The Quark Model

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Abstract I have made this report on Quarks Model based on my reading from "Introduction to elementary particles" by David Griffiths and other online sources.

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1. What is matter made of?

1.1 The Classical Era(1897-1932)

1.2 Discovery of X-rays

X-rays were discovered by Wilhelm Conrad Roentgen on 8 th of November,1895.Discharge tube setup in partial-vaccum condition at high voltage they noticed that cathode rays(high energy electrons) from filament is emitted from cathode and reached anode resulting in fluorescent glow of the tube.When these high energy rays reached 45 degree tilted target they emitted X-rays whose wavelength ranging from 10 nanometers to 10 picometers.Even when covered with black paper this tube caused glow in barium platinocyanide coated paper denoting these rays can even pass through very dense materials.

1.3 Radioactivity

Radioactivity was discovered by Henri Becquerel in 1896 where he noticed photographic plates wrapped with black paper when placed near uranium salts starts blackening.It meant that some sort of invisible rays from the mineral is causing blackenning inspite of black cover then later found out that it was radioactivity of the mineral.Marie Curie along with her husband Pierre Curie and Henry Becquerel discovered radioactive elements radium and polonium.Marie Curie isolated pure radium metal in 1910.This discovery of radioactivity broke the idea of the atoms being indivisible.

1.3.1 Discovery of electron

The very idea of of atom is divisible was in support after the discovery of electron in the year 1897 by Joseph John Thomson.He performed the cathode ray tube experiment in which he observed when electric current passed through cathode ray tube ,a stream of particle travelled from cathode to anode . He then adjusted the electric and magnetic fields to determine the velocity and charge to mass ratio of the particles.The charge to mass ratio turned out to be so large indicating the particle having small mass. He also proposed that atom consist of electrons suspended in heavy positive charge (just like plum in the pudding so it was also called as "The Plum Pudding Model") to account for neutrality and mass of atom.

1.3.2 Discovery of protons(1917)

The Plum Pudding Model was challenged by the α scattering experiment done by Ernest Rutherford.In the experiment α particles were emitted from radio active source directed to thin gold foil and observed that most of the rays passed through and some got scattered at very large angle implying that most of the atom being empty spaced and most of the mass and positive-charge concentrated at the center of the atom.

1.3.3 Discovery of neutron(1932)

In 1914 Niels Bohr proposed a model for hydrogen atom having a single electron orbiting the proton just like our solar-system.The next heavier atom like Helium having 2 electrons and 2 protons weighed 4 times the mass of hydrogen atom ,Lithium having 3 protons and 3 electrons weighed 7 times the mass of hydrogen atom .The dilemma of additional unaccounted mass was resolved with discovery of neutron by Sir James Chadwick.It was then agreed that electron(significantly less mass),proton and neutron which is neutral in charge altogether constitute for the total mass of an atom.

1.3.4 The photon

The issue of light nature being wave and particle was settled by the experiment done by A.K.Compton in 1923. He found that light scattered from a particle at rest is shifted in wavelength,given by
$\lambda' = \lambda + \lambda_c(1 - \cos\theta)$
where λ is the incident wavelength ,λ' is the scattered wavelength,θ is the scattering angle and λ=$\frac{h}{mc}$ is called Compton wavelength.This was the experimental evidence that light behaves as a particle.It was later called to be photon. According to quantum field theory any field or its action of force is the result exchange of particles which we call it to be the quanta of the corresponding field. Photon is the quanta of electromagnetic field.

1.4 The Middle Period(1930-1960)

1.4.1 Mesons(1934-1947)

What holds nucleus together?How does two protons so much close enough not get repelled each other?

THE STRONG FORCE.

Hideki Yukuwa proposed the theory of strong force in 1934 in which he said proton and neutron are attracted by some sort field and the mediator corresponding to the field should be heavier due to its short range.He calculated the weight and found to be less than that of proton and greater than electron. The quanta of this field was named meson(middle-weight),relatively electron(light-weight) and proton and neutron(heavy-weight). By 1937 two seperate groups identified particle matching Yukuwa's discription while studying the cosmic rays which was later found they both are different in terms of weight and lifetime.Also the newly found particle interacted with nuclei weakly which was not expected. Later in 1947 it was resolved that there were actually two middle-weighted particles in cosmic rays π(pion) and μ(muon).It was π which is actual Yukuwa's meson which disintegrated to μ on reaching from atmosphere to sea level.

