Probability Theory 2 – Exercise sheet V

If you wish to submit your solutions to any of these questions, please hand them to your TA during the practical session held in the week starting on 26/12/2021. This deadline is strict!

1. Let $X \sim U([1,2])$ be a random variable with the continuous uniform distribution. Calculate $\mathbb{E}(X^n)$ for every positive integer n.

2. Let $X_1, \ldots, X_n$ be independent exponentially distributed random variables with parameters $\lambda_1, \ldots, \lambda_n$, respectively. Let $X = \min\{X_i : 1 \leq i \leq n\}$ and let $\lambda = \sum_{i=1}^n \lambda_i$. Prove that X is exponentially distributed with parameter λ.

3. A fair 6-sided die is rolled 420 times, all dice rolls being mutually independent. Let S denote the sum of the resulting numbers.

   (a) Use Chebyshev's inequality to find a lower bound on $Pr(1400 \leq S \leq 1540)$.

   (b) Use the central limit theorem (CLT) to estimate $Pr(1400 \leq S \leq 1540)$.

4. Let $X_1, \ldots, X_{1200}$ be mutually independent random variables such that, for every $1 \leq i \leq 1200$, X_i is uniformly distributed over the segment $[0,1)$. For every $1 \leq i \leq 1200$, Let Y_i be a rounding of X_i to the nearest integer, i.e.

$$Y_i = \begin{cases} 0 & \text{if } X_i < 1/2 \\ 1 & \text{if } X_i \geq 1/2 \end{cases}$$

   Use the Central Limit Theorem to estimate the following probabilities:

   (a) $Pr\left(\left|\sum_{i=1}^{1200} X_i - \sum_{i=1}^{1200} Y_i\right| \leq 10\right)$.

   (b) $Pr\left(\sum_{i=1}^{1200} |X_i - Y_i| > 310\right)$.