

STAT 5700 — Quiz 10 SOLUTIONS**Date:** December 11, 2025**FOR BOTH PROBLEMS** Let X, Y be random variables with the following joint pdf:

$$f(x, y) = 6(1 - y), \quad 0 \leq x \leq y \leq 1$$

The following integrals have been evaluated for you and may be useful in the calculations required to solve the problems on this quiz:

- $f_x(x) = \int_x^1 6(1 - y)dy = 3x^2 - 6x + 3$
- $f_y(y) = \int_0^y 6(1 - y)dx = -6y^2 + 6y$
- $E(X) = \int_0^1 x(3x^2 - 6x + 3)dx = 0.25$
- $E(Y) = \int_0^1 y(-6y^2 + 6y)dy = 0.5$
- $E(X^2) = \int_0^1 x^2(3x^2 - 6x + 3)dx = 0.1$
- $E(Y^2) = \int_0^1 y^2(-6y^2 + 6y)dy = 0.3$
- $E(XY) = \int_0^1 \int_x^1 xy6(1 - y)dydx = 0.15$
- $\int_0^{0.5} y6(1 - y)dy = 0.5$

Problem 1 (3pts): Find $V(2X - 3Y)$

$$V(2X - 3Y) = 4V(X) + 9V(Y) + 2(2)(-3)\text{Cov}(X, Y).$$

$$V(X) = E(X^2) - (E(X))^2 = 0.1 - 0.25^2 = 0.0375$$

$$V(Y) = E(Y^2) - (E(Y))^2 = 0.3 - 0.5^2 = 0.05$$

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y) = 0.15 - 0.25(0.5) = 0.025$$

Plugging everything in,

$$V(2X - 3Y) = 4(0.0375) + 9(0.05) - 12(0.025) = 0.30$$

Problem 2 (2pts): Find $E(Y|X = 0.5)$

$$E(Y|X = 0.5) = \int_{0.5}^1 yf(y|x = 0.5)$$

$$f(y|x = 0.5) = \frac{f(0.5, y)}{f_X(0.5)} = \frac{6(1 - y)}{3(0.5)^2 - 6(0.5) + 3} = \frac{1}{0.75}6(1 - y)$$

$$E(Y|X = 0.5) = \frac{1}{0.75} \int_{0.5}^1 y6(1 - y) = \frac{0.5}{0.75} = \frac{2}{3}$$