

Force Sensing

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Abstract

INTRODUCTION

EXPERIMENTAL SETUP

Our experimental system consists of between 20 and 300 $^9\text{Be}^+$ ions confined to a single-plane Coulomb crystal in a Penning trap, described in Fig. 1 and (ref). The trap is characterized by an axial magnetic field $B = 4.45$ T and an axial trap frequency $\omega_z = 2\pi \times 1.57$ MHz. A stack of cylindrical electrodes generates a harmonic confining potential along their axis. Radial confinement is provided by the Lorentz force from $\vec{E} \times \vec{B}$ -induced rotation in the axial magnetic field. Time varying potentials applied to eight azimuthally segmented electrodes generate a rotating wall potential that controls the crystal rotation frequency ω_r , typically between $2\pi \times 172$ kHz and $2\pi \times 190$ kHz. The spin-1/2 system is the $^2S_{1/2}$ ground state of the valence electron spin $|\uparrow\rangle (|\downarrow\rangle) \equiv |m_s = +1/2\rangle (|m_s = -1/2\rangle)$. In the magnetic field of the Penning trap, the ground state is split by 124 GHz. A resonant microwave source provides an effective transverse field, which we use to perform global rotations of the spin ensemble with a Rabi frequency of 8.3 kHz. The T2 spin echo coherence time is 15 ms. Optical transitions to the $^2P_{3/2}$ states are used for state preparation, Doppler cooling, and projective measurement.

By measuring the decrease in the composite Bloch vector length produced by the application of a homogenous spin-dependent force, detailed information about the motional state of the ions is revealed. This spin dephasing is directly measured. The spin-dependent optical dipole force (ODF) is generated from the interference of a pair of detuned lasers, shown in Fig. 1a. The ODF couples the spin and motional degrees of freedom through the interaction

$$H_{ODF} = U \sum_i \sin(\delta k \cdot z_i - \mu t + \phi) \sigma_i^z \quad (1)$$

where z_i is the position operator for ion i and $\mu/2\pi$ is the ODF laser beatnote frequency. This can be approximated as

