

Probability Theory 2 – Exercise sheet IV

If you wish to submit your solutions to any of these questions, please hand them to your TA during the practical session held in the week starting on 12/12/2021. This deadline is strict!

1. Suppose that A is a Monte-Carlo randomized algorithm for some problem P , which, given any input, outputs a correct solution with probability at least $1/2$, and whose running time on inputs of size n is $f(n)$. Suppose also that, for any input x of A , the size of the output of $A(x)$ is not larger than the size of x . Finally, suppose that B is a deterministic algorithm which, given a potential solution for P of size m , verifies its correctness in time $g(m)$, where g is a non-decreasing function. Devise a Las-Vegas algorithm C for P whose expected running time is $O(f(n) + g(n))$.
2. For what values of the parameter C are the following functions probability density functions?

(a) $f : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$f(x) = \begin{cases} C(4x - 2x^2) & \text{if } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(b) $g : \mathbb{R} \rightarrow \mathbb{R}$ is defined by

$$g(x) = \begin{cases} Ce^{-x/100} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

3. Let X be a random variable with probability density function f_X and let $Y = aX$ for some real number $a > 0$.
 - (a) Find f_Y , the probability density function of Y , in terms of f_X .
 - (b) Prove that $f_{-X}(x) = f_X(-x)$ for every $x \in \mathbb{R}$.
 - (c) Prove that X and $-X$ have the same cumulative probability function if and only if $f_X(x) = f_X(-x)$ for every $x \in \mathbb{R}$.