Contents

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RBE 521 Final Exam

```
close all; clear all; clc;

% Booleans to toggle plotting
plot_6_step_pos_and_vel = true; % Needs to be true for part 6
plot_11_step_pos_and_vel = true;
plot_transfer_a_b_g = true;

plot_cycle_pos = true;
plot_cycle_a_b_g = true; % Needs to be true for part 7
plot_cycle_joint_velocities = true; % Needs to be true for part 8

plot_simulation = true; % Needs to be true for part 9
```

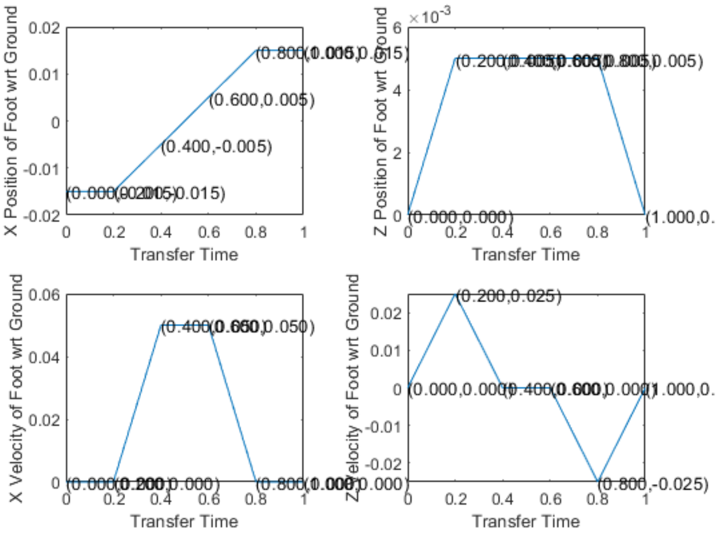
Problem 1

```
% From HW 6 and problem statement:
v = 0.01; % m/s
duty = 4/6;
L = 0.030; % m
leg_lengths = [0, 0.050, 0.100]; % m
liftoff_height = 0.005; % m
body_height = 0.1; % m
body_l = 0.4; % m
body_w = 0.2; % m
body_h = 0.1; % m
D = leg_lengths(1) + leg_lengths(2) + (body_w/2);

% Q1
u_fb = duty / (1 - duty) * v; % m/s

% Q2
u_fg = v / (1 - duty); % m/s

% Q3
num_contact_legs = 4;

% Q4
phi_start = [4/6, 1/6, 2/6, 5/6, 0/6, 3/6]; % start of transfer phase
phi_end = [0/6, 3/6, 4/6, 1/6, 2/6, 5/6]; % end of transfer phase

% Q5
T = L / v; % sec
```

Q6

```
T_s = T * duty; % support time (sec)
T_t = T - T_s; % transfer time (sec)

% Define time steps and initialize generic x and z position vectors
t_steps = [0, (1 * T_t)/5, (2 * T_t)/5, (3 * T_t)/5, (4 * T_t)/5, T_t];
interval = 0.2;
x_pos_generic = zeros(1,size(t_steps,2));
z_pos_generic = zeros(1,size(t_steps,2));

% Calculate generic x and z positions of leg in transfer wrt ground
for i=1:size(t_steps,2)
    if i == 1 % if liftoff start time
        x_pos_generic(i) = 0;
        z_pos_generic(i) = 0;
    elseif i == 2 % if liftoff end time
        x_pos_generic(i) = 0;
        z_pos_generic(i) = liftoff_height;
    elseif i == (size(t_steps,2) - 1) % if touchdown start time
        x_pos_generic(i) = L;
        z_pos_generic(i) = liftoff_height;
    elseif i == size(t_steps,2) % if touchdown end time
```

```
        x_pos_generic(i) = L;
        z_pos_generic(i) = 0;
    else % if between liftoff and touchdown
        x_pos_generic(i) = L / (size(t_steps,2) - 3) * ((t_steps(i) - interval) / interval);
        z_pos_generic(i) = liftoff_height;
        %sqrt(0.015^2 - (x_pos_generic(i) - 0.015)^2) - 0.005;
    end
end

% Center the x trajectory by subtracting its position by L/2
for i = 1:size(t_steps,2)
    x_pos_generic(i) = x_pos_generic(i) - L/2;
end

% Initialize generic x and z velocity vectors
x_vel_generic = zeros(1,size(t_steps,2));
z_vel_generic = zeros(1,size(t_steps,2));

% Calculate generic x and z velocities of one leg in transfer wrt ground
for i=1:size(t_steps,2)
    if i == 1 % if liftoff start time
        x_vel_generic(i) = 0;
        z_vel_generic(i) = 0;
    elseif i == 2 % if liftoff end time
        x_vel_generic(i) = 0;
        z_vel_generic(i) = liftoff_height / (t_steps(i) - t_steps(i-1));
    elseif i == (size(t_steps,2) - 1) % if touchdown start time
        x_vel_generic(i) = 0;
        z_vel_generic(i) = -liftoff_height / (t_steps(end) - t_steps(end-1));
    elseif i == size(t_steps,2) % if touchdown end time
        x_vel_generic(i) = 0;
        z_vel_generic(i) = 0;
    else % if between liftoff and touchdown
        x_vel_generic(i) = (x_pos_generic(i) - x_pos_generic(i-1)) / (t_steps(i) - t_steps(i-1));
        z_vel_generic(i) = (z_pos_generic(i) - z_pos_generic(i-1)) / (t_steps(i) - t_steps(i-1));
    end
end

% Plot generic x and z positions and velocities of one leg in transfer wrt ground
if plot_6_step_pos_and_vel
    figure
    tiledlayout(2,2);

    nexttile
    plot(t_steps,x_pos_generic)
    set(gca, 'xtick', 0:0.2:1);
    xlabel('Transfer Time')
    ylabel('X Position of Foot wrt Ground')
    for point = 1:6
        thisX = t_steps(point);
        thisY = x_pos_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,z_pos_generic)
    set(gca, 'xtick', 0:0.2:1);
    xlabel('Transfer Time')
    ylabel('Z Position of Foot wrt Ground')
    for point = 1:6
        thisX = t_steps(point);
        thisY = z_pos_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,x_vel_generic)
    set(gca, 'xtick', 0:0.2:1);
    xlabel('Transfer Time')
    ylabel('X Velocity of Foot wrt Ground')
    for point = 1:6
        thisX = t_steps(point);
        thisY = x_vel_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,z_vel_generic)
    set(gca, 'xtick', 0:0.2:1);
    xlabel('Transfer Time')
    ylabel('Z Velocity of Foot wrt Ground')
    for point = 1:6
        thisX = t_steps(point);
        thisY = z_vel_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end
end
```