1.4.2 Antiparticles(1930-1956)

The discovery of Dirac's equation in 1927 which gave one positive and other negative energy solution.But the electron does not radiate energy and jump to negative energy levels in general. To account for this he postulated that negative energy levels are filled with infinite sea of electrons.The absence of electron in the sea would be interpreted as particle with net positive charge and positive energy . In support of this theory came the discovery of positron in the year 1931 by Carl Anderson.

1.4.3 Neutrinos(1930-1962)

In β decay process the radioactive nucleus transformed to lighter nucleus with ejection of an electron.

$$A \to B + e^-$$

The energy of the particles was not consistent with expected value.Pauli proposed there is a silent particle which is accounted for missing energy which was later named neutrino.

- Beta decay process

  $$n \to p^+ + e^- + \bar{\nu}$$

- Pion disintegration

  $$\pi \to \mu + \nu$$

  The emerged μ direction was perpendicular to direction π and it can reasoned saying that another particle should have been emitted in the opposite direction which is traceless as it is neutral particle.
- Muon disintegration

  $$\mu \to e + 2\nu$$

  Further the muon disintegrated to give electron in perpendicular direction implying other neutral particle emmited in opposite direction.But as the electron energy in muon decay not was fixed , it was accepted that 2 neutrinos are emitted.
- Inverse beta decay

  $$\bar{\nu} + p^+ \to n + e^+$$

  This experiment performed at Savannah River nuclear reactor in South Carolina in 1950s by Clyde Cowan and Frederick Reines provided the results confirming the existence of neutrino.

Lepton number:
Cross reaction of Cowann and Reines reaction:

$$\nu + n \to p^+ + e^-$$

Raymond Davis replaced neutrinos with anti-neutrinos to check if the reaction occurs.

$$\bar{\nu} + n \to p^+ + e^-$$

But he found this reaction does not occur ,concluding neutrino and anti-neutrino are distinct particles. Later in 1953 ,Konopinski and Mahmoud gave rule in identifying which reaction occurs and which is forbidden.
LAW OF CONSERVATION OF LEPTON NUMBER:
Let us assign lepton number to each particle

1. L= +1 for electron,muon and neutrino
2. L= -1 for positron,the positive muon and the antineutrino
3. L=0 for all other particles

This law says the sum of lepton number before must be equal to sum of lepton number after Now based on this previosly mentioned meson dacays can be altered accordingly

$$\pi^- \to \mu^- + \bar{\nu}$$

Lepton number
LHS:0
RHS:+1-1=0

$$\pi^+ \to \mu^+ + \nu$$

Lepton number
LHS:0
RHS:-1+1=0

$$\mu^- \to e^- + \nu + \bar{\nu}$$

Lepton number
LHS:+1
RHS:+1+1-1=+1

$$\mu^+ \to e^+ + \nu + \bar{\nu}$$

Lepton number
LHS:-1
RHS:-1+1-1=-1
Muon number:

$$\mu^- \to e^- + \gamma$$

The above reaction satisfies the law of lepton number conservation and yet this does not occur. Then muon number was defined supposing there are two different kinds of neutrino ν_e (associated with electron) and ν_μ (associated with muon). Then muon number is given by

1. Muon number
   $L_\mu = +1$ for $\mu_- \; and \; \nu_\mu$
   $L_\mu = -1$ for $\mu_+ \; and \; \bar{\nu}_\mu$
2. Electron number
   $L_e = +1$ for $e^- \; and \; \nu_e$

$$L_e = -1 \text{ for } e^+ \quad and \quad \bar{\nu}_e$$

Based on this previous lepton number conservation law can be refined to conservation of electron and muon number. Now the above mentioned reaction can be altered according to this new assignment.

$$n \to p^+ + e^- + \bar{\nu}_e$$

Muon number:
LHS:0
RHS:0
Electron number
LHS:0
RHS:+1-1=0

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$

Muon number:
LHS:0
RHS:+1-1=0
Electron number:
LHS:0
RHS:0

$$\pi^+ \to \mu^+ + \nu_\mu$$

Muon number:
LHS:0
RHS:-1+1=0
Electron number:
LHS:0
RHS:0

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

Muon number:
LHS:+1
RHS:+1
Electron number:
LHS:0
RHS:+1-1=0

$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

Muon number:
LHS:-1
RHS:-1
Electron number:
LHS:0
RHS:-1+1=0

The first experimental test of two-neutrino hypothesis was conducted at Brookhaven in 1962. Until now lepton family was grown to 8 consisting of :the electron,the muon,their respective neutrinos,and the corresponding antiparticles.

