HW 02 SOLUTIONS

Practice Problems

2.37

a. There are $6! = 720$ possible itineraries.

b. In the 720 orderings, exactly 360 have Denver before San Francisco and 360 have San Francisco before Denver. So, the probability is 0.5.

2.41

If the first digit cannot be zero, there are 9 possible values. For the remaining six digits, there are 10 possible values each. Thus, the total number is $9 \cdot 10^6$.

2.43

The number of ways to choose 3 objects from 9, 6, 5, and 1 is
$\binom{9}{3}\binom{6}{5}\binom{1}{1} = 504$ ways.

2.51

There are $\binom{50}{3} = 19,600$ ways to choose 3 winners. Each of these is equally likely.

a. There are $\binom{4}{3} = 4$ ways for the organizers to win all of the prizes. The probability is $\frac{4}{19600}$.

b. There are $\binom{4}{2}\binom{46}{1} = 276$
ways the organizers can win two prizes and one of the other 46 people to win the third prize. So, the probability is $\frac{276}{19600}$.

c. There are $\binom{4}{1}\binom{46}{2} = 4140$ ways. The probability is $\frac{4140}{19600}$.

d. There are $\binom{46}{3} = 15,180$ ways. The probability is $\frac{15180}{19600}$.

2.57

There are $\binom{52}{2} = 1326$ ways to draw two cards from the deck. The probability is $\frac{4 \cdot 12}{1326} = 0.0362$.

2.59

There are $\binom{52}{5} = 2,598,960$ ways to draw five cards from the deck.

a. $\binom{4}{1}^5 = 1024$ ways. So, the probability is $\frac{1024}{2{,}598{,}960} = 0.000394$.

b. There are 9 different types of "straight" hands. So, the probability is $\frac{9 \cdot 45}{2{,}598{,}960} = 0.00355$. Note that this includes "straight flush" and "royal straight flush" hands.

2.61

a. $\frac{364(364)(364)\cdots(364)}{365^n} = \frac{364^n}{365^n}$

b. With $n = 253$,
$1 - \left(\frac{364}{365}\right)^{253} \approx 0.5005$.

2.71

a. $P(A|B) = \frac{0.1}{0.3} = \frac{1}{3}$.

b. $P(B|A) = \frac{0.1}{0.5} = \frac{1}{5}$.

c. $P(A|A \cup B) = \frac{0.5}{0.5+0.3-0.1} = \frac{5}{7}$.

d. $P(A|A \cap B) = 1$, since A has occurred.

e. $P(A \cap B|B) = \frac{0.1}{0.5+0.3-0.1} = \frac{1}{7}$.

2.75

a. Given the first two cards drawn are spades, there are 11 spades left in the deck. The probability is $\frac{\binom{11}{3}}{\binom{50}{3}} = 0.0084$. Note: this is also equal to $P(S_3S_4S_5|S_1S_2)$.

b. Given the first three cards drawn are spades, there are 10 spades left. The probability is $\frac{\binom{10}{2}}{\binom{49}{2}} = 0.0383$. Note: this is also equal to $P(S_4S_5|S_1S_2S_3)$.

c. Given the first four cards drawn are spades, there are 9 spades left. The probability is $\frac{\binom{9}{1}}{\binom{48}{1}} = 0.1875$. Note: this is also equal to $P(S_5|S_1S_2S_3S_4)$.

2.79

If A and B are mutually exclusive, $P(A \cap B) = 0$. But, $P(A)P(B) > 0$. So they are not independent.

2.85

$P(A \mid \overline{B}) = P(A \cap \overline{B})/P(\overline{B}) = \dfrac{P(\overline{B} \mid A)P(A)}{P(\overline{B})} = \dfrac{[1 - P(B \mid A)]P(A)}{P(\overline{B})} = \dfrac{[1 - P(B)]P(A)}{P(\overline{B})} = \dfrac{P(\overline{B})P(A)}{P(\overline{B})} = P(A)$. So, A and $\overline{B}$ are independent.

