שם הקורס: חשבון אינפיניטסימלי 1

שבוע 12

הנושא : אינטגרל הלא מסוים:

1. חשב את האינטגרלים הבאים בשימוש טבלת אינטגרלים:

1 $\int \left(3x^2-4\right)^2 dx$

2 $\int \left(6x^2+8x-7\right)dx$

3 $\int \left(x^2-1\right)\left(x+1\right)dx$

4 $\int \frac{dx}{\left(5x-2\right)^{5/2}}$

5 $\int \frac{dx}{\left(4x-5\right)^7}$

6 $\int \frac{x^2+4x}{x+2}dx$

7 $\int \frac{xdx}{\left(x-1\right)^3}$

8 $\int \frac{2^x+5^x}{10^x}dx$

9 $\int \frac{dx}{\sqrt{x+1}-\sqrt{x}}$

10 $\int \frac{xdx}{\sqrt{x+1}-1}$

11 $\int \frac{xdx}{\sqrt{x+1}}$

12 $\int \left(e^x+e^{-1}\right)^2 dx$

13 $\int \frac{3x+2}{x+3}dx$

14 $\int \frac{4dx}{x^2+2x-3}$

15 $\int \frac{5dx}{4x^2+8x+7}$

16 $\int \frac{\left(x^2+1\right)dx}{x^3+3x+3}$

17 $\int a^x e^x dx$

18 $\int \left(\frac{1-z}{z}\right)^2 dz$

19 $\int \frac{dx}{\sqrt{3-3x^2}}$

20 $\int \mathrm{ctg}^2\, x dx$

21 $\int \mathrm{tg}^2\, x dx$

22 $\int \frac{\left(1-x\right)^3}{x\sqrt[3]{x}}dx$

23 $\int \frac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}dx$

24 $\int \frac{\sqrt{x^2+1}+\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$

25 $\int \left(\frac{a}{x}+\frac{a^2}{x^2}+\frac{a^3}{x^3}\right)dx$

26 $\int \left(1-\frac{1}{x^2}\right)\sqrt{x\sqrt{x}}\,dx$

27 $\int \frac{\left(\sqrt{2x}-\sqrt[3]{3x}\right)^2}{x}dx$

28 $\int \frac{2^{x+1}-5^{x-1}}{10^x}dx$

29 $\int \frac{\sqrt{x^4+x^{-4}+2}}{x^3}dx$

30 $\int \frac{e^{3x}+1}{e^x+1}dx$

31 $\int \sqrt{1-\sin 2x}dx$

32 $\int \frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx$

תשובות:22. $\left(1+\frac{3}{2}x-\frac{5}{3}x^2+\frac{1}{8}x^3\right)$**, 23.** $-\frac{3}{\sqrt[3]{x}}$**,** $\frac{4}{5}x\sqrt[4]{x}-\frac{24}{17}x\sqrt[12]{x^5}+\frac{4}{3}\sqrt[4]{x^3}$ **.**

$$,-\frac{2}{\ln 5}\left(\frac{1}{5}\right)^x+\frac{1}{5\ln 2}\left(\frac{1}{2}\right)^x \textbf{.28} \quad ,2x-\frac{12}{5}\sqrt[6]{72x^5}+\frac{3}{2}\sqrt[3]{9x^2} \qquad \textbf{.27} \quad ,\frac{4\left(x^2+7\right)}{7\sqrt[4]{x}} \textbf{.26} \quad ,a\ln|x|-\frac{a^2}{x}-\frac{a^3}{2x^2} \textbf{.25}$$

$$,\frac{1}{2}e^{2x}-e^x+x \textbf{.30} \quad ,\ln|x|-\frac{1}{4x^4} \textbf{. 29}$$

$$,\arcsin x+\ln\left(x+\sqrt{1+x^2}\right) \textbf{.32} \quad ,\left(\cos x+\sin x\right)\cdot\operatorname{sgn}\left(\cos x-\sin x\right) \textbf{.31}$$

2. חשב את האנתגרלים הבאים: $\left((a\neq 0) \qquad \int f\left(ax+b\right)dx=\frac{1}{a}F\left(ax+b\right)+C\right)$

