Chapter 2: Probability
STAT 5700: Probability

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Contents

2.1 Introduction

What is Probability?

Colloquially, a measure of one's belief in the occurrence of a future event.

Probability is a mathematical framework to describe and analyze random phenomenon.

Okay.. so what are random phenomenon?

Anything we cannot predict with certainty

- e.g. weather, score of a soccer game, score on a test, arrival time of a flight, protection of vaccine, whether your roommate will wash the dishes, etc

Concept of probability is necessary in work with physical, biological, or social mechanisms that generate observations that cannot be predicted with certainty.

An alternative definition: probability is the mathematics of uncertainty.

Even if random events can't be predicted with certainty, the relative frequency with which they occur in a long series of trials is often remarkably stable.

A goofy (and absurd) example: Random babies

Courtesy of Allan Rossman (2019)
Imagine that a hospital returns newborn babies to their mothers at random...
Use your intuition to arrange the following events in order, from least likely to most likely:

A. None of the four mothers gets the correct baby.
B. At least one of the four mothers gets the correct baby.
C. All of the four mothers gets the correct baby.

How should we investigate the real order? Start by simulating...

1. Write the first names of 4 babies on your 4 index cards
2. Take a blank sheet of paper and divide it into four sections
3. Write the last name of a mother in each section. Make sure you know which baby matches to which mother - try using alliterations (e.g. Bob Barker)
4. Shuffle the cards (babies) and randomly distribute them to the sections of the sheet (mothers)
5. How many matches occurred? Write this number down
6. Repeat Steps 4-5

Random babies (cont'd)

Class by-hand Simulation results:

Matches	Count	Fraction	Decimal
0			
1			
2			
3			
4			

Computer simulations
Let's do this *many* more times:
http://www.rossmanchance.com/applets/randomBabies/RandomBabies.html

- Determine the approximate probability that **at least one** mother gets the correct baby.

  - Indicate two different ways to determine this.

- Interpret this (approximate) probability.

If $P(A)$ = the long-run proportion of times event A will occur, can we determine that exact limiting value?
First need to determine all the possible outcomes:

1234	1243	1324	1342	1423	1432
2134	2143	2314	2341	2413	2431
3124	3142	3214	3241	3412	3421
4123	4132	4213	4231	4312	4321

- Determine the number of matches for each outcome.

- Then count how many outcomes produce 0 matches, 1 match, and so on.

- Finally, divide by the total number of outcomes to determine the exact probabilities. Express these probabilities as fractions and also as decimals, with three decimal places of accuracy

Matches	Count	Fraction	Decimal
0			
1			
2			
3			
4			

- What is the exact (theoretical) probability that at least one mother gets the correct baby?

- Finally, let's return to our original question... order these from least likely to most likely:

  A. None of the four mothers gets the correct baby.
  B. At least one of the four mothers gets the correct baby.
  C. All of the four mothers gets the correct baby.

So, what is Probability?

Probability = the proportion of outcomes of a random experiment that terminate in the event A as the number of trials of that experiment increases without bound.

If an experiment is repeated n times under essentially identical conditions, and if the event A occurs n_A times, then as n grows large, the ratio n_A/n approaches a fixed limit that is the probability of event A. That is, $P(A) = n_A/n$

> Probability interpreted as the proportion of times an event occurs in the long run

2.2 Probability & Inference

What does probability have to do with inference?

Consider a gambler who wishes to make an inference about the balance of a six-sided die.

Hypothesis: the die is balanced (all 6 sides equally likely to occur; not weighted).

Wants to use real-world observations to evaluate the hypothesis (reject the theory if false).

His data: 10 rolls of the die, all 1s. Rejects the hypothesis because even though ten 1s not an *impossible* result, he judges it to be an *improbable* one.

His judgement is probably based on intuition.

What if he got five 1s, and a mix of other numbers? Is that highly improbable? Or four 1s?