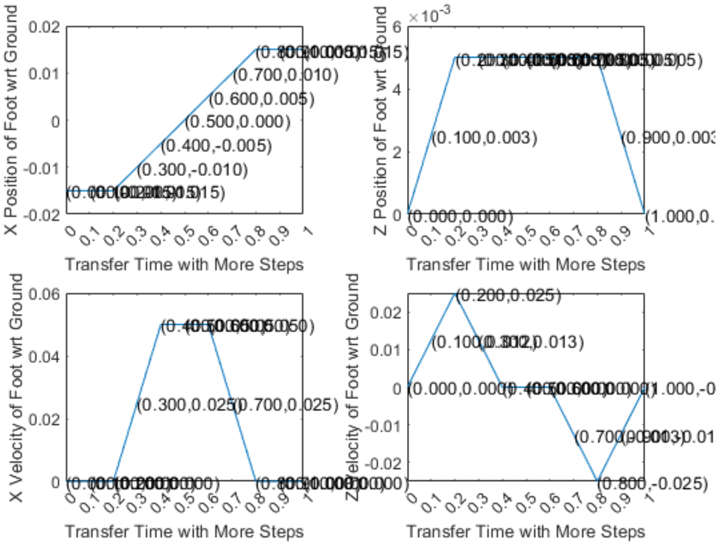


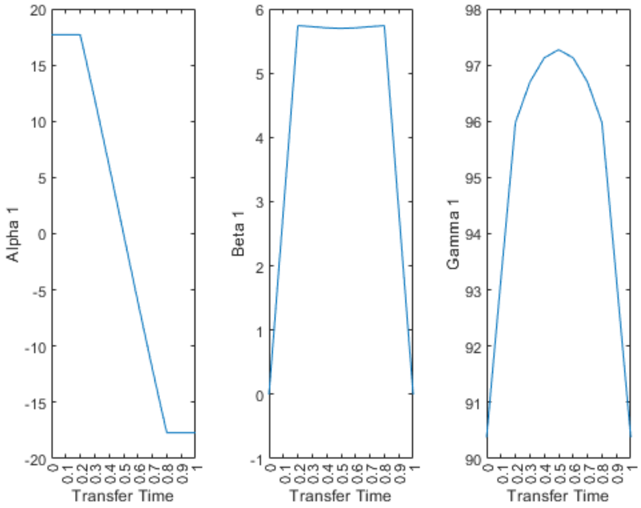
Q7

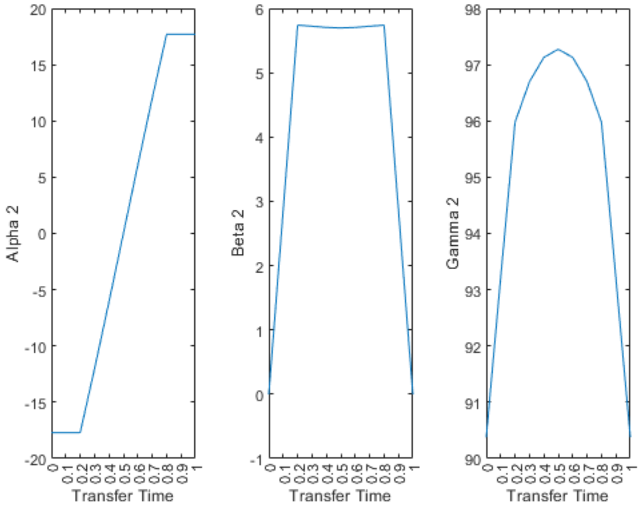
```
% REDEFINE TRAJECTORIES WITH MORE TIME STEPS TO BE MORE FRIENDLY WITH THE
% RELATIVE KINEMATIC PHASE OF EACH LEG (0.2 STEPS WERE TOO BIG, 0.1 OK)

% Define time steps and initialize generic x and z position vectors
t_steps = [0, (1 * T_t)/10, (2 * T_t)/10, (3 * T_t)/10, (4 * T_t)/10, (5 * T_t)/10, (6 * T_t)/10, (7 * T_t)/10, (8 * T_t)/10, (9 * T_t)/10, T_t];
interval = 0.1;
x_pos_generic = zeros(1,size(t_steps,2));
y_pos_generic = zeros(1,size(t_steps,2));
z_pos_generic = zeros(1,size(t_steps,2));

% Calculate generic x and z positions of leg in transfer wrt ground
for i=1:size(t_steps,2)
    if i <= 3 % if liftoff
        x_pos_generic(i) = 0;
        z_pos_generic(i) = liftoff_height/2 * (i-1);
    elseif i > (size(t_steps,2) - 3) && i <= (size(t_steps,2)) % if touchdown
        x_pos_generic(i) = L;
        z_pos_generic(i) = liftoff_height/2 * (size(t_steps,2) - i);
    else % if between liftoff and touchdown
        x_pos_generic(i) = L / 6 * (i - 3);
        z_pos_generic(i) = liftoff_height;
        %sqrt(0.015^2 - (x_pos_generic(i) - 0.015)^2) - 0.005;
    end
end

% Center the x trajectory by subtracting its position by L/2
for i = 1:size(t_steps,2)
    x_pos_generic(i) = x_pos_generic(i) - L/2;
end

% Initialize generic x and z velocity vectors
x_vel_generic = zeros(1,size(t_steps,2));
z_vel_generic = zeros(1,size(t_steps,2));

% Calculate generic x and z velocities of one leg in transfer wrt ground
for i=1:size(t_steps,2)
    if i == 1 % if start of liftoff
        x_vel_generic(i) = 0;
        z_vel_generic(i) = 0;
    elseif i <= 3 % if liftoff
        x_vel_generic(i) = 0;
        z_vel_generic(i) = (((liftoff_height/2) / (t_steps(i) - t_steps(i-1))) * (i-1))/2;
    elseif i > (size(t_steps,2) - 3) && i <= (size(t_steps,2)) % if touchdown
        x_vel_generic(i) = 0;
        z_vel_generic(i) = (-(liftoff_height/2) / (t_steps(end) - t_steps(end-1)) * (size(t_steps,2) - i))/2;
    else % if between liftoff and touchdown
        x_vel_generic(i) = (x_pos_generic(i) - x_pos_generic(i-1)) / (t_steps(i) - t_steps(i-1));
        z_vel_generic(i) = (z_pos_generic(i) - z_pos_generic(i-1)) / (t_steps(i) - t_steps(i-1));
    end
end

% Temporary fix to velocity graphs
x_vel_generic(4) = 0.025;
x_vel_generic(8) = 0.025;
z_vel_generic(4) = 0.0125;
z_vel_generic(8) = -0.0125;

% Plot generic x and z positions and velocities of one leg in transfer wrt ground
if plot_11_step_pos_and_vel
    figure
    tiledlayout(2,2);

    nexttile
    plot(t_steps,x_pos_generic)
    set(gca, 'xtick', 0:interval:T_t);
    xlabel('Transfer Time with More Steps')
    ylabel('X Position of Foot wrt Ground')
    for point = 1:size(t_steps,2)
        thisX = t_steps(point);
        thisY = x_pos_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,z_pos_generic)
    set(gca, 'xtick', 0:interval:T_t);
    xlabel('Transfer Time with More Steps')
    ylabel('Z Position of Foot wrt Ground')
    for point = 1:size(t_steps,2)
        thisX = t_steps(point);
        thisY = z_pos_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,x_vel_generic)
    set(gca, 'xtick', 0:interval:T_t);
    xlabel('Transfer Time with More Steps')
    ylabel('X Velocity of Foot wrt Ground')
    for point = 1:size(t_steps,2)
        thisX = t_steps(point);
        thisY = x_vel_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end

    nexttile
    plot(t_steps,z_vel_generic)
    set(gca, 'xtick', 0:interval:T_t);
    xlabel('Transfer Time with More Steps')
    ylabel('Z Velocity of Foot wrt Ground')
    for point = 1:size(t_steps,2)
        thisX = t_steps(point);
        thisY = z_vel_generic(point);
        labelstr = sprintf('(%.3f,%.3f)', thisX, thisY);
        text(thisX, thisY, labelstr);
    end
end

leg1_joints = calc_joint_angles(1, x_pos_generic, z_pos_generic);
leg2_joints = calc_joint_angles(2, x_pos_generic, z_pos_generic);
leg3_joints = calc_joint_angles(3, x_pos_generic, z_pos_generic);
leg4_joints = calc_joint_angles(4, x_pos_generic, z_pos_generic);
leg5_joints = calc_joint_angles(5, x_pos_generic, z_pos_generic);
leg6_joints = calc_joint_angles(6, x_pos_generic, z_pos_generic);

leg_joints = [leg1_joints; leg2_joints; leg3_joints; leg4_joints; leg5_joints; leg6_joints];

% Plot legs' transfer phase alpha, beta, gamma
if plot_transfer_a_b_g
    for leg = 1:6 % for each leg
        figure
        tiledlayout(1,3);

        nexttile
        plot(t_steps,leg_joints(3*leg-2,:))
        set(gca, 'xtick', 0:interval:T_t);
        xlabel('Transfer Time')
        ylabel(sprintf('Alpha %g', leg))

        nexttile
        plot(t_steps,leg_joints(3*leg-1,:))
        set(gca, 'xtick', 0:interval:T_t);
        xlabel('Transfer Time')
        ylabel(sprintf('Beta %g', leg))

        nexttile
        plot(t_steps,leg_joints(3*leg,:))
```

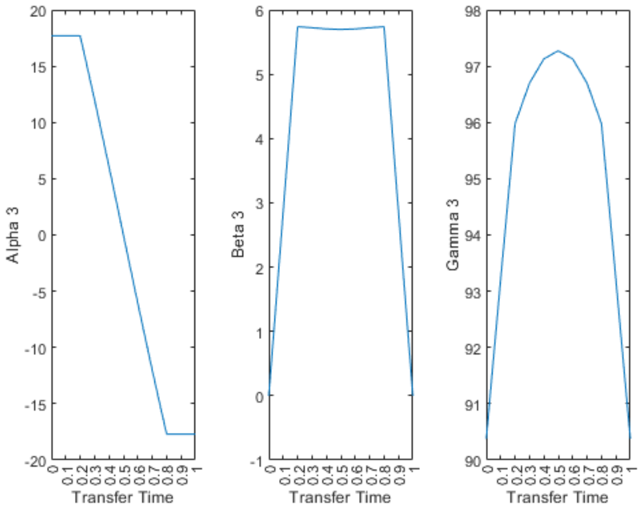
```matlab
        set(gca, 'xtick', 0:interval:T_t);
        xlabel('Transfer Time')
        ylabel(sprintf('Gamma %g', leg))
    end
end

% Applying generic foot trajectory to each leg based on relative
% phase of the leg in the overall cycle
t_cycle = 0:0.1:T-0.1;

transfer_start_time = phi_start * T; % start time of transfer phase of each leg in a total cycle

% Initialize new cycle position matrices
x_pos = zeros(6, 30);
z_pos = zeros(6, 30);

% Initialize support phase trajectories for x
% Don't need z bc the value will be 0 if not in transfer anyway
x_support_pos = zeros(1,20);

for i = 1:size(x_support_pos,2)
    x_support_pos(i) = (L / 2) - (L / 20) * (i - 1);
end

% Leg 1:
x_pos(1,1:20) = x_support_pos(1:20);
x_pos(1,21:30) = x_pos_generic(1:10);
z_pos(1,21:30) = z_pos_generic(1:10);

% Leg 2:
x_pos(2,16:30) = x_support_pos(1:15);
x_pos(2,1:5) = x_support_pos(16:20);
x_pos(2, 6:15) = x_pos_generic(1:10);
z_pos(2, 6:15) = z_pos_generic(1:10);

% Leg 3:
x_pos(3,21:30) = x_support_pos(1:10);
x_pos(3,1:10) = x_support_pos(11:20);
x_pos(3,11:20) = x_pos_generic(1:10);
z_pos(3,11:20) = z_pos_generic(1:10);

% Leg 4:
x_pos(4,6:25) = x_support_pos(1:20);
x_pos(4,26:30) = x_pos_generic(1:5);
x_pos(4,1:5) = x_pos_generic(6:10);
z_pos(4,26:30) = z_pos_generic(1:5);
z_pos(4,1:5) = z_pos_generic(6:10);

% Leg 5:
x_pos(5,11:30) = x_support_pos(1:20);
x_pos(5,1:10) = x_pos_generic(1:10);
z_pos(5,1:10) = z_pos_generic(1:10);

% Leg 6:
x_pos(6,26:30) = x_support_pos(1:5);
x_pos(6,1:15) = x_support_pos(6:20);
x_pos(6,16:25) = x_pos_generic(1:10);
z_pos(6,16:25) = z_pos_generic(1:10);

% Plot the legs' cycle positions
if plot_cycle_pos
    for leg = 1:6
        figure
        tiledlayout(2,1);

        nexttile
        plot(t_cycle,x_pos(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('X Position of Foot %g wrt Ground', leg))

        nexttile
        plot(t_cycle,z_pos(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Z Position of Foot %g wrt Ground', leg))
    end
end

% Calculate joint values at each cycle time step
leg1_joints_cycle = calc_joint_angles(1, x_pos(1,:), z_pos(1,:));
leg2_joints_cycle = calc_joint_angles(2, x_pos(2,:), z_pos(2,:));
leg3_joints_cycle = calc_joint_angles(3, x_pos(3,:), z_pos(3,:));
leg4_joints_cycle = calc_joint_angles(4, x_pos(4,:), z_pos(4,:));
leg5_joints_cycle = calc_joint_angles(5, x_pos(5,:), z_pos(5,:));
leg6_joints_cycle = calc_joint_angles(6, x_pos(6,:), z_pos(6,:));
```

