1

Simpson rule $n=2$

$$\int_0^{2\pi} (x-1)\cos^2 x\,dx = \frac{h}{3}\left(f(0)+4f(1)+f(2)\right)$$

$$h = \frac{b-a}{n} = \frac{2\pi-0}{2} = \pi$$

$$f(0) = (0-1)\cos^2(0) = -1$$

$$f(1) = (\pi-1)\cos^2(\pi) = (\pi-1)$$

$$f(2) = (2\pi-1)\cos^2(2\pi) \quad (2\pi-1$$

$$\frac{\pi}{3}\left(-1+4(\pi-1)+(2\pi-1)\right)$$

$$\frac{\pi}{3}(6\pi-6) = \boxed{13.4560}$$

Simpson rule $n=4$

$$\int_0^{2\pi} (x-1)\cos^2(x)\,dx = \frac{h}{3}\left(f(0)+4f(1)+2f(2)+4f(3)+f(\right.$$

$$h = \frac{(b-a)}{n} = \frac{2\pi-0}{4} \qquad \frac{\pi}{2}$$

$$f(0) = (0-1)\cos^2(0) = -1$$

$$f(1) = \left(\tfrac{\pi}{2}-1\right)\cos^2\left(\tfrac{\pi}{2}\right) = 0$$

$f(x_2) = (\pi - 1)\cos(\pi) = \pi - 1$

$f(x_3) = \left(\frac{3}{2}\pi - 1\right)\cos\left(\frac{3\pi}{2}\right) = 0$

$f(x_4) = (2\pi - 1)\cos(2\pi) = 2\pi - 1$

$\frac{\pi}{b}\left(-1 + 0 + 2(\pi - 1) + 0 + 2\pi - 1\right)$

$\frac{\pi}{b}(4\pi - 4) = 4.4853$

(2) $\int_{-1}^{1} f(x)\,dx = w_1 f(x_1) + w_2 f(x_2)$

$$\int_a^b f(z)\,dz = \int_{-1}^{1} f\left(\frac{(b-a)(x)+(b+a)}{2}\right)\frac{b-a}{2}\,dx$$

from (9.b)

$w_1 = 1$, $w_2 = 1$, $f\left(-\sqrt{\frac{1}{3}}\right)$ $f\left(\sqrt{\frac{1}{3}}\right)$

$$f\left(-\sqrt{\frac{1}{3}}\right) = \left(\frac{2\pi\left(-\sqrt{\frac{1}{3}}\right)+2\pi}{2}-1\right)$$

$$\cos^2\left(\frac{2\pi\left(-\sqrt{\frac{1}{3}}\right)+2\pi}{2}\right)\frac{2\pi}{2} = .05962$$

$$f\left(\sqrt{\frac{1}{3}}\right) = \left(\frac{2\pi\left(\sqrt{\frac{1}{3}}\right)+2\pi}{2}-1\right)$$

$$\cos^2\left(\frac{2\pi\left(\sqrt{\frac{1}{3}}\right)+2\pi}{2}\right)\frac{2\pi}{2} = .71944$$

$$1(.05962) + 1(.71944) = \boxed{.7791}$$

$N=3$

$w_1 = .\overline{55}$, $w_2 = .\overline{88}$ $w_3 = .55$ $f\left(-\sqrt{\frac{3}{5}}\right)$, $f(0)$, $f\left(\sqrt{\frac{3}{5}}\right)$

$$f\left(-\sqrt{\frac{3}{5}}\right) = \left(\frac{2\pi\left(-\sqrt{\frac{3}{5}}\right)+2\pi}{2}-1\right)$$

$$\cos^2\left(\frac{2\pi\left(-\sqrt{\frac{3}{5}}\right)+2\pi}{2}\right)\frac{2\pi}{2} = -.5290$$

$$f(0) = \left(\frac{2\pi(0)+2\pi}{2}-1\right)\cos^2\left(\frac{2\pi(0)+2\pi}{2}\right)\frac{2\pi}{2}$$

$= 6.7280$

$$f\left(\sqrt{\tfrac{3}{5}}\right) = \left(2\pi\left(\sqrt{\tfrac{3}{5}}\right) + 2\pi - 1\right)$$

$$\cos^2\left(\frac{2\pi\left(\sqrt{\tfrac{3}{5}}\right) + 2\pi}{2}\right)\frac{2\pi}{2} = 8.2927$$

$$.55(-.5290) + .88(6.7280)$$
$$+ .55(8.2927) = \boxed{10.2936}$$