

25.04.17

$$A = \begin{pmatrix} 1 & 2 & 1 & 2 \\ 0 & 2 & -1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$BA = B + I: \text{ჩვენ } p \neq n \Rightarrow B \text{ } 3 \times 3 \text{ } N \text{ } 4 \times 3 \text{ } N$$

$$A - I = \begin{pmatrix} 0 & 2 & 1 & 2 \\ 0 & 1 & -1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 \end{pmatrix}$$

$$BA - B = I \\ B(A - I) = I \Rightarrow B = (A - I)^{-1}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & | & 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 \leftrightarrow R_2, R_4 - R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 2 & | & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & | & 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_3 - 2R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 & | & 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -3 & | & 0 & -1 & 0 & 1 \end{pmatrix} \xrightarrow{R_3/3, R_4 - R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 1 & -1 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & | & 1/3 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & -3 & | & -1/3 & 1/3 & 0 & 1 \end{pmatrix} \xrightarrow{R_2 + R_3, R_4 / (-3)} \begin{pmatrix} 1 & 0 & 0 & 0 & | & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & | & 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 1 & 0 & | & 1/3 & -2/3 & 0 & 0 \\ 0 & 0 & 0 & 1 & | & 1/9 & 1/9 & 0 & -1/3 \end{pmatrix} \xrightarrow{R_2 - R_4} B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 2/9 & 2/9 & 0 & 1/3 \\ 1/3 & -2/3 & 0 & 0 \\ 1/9 & 1/9 & 0 & -1/3 \end{pmatrix}$$

$$B = \frac{1}{9} \begin{pmatrix} 0 & 0 & 0 & 9 \\ 2 & 2 & 0 & 3 \\ 3 & -6 & 0 & 0 \\ 1 & 1 & 0 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} X = \begin{pmatrix} 2 & 1 & 6 \\ 1 & 1 & 4 \end{pmatrix}$$

$$AX = B \Rightarrow \\ X = A^{-1}B$$

ჩვენ უნდა

$$\begin{pmatrix} 3 & 1 & | & 2 & 1 & 6 \\ 2 & 1 & | & 1 & 1 & 4 \end{pmatrix} \xrightarrow{R_1 - R_2} \begin{pmatrix} 1 & 0 & | & 1 & 0 & 2 \\ 2 & 1 & | & 1 & 1 & 4 \end{pmatrix} \xrightarrow{R_2 - 2R_1} \begin{pmatrix} 1 & 0 & | & 1 & 0 & 2 \\ 0 & 1 & | & -1 & 1 & 0 \end{pmatrix}$$

$$X = \begin{pmatrix} 1 & 0 & 2 \\ -1 & 1 & 0 \end{pmatrix}$$

תרגילי אלגברה

1. $AB = B$
 $AB = A + 6I$
 B, A $\in M_n$

$$AB = B \quad | \cdot B^{-1} \Rightarrow AB \cdot B^{-1} = BB^{-1} \Rightarrow A = I$$

$$I \cdot B = I + 6I \Rightarrow B = 7I$$

2. $B = I - A$ $A^4 = 0$ A 3×3 matrix
 $B^{-1} = I + A + A^2 + A^3$

$$B \cdot B^{-1} = B(I + A + A^2 + A^3)$$

$$I = (I - A)(I + A + A^2 + A^3) = I + A + A^2 + A^3 - A - A^2 - A^3 - A^4 = I$$

3. $\begin{pmatrix} x+y-1 & -2 \\ y & 3 \end{pmatrix}$ 2×2 matrix x, y are parameters

$$\left(\begin{array}{cc|cc} x+y-1 & -2 & 1 & 0 \\ y & 3 & 0 & 1 \end{array} \right) \xrightarrow{(*)} \left(\begin{array}{cc|cc} x+y-1 & -2 & 1 & 0 \\ 0 & 3x+5y-3 & -y & x+y-1 \end{array} \right) \xrightarrow{2R_2 + (3x+5y-3)R_1}$$

$$\left(\begin{array}{cc|cc} x+y-1 & 0 & 3x+5y-3 & 2x+2y-2 \\ 0 & 3x+5y-3 & -y & x+y-1 \end{array} \right)$$

$$3x+5y-3$$

$$(*) \quad \begin{matrix} x+y-1=0 \\ y=1-x \end{matrix} \quad \left(\begin{array}{cc|cc} 0 & -2 & 1 & 0 \\ y & 3 & 0 & 1 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|cc} y & 3 & 0 & 1 \\ 0 & -2 & 1 & 0 \end{array} \right) \xrightarrow{R_1 - 3R_2} \left(\begin{array}{cc|cc} y & 0 & -3 & 1 \\ 0 & 1 & -1/2 & 0 \end{array} \right)$$

