STAT 5700 — Quiz 6
Date: October 30, 2025

SOLUTIONS

Problem 1 (3pts) Let $f(y) = 3y^2$ for $0 \le y \le 1$.

   a. Find $F(y)$
   b. Find $P(0.4 < Y < 0.7)$
   c. Find the mean of Y

Solution

$f(y) = 3y^2, \ 0 \le y \le 1$.

   (a)

$$F(y) = \begin{cases} 0, & y < 0, \\ \int_0^y 3t^2 \, dt = y^3, & 0 \le y \le 1, \\ 1, & y > 1. \end{cases}$$

   (b)
$$P(0.4 < Y < 0.7) = F(0.7) - F(0.4) = 0.7^3 - 0.4^3 = 0.343 - 0.064 = 0.279.$$

   (c)
$$E[Y] = \int_0^1 y \cdot 3y^2 \, dy = \int_0^1 3y^3 \, dy = \left[\frac{3y^4}{4}\right]_0^1 = \frac{3}{4}.$$

Problem 2 (2pts) Consider a new random variable with the following cdf

$$F(y) = \begin{cases} 0, & y < 0, \\ 1 - e^{-y}, & y \ge 0 \end{cases}$$

   a. Find $f(y)$
   b. Find the 90th percentile of Y

Solution

(a)

$$f(y) = F'(y) = \begin{cases} 0, & y < 0, \\ e^{-y}, & y > 0. \end{cases}$$

(b) 90th percentile: set $F(y) = 0.9$ and solve for y:

$$1 - e^{-y} = 0.9 \implies e^{-y} = 0.1 \implies y = -\ln(0.1) = 2.3026.$$