Many events are in a grey area that intuition alone makes it difficult to guess the probability. We need a rigorous method for finding a number (a probability) that will agree with the actual relative frequency of occurrence of an event in a long series of trials.

We need a **theory of probability** that will permit us to calculate the probability of observing specified outcomes, assuming that our hypothesized model is correct.

Set theory is a first set of tools that will help us in constructing probabilistic models for experiments.

2.3 A Review of Set Notation
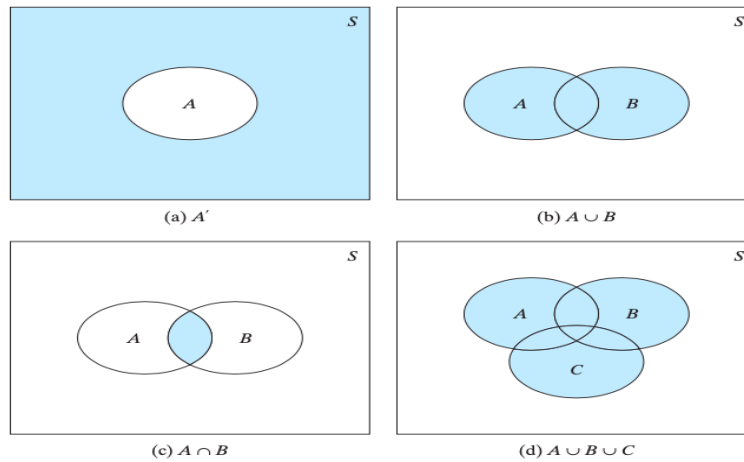
Set Theory / Activity

(a) A'

(b) $A \cup B$

(c) $A \cap B$

(d) $A \cup B \cup C$

Figure 1.1-1 Algebra of sets

Notation / terminology

$A \cup B$ is the **union** of A and B

   Union = event A occurs OR event B occurs OR both

$A \cap B$ is the **intersection** of A and B

   Intersection = event A occurs AND event B occurs

$\emptyset$ denotes the **null** or **empty** set

A' is the **complement** of A (our book uses $\overline{A}$, others use A^c)

   i.e. all elements in S that are not in A

$A \subset B$ means A is a **subset** of B (all elements in A are also in B)

Mutually Exclusive = **Disjoint** = Events that cannot occur simultaneously

$$A \cap B = \emptyset$$

Mutually Exhuastive= a set of events that completely cover the sample space

$$A \cup B \cup C = S$$

Example:

    Event A = stock market closes up today

    Event B = stock market closes down today

    A and B cannot both happen $\implies$ A & B are disjoint

Example:

    Event A = patient's temperature increases

    Event B = patient's temperature decreases

    Event C = patient's temperature stays the same

    $\{A, B, C\}$ encompasses all posibilities $\implies$ mutually exhaustive

Laws of Set Algebra

Commutative Laws

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associative Laws

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributive Laws

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

DeMorgan's Laws

$$(A \cup B)' = A' \cap B'$$
$$(A \cap B)' = A' \cup B'$$

2.4 A Probabilistic Model for an Experiment: The Discrete Case

Some definitions

Experiment = process by which an observation is made.

Random experiment = process of observing the outcome of a chance event (i.e. the outcome of the experiment cannot be predicted with certainty).

> Random babies: process of randomly delivering 4 babies to 4 mothers (1 shuffle in our simulation = 1 random experiment)

Experiments can result in one or more outcomes, which are called **events**, denoted by capital letters (eg. A, B)

> Random babies example outcomes/events:

- A: At least one baby delivered to its correct mother
- B: Less than 4 babies delivered to their correct mothers
- E_0: 0 babies delivered correctly
- E_1: Exactly 1 baby delivered correctly
- E_2: Exactly 2 babies delivered correctly
- E_4: All 4 babies delivered correctly

Note that events can either be **compound** or **simple**. For example, if you observe A (*at least one baby*), you will have also observed either E_1, E_2, or E_4. A is an example of a compound event, because it can be decomposed into other simple events.