1.4.4 Strange particles(1947-1960)

In 1947 Stuart Butler and Robert E.L.Rochester published a cloud chamber photograph depicting the below reaction where Kaon the neutral particle disintegrated to give $\pi^+ and \pi^-$

$$K^0 \to \pi^+ + \pi^-$$

Kaon was included in meson family.
In 1950 ,Carl David Anderson and his group discovered another neutral particle particle by following reaction.

$$\lambda \to p^+ + \pi^-$$

Baryon number: A=+1 for all baryons
A=-1 for all anti-baryons

Lambda belongs to baryon family along with proton and neutron.Following this many baryons were discovered and termed as Strange particles as their formation rate was relatively faster than decay rate,which implied that different mechanisms governed them. That is strange particles are produced by strong force and decay by weak force.This lead to a result of strange particles be produced in pairs(Associated production). Murray Gell-Mann and Kazuhiko Nishijima in 1953 assigned each particle a new property called strangeness that is conserved in strong interaction but not conserved in weak interaction. In pion-proton collision

$$\pi^- + p^+ \to K^+ + \Sigma^-$$

$$\pi^- + p^+ \to K^0 + \Sigma^0$$

$$\pi^- + p^+ \to K^0 + \lambda$$

Strangeness:
K :S=+1 ; Σ and λ:S=-1 When these particles decay ,strangeness is not conserved.

2. The Eightfold Way(1961-1964)

By 1960, there were lot of elementary particles and there was a need its own periodic table.Murray Gell-Mann introduced the "Eightfold Way" in 1961.It arranged the baryons and mesons in geometrical patterns ,according to their charge and strangeness. In Figure 1 the particles with like charges are placed along downward-sloping diagonals and horizontal lines associate particles of like strangeness. In case of mesons the anti-particles lie in the same supermultiplets in diametrically opposite directions. The bottom most particle in Figure 3 was not known then and Gell-Mann predicted that a particle will be found having charge of -1 and strangeness of -3 and later in 1964 omega minus particle was discovered satisfying the proposed properties.

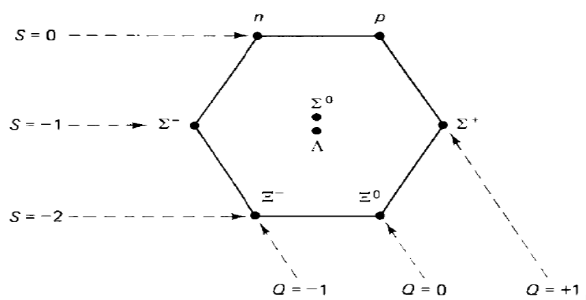

FIGURE 1. Baryon octet

3. The Quark Model(1964)

Why do the hadrons fit in these patterns? Murray Gell-Mann and George Zweig in 1964 proposed that all hadrons are composed of even more independant particles ,which was called quarks.

- Up quark
  (charge:2/3,strangeness:0)
- down quark
  (charge:-1/3,strangeness:0)
- strange quark
  (charge:-1/3,strangeness:-1)

Quarks form a triangular pattern similar to Eightfold way pattern.For each quark there corresponds an antiquark having opposite charge and strangeness. And there are two rules of composition to form hadrons:

1. Every baryon is composed of three quarks and every antibaryon is composed of three anti-quarks.
2. Every meson is composed of a quark and an anti-quark

With the composition rules in mind we can try various combinations of quarks to give integral charge and strangeness.

