Q no 1

Given points

$(1,3), (2,7), (-1,1)$
$(x_0,y_0)\ (x_1,y_1)\ (x_2,y_2)$

Now

we have the standard form

Given by

$P_n(x) = a_0 + a_1 x + \cdots + a_n x^n$

since we only have 3 points,

our standard form will be

$P_2(x) = a_0 + a_1 x + a_2 x^2$ where a_0, a_1 & a_2

are to be determined.

Now,

$P_2(x_0) = a_0 + a_1 x_0 + a_2 x_0^2 = y_0$

$P_2(x_1) = a_0 + a_1 x_1 + a_2 x_1^2 = y_1$

$P_2(x_2) = a_0 + a_1 x_2 + a_2 x_2^2 = y_2$

Then, we can obtain.

$$M = \begin{bmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \end{bmatrix}, \quad a = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix}$$

$$Ma = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \end{bmatrix}$$

we can solve the above equation to obtain a,

Hence,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ 1 \end{bmatrix}$$

or, $$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Hence, $P_2(x) = 1 + x + x^2$, is required system of polynomial of degree 2.

Again writing the polynomial in terms Lagrange interpolants $L_k(x)$

Now,

we have,

$$Pn(x) = y_0 L_0(x) + y_1 L_1(x) - - - y_n L_n(x)$$

For our case,

$$P_2(x) = y_0 L_0(x) + y_1 L_1(x) + y_2 L_2(x)$$

Solving for Langrage interpolants,

where,

$$L_0(x_0) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$\Rightarrow \frac{(x-2)(x+1)}{(1-2)(1+1)}$$

$$= \frac{x^2-x-2}{-2}$$

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}$$

$$= \frac{(x-1)(x+1)}{(2-1)(2+1)} = \frac{x^2-1}{3}$$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{x^2-3x+2}{6}$$

putting back to original equation.

$$P_2(x) = \frac{3(x^2-x-2)}{-2} + \frac{7(x^2-1)}{3} + \frac{(x^2-3x+2)}{6}$$

$$\Rightarrow x^2\left(-\tfrac{3}{2}+\tfrac{7}{3}+\tfrac{1}{6}\right)+x\left(\tfrac{3}{2}-\tfrac{1}{2}\right)+$$

$$\frac{6}{2}-\tfrac{7}{3}+\tfrac{1}{3}$$

$$\Rightarrow x^2+x+1$$

Hence, Both methods are giving the same polynomial system of degree 2.

Q no 3

problem 3

Given data set

k	0	1	2	3
x_k	0.1	0.3	0.5	0.7
y_k	1.1	1.25	1.17	1.31

solution

we have,

$$Q_{i,j}(x) = \frac{(x - x_{i-j})\, Q_{i,j-1}(x) - (x - x_i)\, Q_{i-1,j-1}(x)}{x_i - x_{i-j}}$$

$j = 1, 2, \ldots i$ and $0 \le j \le i$

where, $Q_{i,j}(x)$ is degree j interpolating polynomial on the $(j+1)$ positions

Now, $Q_{k,0}(x) = y_k$, so $(Q_{0,0})(x) = y_0 = 1.1$

$Q(1,0)(x) = y_1 = 1.25$

$Q(2,0)(x) = y_2 = 1.17$

$Q(3,0)(x) = y_3 = 1.31$

Again Using recursive point wise evaluation of the interpolation polynomials :

Q: The approximatiomation are

$$Q_{2,1}(0.45) = \frac{(0.45-0.3)(1.17)-(0.45-0.5)(1.25)}{0.5-0.3}$$

where $(Q_{i,j-1})(x) = 1.17$

$Q_{i-1,j-1}(x) = 1.25$

$$\Rightarrow \frac{0.238}{0.2}$$

$\Rightarrow 1.19$

$$Q_{3,1}(0.45) = \frac{(0.45-0.5)(1.31)-(0.45-0.7)(1.17)}{0.7-0.5}$$

$$\approx \frac{0.227}{0.2}$$

≈ 1.135

$$Q_{(2,2)}(0.45) = \frac{(0.45-0.1)(1.19)-(0.45-0.5)(1.0362)}{0.5-0.1}$$

$Q_{(1,1)}(0.45)$ is calculated in next page

$$\Rightarrow \frac{0.4846}{0.4}$$

$\Rightarrow 1.211$

$$Q_{(3,2)}(0.45) = \frac{(0.45-0.3)(1.135)-(0.45-0.7)(1.19)}{0.7-0.3}$$

$$= \frac{0.46775}{0.4}$$

$\Rightarrow 1.169$

$$Q(1,1)(0.45) = \frac{(0.45-0.1)(1.25)-(0.45-0.3)(1.1)}{0.3-0.1}$$

$$= \frac{0.2725}{0.2}$$

$$\Rightarrow 1.362$$

Hence

we have,

$Q(1,1)(0.45$	1.362
$Q(2,1)(0.45)$	1.19
$Q(3,1)(0.45)$	1.135
$Q(2,2)(0.45)$	1.211
$Q(3,2)(0.45)$	1.169