A **simple event** is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point.

Sample space = the set of all possible simple outcomes/events for an experiment

> Random babies: {0,1,2,4 matches}

> Sample space = S (or sometimes Ω)

Discrete sample space is one that contains either a finite or countable number of distinct sample points.

An *event* in a discrete sample space S is a collection of sample points - that is, any subset of S.

What is Probability? (more formally)

> **Definition 2.6** Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S), we assign a number $P(A)$, called the **probability** of A, so that the following axioms hold:
>
> 1. $P(A) \geq 0$;
> 2. $P(S) = 1$;
> 3. if $A_1, A_2, A_3, \ldots$ are a series of pairwise mutually exclusive events (i.e., $A_i \cap A_j = \emptyset$, if $i \neq j$), then
>
> $$P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) \cup P(A_2) \cup \ldots \cup P(A_k) = \sum_{i=1}^{k} P(A_i)$$
>
> for each positive integer k, and
>
> $$P(A_1 \cup A_2 \cup A_3 \cup \ldots) = P(A_1) \cup P(A_2) \cup P(A_3) \cup \ldots = \sum_{i=1}^{\infty} P(A_i)$$
>
> for an infinite, but countable, number of events

Note: the definition states only the conditions an assignment of probabilities must satisfy; it does not tell us *how* to assign specific probabilities of events.

Example: Suppose a coin yielded 800 heads in 1000 tosses. Consider the experiment of one more toss of the same coin.

- Two possible outcomes (simple events): heads or tails
- Definition of probability allows us to assign to these two simple events any two non-negative numbers that add to 1.
- Could do 1/2 for each simple event.
- Past data suggests P(heads) = 0.8 and P(tails) = 0.2 might be a more reasonable assignment.

Specific assignments of probabilities must be consistent with reality in order for the probabilistic model to be useful.

Some Probability Laws

> - $P(A) = 1 - P(A')$
>
> - $P(\emptyset) = 0$
>
> - If $A \subset B$, then $P(A) \leq P(B)$
>
> - $0 \leq P(A) \leq 1$
>
> - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Addition rule)

The last law is known as the "Additional Rule"

Must avoid "double counting" the intersection of the two events

Special case: If A and B are disjoint, then $P(A \cup B) = P(A) + P(B)$ because $P(A \cap B) = 0$

2.5 Calculating the Probability of an Event: The Sample-Point Method

For discrete sample spaces, it suffices to assign probabilities to each simple event.

Example: Toss a balanced six-sided die once.

- What is one possible simple event?

- What is the sample space?

- What is a reasonable probability for each sample point in S? Confirm that the first two properties of Definition 2.6 hold.

The sample-point method for calculating probability

1. Define the experiment and clearly determine how to describe one simple event.
2. Define the sample space by listing the simple events of the experiment
3. Assign probabilities to the sample points in S, making certain that $P(E_i) \geq 0$ and $\sum P(E_i) = 1$
4. Define the event of interest, A, as a specific collection of sample points.
5. Find $P(A)$ by summing the probabilities of the sample points in A.

Example:
A fair coin is tossed 3 times, and the sequence of heads and tails is observed. What is the probability that at least two heads is observed?

Example (cont'd): Using the same scenario as above (three coin tosses), define the following additional events (A listed again for completeness):

- $A = \{\text{at least two heads}\}$
- $B = \{\text{at most two heads}\}$
- $C = \{\text{heads on the 2nd toss}\}$
- $D = \{1 \text{ head and } 2 \text{ tails}\}$

Find:

a) $P(B)$

b) $P(A \cap B)$

c) $P(A \cap C)$

d) $P(D)$

e) $P(A \cup C)$

f) $P(B \cap D)$

The sample-point method is direct and powerful, but is a bit of a bulldozer approach. It can be cumbersome to list all the events, and it can be susceptible to human error. Orderliness, computers, and the mathematical theory of counting (combinatorics) can help reduce effort and human error in carrying out the sample-point method for calculating probabilities. (2.6)

2.9 Calculating the Probability of an Event: The Event-Composition Method

We've seen that sets (events) can often be expressed as unions, intersections, or complements of other events. The event-composition method for calculating the probability of an event A, expresses A as a composition involving unions and/or intersections of other events. The laws of probability are then applied to find $P(A)$.