$$x=1 \Leftrightarrow y=0$$

4. $AB = -BA$ A, B $n \times n$ matrices

show that $(A-B)^2 = A^2 + B^2$

$$\begin{aligned} (A-B)^2 &= A^2 - AB - BA + B^2 \\ AB^2 &= \dots \\ AB^3 &= \dots \end{aligned}$$

$$(*) \quad (A-B)^2 = A^2 - AB - BA + B^2 = A^2 + B^2 - AB - BA = A^2 + B^2$$

$$((A-B)^2)^t = (A^2 + B^2)^t = (A^2)^t + (B^2)^t = (A^t)^2 + (B^t)^2 = A^2 + B^2 = (A-B)^2$$

$$2) \quad (AB^2)^t = (AB \cdot B)^t = (B^t \cdot A^t \cdot B^t)^t = B^2 \cdot A \cdot (-B) = -AB(-B) = AB^2$$

$$3) \quad (AB^3)^t = (AB \cdot B^2)^t = (B^2 \cdot A^t \cdot B^t)^t = B^2 \cdot A \cdot (-B) = B \cdot BA \cdot (-B) = B \cdot A \cdot B^2 = -AB^3$$

5.

A, B ממשפחה ליניארית ממוננת S'3

(א) אם B הפיכה אז $A+B$ הפיכה $\Leftrightarrow I+B^{-1}A$ הפיכה.(א) נניח: B הפיכה ואם $I+B^{-1}A$ הפיכה

$$B \cdot (I+B^{-1}A) = B+B^{-1}AB = B+A = A+B$$

(ב) נניח: B הפיכה ואם $A+B$ הפיכה

$$B(A+B) = B^{-1}A + B^{-1}B = B^{-1}A + I$$

(א) אם $I+AB$, $I+BA$ הפיכה אז

$$(I+AB)^{-1}A = A(I+BA)^{-1}$$

$$A(I+BA) = (I+AB)A$$

$$A+ABA = A+ABA$$

(ב) אם $A+BB^t$, $I+B^tA^{-1}B$ הפיכה אז

$$(A+BB^t)^{-1}B = A^{-1}B(I+B^tA^{-1}B)^{-1}$$

$$B(I+B^tA^{-1}B) = (A+BB^t) \cdot A^{-1}B$$

$$B + BB^tA^{-1}B = AA^{-1}B + BB^tA^{-1}B$$

$$A^tB = A \cdot A^t$$

אם $A^tB = A \cdot A^t$ אז $AB+BA=0$ (א)

$$A^tB = A \cdot AB = A(-BA) = (-AB)A = -BA \cdot A = -BA^t$$

$$AB = -BA$$

6.

A ממשפחה ליניארית ממוננת S'3, הפיכה.

$$A = \begin{pmatrix} * & -1 & * \\ * & + & 4 \\ 0 & * & * \end{pmatrix} \quad A^{-1} = \begin{pmatrix} * & -3 & * \\ * & + & 1 \\ 1 & * & * \end{pmatrix}$$

$$A = \begin{pmatrix} a & -1 & 0 \\ -1 & b & 4 \\ 0 & 4 & c \end{pmatrix} \quad A^{-1} = \begin{pmatrix} x & -3 & 1 \\ -3 & y & 1 \\ 1 & * & z \end{pmatrix}$$

$$AA^{-1} = I \Rightarrow \begin{pmatrix} a & -1 & 0 \\ -1 & b & 4 \\ 0 & 4 & c \end{pmatrix} \cdot \begin{pmatrix} x & -3 & 1 \\ -3 & y & 1 \\ 1 & * & z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$(1,2) \quad -3a - y = 0$$

$$(1,3) \quad a - 1 = 0 \Rightarrow a = 1 \quad y = -3$$

$$(2,2) \quad 3 + yb + 4 = 1 \Rightarrow -3b = -6 \quad b = 2$$

$$(2,3) \quad -1 + b + 4z = 0 \Rightarrow z = -1/4$$

$$(1,1) \quad ax + 3 = 1 \Rightarrow x = -2$$

$$(3,2) \quad 4y + c = 0 \Rightarrow c = 12$$

נ.

$$X \in S'3 \text{ ממוננת} \quad B^{-1}X(B^{-1})^t = A^tA$$

$$X = BA^tA \cdot B^t \Rightarrow X^t = (BA^tAB^t)^t = (B^t)^t \cdot A^t \cdot (A^t)^t \cdot B^t = BA^tAB^t$$

נניח