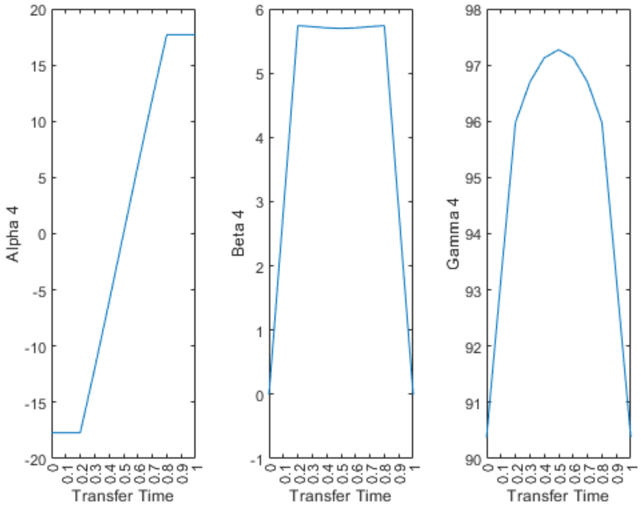
```
leg_joints_cycle = [leg1_joints_cycle; leg2_joints_cycle; leg3_joints_cycle; leg4_joints_cycle; leg5_joints_cycle; leg6_joints_cycle];

% Build master alpha, beta, gamma matrices
for leg = 1:6
    alpha(leg,:) = leg_joints_cycle(3*leg-2,:);
    beta(leg,:) = leg_joints_cycle(3*leg-1,:);
    gamma(leg,:) = leg_joints_cycle(3*leg,:);
end

% Plot legs' transfer phase alpha, beta, gamma
if plot_cycle_a_b_g
    for leg = 1:6 % for each leg
        figure
        tiledlayout(1,3);

        nexttile
        plot(t_cycle,alpha(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Alpha %g', leg))

        nexttile
        plot(t_cycle,beta(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Beta %g', leg))

        nexttile
        plot(t_cycle,gamma(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Gamma %g', leg))
    end
end
```

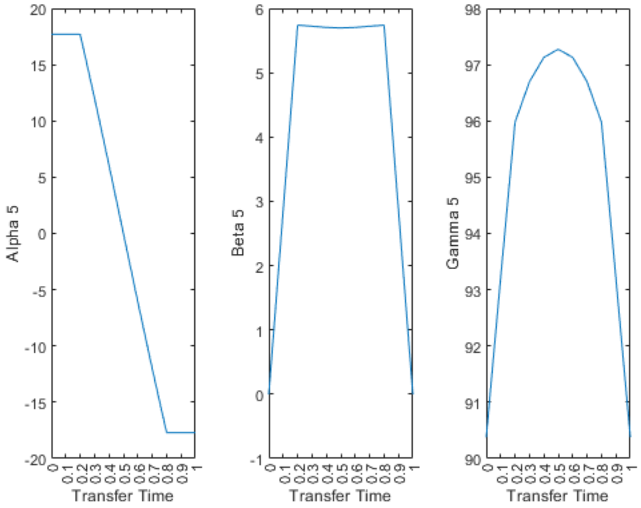

X Position of Foot wrt Ground
Z Position of Foot wrt Ground
×10^-3
X Velocity of Foot wrt Ground
Z Velocity of Foot wrt Ground
Transfer Time with More Steps
(0.700,0.010)
(0.600,0.005)
(0.500,0.000)
(0.400,-0.005)
(0.300,-0.010)
(0.100,0.003)
(0.900,0.003
(0.000,0.000)
(1.000,0.
(0.300,0.025)
(0.700,0.025)
(0.200,0.025)
(0.100,0.012)
(0.300,0.013)
(0.000,0.000)
(1.000,-0
(0.800,-0.025)

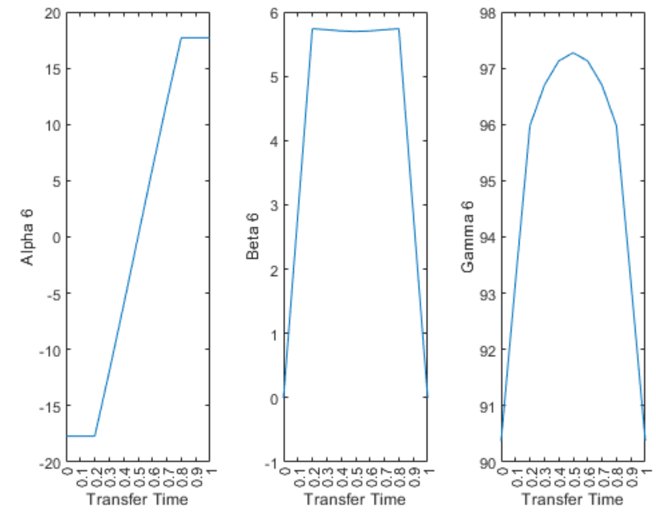

Alpha 1
Beta 1
Gamma 1
Transfer Time

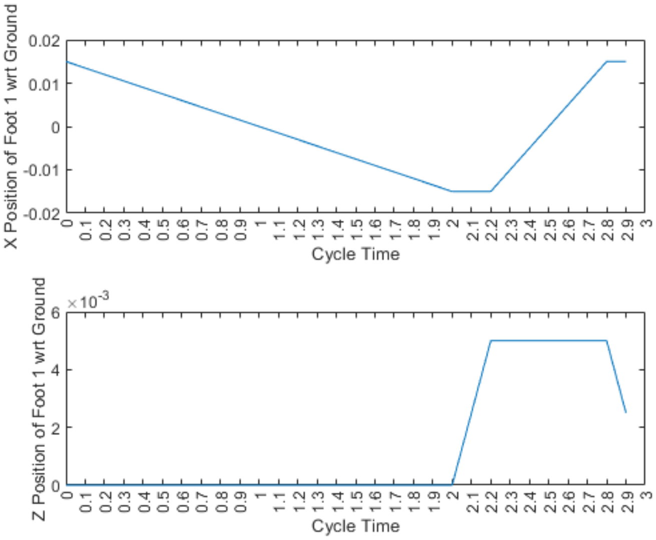

Alpha 2
Beta 2
Gamma 2
Transfer Time

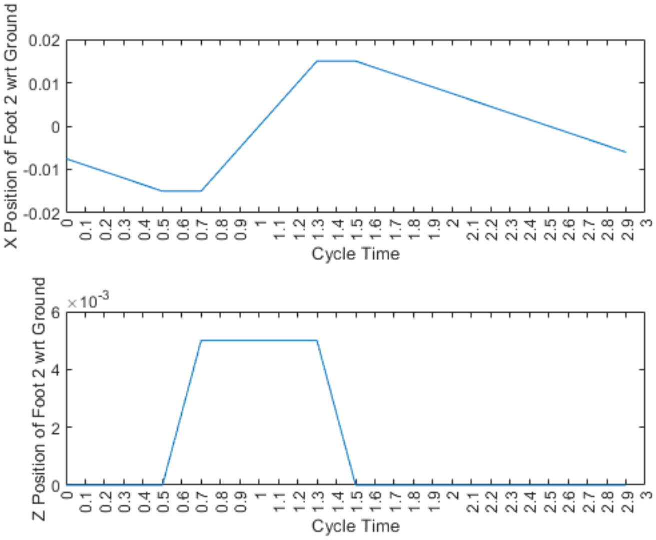

Alpha 3
20
15
10
5
0
-5
-10
-15
-20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Beta 3
6
5
4
3
2
1
0
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Gamma 3
98
97
96
95
94
93
92
91
90
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time

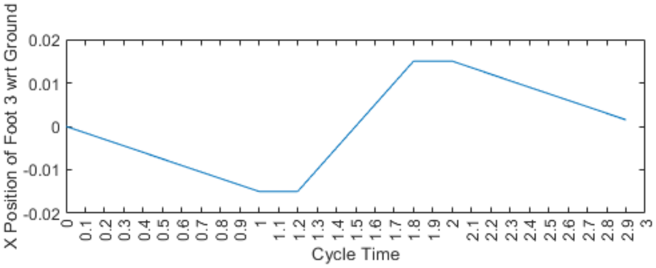

Alpha 4
20
15
10
5
0
-5
-10
-15
-20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Beta 4
6
5
4
3
2
1
0
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Gamma 4
98
97
96
95
94
93
92
91
90
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time