Example: Of a group of patients having injuries, 28% visit both a physical therapist and a chiropractor and 8% visit neither. Say that the probability of visiting a physical therapist exceeds the probability of visiting a chiropractor by 16%. What is the probability of a randomly selected person from this group visiting a physical therapist?

2.6 Tools for Counting Sample Points

Combinatorics can help you calculate the total number of sample points in a sample space S or an event of interest A, which can provide a confirmation (or avoidance) of your listing of simple events. Sometimes counting methods are the only way to calculate a probability if the sample space is very large.

If a sample space S contains N equiprobable sample points and an event A contains exactly n_A sample points, then
$$P(A) = \frac{n_A}{N}$$

Multiplication Rule or "mn" Rule

Example: A sandwich shop offers 3 types of bread and 4 meats. How many different sandwiches are possible?

Hint: think in terms of "slots" for each element of the sandwich, and ask how many ways to fill 1st slot, then 2nd etc.

OR use a "TREE DIAGRAM" to enumerate all possibilities

General Rule

If procedure 1 has m possible outcomes $(a_1, a_2, \ldots a_m)$, procedure 2 has n possible outcomes $(b_1, b_2, \ldots b_n)$, it is possible to form $mn = m \times n$ composite outcomes, containing one element from each group.

This multiplication rule can be extended to any number of sets.

Example: Refer back to the experiment of tossing 3 coins. Use the extension of the mn rule to confirm the number of sample points.

Suppose you randomly select 20 people from a population and record their birthdays $(b_1, b_2, \ldots, b_{19}, b_{20})$. Ignoring leap years and assuming there are only 365 possible distinct birthdays, find the number of points in the sample space S for this experiment.

If we assume each of the possible sets of birthdays is equiprobable, what is the probability that each person in the 20 has a different birthday?

Hint: Define A = set of all 20-tuples where there are no repeat numbers. To answer this question, we need to find the number of sample points in A.

Permutations

We've seen that sample points are often represented by a sequence of numbers or symbols (e.g., HHH). Often, it will be clear that the total number of sample points equals the number of distinct ways that the representative symbols can be arranged in sequence.

Examples

In how many ways can you arrange Art, Bud and Carl in 3 seats?

In how many ways can you arrange Art, Bud, Carl and Dave in 4 seats?

What if there are 10 people and 10 seats - how many arrangements?

Hint: think in terms of "slots" for the seats _ _ _ _ _ _ _ _ _ _ and ask how many ways to fill 1st seat, then 2nd given someone is in 1st etc.

Factorial = Total number of ways to order n (unique) subjects

$$n! = \text{"n factorial"} = n \times (n-1) \times \cdots \times 2 \times 1$$

Example:
What if there are 10 people and only 3 seats - how many arrangements?

Hint: think in terms of "slots" again. . . start with first slot

---

Permutation = an ordered arrangement of r distinct objects. The number of ways of ordering n distinct objects, taken r at a time, is given by

$$_nP_r = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$

In our example, 3 seats (slots) for 10 people: 10 x 9 x 8 x _____ we need to "remove" slots 7, 6, 5,. . .,1 from the factorial:

$$_{10}P_3 = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7 \times \cdots \times 1}{7 \times \cdots \times 1} = 10 \times 9 \times 8$$

The Permutation formula $_nP_r = \frac{n!}{(n-r)!}$ is based on **sampling WITHOUT replacement**

Example:
What if there are 10 people and only 3 seats – but it is a RAFFLE of tickets to a show and each person can win more than one ticket (or all three). How many ways are there to choose 3 tickets in this scenario?

