- Consider a married woman making labor supply decisions between the ages of 45 and 64.

- Husband's income stream $\{y_t\}$ is observed with perfect foresight, and the husband works each period.

- The wife's income if she works is given by

$$w_t = \exp(r_0 + r_1 s + r_2 k + r_3 k_t^2 + \xi_t),$$

  Where s_t is the wife's years of school, k_t is the years of experience, and ξ_t is randomly distributed $\mathrm{N}(0, \sigma_\xi^2)$.

- The husband and wife face a two-part tax system over joint income. Up to income amount L, the couple pays taxes at rate τ_1, and any income above L is taxed at rate τ_2. Assume $\tau_2 \geq \tau_1$.

- Define $L_t^W = \max(\{L - y_t,\ 0\})$, which is the income for the wife at which second tax rate τ_2 is used for the wife's income in the for the wife.

- Define tax functions for the husband and wife as follows:

$$T_t^H(y_t) = \begin{Bmatrix} (1-\tau_1)y_t & \text{if} \quad y_t < L \\ (1-\tau_1)L + (1-\tau_2)(y_t - L) & \text{if} \quad y_t \geq L \end{Bmatrix}$$

$$T_t^W(w_t) = \begin{Bmatrix} (1-\tau_1)w_t & \text{if} \quad w_t < L_t^W \\ (1-\tau_1)L_t^W + (1-\tau_2)(w_t - L_t^W) & \text{if} \quad w_t \geq L_t^W \end{Bmatrix}$$

- If the wife works at time t, the utility is given by

$$U_1(y_t, w_t, P_{t-1}, s, k_t) = T_t^H(y_t) + T_t^W(w_t) + \beta \mathbb{E} V_{t+1}(k_t + 1, 1)$$

- If the wife does not work, the utility is given by

$$U_0(y_t, w_t, P_{t-1}, s, k) = \alpha_1 + (1 + \alpha_2)T_t^H(y_t) + \alpha_3 s + \alpha_4 k + \alpha_5 P_{t-1} + \beta \mathbb{E} V_{t+1}(k_t, 0).$$

- The wife will decide to work if

$$U_1(y_t, w_t, P_{t-1}, s, k_t) \geq \; U_0(y_t, w_t, P_{t-1}, s, k_t),$$

Which implies

$$T_t^W(w_t) \geq U_0(y_t, w_t, P_{t-1}, s, k_t) - \beta \mathbb{E} V_{t+1}(k_t + 1, 1) - T_t^H(y_t).$$

- The cut-off for whether the wife works or not ξ_t^* gives wife's wage:

$$w_t^* = \exp(r_0 + r_1 s + r_2 k + r_3 k_t^2 + \xi_t^*)$$

And solves

$$T_t^W(w_t^*) = U_0(y_t, w_t, P_{t-1}, s, k_t) - \beta \mathbb{E} V_{t+1}(k_t + 1, 1) - T_t^H(y_t).$$

- We can solve for ξ_t^* with the following two-step process.

1. First assume, ξ_t^* is low enough such that $w_t^* \leq L_t^W$, and solve for

$$\xi_t^* = \log\left(\frac{U_0(y_t, w_t, P_{t-1}, s, k_t) - \beta \mathbb{E} V_{t+1}(k_t + 1, 1) - T_t^H(y_t)}{1 - \tau_1}\right) - r_0 + r_1 s + r_2 k + r_3 k_t^2.$$

   Then, check whether the assumption that $w_t^* \leq L_t^W$ is true. If true, then this is ξ_t^*. If not true, then go on to next step.

2. If ξ_t^* as calculated before forces $w_t^* \geq L_t^W$, then calculate ξ_t^* using the other part of the tax formula:

$$\xi_t^* = \log\left(\frac{U_0(y_t, w_t, P_{t-1}, s, k_t) - \beta \mathbb{E} V_{t+1}(k_t + 1, 1) - T_t^H(y_t) - (\tau_2 - \tau_1) L_t^W}{1 - \tau_2}\right)$$
$$- r_0 + r_1 s + r_2 k + r_3 k_t^2.$$

This two step procedure only works because $\tau_2 \geq \tau_1$.

- Let ξ_t^0 be such that $L_t^W = \exp(r_0 + r_1 s + r_2 k + r_3 k_t^2 + \xi_t^0)$.
- Furthermore, let $\xi_t^{**} = \max\{(\xi_t^0,\ \xi_t^*)\}$.
- The expected value function is then as follows:

$$
\begin{aligned}
&\mathbb{E}V_t(k_t, P_{t-1}) = \mathbb{E}\left[\mathbf{1}(\xi_t > \xi_t^*)\left(T_t^H(y_t) + T_t^W(w_t) + \beta\mathbb{E}V_{t+1}(k_t+1, 1)\right) + \mathbf{1}(\xi_t \leq \xi_t^*)U_0(y_t, w_t, P_{t-1}, s, k)\right] \\
&= \mathbb{P}(\xi_t > \xi_t^*)\left(T_t^H(y_t) + \beta\mathbb{E}V_{t+1}(k_t+1, 1)\right) \\
&\qquad\qquad + \mathbb{P}(\xi_t > \xi_t^*)\mathbb{E}[T_t^W(w_t) | \xi_t > \xi_t^*] \\
&\qquad\qquad + \mathbb{P}(\xi_t \leq \xi_t^*)U_0(y_t, w_t, P_{t-1}, s, k) \\
&= \mathbb{P}(\xi_t > \xi_t^*)\left(T_t^H(y_t) + \beta\mathbb{E}V_{t+1}(k_t+1, 1)\right) + \mathbb{P}(\xi_t > \xi_t^*)\mathbb{E}[(1-\tau_1)w_t | \xi_t > \xi_t^*] \\
&\qquad\qquad - \mathbb{P}(\xi_t > \xi_t^{**})\mathbb{E}[(\tau_2 - \tau_1)(w_t - L) | \xi_t > \xi_t^{**}] \\
&\qquad\qquad + \mathbb{P}(\xi_t \leq \xi_t^*)U_0(y_t, w_t, P_{t-1}, s, k) \\
&= \left(1 - \Phi(\xi_t^*)\right)\left(T_t^H(y_t) + \beta\mathbb{E}V_{t+1}(k_t+1, 1)\right) \\
&\qquad\qquad + \Phi\left(0.5\sigma_\xi^2\right)\left(1 - \Phi\left(\left(\xi_t^* - \sigma_\xi^2\right)\sigma_\xi^{-1}\right)\right)(1-\tau_1)\exp(r_0 + r_1 s + r_2 k + r_3 k_t^2) \\
&\qquad\qquad - \Phi\left(0.5\sigma_\xi^2\right)\left(1 - \Phi\left(\left(\xi_t^{**} - \sigma_\xi^2\right)\sigma_\xi^{-1}\right)\right)(\tau_2 - \tau_1)\exp(r_0 + r_1 s + r_2 k + r_3 k_t^2) \\
&\qquad\qquad + \left(1 - \Phi(\xi_t^{**})\right)(\tau_2 - \tau_1)L_t^W \\
&\qquad\qquad + \Phi(\xi_t^*)U_0(y_t, w_t, P_{t-1}, s, k).
\end{aligned}
$$