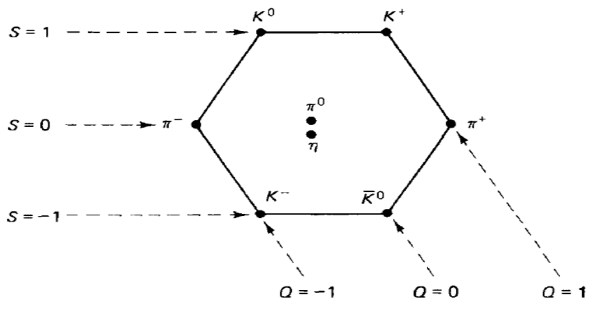

FIGURE 2. Meson octet

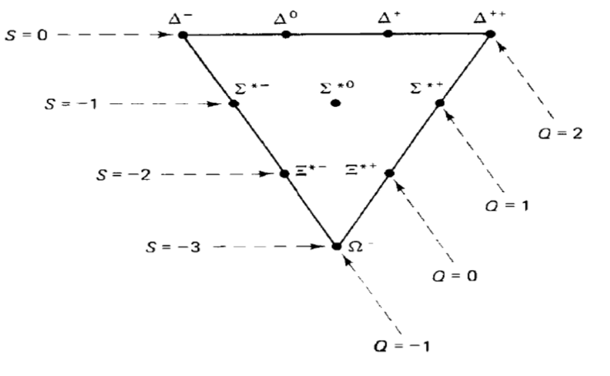

FIGURE 3. Baryon decuplet

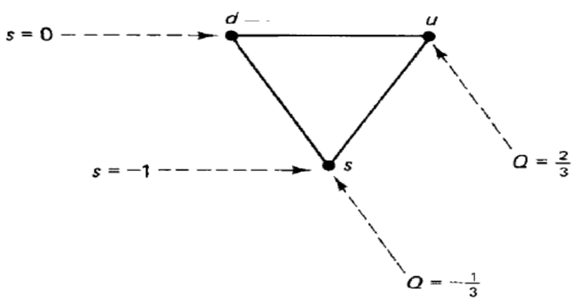

FIGURE 4. The quarks

For baryons combination of three quarks to give various particles is given in Figure 6.We get 10 combinations of three quarks

For mesons similary the combination of a quark and an anti-quark to give various particle is given in Figure 7.We get nine combinations but only 8 were given by meson octet ,then quark model implied that three is a other combination having Q=0 and S=0 and composition $s\bar{s}$ similar to $\pi^0(u\bar{u})$ and $\eta(d\bar{d})$

Some problems agianst Quark Model

- Quark confinement The failure of experiments to produce isolated quarks in 1960s and 1970s was one major problem faced.People who clung to quarks model proposed quarks are absolutely confined within baryons and mesons for no reason and you cannot get them out.
- Violation of Pauli exclusion principle: Δ^{++} which is compostion of three identical u quarks in the same state appear to violate the pauli exclusion principle.

Solution Though quarks are confined it can be studied like how atom was bombarded with particle from outside,and result of bombarding particles on proton showed most of them passed right through while some bounced back sharply implying charge concentrated in lumps.Also it provided evidence for quarks by giving three lumps.

Color hypothesis proposed by O.W.Greenberg in 1964 which suggested that quarks posses additonal property of three different color(red,green and blue) in addition to three

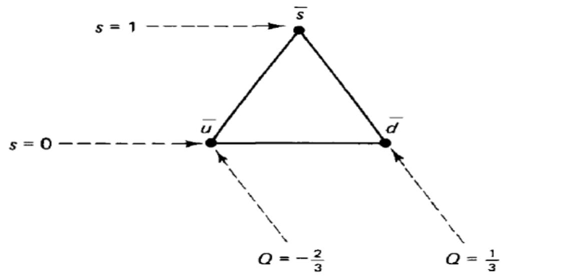

FIGURE 5. The antiquarks

qqq	Q	S	Baryon
uuu	2	0	Δ^{++}
uud	1	0	Δ^{+}
udd	0	0	Δ^{0}
ddd	−1	0	Δ^{-}
uus	1	−1	Σ^{*+}
uds	0	−1	Σ^{*0}
dds	−1	−1	Σ^{*-}
uss	0	−2	Ξ^{*0}
dss	−1	−2	Ξ^{*-}
sss	−1	−3	Ω^{-}

FIGURE 6. The baryon decuplet

$q\bar{q}$	Q	S	Meson
$u\bar{u}$	0	0	π^0
$u\bar{d}$	1	0	π^+
$d\bar{u}$	−1	0	π^-
$d\bar{d}$	0	0	η
$u\bar{s}$	1	1	K^+
$d\bar{s}$	0	1	K^0
$s\bar{u}$	−1	−1	K^-
$s\bar{d}$	0	−1	$\bar{K}^0$
$s\bar{s}$	0	0	??

