Assignment 4

If you wish to submit your solutions to any of these questions, please send them via email to your TA by 23/05/2021. This deadline is strict!

Exercise 1 Let $A_1, \ldots, A_n$ be events in some probability space $(\Omega, \mathbb{P})$. For every $1 \leq i \leq n$, let 1_{A_i} denote the indicator of A_i (i.e., $1_{A_i} = 1$ if A_i occurs and $1_{A_i} = 0$ otherwise). Prove that the events $A_1, \ldots, A_n$ are mutually independent if and only if the random variables $1_{A_1}, \ldots, 1_{A_n}$ are mutually independent.

Exercise 2 Prove that the Hypergeometric distribution is in fact a probability distribution.

Exercise 3 The number of cars crossing a particular bridge is a random variable with Poisson distribution with an average of $\lambda = 0.3$ cars per minute, i.e., if X counts the number of cars crossing the bridge in any given minute, then $X \sim \mathrm{Poi}\,(0.3)$. Calculate the probability that within 5 minutes:

1. No cars have crossed the bridge.
2. More than one car has crossed the bridge.
3. The number of cars that have crossed the bridge is between 1 and 3.
4. Exactly 3 cars have crossed the bridge.

Exercise 4 Let $X \sim \mathrm{Geom}\,(p)$ and $Y \sim \mathrm{U}\,(\{0, 1, \ldots, n\})$ be independent random variables. Find the probability distribution of $Z := X + Y$.

Exercise 5 A fair coin with 0 on one side and 1 on the other side is tossed. If the outcome of the coin toss is 0, a fair die is rolled, otherwise the coin is tossed again. Let X and Y be the outcomes of the first and second experiments, respectively. Calculate the joint distribution of X and Y.

Exercise 6 A coin with probability p to come up heads, is tossed indefinitely, all coin tosses being mutually independent. For every positive integer r, let X_r be the number of tosses until the rth time the outcome of the coin toss is heads.

1. Calculate the distribution of X_r for every positive integer r.
2. Calculate the joint distribution of X_r and X_s for every $1 \leq r < s$.