$P(\overline{B} \mid \overline{A}) = P(\overline{B} \cap \overline{A})/P(\overline{A}) = \dfrac{P(\overline{A} \mid \overline{B})P(\overline{B})}{P(\overline{A})} = \dfrac{[1 - P(A \mid \overline{B})]P(\overline{B})}{P(\overline{A})}$. From the above, A and $\overline{B}$ are independent. So $P(\overline{B} \mid \overline{A}) = \dfrac{[1 - P(A)]P(\overline{B})}{P(\overline{A})} = \dfrac{P(\overline{A})P(\overline{B})}{P(\overline{A})} = P(\overline{B})$. So, $\overline{A}$ and $\overline{B}$ are independent.

2.91

If A and B are mutually exclusive, $P(A \cup B) = P(A) + P(B)$. This value is greater than 1 if $P(A) = 0.4$ and $P(B) = 0.7$. So they cannot be mutually exclusive. It is possible if $P(A) = 0.4$ and $P(B) = 0.3$.

Submitted Problems

2.38

By the mn rule, $4 \cdot 3 \cdot 4 \cdot 5 = 240$.

2.42

There are three different positions to fill using ten engineers. Then, there are ${}_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 720$ different ways to fill the positions.

2.46

There are $\binom{10}{2}$ ways to choose two teams for the first game, $\binom{8}{2}$ for the second, etc. So, $\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{10!}{(2!)^5} = 113,400$ ways to assign the ten teams to five games.

2.48

Same answer: $\binom{8}{5} = \binom{8}{3} = 56$.

2.50

Two numbers, 4 and 6, are possible for each of the three digits. So, there are $2 \cdot 2 \cdot 2 = 8$ potential winning three-digit numbers.

2.58

There are $\binom{52}{5} = 2{,}598{,}960$ ways to draw five cards from the deck.

a. To draw three Aces and two Kings: $\binom{4}{3}\binom{4}{2} = 24$ So the probability is $\frac{24}{2{,}598{,}960}$.

b. There are $13 \cdot 12 = 156$ types of full house hands. From part (a), each type can be made in 24 different ways.
So the probability is $\frac{156 \cdot 24}{2{,}598{,}960} \approx 0.00144$

2.64

$6! \left(\frac{1}{6}\right)^5 = \frac{5}{324}$

2.68a–c

a. $\binom{n}{n} = \dfrac{n!}{n!(n-n)!} = 1$. There is only one way to choose all of the items.

b. $\binom{n}{0} = \dfrac{n!}{0!(n-0)!} = 1$. There is only one way to choose none of the items.

c. $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \frac{n!}{(n-r)!(n-(n-r)!)} = \binom{n}{n-r}$. There are the same number of ways to choose r out of n objects as there are to choose $n-r$ out of n objects.

2.72

Note that $P(A) = 0.6$ and $P(A \mid M) = \frac{0.24}{0.4} = 0.6$. So, A and M are independent. Similarly, $P(A' \mid F) = \frac{0.24}{0.6} = 0.4 = P(A')$, so A' and F are independent.

2.74

a. $P(A) = 0.61$, $P(D) = 0.30$, $P(A \cap D) = 0.20$. Dependent.

b. $P(B) = 0.30$, $P(D) = 0.30$, $P(B \cap D) = 0.09$. Independent.

c. $P(C) = 0.09$, $P(D) = 0.30$, $P(C \cap D) = 0.01$. Dependent.

2.80

If $B \subset A$, then $P(A \cap B) = P(A) \neq P(A)P(B)$, unless $B = S$ (in which case $P(B) = 1$).

2.86

a. No. It follows from $P(A \cup B) = P(A) + P(B) - P(A \cap B) \leq 1$.

b. $P(A \cap B) \geq 0.5$

c. No.

d. $P(A \cap B) \leq 0.70$

2.90

a. $(1/50)(1/50) = 0.0004$

b. $P(\text{at least one injury}) = 1 - P(\text{no injuries in 50 jumps}) = 1 - \left(\frac{49}{50}\right)^{50} \approx 0.636$. Your friend is not correct.

2.94

Define the events A: device A detects smoke, B: device B detects smoke.

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.95 + 0.90 - 0.88 = 0.97$

b.$ P(smoke undetected) = 1 - P(A B) = 1 - 0.97 = 0.03$

2.110

Define the events:

- I: item is from line I
- II: item is from line II
- N: item is not defective

Then, $P(N) = P(N \cap (I \cup II)) = P(N \cap I) + P(N \cap II) = 0.92(0.4) + 0.90(0.6) = 0.908$