Hint: We choose 1st winner THEN REPLACE THAT NAME IN THE HAT before picking second winner.

For **sampling WITH replacement**: use n^r

$$n^3 = 10 \times 10 \times 10 = 1000$$

Theorem 2.3 The number of ways of partitioning n distinct objects into k distinct groups containing $n_1, n_2, \ldots n_k$ objects, respectively, where each object appears in exactly one group and $\sum_{i=1}^{k} n_i = n$ is

$$N = \binom{n}{n_1 \ \ n_2 \ \ \cdots \ \ n_k} = \frac{n!}{n_1! n_2! \cdots n_k!}$$

Example: A labor dispute has arisen concerning the distribution of 20 laboreres to four different construction jobs. The first job (considered to be very undesireable) requires 6 laborers; the other three jobs required 4, 5, and 5 laborers respectively. The dispute arose over an alleged random distribution of the laborers to jobs that placed all 4 members of a particular ethnic group on job 1. In considering whether the assignment represented injustice, a mediation panel desired the probability of the observed event. Determine the number of sample points in the sample space S for this experiment. That is, determine the number of ways the 20 laborers can be divided into groups of the appropriate sizes to fill all the jobs. Then, find the probability of the observed event if it is assumed that the laborers are randomly assigned to jobs.

Combinations

Sometimes the *order* of sample points in a subset is unimportant, and all that matters is which sample points are chosen.

> **Example**: What if there are 10 people and 3 identical prizes - how many arrangements of prize winners if we don't care about order (i.e we just want to pick 3 people)?
>
> *Hint: for one set of 3 people, how many extra sets do we get if order mattered? That is, think of how many ways to order the letters ABC*

Combination = Total number of ways pick to r (unique) subjects selected from n subjects (also known as a "distinguishable permutation")

$$_nC_r = \frac{_nP_r}{r!} = \frac{n!}{(n-r)!r!}$$

We "divide out" the extra cases where order changes. In our example, choosing 3 of 10 people for each combination of 3 we get 3! "extra" if we allow order to matter. Example, combination ABC: ABC, ACB, BAC, BCA, CAB, CBA = 3! = 6 times counted in Permutation

$$_{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10 \times 9 \times 8}{6}$$
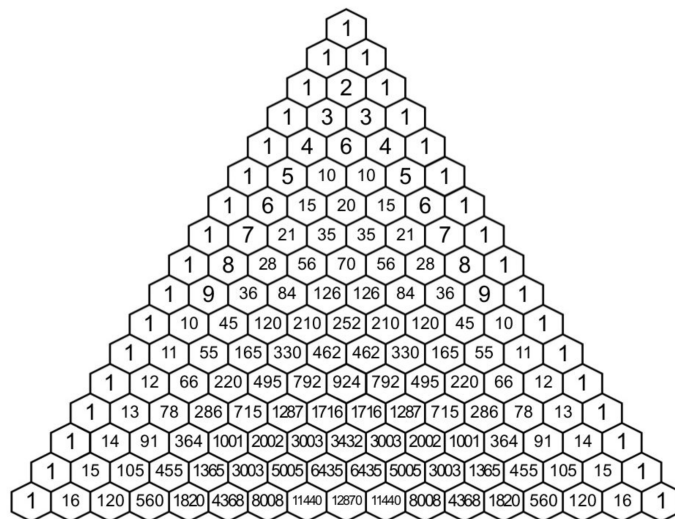
Helpful to "read" the mathematical notation

$$_nC_r = \text{"n choose r"}$$

> Example: Find the number of ways of selecting two applicants out of five.

> Example: How many ways are there to have a 3 daughters among a family of 5 children? In other words, how many ways can 3 "Fs" be placed in 5 "slots"?

