Problem 1

Given,

$$A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}.$$

To show,

$$A^2 - 3A = 10I$$

Now,

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 4 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times 3} \begin{bmatrix} 2 & 4 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \begin{bmatrix} 4+12 & 8+4 & 0 \\ 6+3 & 12+1 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16 & 12 & 0 \\ 9 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix}$$

And

$$3A = \begin{bmatrix} 6 & 12 & 0 \\ 9 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\therefore A^2 - 3A = \begin{bmatrix} 16 & 12 & 0 \\ 9 & 13 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 6 & 12 & 0 \\ 9 & 3 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$\Rightarrow 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow 10I$$

where $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we have,

$$A^2 - 3A = 10I$$

or, $A(A - 3I) = 10I$

or, $\cancel{A}(A - 3I) = 10 \cancel{A} A^{-1}$

or, $\frac{(A - 3I)}{10} = A^{-1}$

or, $A^{-1} = \cancel{A} \frac{1}{10}[A] - \frac{3}{10}[I]$

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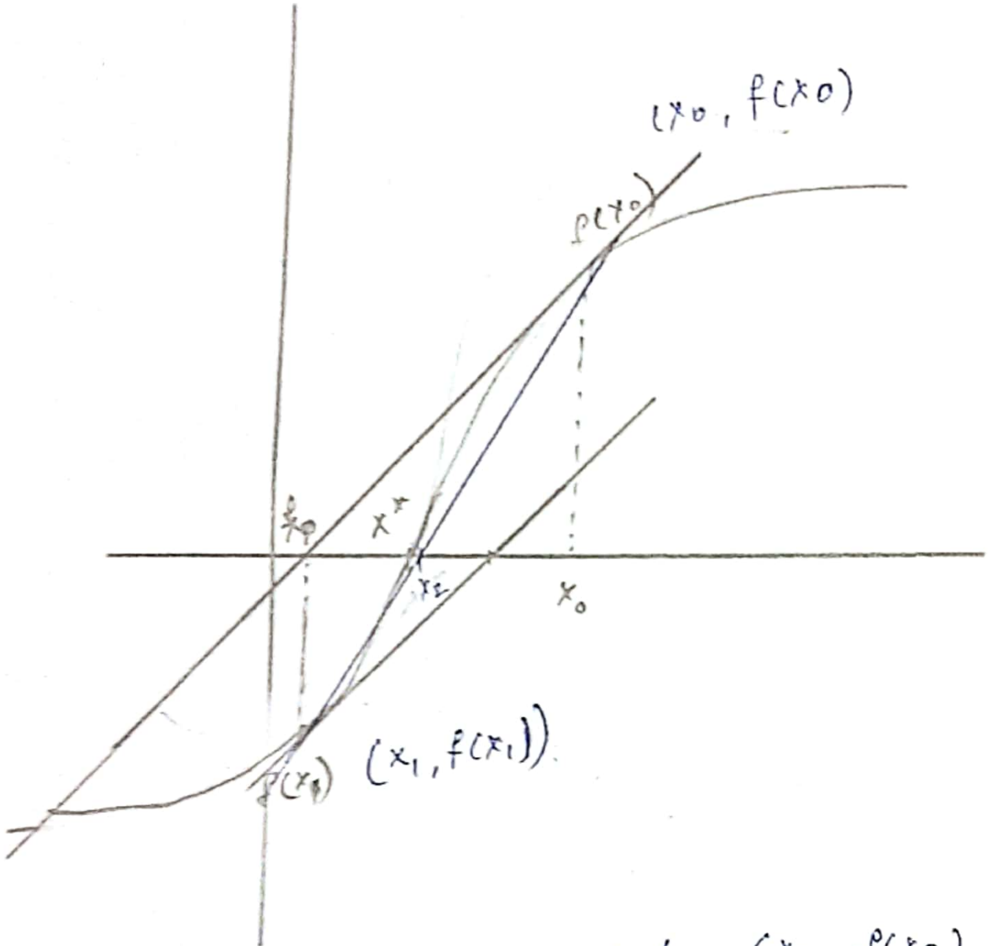
$$A^{-1} = \begin{bmatrix} \frac{2}{10} & \frac{4}{10} & 0 \\ \frac{3}{10} & \frac{1}{10} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} \frac{3}{10} & 0 & 0 \\ 0 & \frac{3}{10} & 0 \\ 0 & 0 & \frac{3}{10} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \frac{2}{10} - \frac{3}{10} & \frac{4}{10} & 0 \\ \frac{3}{10} & \frac{1}{10} - \frac{3}{10} & 0 \\ 0 & 0 & \frac{1}{2} - \frac{3}{10} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-1}{10} & \frac{2}{5} & 0 \\ \frac{3}{10} & -\frac{2}{10} & 0 \\ 0 & 0 & \frac{2}{10} \end{bmatrix}$$

$$= \begin{bmatrix} -0.1 & 0.4 & 0 \\ 0.3 & -0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} = A^{-1}$$

Qno 8



Now we have two point $(x_0, f(x_0))$ and $(x_1, f(x_1))$ as given in the figure above.

Now, by slope-intercept for, we will develop the equation for the line above,

we have, when $x = x_2$

$$f(x_2) - f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_1)$$

Now, we have root at $f(x_2)$, so $f(x_2) = 0$

Here, $$0 - f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x_2 - x_1)$$

$$\text{or,} \quad -f(x_1)(x_1-x_0) = f(x_1)-f(x_0)(x_2-x_1)$$

$$\text{or,} \quad \frac{+f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)} = -x_2+x_1$$

$$\text{or,} \quad x_2 = x_1 - \frac{f(x_1)(x_1-x_0)}{f(x_1)-f(x_0)}$$

Hence, we have the new value of x_2:

we can also generalize the above ~~value of~~ x by

$$x_3 = x_2 - \frac{f(x_2)(x_2-x_1)}{f(x_2)-f(x_1)}$$

$$x_4 = x_3 - \frac{f(x_3)(x_3-x_2)}{f(x_3)-f(x_2)}$$

$$\vdots$$

$$x_{k-1} = x_{k-2} - \frac{f(x_{k-2})(x_{k-2}-x_{k-3})}{f(x_{k-2})-f(x_{k-3})}$$

$$\vdots$$

$$x_k = x_{k-1} - \frac{f(x_{k-1})(x_{k-1}-x_{k-2})}{f(x_{k-1})-f(x_{k-2})}$$

$$\boxed{x_{k+1} = x_k - \frac{f(x_k)\cdot(x_k-x_{k-1})}{f(x_k)-f(x_{k-1})}}$$

is the required generalized form.