$$H_{ODF} = U \sum_i \sin(\delta k \cdot z_i) \cos(\mu t + \phi) \sigma_i^z$$

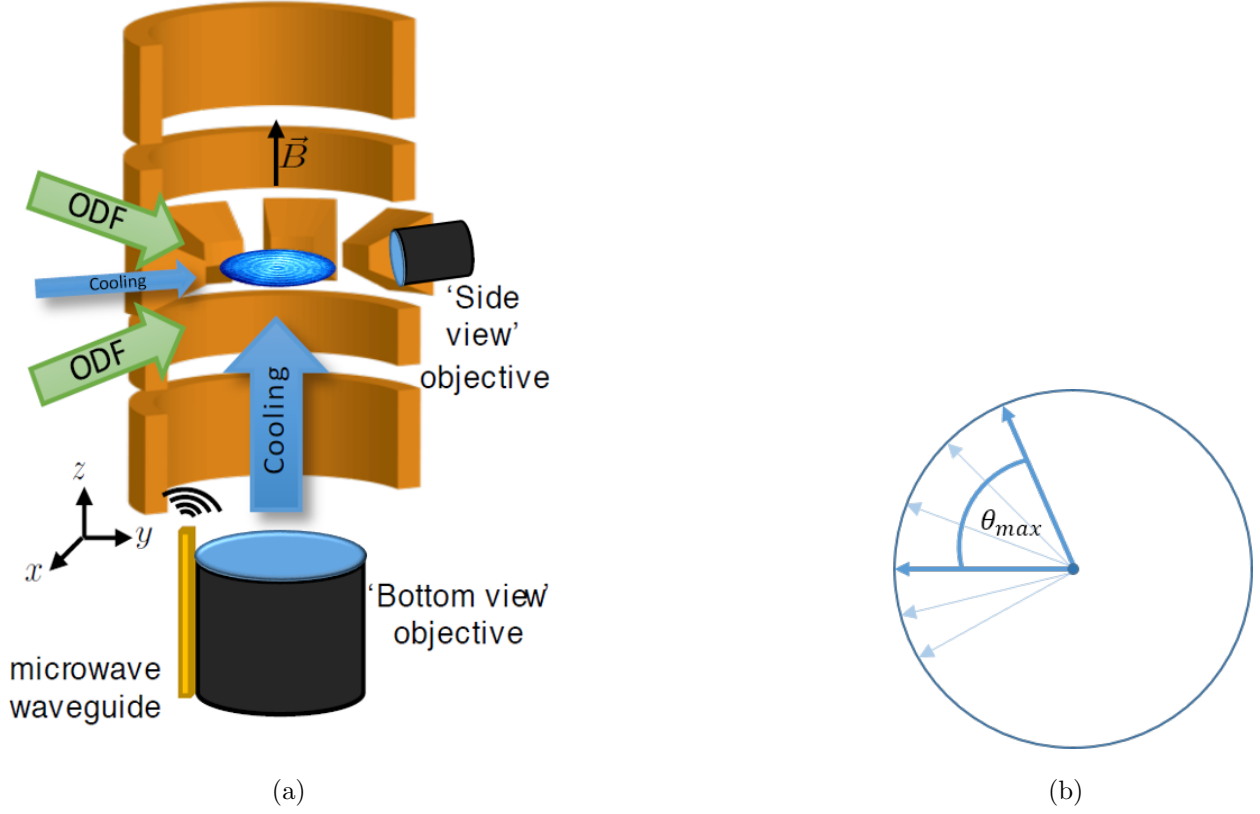


FIG. 1

To find the precession due to an axial oscillation, consider a classical driven motion of constant amplitude and phase, $z_i \rightarrow z_i + z_0 \cos(\omega t + \delta)$.

Then,

$$H_{ODF} = DWF \cdot U \cdot \delta k \cdot z_0 \cos((\omega - \mu)t + \delta + \phi) \sum_i \frac{\sigma_i^z}{2}$$

,

where $DWF = \exp(-\delta k^2 \langle z_i^2 \rangle / 2)$.

For $\omega = \mu$,

$$\theta = \Delta \tau = DWF \cdot U \cdot \delta k \cdot z_0 \tau \cos(\delta - \phi) = \theta_{max} \cos(\delta - \phi)$$

where Δ is the energy difference between spin-up and spin-down for each ion.

We do our experiments phase incoherently (provide some motivation, ala Ozeri paper? or just explain that this is a limitation? probably both), but can extract the maximum precession angle, θ_{max} . In a Ramsey sequence where the Bloch vector with no precession

is rotated down to the dark state, the component along $-\hat{z}$ has length $\cos(\theta)$. Thus the probability for measuring spin-up is $P_{\uparrow} = \frac{1}{2}[1 - e^{-\Gamma\tau} \cos(\theta)]$, where τ is the total ODF interaction time and $\Gamma = \frac{1}{2}(\Gamma_{el} + \Gamma_{ram})$ is the spontaneous decay rate where Γ_{el} (Γ_{ram}) is the elastic (Raman) scattering decoherence rate (Uys PRL ref). To model the phase incoherent experiment, one must average over this length, $\langle \cos(\theta) \rangle$. It can be shown that $\langle \cos(\theta) \rangle = J_0(\theta_{max})$, where J_0 is the zeroth order Bessel function of the first kind. Thus,

$$\langle P_{\uparrow} \rangle = \frac{1}{2}[1 - e^{-\Gamma\tau} J_0(\theta_{max})]. \quad (2)$$

To create an axial oscillation, we apply an AC voltage (RF potential?) to the endcap of the Penning trap. This can be done either on-resonance or off-resonance with the center of mass mode of the ion crystal at $2\pi \times 1.57$ MHz. To calibrate the displacement of the ions as a result of this applied drive, we apply a static voltage to the endcap and measure the deflection of the ion crystal. From this calibration, we find 1 mV results in 1 nm of displacement.

OFF-RESONANCE AMPLITUDE SENSING

By applying a 400kHz drive to the endcap electrode, we can study the response of the ion harmonic oscillator to an off resonant force. A spin echo sequence is used to decouple from magnetic field fluctuations over the course of the experimental sequence. To extend the sequence out to long time in order to detect more sensitively, a CPMG style sequence is used. Our experimental sequence makes use of the quantum lock-in technique wherein the phase accumulated in each arm of the sequence is added coherently. Figure () shows a CPMG sequence with 2 π pulses and phase jumps appropriate for adding the phases. Adding in 6 additional π pulses and ODF pulses allows us to push the sequence time out to 20 ms and maintain a manageable background. For this experiment, the drive is left on continuously.