Binomial Coefficients & Pascal's triangle

$$_nC_r = \binom{n}{r} = \frac{n!}{(n-r)!r!}$$

Video on Pascal's triangle: https://ed.ted.com/lessons/the-mathematical-secrets-of-pascal-s-triangle-wajdi-mohamed-ratemi

Seeing Theory website: https://seeing-theory.brown.edu/compound-probability/index.html#section2

2.7 Conditional Probability and the Independence of Events

Example:
Roll two dice. Let Event B = the dice add to 4.

$$S =$$

Q1: Before rolling the dice, what is P(B)?

Q2: Suppose I roll one dice and get a 1 (i.e. Event A = get a 1 on first dice). What is the probability of B now that A has occured?

In Q2, we wanted the **conditional probability** - the probability that B will occur given A has already occurred.

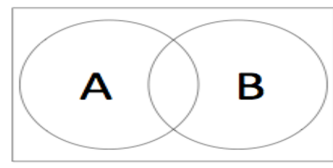$$P(B|A) = \text{probability of } B \textbf{ given } A$$

Rolling a 1 reduced the sample space from 36 possible outcomes to 6:

$$S = \left\{ \; \square\square \; \square\square \; \square\square \; \square\square \; \square\square \; \square\square \; \right\}$$

With only 1 of these being a sum of 4 we get:

$$P(B|A) = \frac{1}{6}$$

What does Conditional Probability look like in a Venn Diagram?

The **conditional probability** of an event B, given that event A has occured, is defined by:
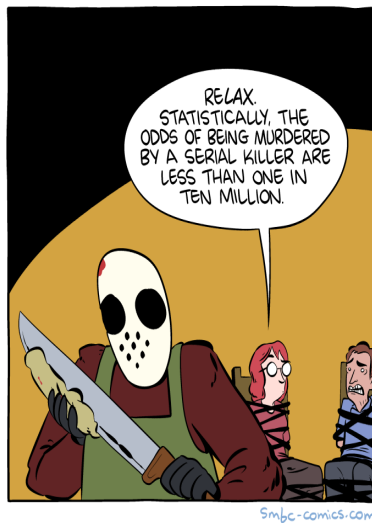
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

Note that $P(A) > 0$

Conditional probabilities satisfy the axioms of Probability:

- $P(B|A) \geq 0$;
- $P(A|A) = 1$;
- $P(B'|A) = 1 - P(B|A)$
- if $B_1, B_2, ...B_k$ are mutually exclusive events then

$$P(B_1 \cup B_2 \cup ... \cup B_k|A) = P(B_1|A) + P(B_2|A) + ... + P(B_k|A)$$

Independence Day Activity (courtesy of Allan Rossman)

In 2025, the New York times released a list of the 100 best movies from the 20th century so far. See their list here. Below is some real data of which of the 100 movies my sister and I have seen.

	Anna has seen the film	Anna has NOT seen the film	Total
Katie has seen the film	27	9	36
Katie has NOT seen the film	20	44	64
Total	47	53	100

Suppose a movie is chosen at random (meaning each movie is equally likely to be chosen)

a) What is the probability that Anna has seen the film?

b) Given the partial information that Katie has seen the film, what is the updated (conditional) probability that Anna has seen it?

c) Does learning that Katie has seen the randomly selected film change the probability that Anna has seen it? In which direction? Why might this make sense?

d) Repeat this analysis based on the following table (of made-up data) for two other people, Nathalia and Joe:

	Joe has seen the film	Joe has NOT seen the film	Total
Nathalia has seen the film	42	28	70
Nathalia has NOT seen the film	18	12	30
Total	60	40	100

e) In which case (Katie-Anna) or (Nathalia-Joe) would it make sense to say that the events are *independent*?

f) Based on these data, would you still say that (Katie having seen the film) and (Anna having seen the film) are dependent events, even if they never saw any films together and perhaps did not even know each other?