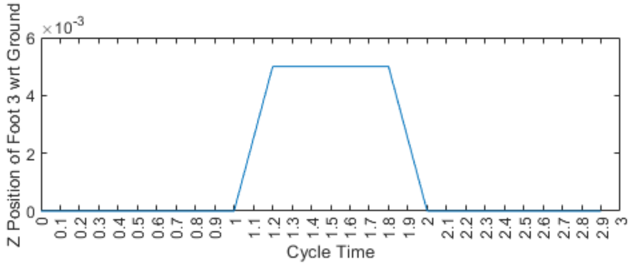

Alpha 5
20
15
10
5
0
-5
-10
-15
-20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Beta 5
6
5
4
3
2
1
0
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Gamma 5
98
97
96
95
94
93
92
91
90
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time

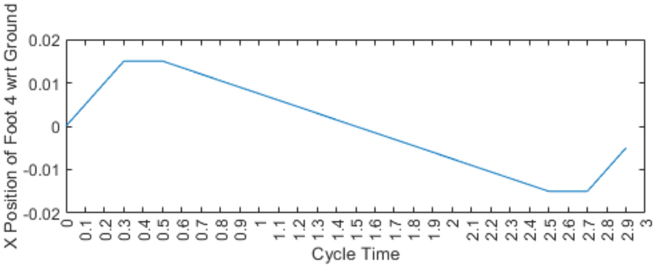

Alpha 6
20
15
10
5
0
-5
-10
-15
-20
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Beta 6
6
5
4
3
2
1
0
-1
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time
Gamma 6
98
97
96
95
94
93
92
91
90
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Transfer Time

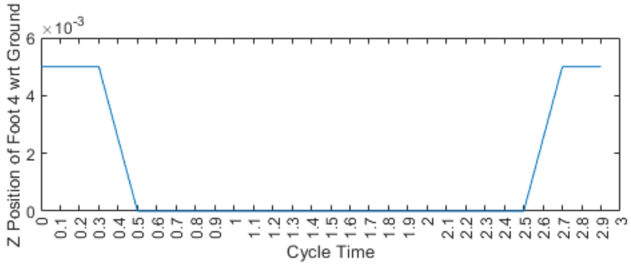

X Position of Foot 1 wrt Ground
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time
Z Position of Foot 1 wrt Ground
$\times 10^{-3}$
6
4
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

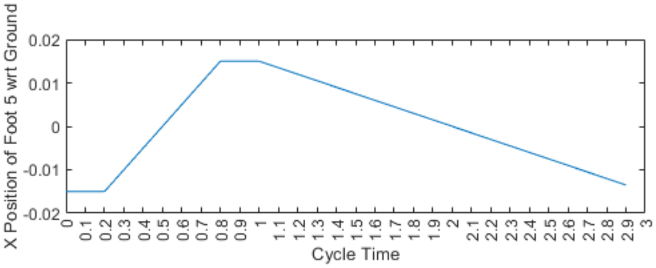

X Position of Foot 2 wrt Ground
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time
Z Position of Foot 2 wrt Ground
$\times 10^{-3}$
6
4
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

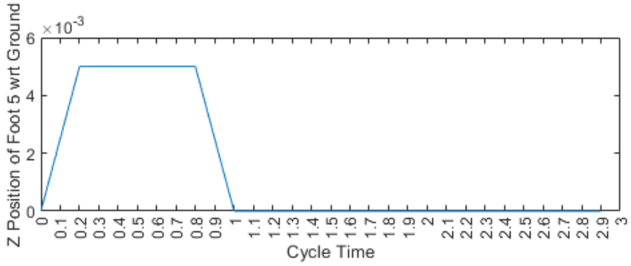

X Position of Foot 3 wrt Ground
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

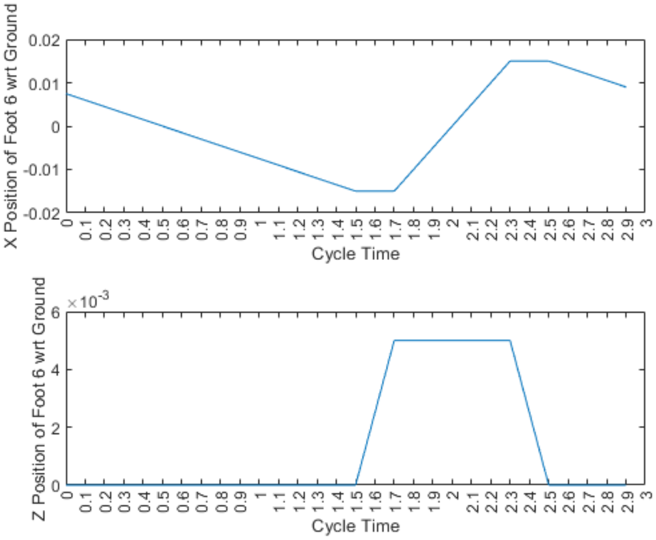

Z Position of Foot 3 wrt Ground
×10^-3
6
4
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

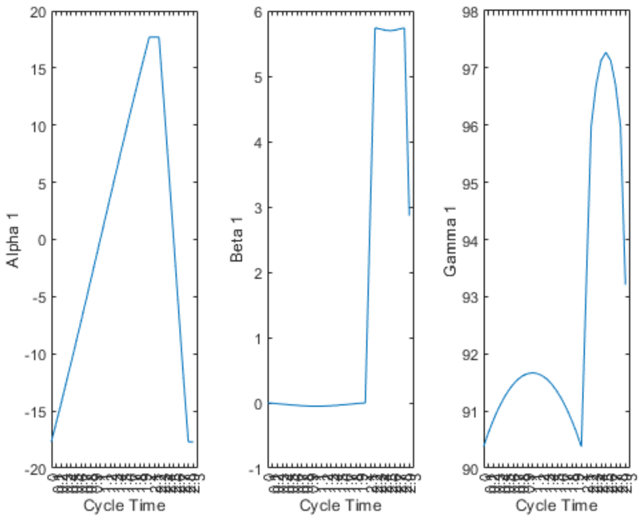

X Position of Foot 4 wrt Ground
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9
Cycle Time

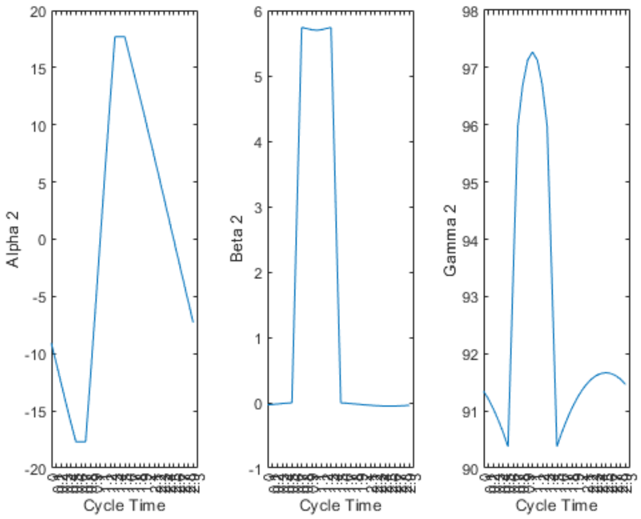

Z Position of Foot 4 wrt Ground
×10^-3
6
4
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

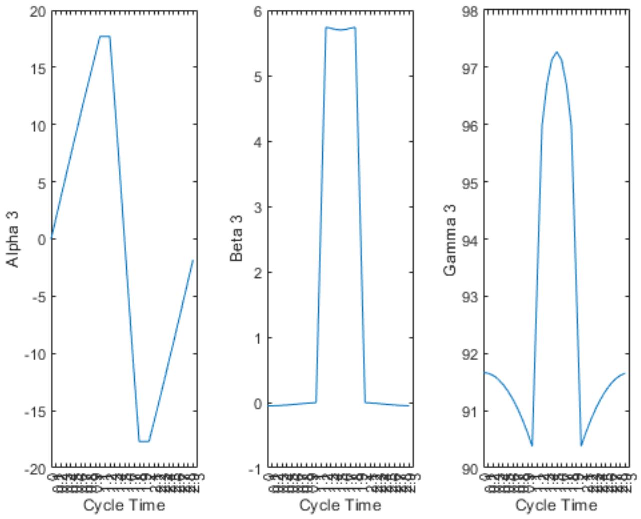

X Position of Foot 5 wrt Ground
0.02
0.01
0
-0.01
-0.02
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time


Z Position of Foot 5 wrt Ground
×10^-3
6
4
2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9 3
Cycle Time

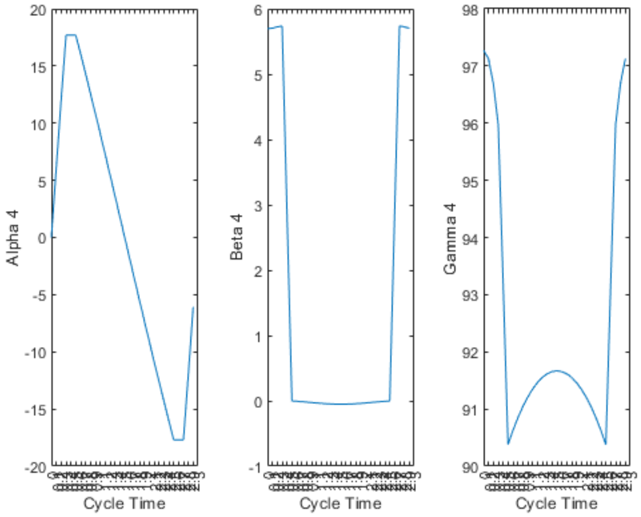

X Position of Foot 6 wrt Ground
0.02
0.01
0
-0.01
-0.02
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3
Cycle Time
Z Position of Foot 6 wrt Ground
×10^-3
6
4
2
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9
3
Cycle Time


