ = number of accounts audited until the first with substantial errors is found.

(a)

$$P(Y = 3) = (0.12)(0.9)^2 = 0.009$$

(b)

$$P(Y \geq 3) = P(Y > 2) = (0.9)^2 = 0.81$$

## 3.77

$$P(Y = odd) = P(Y = 1, 3, 5, ...) = P(Y = 2k + 1 \text{ for integers } k = 1, 2, ...) = \sum_{k=1}^{\infty} q^{2k+1-1} p$$
$$= p \sum_{k=1}^{\infty} q^{2k} = p \sum_{k=1}^{\infty} (q^2)^k$$
$$= \frac{p}{(1 - q^2)}$$

## 3.93

From Ex. 3.92:

(a)

$$P(Y = 5) = \binom{4}{2}(0.9^3)(0.1^2) = 0.04374$$

(b)

$$P(Y \leq 5) = P(Y = 3) + P(Y = 4) + P(Y = 5) = 0.729 + 0.2187 + 0.04374 = 0.99144$$

## 3.97

(a) Geometric probability:

$$(0.8)^2(0.2) = 0.128$$

(b) Negative binomial probability:

$$\binom{6}{2}(0.2^3)(0.8^4) = 0.049$$

(c) The trials are independent and the probability of success is the same from trial to trial.

(d)

$$\mu = \frac{3}{0.2} = 15, \quad \sigma^2 = \frac{3(0.8)}{0.2^2} = 60$$

## 3.103

Use the hypergeometric distribution with $N = 10, r = 4, n = 5$:

$$P(Y = 0) = \frac{\binom{6}{5}}{\binom{10}{5}} = 0.0238$$

## 3.105

(a) $Y$ follows a hypergeometric distribution. The probability of being chosen on a trial is dependent on previous outcomes.

(b)

$$P(Y \geq 2) = P(Y = 2) + P(Y = 3) = 0.5357 + 0.1786 = 0.7143$$

(c)

$$\mu = 3 \cdot \frac{5}{8} = 1.875, \quad \sigma^2 = 3 \cdot \frac{5}{8} \cdot \frac{3}{8} \cdot \frac{5}{7} = 0.5022, \quad \sigma = 0.7087$$

## 3.121

(a)

$$P(Y = 4) = \frac{\lambda^4 e^{-\lambda}}{4!} = 0.090$$

(b)

$$P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - 0.857 = 0.143$$

(c)

$$P(Y < 4) = P(Y \leq 3) = 0.857$$

(d)

$$P(Y \geq 4 \mid Y \geq 2) = \frac{P(Y \geq 4)}{P(Y \geq 2)} = \frac{0.143}{0.594} = 0.241$$

## 3.123

If $p(0) = p(1)$, then

$$\frac{\lambda^0 e^{-\lambda}}{0!} = \frac{\lambda^1 e^{-\lambda}}{1!}$$

So $\lambda = 1$.

Then

$$p(2) = \frac{1^2 e^{-1}}{2!} = 0.1839$$