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Problem 1:

Code for Gauss Quadrature:

internalGx.m

```
function [gx] = internalGx(curr, xi, wi, N, f)
a = -sqrt(1-curr^2);
b = sqrt(1-curr^2);
sum = 0;
for k = 1:N
    x = 0.5*(b-a)*xi(k) + 0.5*(b+a);
    sum = sum + wi(k) * f(x, curr);
end

%I was getting wrong value because I was mutliplying 0.5*(b-a) inside the
%loop
gx = 0.5*(b-a) * sum;
end
```

scriptProblem01

```
clear all;
f  = @(x,y) 0.5*(exp(-x.^2) + exp(-y.^2));

prompt = "Choose the value of N for Gauss Quadrature:";

N = input(prompt);

if(N==3)
    xi= [0.77459667, 0, -0.77459667];
    wi= [0.55555556,0.88888889,0.55555556];
end
```

```
if(N==4)
    xi =[0.33998104,0.86113631,-0.33998104,-0.86113631];
    wi = [0.65214515,0.34785485,0.65214515,0.3478548];
end
if(N==5)
    xi = [0.90617985, 0.53846931, 0.00000000,-0.53846931, -0.90617985];
    wi = [0.23692689, 0.47862867, 0.56888889, 0.47862867, 0.23692689];
end
Approx = 0;
for j = 1:N
    Approx = Approx + wi(j)*internalGx(xi(j), xi, wi, N, f);
end

Approx
```

Code for Monte-Carlo

MC2D.m

```
function[integralMC2D] = MC2D(M)
f  = @(x,y) 0.5*(exp(-x.^2) + exp(-y.^2));
a = -1;
b = 1;
c= -1;
d = 1;
sum = 0;



%Randomizing
x = a+ (b-a)*rand(1,M);
y = c+ (d-a)*rand(1,M);

for i = 1:M
    sum = sum + fExt(x(i), y(i), f);
```

```
end

fbar = sum/M;
integralMC2D = pi *fbar;



end
```

fExt.m

```
function [val] = fExt(x,y, f)
    val = 0;
    if x^2 + y^2 <=1
        val= f(x, y);
    end
end
```

scriptM2CD.m

```
clear all;
format long;
M = 10^6;
IntregalV= 0;

for i = 1: 10
    IntregalV = IntregalV + MC2D(M);
end

IntregalV = IntregalV/10
```

Calculating the difference:

N	Value Using Monte-Carlo	Value Using Gauss Quadrature	Absolute Value
3	2.517621439781281	2.560061128586483	0.042439688805202
4	2.517495391527651	2.530258937959873	0.012763546432222
5	2.518361697830591	2.524923343620527	0.006561645789936

Problem02:

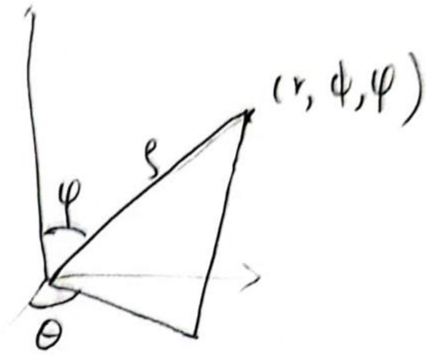
Solving Analytically:

• Spherical coordinates

Problem 2 Analytically

Given,

$f(x,y,x) = 0.7(x^2+y^2+z^2)$



Expressing the function in spherical coordinate system:

we have,

$x = r\sin\phi\cos\theta$

$y = r\sin\phi\sin\theta$

$z = r\cos\phi$

so, $0 \le r \le 1$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

so, $f(x,y,x) = 0.7\left((r\sin\phi\cos\theta)^2 + (r\sin\phi\sin\theta)^2 + r^2\cos^2\phi\right)$

$= 0.7\left(r^2\sin^2\phi\cos^2\theta + r^2\sin^2\phi\cdot\sin^2\theta + r^2\cos^2\phi\right)$

$\Rightarrow 0.7r^2\left(\sin^2\phi(\cos^2\theta+\sin^2\theta) + \cos^2\phi\right)$

$\Rightarrow 0.7r^2\left(\sin^2\phi + \cos^2\phi\right)$

$\Rightarrow 0.7r^2$

Now

$$\int_0^\theta\int_0^\phi\int_0^r r^2\sin\phi\cdot f(r,\phi,\theta)\cdot dr\,d\phi\,d\theta$$

$$\cdot \int_0^1 0.7r^4\sin\phi\cdot dr$$

$$= \left.\frac{0.7r^5\sin\phi}{5}\right|_0^1 \Rightarrow \frac{0.7}{5}\sin\phi$$

$$\int_0^{\pi} \frac{0.7}{5} \sin\phi \, d\phi$$

$$= -\frac{0.7}{5} \left(\cos\phi\right)\Big|_0^{\pi}$$

$$\Rightarrow -\frac{0.7}{5}(-1-1)$$

$$\Rightarrow -\frac{0.7}{5}(-2)$$

$$\Rightarrow \frac{0.7 \times 2}{5}$$

$$\int_0^{2\pi} \frac{0.7 \cdot 2}{5} \, d\theta$$

$$\Rightarrow \frac{0.7 \cdot 2}{5} \theta \Big|_0^{2\pi}$$

$$\Rightarrow \frac{0.7 \times 2 \times 2\pi}{5}$$

$$\Rightarrow 1.759291886$$

MathLab Implementation using Monte Carlo:

MC3D.m

```
function[integralMC3D] = MC3D(M)
fx = @(x,y,z) 0.7 * (x.^2 + y.^2 + z.^2);

sum = 0;
x = -1+ 2*rand(1,M);
y = -1+ 2*rand(1,M);
z = -1+ 2*rand(1,M);

for i = 1:M
   sum = sum + (fExt3D(x(i), y(i),z(i), fx));
end

fbar = sum/M;
Volume = (4*pi)/3;

integralMC3D = Volume*(fbar);
```

fExt3D.m

```
function [val] = fExt3D(x,y, z,fx)
   val = 0;
   if (x^2 + y^2 + z^2) <=1
      val= feval(fx,x, y,z);
   end
end
```

scriptMonteCarlo.m

```
clear all;

format long;

M = 10^6;

IntregalV= 0;


for i = 1: 10

   IntregalV = IntregalV + MC3D(M);

end


IntregalV = IntregalV/10
```

Exact Answer solving Analytically : 1.759291886
From MonteCarlo: 0.920943486960326

Absolute Difference: 0.838348399039674