Alpha 1
20
15
10
5
0
-5
-10
-15
-20
Cycle Time
Beta 1
6
5
4
3
2
1
0
-1
Cycle Time
Gamma 1
98
97
96
95
94
93
92
91
90
Cycle Time

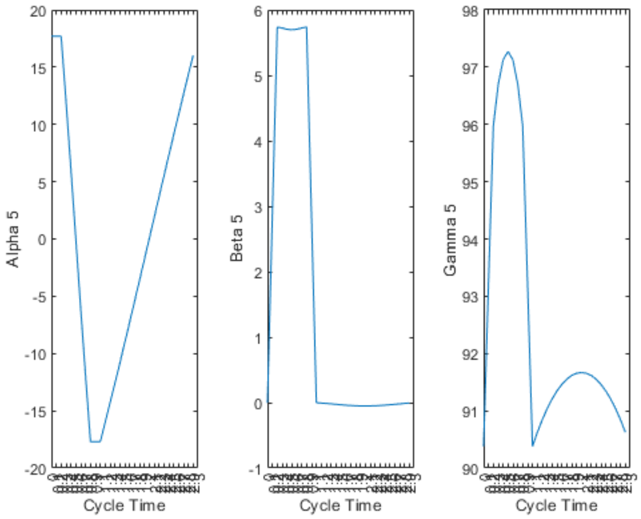

Alpha 2
20
15
10
5
0
-5
-10
-15
-20
Cycle Time
Beta 2
6
5
4
3
2
1
0
-1
Cycle Time
Gamma 2
98
97
96
95
94
93
92
91
90
Cycle Time

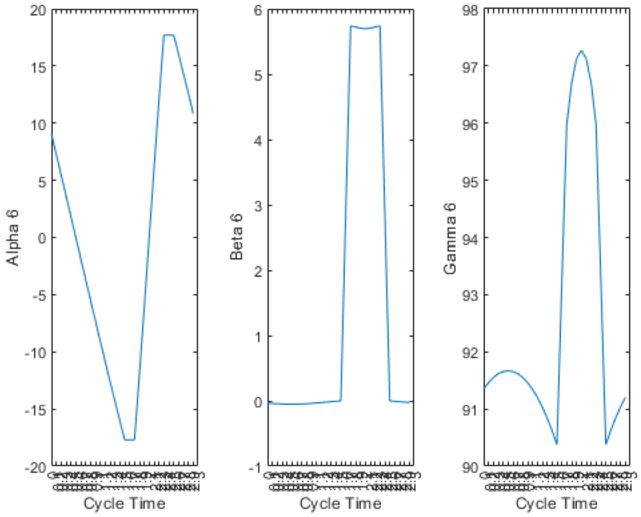


Alpha 3
20
15
10
5
0
-5
-10
-15
-20
Cycle Time
Beta 3
6
5
4
3
2
1
0
-1
Cycle Time
Gamma 3
98
97
96
95
94
93
92
91
90
Cycle Time



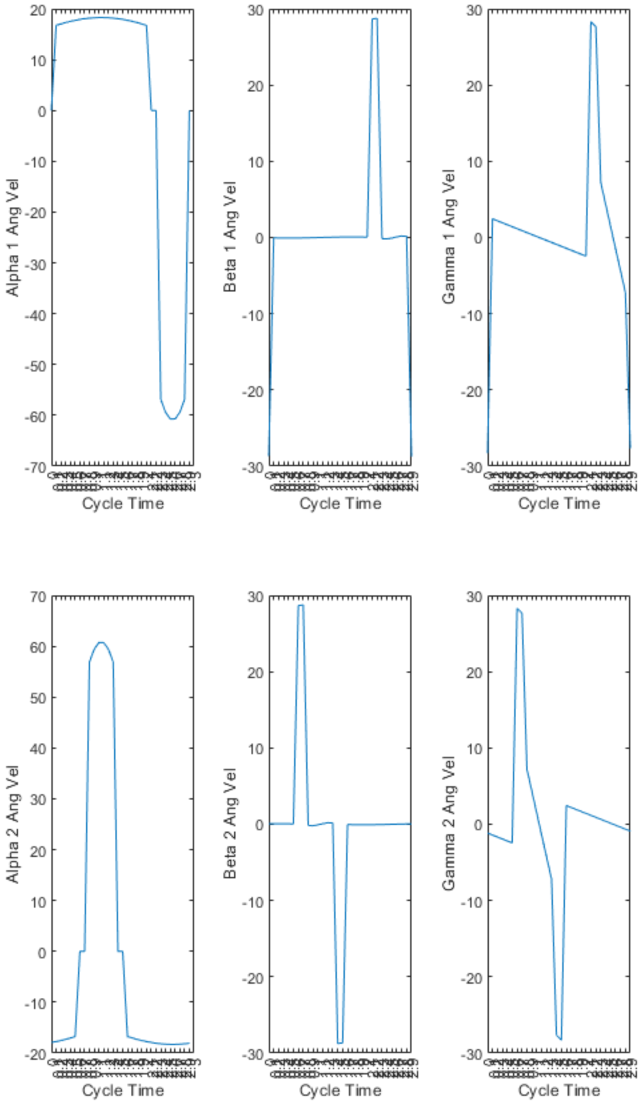
Alpha 4
20
15
10
5
0
-5
-10
-15
-20
Cycle Time
Beta 4
6
5
4
3
2
1
0
-1
Cycle Time
Gamma 4
98
97
96
95
94
93
92
91
90
Cycle Time

