

# Legged Robotics

## HW #2



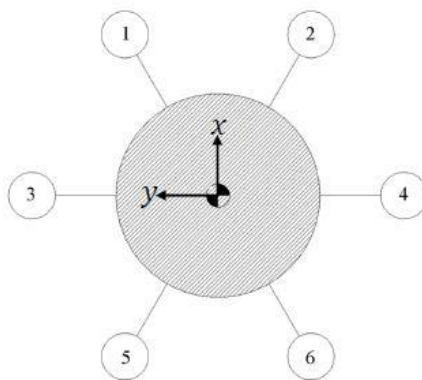
# WPI

### Problem 1 – 35 pts.

Consider a hexapod machine tool (hexapod robot) with a base frame located on the ground (X-Y-Z) and a local frame located in the middle of the moving platform (x-y-z) and six prismatic legs. Assume that the legs are evenly distributed around the top platform with angle of 60 degrees (radially symmetric) as shown in the figure below. Let  $\vec{u}_i$  be the distance between the origin of the base frame and the  $i$ th joint on the fixed platform. Let  $\vec{s}_i$  be the distance vector between the origin of the local frame and the  $i$ th joint on the moving platform where  $i = 1, \dots, 6$ . Define  $\vec{l}_i$  as  $i$ th leg vector and vector  $\vec{O} = [x, y, z]^T mm$  connecting the origin of the base frame to the origin of the local frame. Define  $\vec{\alpha} = [a, b, c]^T$  as vector of Euler angles representing the rotation (orientation) of the local frame w.r.t. the base frame. Assume that the diameter of the fixed platform is 500 mm and the diameter of the moving platform is 300 mm. For the given pose  $\vec{P} = [x, y, z, a, b, c]^T = [10, 0, 100, 5, 5, 0]^T$ , calculate the leg lengths,  $l_i$ , if  $\vec{\alpha}$  represents:

- ZZZ Euler angles in degrees.
- XYZ Euler angles in degrees.
- Compare the result from a) with the result from b). Are they the same? Why? Please explain your answer clearly. What do you conclude?

Note that in the home position, the upper platform has no angle (is horizontal) and z-axis is along Z-axis, and x-axis and y-axis are parallel to X-axis and Y-axis, respectively.

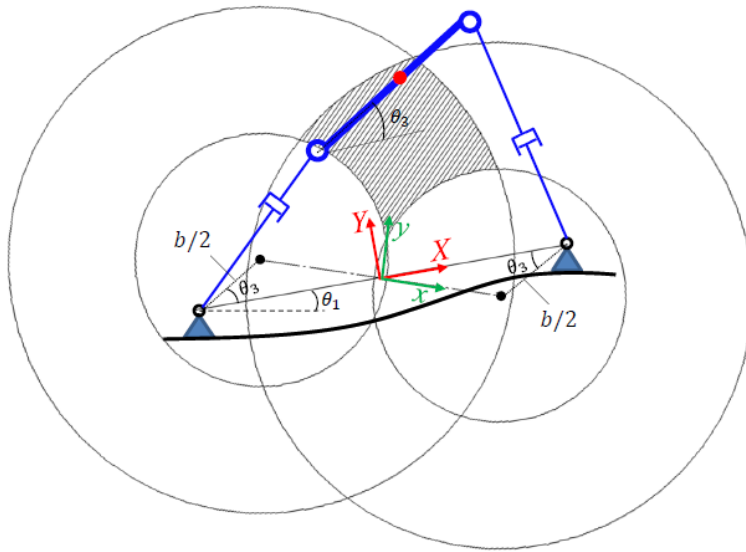


### Problem 2 – 30 pts.

- For the planar parallel robot of HW1, shown below, where  $l_{min} = 100$ ,  $l_{max} = 200$ ,  $b = 300$ ,  $d = 500$ , write up the Velocity Jacobian and report if there is any singularities (singular configurations) within the workspace of this 2D robot when  $\theta_1 = \theta_3 = 0$ . Show your work and discuss your result.

*Hint:* “ $\det(J)=0$ ” indicates that  $J$  is NOT full rank ( $J$  is rank-deficient). You can always use both concepts interchangeably. In other words, in a singular configuration where  $\det(J)=0$ , we can conclude that  $J$  is rank-deficient. This concept is particularly useful when you have a robot whose Jacobian,  $J$ , is a non-square matrix where you cannot solve for  $\det(J)=0$  to find singularities. In this case, you can instead find conditions (configurations) under which the Jacobian,  $J$ , becomes rank-deficient.

*Definition:* A “configuration” is defined as a set of joint variables (leg lengths in this robot). A set of joint variables (leg lengths) corresponds to a specific pose (position and orientation) of the end-effector.



### **Problem 3 – 35 pts.**

Derive the matrix  $B(\alpha)$  that corresponds to XYZ and ZYX Euler angles. The one that we did in the lecture corresponds to ZYZ Euler angles. For credit, you need to show your work for deriving  $B(\alpha)$  corresponding to both XYZ and ZYX Euler angles and include your work in your submission.

Good Luck!