

Assignment 2

If you wish to submit your solutions to any of these questions, please send them via email to your TA by 18/04/2021. This deadline is strict!

Exercise 1 Three identical chests, each with two drawers are examined. One has a gold coin in each of its two drawers, another has a silver coin in each drawer and the third has a silver coin in one drawer and a gold coin in the other. A chest is chosen uniformly at random, and in that chest a drawer is chosen uniformly at random. Given that the chosen coin is gold, what is the probability that the second drawer of the chosen chest contains a gold coin as well?

Exercise 2 The weather at a given planet can be either cloudy or clear, with a constant probability. In 60% of the cloudy days, the next day was clear and in 30% of the clear days, the next day was cloudy. What percent of the days are cloudy?

Exercise 3 Let S be a set with n elements. Out of the 2^n subsets of S two sets A and B are randomly chosen with replacement.

1. What is the probability that $|A| = k$?
2. Use your solution to Part 1 of this question to find the probability that A is a subset of B .

Exercise 4 Bowl I contains 2 black balls and a single white ball. Bowl II contains a single black ball and 3 white balls. A bowl is chosen uniformly at random and then a ball is chosen uniformly at random from that bowl.

1. What is the probability that the chosen ball is white?
2. What is the probability that Bowl I was chosen, given that the chosen ball is white?
3. The chosen ball is placed back into its bowl and a new ball is chosen uniformly at random from the same bowl. What is the probability that the second chosen ball is white, given that the first one was white?
4. Same as Part 3 except that the first ball that was chosen is not placed back into any of the bowls.

Exercise 5 Prove that an event is independent of all other events if and only if its probability is either 0 or 1.

Exercise 6 Prove the following statements:

1. Event A is independent of itself if and only if $\mathbb{P}(A) \in \{0, 1\}$.
2. If the events A and B are mutually exclusive and independent then either $\mathbb{P}(A) = 0$ or $\mathbb{P}(B) = 0$.
3. If the events A and B are independent it follows that A and B^c are independent.

Exercise 7 For each of the four equations listed below, give an example of a probability space and three events A, B, C in this space which will uphold the other three equations but not this one.

1. $\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B)$
2. $\mathbb{P}(A \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(C)$
3. $\mathbb{P}(B \cap C) = \mathbb{P}(B) \cdot \mathbb{P}(C)$
4. $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A) \cdot \mathbb{P}(B) \cdot \mathbb{P}(C)$