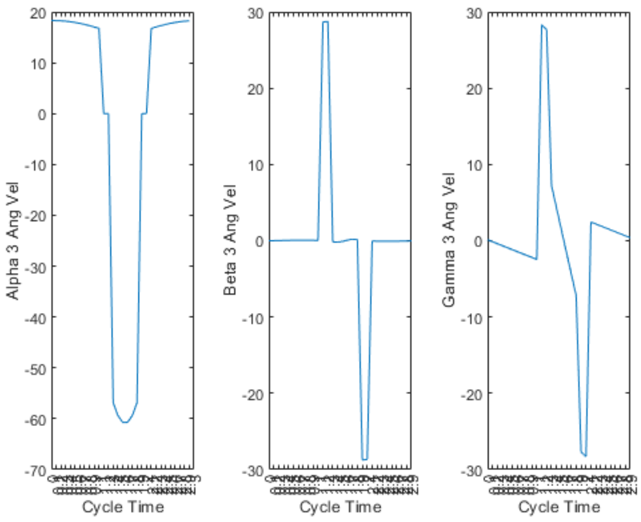


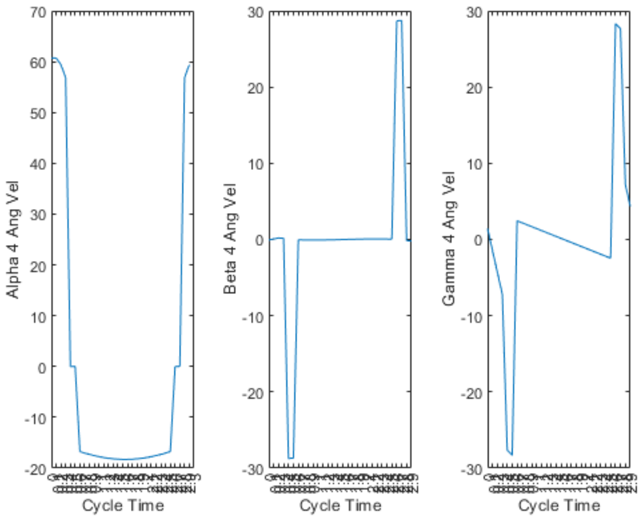


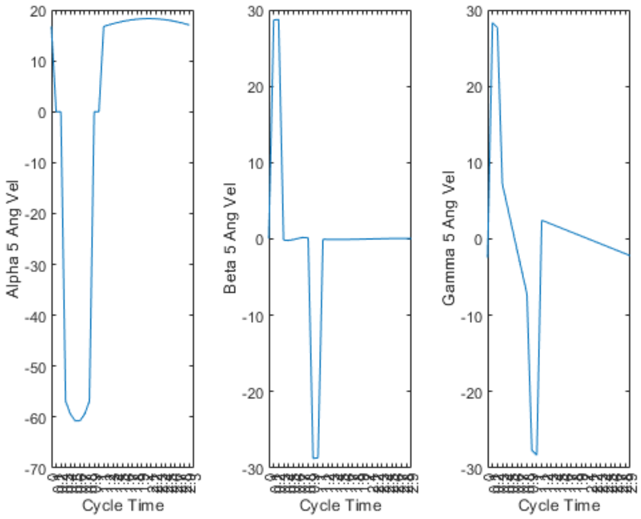
Q8

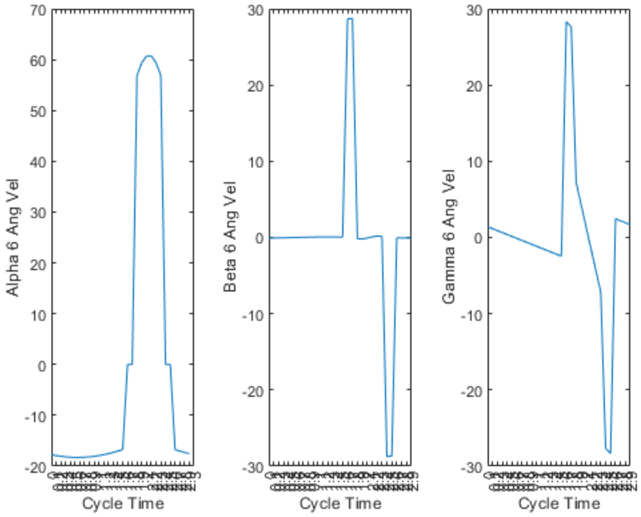
```
% Initialize angular velocity matrices for alpha, beta, and gamma
alpha_ang_vel = zeros(6, size(t_cycle,2));
beta_ang_vel = zeros(6, size(t_cycle,2));
gamma_ang_vel = zeros(6, size(t_cycle,2));

for leg = 1:6
    for t = 1:size(t_cycle,2)
        if t == 1
            alpha_ang_vel(leg, t) = ((alpha(leg, t) - alpha(leg, end))/interval);
            beta_ang_vel(leg, t) = ((beta(leg, t) - beta(leg, end))/interval);
            gamma_ang_vel(leg, t) = ((gamma(leg, t) - gamma(leg, end))/interval);
        else
            alpha_ang_vel(leg, t) = ((alpha(leg, t) - alpha(leg, t-1))/interval);
            beta_ang_vel(leg, t) = ((beta(leg, t) - beta(leg, t-1))/interval);
            gamma_ang_vel(leg, t) = ((gamma(leg, t) - gamma(leg, t-1))/interval);
        end
    end
end

% Plot joint angular velocities
if plot_cycle_joint_velocities
    for leg = 1:6 % for each leg
        figure
        tiledlayout(1,3);

        nexttile
        plot(t_cycle,alpha_ang_vel(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Alpha %g Ang Vel', leg))

        nexttile
        plot(t_cycle,beta_ang_vel(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Beta %g Ang Vel', leg))

        nexttile
        plot(t_cycle,gamma_ang_vel(leg,:))
        set(gca, 'xtick', 0:0.1:T);
        xlabel('Cycle Time')
        ylabel(sprintf('Gamma %g Ang Vel', leg))
    end
end
```




Alpha 3 Ang Vel
20
10
0
-10
-20
-30
-40
-50
-60
-70
Cycle Time
Beta 3 Ang Vel
30
20
10
0
-10
-20
-30
Cycle Time
Gamma 3 Ang Vel
30
20
10
0
-10
-20
-30
Cycle Time


Alpha 4 Ang Vel
70
60
50
40
30
20
10
0
-10
-20
Cycle Time
Beta 4 Ang Vel
30
20
10
0
-10
-20
-30
Cycle Time
Gamma 4 Ang Vel
30
20
10
0
-10
-20
-30
Cycle Time

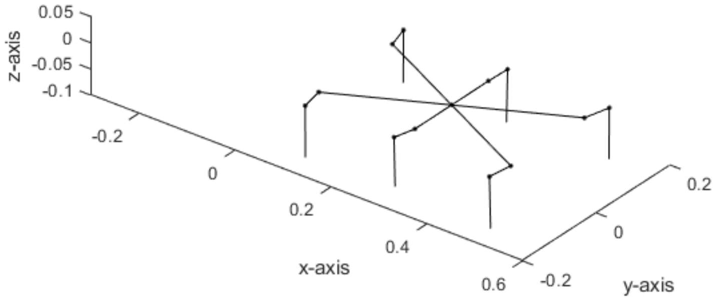




Q9

```
if plot_simulation
    % Simulate the robot moving
    figure

    % Initialize robot parts in starting configuration at t = 0
    body_origin(:,1) = [0; 0; 0];

    hip(1:18,1) = [body_origin(1,1) + body_l/2; body_origin(2,1) + body_w/2; body_origin(3,1) + 0;
                   body_origin(1,1) + body_l/2; body_origin(2,1) - body_w/2; body_origin(3,1) + 0;
                   body_origin(1,1);            body_origin(2,1) + body_w/2; body_origin(3,1) + 0;
                   body_origin(1,1);            body_origin(2,1) - body_w/2; body_origin(3,1) + 0;
                   body_origin(1,1) - body_l/2; body_origin(2,1) + body_w/2; body_origin(3,1) + 0;
                   body_origin(1,1) - body_l/2; body_origin(2,1) - body_w/2; body_origin(3,1) + 0];

    knee(1:18,1) = hip(:,1);

    ankle(1:18,1) = [knee(1,1) - leg_lengths(2)*sind(alpha(1,1))*cosd(beta(1,1)); knee(2,1) + leg_lengths(2)*cosd(alpha(1,1))*cosd(beta(1,1)); knee(3,1) + leg_lengt
                     knee(4,1) + leg_lengths(2)*sind(alpha(2,1))*cosd(beta(2,1)); knee(5,1) - leg_lengths(2)*cosd(alpha(2,1))*cosd(beta(2,1)); knee(6,1) + leg_lengt
                     knee(7,1) - leg_lengths(2)*sind(alpha(3,1))*cosd(beta(3,1)); knee(8,1) + leg_lengths(2)*cosd(alpha(3,1))*cosd(beta(3,1)); knee(9,1) + leg_lengt
                     knee(10,1) + leg_lengths(2)*sind(alpha(4,1))*cosd(beta(4,1)); knee(11,1) - leg_lengths(2)*cosd(alpha(4,1))*cosd(beta(4,1)); knee(12,1) + leg_le
                     knee(13,1) - leg_lengths(2)*sind(alpha(5,1))*cosd(beta(5,1)); knee(14,1) + leg_lengths(2)*cosd(alpha(5,1))*cosd(beta(5,1)); knee(15,1) + leg_le
                     knee(16,1) + leg_lengths(2)*sind(alpha(6,1))*cosd(beta(6,1)); knee(17,1) - leg_lengths(2)*cosd(alpha(6,1))*cosd(beta(6,1)); knee(18,1) + leg_le

    foot(1:18,1) = [hip(1,1) + x_pos(1,1); hip(2,1) + 0.047; (hip(3,1) + z_pos(1,1)) - body_height;
                    hip(4,1) + x_pos(2,1); hip(5,1) - 0.047; (hip(6,1) + z_pos(2,1)) - body_height;
                    hip(7,1) + x_pos(3,1); hip(8,1) + 0.047; (hip(9,1) + z_pos(3,1)) - body_height;
                    hip(10,1) + x_pos(4,1); hip(11,1) - 0.047; (hip(12,1) + z_pos(4,1)) - body_height;
                    hip(13,1) + x_pos(5,1); hip(14,1) + 0.047; (hip(15,1) + z_pos(5,1)) - body_height;
                    hip(16,1) + x_pos(6,1); hip(17,1) - 0.047; (hip(18,1) + z_pos(6,1)) - body_height];

    % Initial Plot
    num_cycles = 10;
```

```
hold on
axis equal
view([37.5 30])
xlim([-0.3 ((L*num_cycles)+0.3)])
xlabel('x-axis')
ylim([-0.2 0.2])
ylabel('y-axis')
zlim([(-body_height) body_h/2])
zlabel('z-axis')

for leg = 1:6
    O_h(leg) = quiver3(body_origin(1,1), body_origin(2,1), body_origin(3,1), hip(3*leg-2,1) - body_origin(1,1), hip(3*leg-1,1) - body_origin(2,1), hip(3*leg,1)
    h_a(leg) = quiver3(hip(3*leg-2,1), hip(3*leg-1,1), hip(3*leg,1), ankle(3*leg-2,1) - hip(3*leg-2,1), ankle(3*leg-1,1) - hip(3*leg-1,1), ankle(3*leg,1) - hip(
    a_f(leg) = quiver3(ankle(3*leg-2,1), ankle(3*leg-1,1), ankle(3*leg,1), foot(3*leg-2,1) - ankle(3*leg-2,1), foot(3*leg-1,1) - ankle(3*leg-1,1), foot(3*leg,1)

    O_h(leg).Marker = '.';
    h_a(leg).Marker = '.';
    a_f(leg).Marker = '.';
end

for n = 1:num_cycles
    %body_origin(:,1) = [(n-1)*L; 0; 0];
    for t = 1:size(t_cycle,2)

        % Recalculate Positions
        if t == 1 && n == 1
            % Don't change the very first instance of body_origin
        elseif t == 1 && n > 1
            body_origin(1,t) = body_origin(1,end) + v*interval;
        else
            body_origin(1,t) = body_origin(1,t-1) + v*interval;
        end

        hip(1:18,t) = [body_origin(1,t) + body_l/2; body_origin(2,t) + body_w/2; body_origin(3,t) + 0;
                       body_origin(1,t) + body_l/2; body_origin(2,t) - body_w/2; body_origin(3,t) + 0;
                       body_origin(1,t);            body_origin(2,t) + body_w/2; body_origin(3,t) + 0;
                       body_origin(1,t);            body_origin(2,t) - body_w/2; body_origin(3,t) + 0;
                       body_origin(1,t) - body_l/2; body_origin(2,t) + body_w/2; body_origin(3,t) + 0;
                       body_origin(1,t) - body_l/2; body_origin(2,t) - body_w/2; body_origin(3,t) + 0];

        knee(:,t) = hip(:,t);

        ankle(1:18,t) = [knee(1,t) - leg_lengths(2)*sind(alpha(1,t))*cosd(beta(1,t)); knee(2,t) + leg_lengths(2)*cosd(alpha(1,t))*cosd(beta(1,1)); knee(3,t) + l
                         knee(4,t) + leg_lengths(2)*sind(alpha(2,t))*cosd(beta(2,t)); knee(5,t) - leg_lengths(2)*cosd(alpha(2,t))*cosd(beta(2,1)); knee(6,t) + l
                         knee(7,t) - leg_lengths(2)*sind(alpha(3,t))*cosd(beta(3,t)); knee(8,t) + leg_lengths(2)*cosd(alpha(3,t))*cosd(beta(3,1)); knee(9,t) + l
                         knee(10,t) + leg_lengths(2)*sind(alpha(4,t))*cosd(beta(4,t)); knee(11,t) - leg_lengths(2)*cosd(alpha(4,t))*cosd(beta(4,1)); knee(12,t)
                         knee(13,t) - leg_lengths(2)*sind(alpha(5,t))*cosd(beta(5,t)); knee(14,t) + leg_lengths(2)*cosd(alpha(5,t))*cosd(beta(5,1)); knee(15,t)
                         knee(16,t) + leg_lengths(2)*sind(alpha(6,t))*cosd(beta(6,t)); knee(17,t) - leg_lengths(2)*cosd(alpha(6,t))*cosd(beta(6,1)); knee(18,t)

        foot(1:18,t) = [hip(1,t) + x_pos(1,t); hip(2,t) + 0.047; (hip(3,t) + z_pos(1,t)) - body_height;
                        hip(4,t) + x_pos(2,t); hip(5,t) - 0.047; (hip(6,t) + z_pos(2,t)) - body_height;
                        hip(7,t) + x_pos(3,t); hip(8,t) + 0.047; (hip(9,t) + z_pos(3,t)) - body_height;
                        hip(10,t) + x_pos(4,t); hip(11,t) - 0.047; (hip(12,t) + z_pos(4,t)) - body_height;
                        hip(13,t) + x_pos(5,t); hip(14,t) + 0.047; (hip(15,t) + z_pos(5,t)) - body_height;
                        hip(16,t) + x_pos(6,t)-0.001; hip(17,t) - 0.047; (hip(18,t) + z_pos(6,t)) - body_height];

        % Replot
        hold on

        for leg = 1:6
            O_h(leg).XData = body_origin(1,t);
            O_h(leg).YData = body_origin(2,t);
            O_h(leg).ZData = body_origin(3,t);
            O_h(leg).UData = hip(3*leg-2,t) - body_origin(1,t);
            O_h(leg).VData = hip(3*leg-1,t) - body_origin(2,t);
            O_h(leg).WData = hip(3*leg,t) - body_origin(3,t);

            h_a(leg).XData = hip(3*leg-2,t);
            h_a(leg).YData = hip(3*leg-1,t);
            h_a(leg).ZData = hip(3*leg,t);
            h_a(leg).UData = ankle(3*leg-2,t) - hip(3*leg-2,t);
            h_a(leg).VData = ankle(3*leg-1,t) - hip(3*leg-1,t);
            h_a(leg).WData = ankle(3*leg,t) - hip(3*leg,t);

            a_f(leg).XData = ankle(3*leg-2,t);
            a_f(leg).YData = ankle(3*leg-1,t);
            a_f(leg).ZData = ankle(3*leg,t);
            a_f(leg).UData = foot(3*leg-2,t) - ankle(3*leg-2,t);
            a_f(leg).VData = foot(3*leg-1,t) - ankle(3*leg-1,t);
            a_f(leg).WData = foot(3*leg,t);% - ankle(3*leg,t);
        end

        pause(interval);
    end
    end
end
```