Statistical **Independence** = the occurance of another event (A) has **no effect** on the probability of B occurring:

$$P(B|A) = P(B)$$

Suppose we roll two fair, six-sided die. Let's assume one is red and the other is green, so we can tell them apart. Consider these four events:

- A = {green die lands on 6}
- B = {red die lands on 5}
- C = {sum equals 11}
- D = {sum equals 7}.

For each pair of events, determine whether or not the events are independent. Justify your answers with appropriate probability calculations.

Recall that

$$\frac{P(A \cap B)}{P(A)} = P(B|A) \implies P(A \cap B) = P(B|A) \times P(A)$$

IF A and B are independent, this becomes:

$$P(A \cap B) = P(B|A) \times P(A) = P(B) \times P(A)$$

Definition 2.10   Events A and B are said to be **independent** if and only if any of the following holds:

- $P(A \cap B) = P(A)P(B)$
- $P(A|B) = P(A)$
- $P(B|A) = P(B)$

Otherwise, A and B are called **dependent** events.

A few theorems & laws of probability

Theorem 2.5: The Multiplicative Law of Probability The probability of the intersection of two events A and B is

$$P(A \cap B) = P(A)P(B|A)$$
$$= P(B)P(A|B)$$

If A and B are independent, then

$$P(A \cap B) = P(A)P(B)$$

If A and B are independent events, then the following pairs of events are also independent:

  (a) A and B'
  (b) A' and B
  (c) A' and B'

PROOF:

Example: Suppose that you have applied to two internship programs E and F. Based on your research about how competitive the programs are and how strong your application is, you believe that you have a 60% chance of being accepted for program E and an 80% chance of being accepted for program F. Assume that your acceptance into one program is independent of your acceptance into the other program.

a) What is the probability that you will be accepted by both programs?

b) What is the probability that you will be accepted by at least one of the two programs? Show two different ways to calculate this.

c) Suppose you also apply to programs G and H, and you believe you have a 70% and 20% chance of being accepted into each program, respectively. What is the probability that you will be accepted by all four programs? Is this pretty unlikely?

d) What is the probability that you will be accepted by at least one of the four programs? Is this very likely?

e) Explain why the assumption of independence is probably not reasonable in this situation.

2.10 Law of Total Probability & Bayes Rule

Example:
TRUE STORY (from a former colleague)
After giving blood one year, Dr. S. received a letter saying the blood was rejected due to a positive test result for a Hepatitus A virus. The letter further stated this was "nothing to worry about"...?!? Suppose some research reveals that only 1% of people in the U.S. have HepA. The blood screening test is known to be correct 99% of the time when the person has the disease and 95% when the person does not. Should Dr. S. have worried?

Define (two) events:

What probabilities do we know?

What probabilities do we want to know?

Create a tree diagram:

Bayes' Rule

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

Definition 2.10: For some positive integer k, let the sets $B_1, B_2, \ldots B_k$ be such that

1. $S = B_1 \cup B_2 \cup \cdots \cup B_k$
2. $B_i \cap B_j = \emptyset$, for $i \neq j$

Then the collection of sets $\{B_1, B_2, \ldots B_k\}$ is said to be a **partition** of S

Theorem 2.8: Assume that $\{B_1, B_2, \ldots B_k\}$ is a partition of S such that $P(B_i) > 0$ for $i = 1, 2, \ldots, k$. Then for any event A

$$P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$$

Theorem 2.9 (Bayes' Rule, more generally): Assume that $\{B_1, B_2, \ldots B_k\}$ is a partition of S such that $P(B_i) > 0$ for $i = 1, 2, \ldots, k$. Then

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{P(A)} = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}$$

- Numerator is simply re-writing the intersection REVERSING THE CONDITIONAL PROBABILITY
- Denominator is known as the **Law of Total Probability**

Bayes' Terminology

Prior probability - $P(B_j)$ the probability BEFORE we observe information A

Posterior probability - $P(B_j|A)$ the probability AFTER we observe A ... "updated" probability of the event based on "new data/information"

Chapter 2 Group Work Practice

1. How many four-letter code words are possible using the letters in IOWA if:

   a. The letters may not be repeated?
   b. The letters may be repeated?