$\int e^{4-3x}dx$ **.1**	$\int\cos\left(5-3x\right)dx$ **.2**	$\int 5^{2x-3}dx$ **.3**
$\int\frac{xdx}{a+bx}$ **.4**	$\int\frac{\sqrt[5]{1-2x+x^2}}{1-x}dx$ **.5**	$\int\frac{dx}{2+3x^2}$ **.6**
$\int\frac{dx}{2-3x^2}$ **.7**	$\int\frac{dx}{\sqrt{2-3x^2}}$ **.8**	$\int\frac{dx}{\left(5x-2\right)^{\frac{5}{2}}}$ **.9**
$\int\frac{dx}{\sqrt{3x^2-2}}$ **.10**	$\int\left(\sin 5x-\sin 5a\right)dx$ **.11**	$\int\left(e^{-x}+e^{-2x}\right)dx$ **.12**
$\int\frac{dx}{\sin^2\left(2x+\frac{\pi}{4}\right)}$ **.13**	$\int\left[\sinh\left(2x+1\right)+\cosh\left(2x-1\right)\right]dx$ **.14**	$\int\frac{dx}{\cosh^2\frac{x}{2}}$ **.15**

תשובות: 5. $,-\frac{5}{2}\sqrt[5]{\left(1-x\right)^2}$ **.6** $,\frac{1}{\sqrt{6}}ar\operatorname{ctg}\left(x\sqrt{\frac{3}{2}}\right)$ **.7** $,\frac{1}{2\sqrt{6}}\ln\left|\frac{\sqrt{2}+x\sqrt{3}}{\sqrt{2}-x\sqrt{3}}\right|$ **.8** $,\frac{1}{\sqrt{3}}\arcsin\left(x\sqrt{\frac{3}{2}}\right)$

$,-\frac{2}{15\left(5x-2\right)^{3/2}}$ **.9** $,\frac{1}{\sqrt{3}}\ln\left|x\sqrt{3}+\sqrt{3x-2}\right|$ **.10** $,-x\sin 5a-\frac{1}{5}\cos 5x$ **.11** $,-\left(e^{-x}+\frac{1}{2}e^{-2x}\right)$ **.12**

$,-\frac{1}{2}\operatorname{ctg}\left(2x+\frac{\pi}{4}\right)$ **.13** $,\frac{1}{2}\left|\cosh\left(2x+1\right)+\sinh\left(2x-1\right)\right|$ **.14** $2\operatorname{tg}h\frac{x}{2}$ **.15**

3. חשב את האינטגרלים הבאים (שיטת ההצבה):

$\int\frac{\left(\ln x\right)^3dx}{x}$ **1**	$\int\sin^3\cos xdx$ **2**	$\int\operatorname{tg}xdx$ **3**

4 $\int \operatorname{ctg} x dx$	**5** $\int \frac{dx}{1+\sqrt{x+1}}$	**6** $\int \frac{dx}{\sqrt{x}\left(1+\sqrt[3]{x}\right)}$
7 $\int \frac{e^x dx}{\sqrt{e^x-1}}$	**8** $\int \frac{dx}{\sqrt{8+6x-9x^2}}$	**9** $\int \frac{dx}{\sqrt{4x+3-x^2}}$
10 $\int e^{\arccos} \cdot \frac{dx}{\sqrt{1-x^2}}$	**11** $\int \sin^3 2x dx$	**12** $\int \frac{ar\operatorname{ctg} x dx}{1+x^2}$
13 $\int x^2 \sqrt[3]{1-x} dx$	**14** $\int x^2\left(1-5x^2\right)^{10} dx$	**15** $\int \frac{x^2}{\sqrt{2-x}} dx$
16 $\int \frac{x^5}{\sqrt{1-x^2}} dx$	**17** $\int x^5\left(2-5x^3\right)^{2/3} dx$	**18** $\int \cos^5 x \cdot \sqrt{\sin x} dx$
19 $\int \frac{\sin x \cos^3 x}{1+\cos^2 x} dx$	**20** $\int \frac{\sin^2 x}{\cos^6 x} dx$	**21** $\int \frac{\ln x dx}{x\sqrt{1+\ln x}}$
22 $\int \frac{dx}{e^{x/2}+e^x}$	**23** $\int \frac{dx}{\sqrt{1+e^x}}$	**24** $\int \frac{ar\operatorname{ctg}\sqrt{x}}{\sqrt{x}} \cdot \frac{dx}{1+x}$

תשובות: **13.** $-\frac{3}{140}\left(9+12x+14x^2\right)(1-x)^{4/3}$, **14.** $-\frac{1+55x^2}{6600}\left(1-5x^2\right)^{11}$, **15.** $-\frac{2}{15}\left(32+8x+3x^2\right)\sqrt{2-x}$,

