

**STAT 5700 — Quiz 8**  
**Date:** November 13, 2025

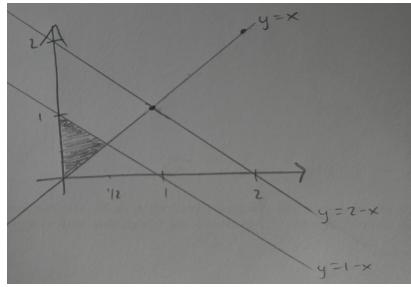
**SOLUTIONS**

**FOR ALL 3 PROBLEMS:** Let  $X$  and  $Y$  be two continuous random variables with joint pdf

$$f(x, y) = 3xy, \quad 0 < x < y, \quad x + y \leq 2$$

**Problem 1 (3pts)** Find  $P(X + Y < 1)$

**SOLUTION**



$$\begin{aligned} P(X + Y < 1) &= \int_0^{1/2} \int_x^{1-x} 3xy \, dy \, dx \\ &= \int_0^{1/2} 3x \left[ \frac{y^2}{2} \right]_{y=x}^{1-x} \, dx \\ &= \int_0^{1/2} \frac{3x}{2} ((1-x)^2 - x^2) \, dx \\ &= \int_0^{1/2} \frac{3x}{2} (1 - 2x) \, dx \\ &= \int_0^{1/2} \left( \frac{3}{2}x - 3x^2 \right) \, dx \\ &= \left[ \frac{3}{4}x^2 - x^3 \right]_0^{1/2} \\ &= \frac{3}{16} - \frac{1}{8} \\ &= \frac{1}{16}. \end{aligned}$$

**Problem 2 (1pt):** Find an expression for the marginal distribution of  $X$ . You can leave your answer written as an integral, but make clear what function you are integrating and what the appropriate bounds are.

$$f_X(x) = \int_x^{2-x} 3xy \, dy$$

**Problem 3 (1pt):** Find an expression for  $f(y|x = \frac{1}{2})$

$$f(y | x = \frac{1}{2}) = \frac{f(0.5, y)}{\int_{0.5}^{1.5} 3(0.5)y \, dy} = \frac{3(0.5)y}{\int_{0.5}^{1.5} 3(0.5)y \, dy} = y, \quad 0.5 < y < 1.5$$

$$f_X(\frac{1}{2}) = \int_{y_{\min}}^{y_{\max}} f_{X,Y}(\frac{1}{2}, t) \, dt$$

$$f(y | x = \frac{1}{2}) = \frac{f_{X,Y}(\frac{1}{2}, y)}{\int_{y_{\min}}^{y_{\max}} f_{X,Y}(\frac{1}{2}, t) \, dt}$$