2. Three students (S) and six faculty members (F) are on a panel discussing a new college policy.

   a. In how many different ways can the nine participants be lined up at a table in front of the auditorium?
   b. How many lineups are possible if you only care about where the students are placed in relation to faculty? In other words, how many ways are there to choose 3 seats to place the students in (or alternatively, choose 6 seats to place the faculty in)?
   c. For each of the nine participants, you are to decide whether the participant did a good job or a poor job stating their opinion of the new policy; that is, give each of the nine participants a grade of G or P. How many different "scorecards" are possible?

3. The NBA Finals continues until either the Western Conference team or the Eastern Conference team wins four games. How many different orders are possible (e.g. WEEWWW means the Western Conferences team wins in six games) if the series lasts. . . .

   a. four games?
   b. five games? *Hint: start by focusing on the number of ways the Western Conference can win in 5. And note that if the West wins in 5, the results are of the form _ _ _ _ W*
   c. six games?
   d. seven games?

4. Suppose that 78% of the students at a particular college have a TikTok account and 43% have a Facebook account. Using only this information,

   a. what is the largest possible value for the percentage who have both a TikTok account and a Facebook account? Describe the (unrealistic) situation in which this occurs.
   b. what is the smallest possible value for the percentage who have both a TikTok account and a Facebook account? Describe the (unrealistic) situation in which this occurs.

	Facebook	No Facebook	Total
TikTok			0.78
No TikTok			0.22
Total	0.43	0.57	1.00

Now assume you know that 36% of students have both a TikTok and a Facebook account.

   c. What percentage of students have at least one of these accounts?
   d. What percentage of students have neither of these accounts?
   e. What percentage of students have one of these accounts but not both?

5. Let A_1 and A_2 be the events that a person is left- eye dominant or right-eye dominant, respectively. When a person folds their hands, let B_1 and B_2 be the events that the left thumb and right thumb, respectively, are on top. A survey in one statistics class yielded the following table:

	B_1	B_2	Totals
A_1	5	7	12
A_2	14	9	23
Totals	19	16	35

If a student is selected randomly, find the following probabilities:
(a) $P(A_1 \cap B_1)$,
(b) $P(A_1 \cup B_1)$,
(c) $P(A_1|B_1)$,
(d) $P(B_2|A_2)$.
(e) If the students had their hands folded and you hoped to select a right-eye-dominant student, would you select a "right thumb on top" or a "left thumb on top" student? Why?

6. Let A and B be independent events. Prove that A' and B are also independent.

7. Suppose that every day you play a lottery game in which a three-digit number is randomly selected. Your probability of winning for each day is 1/1000.

a) Is it reasonable to assume that whether you win or lose is independent from day to day?
b) Determine the probability that you win at least once in a 7-day week. Report your answer with five decimal places. Why is this probability is not exactly equal to 7/1000?
c) Determine the probability that you win at least once in a 365-day year.
d) Suppose that your friend says that because there are only 1000 three-digit numbers, you're guaranteed to win once if you play for 1000 days. How would you respond?
e) Express the probability of winning at least once as a function of the number of days that you play. Also produce a graph of this function, from 1 to 3652 days (about 10 years). How would you describe the function's behavior?
f) For how many days would you have to play in order to have at least a 90% chance of winning at least once? How many years is this?
g) Suppose that the lottery game costs $1 to play and pays $500 when you win. If you were to play for that many days (your answer to the previous part), is it likely that you would end up with more or less money than you started with?

8. Two processes of a company produce rolls of materials: The rolls of Process I are 3% defective and the rolls of Process II are 1% defective. Process I produces 60% of the company's output, Process II 40%. A roll is selected at random from the total output. Given that this roll is defective, what is the conditional probability that it is from Process I?