16. $-\frac{1}{15}\left(8+4x^2+3x^4\right)\sqrt{1-x^2}$, **17.** $-\frac{6+25x^3}{1000}\left(2-5x^3\right)^{5/3}$, **18.** $\left(\frac{2}{3}-\frac{4}{7}\sin^2 x+\frac{2}{11}\sin^4 x\right)\sqrt{\sin^3 x}$,

19. $-\frac{1}{2}\cos^2 x+\frac{1}{2}\ln\left(1+\cos^2 x\right)$, **20.** $\frac{1}{3}\operatorname{tg}^3 x+\frac{1}{5}\operatorname{tg}^5 x$, **21.** $\frac{2}{3}\left(-2+\ln x\right)\sqrt{1+\ln x}$,

22. $-x-2e^{-x/2}+2\ln\left(1+e^{x/2}\right)$, **23.** $x-2\ln\left(1+\sqrt{1+e^x}\right)$, **24.** $\left(ar\operatorname{ctg}\sqrt{x}\right)^2$

4. חשב את האנתגרלים הבאים (אינטגרציה בחלקים):

1 $\int xe^{-x} dx$	**2** $\int x\cos x dx$	**3** $\int x^n \ln x dx$
4 $\int \arccos x dx$	**5** $\int x \cdot ar\operatorname{ctg} x dx$	**6** $\int x^3 e^x dx$
7 $\int e^x \cdot \cos x dx$	**8** $\int \sin\ln x dx$	**9** $\int x^3 e^{x^2} dx$

$\int \left(\frac{\ln x}{x}\right)^2 dx$ **10** $\int \sqrt{x}\ln^2 x dx$ **11** $\int x^2 \sin 2x dx$ **12**

$\int x^2 \arccos x dx$ **13** $\int \frac{\arcsin x}{x^2} dx$ **14** $\int \ln\left(x+\sqrt{1+x^2}\right) dx$ **15**

$\int x \ln\frac{1+x}{1-x} dx$ **16** $\int \operatorname{arctg}\sqrt{x} dx$ **17** $\int \sin x \cdot \ln(\operatorname{tg} x) dx$ **18**

תשובות: **10.** $-\frac{1}{x}\left(\ln^2 x + 2\ln x + 2\right)$, **11.** $\frac{2}{3}x^{3/2}\left(\ln^2 x - \frac{4}{3}\ln x + \frac{8}{9}\right)$, **12.** $-\frac{2x^2-1}{4}\cos 2x + \frac{x}{2}\sin 2x$,

13. $-\frac{2+x^2}{9}\sqrt{1-x^2} + \frac{x^3}{3}\arccos x$, **14.** $-\frac{\arcsin x}{x} - \ln\left|\frac{1+\sqrt{1-x^2}}{x}\right|$, **15.** $x\ln\left(x+\sqrt{1+x^2}\right) - \sqrt{1+x^2}$,

16. $x - \frac{1-x^2}{2}\ln\frac{1+x}{1-x}$, **17.** $-\sqrt{x} + (1+x)\,arctg\sqrt{x}$, **18.** $\ln\operatorname{tg}\frac{x}{2} - \cos x \cdot \ln\operatorname{tg} x$

5. חשב את האנטגרלים הבאים (פונקציות רציונליות):

$\int \frac{dx}{4x^2+4x+3}$ **1** $\int \frac{dx}{4x^2-9}$ **2** $\int \frac{dx}{(3+x)(4-x)}$ **3**

$\int \frac{dx}{4x^2+20x+25}$ **4** $\int \frac{xdx}{2x^2-3x-2}$ **5** $\int \frac{x^5+x^4-8}{x^3-4x} dx$ **6**

$\int \frac{3x+2}{2x^2+3x+4} dx$ **7** $\int \left(\frac{x+2}{x-1}\right)^2 \frac{dx}{x}$ **8** $\int \frac{dx}{x^2-x^4}$ **9**

$\int \frac{dx}{x(x^2+1)}$ **10** $\int \frac{7-8x}{2x^2-3x+1} dx$ **11** $\int \frac{dx}{x^3-2x^2+x}$ **12**

$\int \frac{(7x^2+26x-9)dx}{(x^2+2x+3)(x^2+2x-3)}$ **13** $\int \frac{dx}{\sqrt{5-2x+x^2}}$ **14** $\int \frac{(8x-11)dx}{\sqrt{5+2x-x^2}}$ **15**