To model the lineshape of the signal, it is necessary to account for the accumulated phase due to the spin-dependent ODF potential without making the simplification that $\omega = \mu$. This results in a characteristic response function for each sequence. For the 8 π pulse CPMG sequence

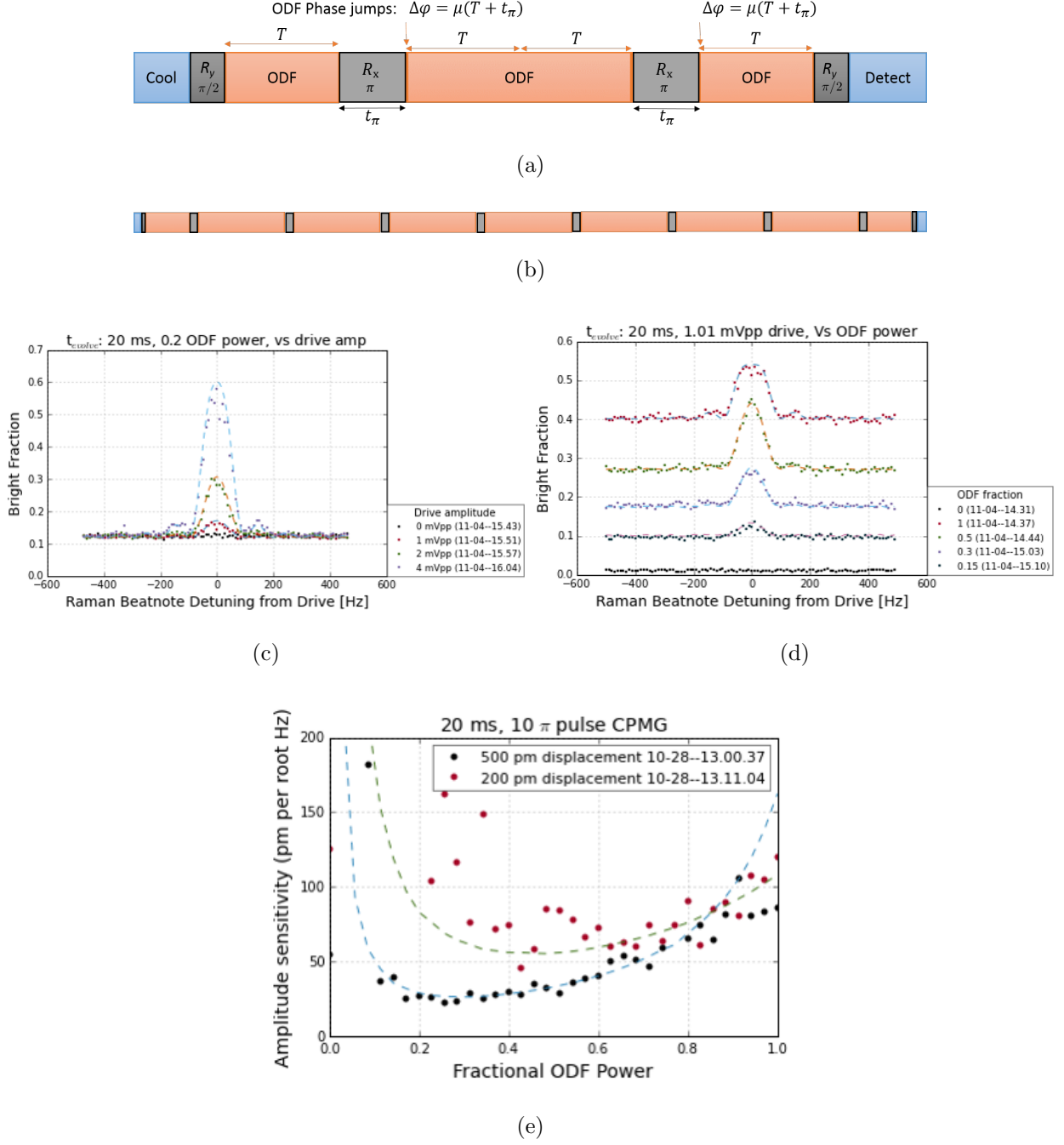


FIG. 2

$$\theta_{tot} = \theta_{max} \left[\frac{2 \sin(\frac{1}{2}[(\omega - \mu)T])}{\omega - \mu} \right] \left[\sin(\frac{\omega}{2}(T + t_\pi)) \right] \left[\sin(\frac{1}{2}[\omega(T + t_\pi) + (\omega - \mu)T]) \right] \left[\cos(\omega(T + t_\pi) + (\omega - \mu)T) \right] \left[\cos(2\omega(T + t_\pi) + (\omega - \mu)T) \right]. \quad (3)$$

