HW 06 SOLUTIONS

Practice Problems

3.145

$m(t) = E(e^{ty}) = \sum_{y=0}^{n} \binom{n}{y}(pe^t)^y(1-p)^{n-y} = (pe^t + 1 - p)^n$

3.147

$m(t) = E(e^{ty}) = \sum_{y=1}^{\infty} pe^{ty}q^{y-1} = pe^t \sum_{(y-1)=0}^{\infty}(qe^t)^{y-1} = \frac{pe^t}{1-qe^t}$

Note the second-to-last step is because $e^{ty} = (e^t)^{y-1}e^t$ and the final step is recognizing the geometric series $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$.

3.149

This is the moment–generating function for the binomial with n = 3 and p = .6.

3.155

Differentiate to find the necessary moments:

a. $E(Y) = \frac{7}{3}$

b. $V(Y) = E(Y^2) - [E(Y)]^2 = 6 - \left(\frac{7}{3}\right)^2 = \frac{5}{9}$

c. Since $m(t) = E(e^{tY})$, and Y can only take on values 1, 2, and 3 with probabilities $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$ respectively.

Submitted Problems

3.146

$m'(t) = npe^t(pe^t + q)^{n-1}$
At $t = 0$: $E(Y) = np$

$m''(t) = n(n-1)(pe^t + q)^{n-1}(pe^t)^2 + n(pe^t + q)^{n-1}pe^t$
At $t = 0$: $E(Y^2) = n(n-1)p^2 + np$

$V(Y) = E(Y^2) - [E(Y)]^2 = n(n-1)p^2 + np - (np)^2 = np(1-p)$

3.148

$m'(t) = \dfrac{pe^t}{(1-qe^t)^2}$
At $t = 0$: $E(Y) = \dfrac{1}{p}$

$m''(t) = \frac{(1-qe^t)^2pe^t - 2pe^t(1-qe^t)(-qe^t)}{(1-qe^t)^4}$.
At $t = 0$: $E(Y^2) = \dfrac{1+q}{p^2}$

$V(Y) = E(Y^2) - [E(Y)]^2 = \dfrac{1+q}{p^2} - \dfrac{1}{p^2} = \dfrac{q}{p^2}$

3.150

This is the mgf of a geometric distribution with $p = 0.3$.

3.158

If $m_Y(t)$ is the mgf of Y, then
$m_W(t) = E(e^{tW}) = E(e^{t(aY+b)}) = E(e^{bt}e^{(at)Y}) = e^{bt}m_Y(at)$.