$\int \frac{x^{10}dx}{x^2+x-2}$ **16** $\int \frac{x^3+1}{x^3-5x^2+6x} dx$ **17** $\int \frac{x^2+1}{(x+1)^2(x-1)} dx$ **18**

$\int \frac{x^2+5x+4}{x^4+5x^2+4} dx$ **19** $\int \frac{dx}{(x+1)(x^2+1)}$ **20** $\int \frac{xdx}{(x-1)^2(x^2+2x+2)}$ **21**

$$\int \frac{dx}{x^4+1} \quad \textbf{24} \qquad \int \frac{dx}{x^4-1} \quad \textbf{23} \qquad \int \frac{xdx}{x^3-1} \quad \textbf{22}$$

תשובות: **16.** $\frac{x^9}{9}-\frac{x^8}{8}+\frac{3x^7}{7}-\frac{5x^6}{6}+\frac{11x^5}{5}-\frac{21x^4}{4}+\frac{43x^3}{3}-\frac{85x^2}{2}+171x+\frac{1}{3}\ln\left|\frac{x-1}{(x+2)^{1024}}\right|$,

17. $x+\frac{1}{6}\ln|x|-\frac{9}{2}\ln|x-2|+\frac{28}{3}\ln|x-3|$, **18.** $\frac{1}{x+1}+\frac{1}{2}\ln|x^2-1|$, **19.** $ar\operatorname{ctg}x+\frac{5}{6}\ln\frac{x^2+1}{x^2+4}$,

20. $\frac{1}{2}arc\operatorname{ctg}x+\frac{1}{4}\ln\frac{(x+1)^2}{x^2+1}$, **21.** $-\frac{1}{5(x-1)}+\frac{1}{50}\ln\frac{(x-1)^2}{x^2+2x+2}-\frac{8}{25}ar\operatorname{ctg}(x+1)$,

22. $\frac{1}{6}\ln\frac{(x-1)^2}{x^2+x+1}+\frac{1}{\sqrt{3}}ar\operatorname{ctg}\frac{2x+1}{\sqrt{3}}$, **23.** $\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}ar\operatorname{ctg}x$,

24. $\frac{1}{4\sqrt{2}}\ln\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}+\frac{1}{2\sqrt{2}}ar\operatorname{ctg}\frac{x\sqrt{2}}{1-x^2}$

6. מצא את האינטגרלים הבאים (פונקציות רציונליות מהצורה $R(\cos x,\sin x)$):

1 $\int\cos^2 xdx$	**2** $\int\cos x\cdot\sin 3xdx$	**3** $\int\cos x\cdot\cos 2xdx$
4 $\int\sin x\cdot\sin 5xdx$	**5** $\int\frac{dx}{1-\cos x}$	**6** $\int\frac{dx}{1+\cos 2x}$
7 $\int\frac{\cos^3 x}{\sin^4 x}dx$	**8** $\int\sin^5 xdx$	**9** $\int\sin^4 xdx$
10 $\int\sin^{10}x\cdot\cos^3 xdx$	**11** $\int\tan^5 xdx$	**12** $\int\frac{\sin^5 x}{\cos^4 x}dx$
13 $\int\sin^2 x\cdot\cos^3 xdx$	**14** $\int\frac{dx}{\sin^4 x\cdot\cos^2 x}$	**15** $\int\frac{dx}{1-\sin x}$
16 $\int\frac{dx}{5-4\sin x+3\cos x}$	**17** $\int\frac{dx}{\sin^2 x+\sin 2x-\cos^2 x}$	**18** $\int\frac{dx}{4\sin x+3\cos x+5}$
19 $\int\frac{dx}{3+5\cos x}$	**20** $\int\sin x\cdot\sin 2x\cdot\sin 3xdx$	**21** $\int\frac{dx}{1+\sin x+\cos x}$
22 $\int\frac{dx}{\sqrt[4]{5-x}+\sqrt{5-x}}$	**23** $\int\sqrt{2+x^2}dx$	**24** $\int\sqrt{3-2x-x^2}dx$

25. נתון $g(0)=\frac{1}{5}\ ,g'(x)=\sin^3 x\cos^2 x$

מצא: $g\left(\frac{\pi}{2}\right)$

26. נתון $g(0)=\frac{1}{2}\ln 3\ ,g'(x)=\frac{e^x+1}{e^x+2}$

מצא: $g(1)$

27. נתון $h(1)=1\ ,h'(x)=\frac{1}{x\left(x^2+1\right)}$

מעא: $h(2)$