Q10

```
% On other attachment
```

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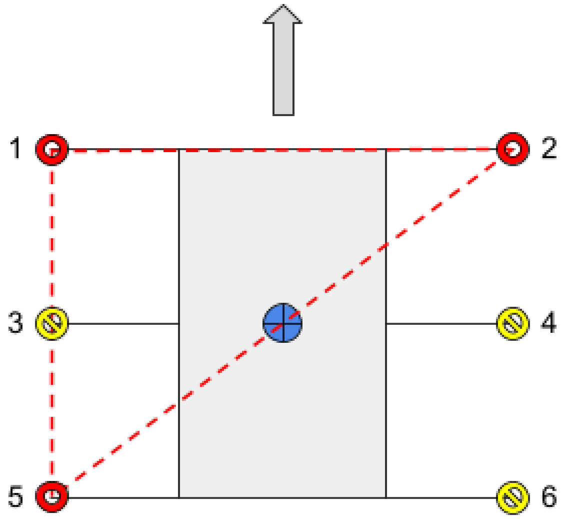
```
function joint_angles = calc_joint_angles(leg_number, x_pos, z_pos)
    % Initialize known values
    leg_lengths = [0, 0.050, 0.100]; % m
    body_height = 0.1; % m
    liftoff_height = 0.005; % m
    body_l = 0.4; % m
    body_w = 0.2; % m
    body_h = 0.1; % m
    num_timesteps = size(x_pos,2);

    % Initialize blank values
    alpha = zeros(1,num_timesteps); % -90 <= alpha <= 90
    beta = zeros(1,num_timesteps); % -30 <= beta <= 90
    gamma = zeros(1,num_timesteps); % 0 <= gamma <= 120

    % Calculate y linear trajectory appropriate for the leg
    % based on the leg's side of the body (left or right)
    %y_pos_from_joint = ((-1)^(leg_number+1)) * (leg_lengths(2) * cosd(asind(x_pos(1) / leg_lengths(2))));
    y_pos_from_joint = ((-1)^(leg_number+1)) * 0.047;
    y_pos = y_pos_from_joint * ones(1,num_timesteps);
    %y_pos_from_body_origin = ((-1)^(leg_number+1)) * ((body_w / 2) + leg_lengths(2) * cosd(asind(x_pos(1) / leg_lengths(2))));


    % Calculate hip angles, 0 deg is outwards from chassis
    % Left legs transfer from +alpha to -alpha
    % Right legs transfer from -alpha to +alpha
    alpha = atan2d(y_pos,x_pos) + ((-1)^(leg_number)) * 90; % hip joint angles

    % Calculate knee and ankle joint angles
    % Knee joint is same point as hip joint
    h = body_height * ones(1,num_timesteps);
    Li = [(-x_pos); (-y_pos); (h - z_pos)]; % vector from foot to knee joint (and hip too)

    for i = 1:num_timesteps % for each time step
        li(i) = norm(Li(:,i),2); % magnitude of vector from foot to knee joint

        phi(i) = 0; % hip and knee are on top of heachother, so asind((h_h - h_k)/2) = 0
        rho(i) = asind(Li(3,i) / li(i));

        % Knee joint angles, 0 is horizontally out from robot, + is up
        beta(i) = loc(leg_lengths(2), li(i), leg_lengths(3)) - (rho(i) + phi(i));

        % Ankle joint angles, 90 is straight down, + is into robot
        gamma(i) = 180 - loc(leg_lengths(2), leg_lengths(3), li(i));
    end

    joint_angles = real([alpha; beta; gamma]);
end
```

```matlab
function loc_angle = loc(a, b, c)
    loc_angle = acosd((a^2 + b^2 - c^2)/(2 * a * b));
end
```

Kyle Mitchell
RBE 521 Final Exam Responses

Problem 1)
Parts 6-9)
Please note that the figures and simulation in the main .pdf submission are compressed for size and do not show the axes well. I have saved and labeled each plot and stored them in a folder in the zip folder of the submission. Please look at these graphs should you need a clearer plot to view. Included in this folder is a video of the simulation running, as well. Running Final_Prob1.m will generate all figures and the simulation, as well.
Part 10)
"If the only information that we have is that legs 1, 2, and 5 are on the ground, would the robot be statically and dynamically stable? Explain your answer clearly? (1 points)"

Assuming an equal weight distribution of the robot, where the center of mass is at the geometric center of the robot, the robot would be statically and dynamically stable. This is because legs 1, 2, and 5 form a support polygon (triangle) under the robot. The robot will be stable if the center of mass's perpendicular projection is within the support polygon. Because the center of mass falls on the edge of the support polygon, it is stable, but highly susceptible to tipping due to external forces. Below is a diagram of the robot with its legs in its home configuration, but legs 3, 4, and 6 are not touching the ground (denoted by the yellow crossed out circles). Legs 1, 2, and 5 are contacting the ground, denoted with red circles. The center of mass is the blue circle with a cross in it. The red dotted lines denote the perimeter of the support polygon.



However, the stability of the robot as it continues moving is dependent on its footfall pattern. For example, if leg 4 was to touch down next the robot would be much more stable, as the center of mass is closer to the centroid of the support polygon. However, if leg 5 then lifts up, the support polygon would no longer contain the center of mass, thus resulting in the robot tipping over. Therefore, when designing gaits for a walking robot, keeping in mind its center of mass and leg positions is critical to make sure that the robot is stable throughout its entire stride cycle.