With the drive frequency chosen to correspond to the peak signal, scanning the power of

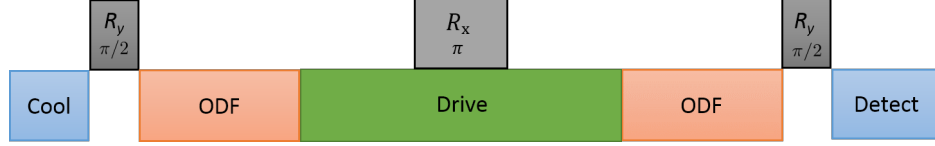
the ODF laser varies the strength of the measurement. For particular drive amplitudes, by comparing the curve to the background, the signal and the signal-to-noise can be extracted. From this, the sensitivity of the sequence is found.

To calculate the signal-to-noise ratio, a value for θ_{max} needs to be extracted. Taking the difference between $\langle P_{\uparrow} \rangle$ with the classical drive applied and $\langle P_{\uparrow} \rangle_{bckgnd}$ with no drive and solving for $J_0(\theta_{max})$ yields

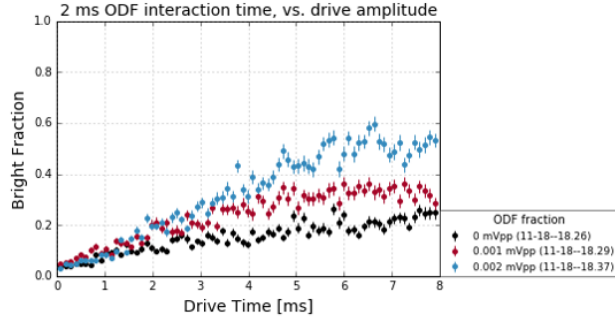
$$J_0(\theta_{max}) = 1 - 2e^{\Gamma\tau}[\langle P_{\uparrow} \rangle - \langle P_{\uparrow} \rangle_{bckgnd}]$$

The signal-to-noise for a single experiment is given by $S/N = \theta_{max}/\delta\theta_{max}$, where $\delta\theta_{max} \equiv \delta J_0(\theta_{max})/(\frac{dJ_0(\theta_{max})}{d\theta_{max}})$. To get the per-root-Hz sensitivity, the amplitude of the displacement - gotten from the calibration with a known applied drive voltage - and the measurement bandwidth is used. $sense = z_d\sqrt{\tau}/SNR$.

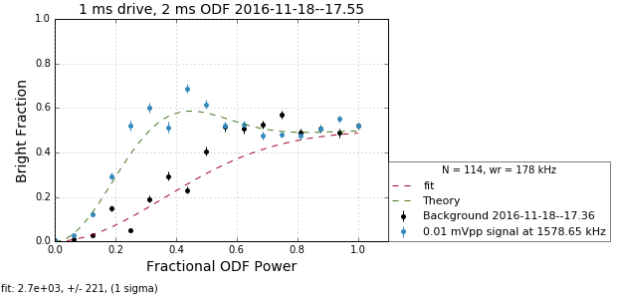
ON-RESONANCE AMPLITUDE SENSING



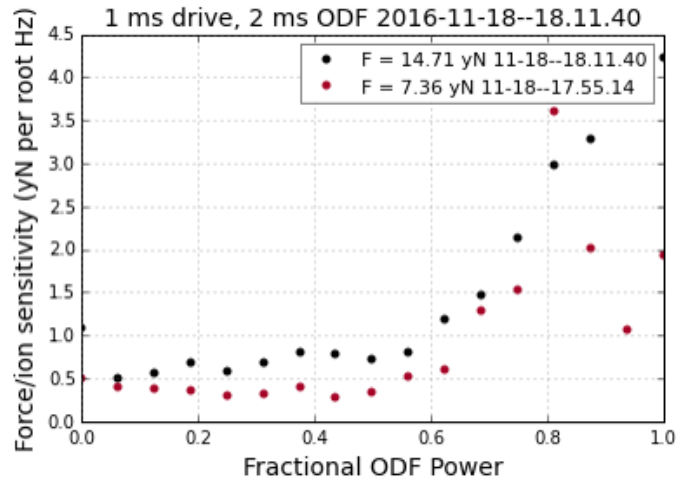
(a)



(b)



(c)



(d)

FIG. 3