## Dear Dr. Narducci,

Thank you very much for your correspondence on our manuscript (LC16484) "Amplitude sensing below the zero-point fluctuations with a two-dimensional trapped-ion mechanical oscillator". We thank the referees for their valuable comments and questions. We are pleased to receive feedback from the referees and their support for the publication of our work. With the comments of both referees in mind, we have revised our manuscript accordingly. In general, we have clarified language used in the manuscript that proved to be confusing. We have added elaboration and clarification regarding the scheme we present, in particular concerning the spin-motion coupling. We have added a brief discussion at the end of the manuscript addressing back action. We detail these changes below in our direct response to Referee A's numbered comments. Additionally, following Referee B's suggestion, we have added a reference to Non-Neutral Plasma Physics VIII, AIP Conf. Proc. **1521**, 200-209 (2013) in the introduction. In the following we provide a point-by-point response to Referee A's comments and questions. Referee A's comments are italicized.

The authors could elaborate more on the measurement principle in general, and specifically on the coupling between the spin and the position of the atoms.

- 1. As explained above, please provide more information about the scheme itself.
- 2. The spins couple to the position of the ions, not to their motion, correct?

We have made changes to better elucidate our measurement technique in response to the first two points, as well as the initial general suggestion. On page 2, we have modified the paragraph leading into Eq. (2) to the following:

"Equation (1) describes a dependence of the spin transition frequency on the axial position of the ions and the ODF frequency  $\mu$ . We excite a small, classically driven COM motion of constant amplitude  $\hat{z}_i \to \hat{z}_i + Z_c \cos(\omega t + \delta)$  with a weak RF drive on a trap endcap electrode (see Fig. 1(a)) at a frequency  $\omega$  far from  $\omega_z$ . If  $\omega \sim \mu$ , Eq. (1) produces an approximately constant shift in the spin transition frequency. With  $\delta k Z_c \ll 1$ , this shift is given by"

Yes, the ions couple to the position of the ions and not their velocity. This is explicit in Eq. (1), and we now have adapted Referee A's language from their report and included this in the above paragraph.

3. Why do you stress in the abstract that the scheme enables a 'discrete' measurement of one quadrature?

Our choice of 'discrete' was intended to draw a distinction between the weak, continuous measurements typically used in cavity-optomechanics style experiments and our measurement technique, which involves accumulating phase during a coupling between spin and axial position, and subsequently reading out this phase with a measurement of the spin state.

We agree with the referee that our use of the term 'discrete' was not clear. We have removed this term from the abstract and introduction and now offer the following clarification in the introduction:

"In contrast to the continuous measurement typical of optomechanics experiments, we measure

the spin precession only at the end of the experimental sequence, with a precision imposed by spin projection noise [21]."

4. When talking about 'single measurements' and 'measurement trials' it is unclear what is meant. Consecutive measurements of the same system, or applying once the experimental CPMG-sequence?

We appreciate that our choice of language to describe iterations of the experimental sequence was confusing and not well-defined. We have removed the phrases 'single measurement' and 'measurement trials'. In particular, we clarify that we previously meant by 'single measurement' is **applying once the experimental CPMG sequence**. We have altered the start of the last paragraph on page 2:

"We allow the phase  $\delta$  to randomly vary from one iteration of the CPMG sequence to the next, effectively measuring a random quadrature of the motion for each experimental trial."

We now use 'experimental trial' consistently through the paper to mean an application of the experimental sequence.

5. Measurement schemes operating close to quantum limits should show the transition from being dominated by imprecision to being dominated by back action noise. As the stated aim of this work is approach quantum limits, I was missing the corresponding discussion in the manuscript (except for the one sentence hinting at reference 31). Which processes lead to the imprecision, where is the current measurement positioned when comparing imprecision, back action and technical noise as a function of measurement strength?

As we now explain more clearly in the text, with the off-resonant sensing experiment we describe, back action is a negligible effect. The imprecision of our measurement is set by spin projection noise and off-resonant light scattering that reduces the length of the Bloch vector - we document this in Fig. 4 and Eq. (5) and subsequent discussion in the text. In the future we plan to pursue experiments that sense COM motion resonant with the trap axial frequency  $\omega_z$ . With the measurement strength that we demonstrate, this on-resonance sensing will be subject to back action. We believe the control and measurement capabilities of the trapped ion platform make it an ideal platform to explore quantum limits and the interplay between back action and measurement imprecision. We thank the referee for bringing up this potentially very interesting future direction of this work. We are sure there will be surprises, but we have added a short discussion at the end of the manuscript that summarizes our current understanding of a source of back action:

"By sensing COM motion far from resonance, we are able to calibrate the measurement imprecision of our protocol in the absence of thermal noise and back action. Probing on resonance with a measurement imprecision below  $z_{ZPT}$  will be sensitive to thermal fluctuations and back action due to spin-motion entanglement [19]. This motivates the investigation of potential back-action-evading protocols with trapped ion set-ups. For the phase coherent measurement of a single quadrature, back action due to spin-motion entanglement can be evaded through the introduction of the appropriate correlations between spin and motion [34]."

Additionally, we have made a few small wording changes for clarity and brevity's sake, and to stay

within the word limit. In particular, we have made Fig. (4) smaller and rephrased our discussion on page two regarding the Debye-Waller factor:

"The Debye-Waller factor  $DWF=\exp(-\delta k^2\left<\hat{z}_i^2\right>/2)$  reduces  $F_0$  due to the departure from the Lamb-Dicke confinement regime [24];  $DWF\approx 0.86$  for the conditions of this work."

We thank the editor and referees for their consideration of our revised manuscript.

Sincerely,

Kevin Gilmore

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