Problem 2)

Part 1)

Human Approximated Walking Gait Kinematic Phase Diagram

Left						▨	▨	▨	▨	
Right	▨	▨	▨	▨						

White = Support Phase

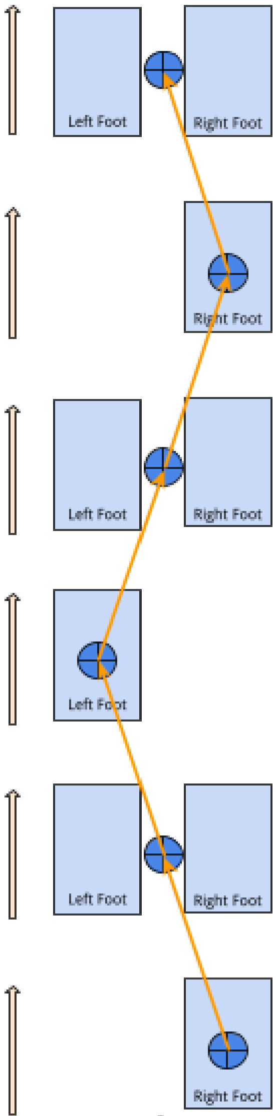
Grey = Transfer Phase

Part 2)

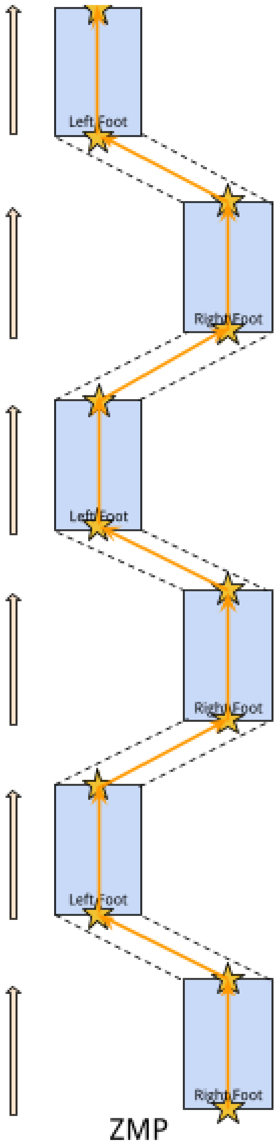
Duty Factor = 6/10 = 3/5 = 60%

Number of legs in contact with ground = # total legs * duty cycle = 2 * 0.6 = 1.2 => 1 or 2 legs

Part 3)



Center of Mass Trajectory For Walking



ZMP Trajectory For Walking

Part 4)

Human Approximated Running Gait Kinematic Phase Diagram

Left	□	□	▨	▨	▨	▨	▨	▨	▨	▨
Right	▨	▨	▨	▨	▨	□	□	▨	▨	▨

White = Support Phase

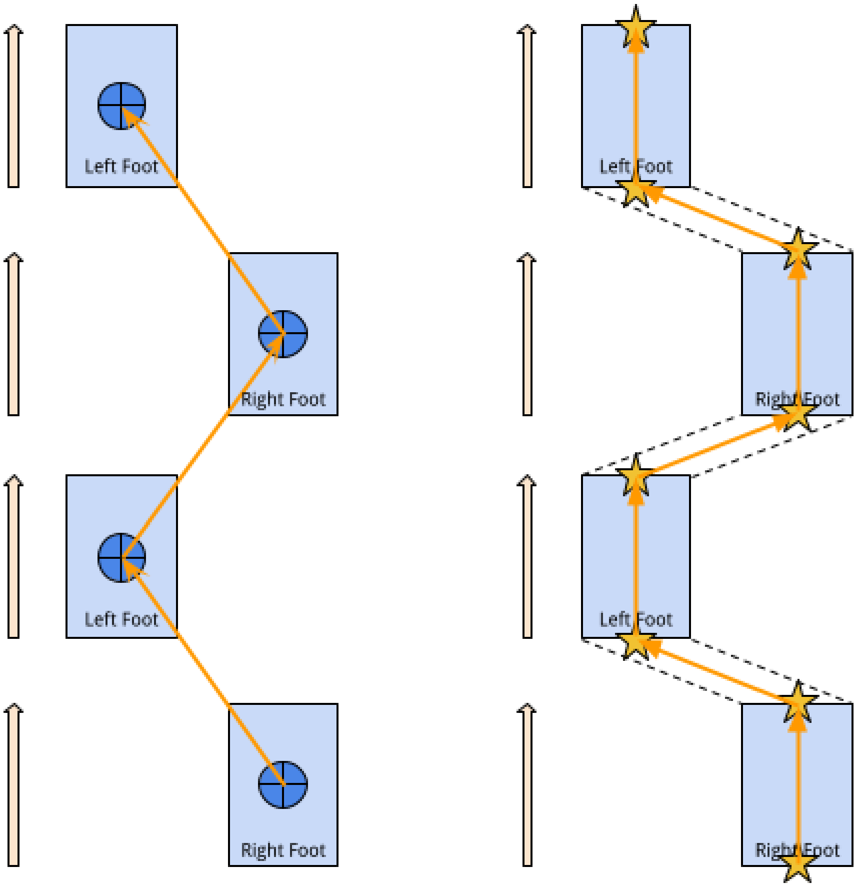
Grey = Transfer Phase

Part 5)

Duty Factor = 2/10 = 1/5 = 20%

Number of legs in contact with ground = # total legs * duty cycle = 2 * 0.2 = 0.4 => 0 or 1 leg

Part 6)



Center of Mass Trajectory For Running

ZMP Trajectory For Running

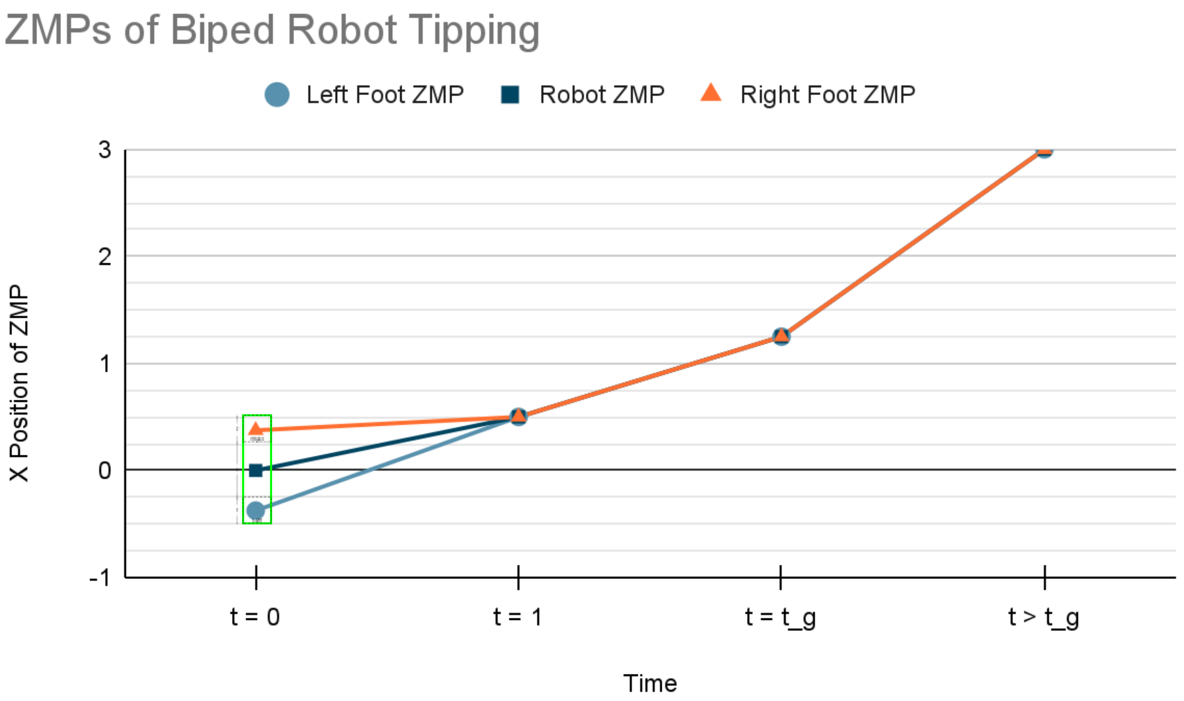
Problem 3)
For this problem, I assumed l = 0.5 meters and f = 0.25 meters from the diagram provided. I chose these values to make the graphing of the three trajectories easier. Please note that while l = 0.5 meters and f = 0.25 meters are arbitrary values, their value in relation to each other is accurate, assuming the diagram is to scale (l appears to be two times larger than f, or 2*f = l).

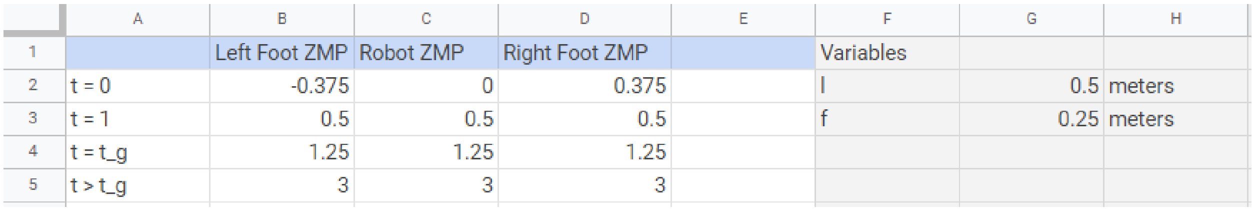
From t = 0 to t = 1, all three ZMPs move in the positive x direction, where at t = 1, they meet at the tipping edge of the robot, the right side of the right foot (0.5 meters using the values assigned to the problem). The Left ZMP has the largest velocity, as it travels the farthest distance of the three ZMPs.

From t = 1 to t = t_g, the robot is tipping over. This is because the ZMP has left the stability region of the robot. Mathematically, this is because the net value of the resultant moment of gravity, inertial force of the robot, and moment due to external forces must equal zero. In order to make this statement correct, the ZMP is in a position where the robot would need additional support to remain stable.

For t > t_g, the robot has tipped all the way over. In this configuration, the relation between the resultant moment of gravity, inertial force of the robot, and moment due to external forces is unknown (in the scope of this problem; new height of COM of the robot, height of where force is being applied, direction of the force acting on COM, whether the robot has other components moving that is causing inertial moments on the system, etc). For this reason, I show the ZMP locations going to a random position farther from the origin of the problem (equidistant between the feet at t = 0).

Below is the graph and cells used to create the graph. please note that at t > 1, the locations of all three ZMPs are assumed to be equal.





	A	B	C	D	E	F	G	H
1		Left Foot ZMP	Robot ZMP	Right Foot ZMP		Variables		
2	t = 0	-0.375	0	0.375		l	0.5	meters
3	t = 1	0.5	0.5	0.5		f	0.25	meters
4	t = t_g	1.25	1.25	1.25				
5	t > t_g	3	3	3				

(Link to Sheet: here)