FIGURE 7. The meson nonet

flavours(u,d and s).Hence to make a quark we take one from each color making it different and resolving the problem as pauli principle only applies for identical particles.
The introduction of color also showed why only particular combination of quarks and anti-quarks makes up particle.
In analogous to white light equal amount of all three colors make the resultant colorless.And anti-quark having anti-color and in combination with corresponding quarks gives again the resultant colorless.This is the reason for particlar composition of mesons,baryons and anti-baryons.

4. The November Revolution

This started with discovery of ψ meson in 1974 Chao Chung Ting.It was electrically neutral, extremely heavy and extrodinary long lifetime.In the following month nature of this meson was on debate and quarks model provided explanation stating $\psi(c\bar{c})$ meson is a bound state of a new fourth quark named charm(c).This was welcomed with previuosly expected parallel between leptons and quarks by Glashow compelling for four leptons and four quarks.With the discovery of new quark new baryons and mesons were expected carrying various amount of charms.The first charmed baryons (λ^+ and Σ_c^{++}) appeared in 1975 and first charmed mesons ($D^0 = c\bar{u}$ and $D^+ = c\bar{d}$) were discovered in 1976. Then in 1975 a new lepton τ(tau) was discovered spoiling the Glashow's symmetry .It has its own neutrino thus reaching to total of 6 leptons.Also following this new heavy meson named the upsilon was disovered said to have new fifth quark called beauty or bottom.The first bottom baryon $\lambda_b^0 = udb$ and the first baryon mesons $\bar{B} = b\bar{d}$ and $B^- = b\bar{u}$ were found in 1983.With all these it was obvious to predict the sixth quark to restore the symmetry that is top quark whose existence was definitely established by 1995.

5. Intermediate Vector Bosons

In beta decay process where weak force is responsible at high energies it was proposed that interaction is mediated by the exchange of some particle came to be known as intermediate vector bosons.Electro weak theory of Sheldon Glashow, Abdus Salam, and Steven Weinberg predicted three intermediate vector bosons($W^+, W^- and Z$) and predicted mass.Later in 1983 Carlo Rubbia's group reported the discovery of W and Z having the nearly the same energy as predicted.

6. The Standard Model

There are 6 leptons as given in lepton classification table and corresponding 6 more antileptons making total of 12 leptons. There are 6 quarks as given in quark classification table and corresponding 6 more antiquarks . Total of 12 quarks each having three colors making the total 36. For mediators of every interaction -the photon for electromagnetic force ,two W's and a Z for the weak force and 8 gluons for strong force making total mediators count of 12.Gluons are the mediators between two quarks.The eight gluons are same except for the colors.Including the Higgs particle in total we are left with of total 61 particles. Anyways standard model does not tell us how to calculate quarks and lepton masses.

7. Symmetries

7.1 Flavour symmetry

Werner Karl Heisenberg proposed that we regard proton and nuetron as two states of a single particle nucleon as they had identical mass and differ in charge.

p= $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$; n= $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ With anology of spin of electron we introduce isospin denoted by I and having components I_1, I_2, I_3 in an abstract isospin space.Now we say nucleon carries isospin $1/2$ and having the third component I_3 as $+1/2$ for the proton and $-1/2$ for the neutron. p=$|1/2 \quad 1/2\rangle$ and n=$|1/2 \quad -1/2\rangle$ where proton is said to have isospin up and neutron to have isospin down.Heisenbery also proposed that strong interactions are invariant under rotations in isospin space which we call internal symmetry.Then it follows from Noether's theorem that isospin is conserved in all strong interactions and also that the strong interactions are invariant under an internal symmetry group SU(2) where the nucleons belong to two-dimensional representation(isospin 1/2).
Now we can assign isospin to horizontal rows in Eightfold way diagram as they display same mass and differ by charge.Knowing the multicipility we can calculate isospin as follows:
Multicipility=2I+1
The third component of isospin is given by Gell-Mann-Nishijima formula:
Q=$I_3 + 1/2(A + S)$
For pions,I=1:
$\pi^+ = |1 \quad 1\rangle$,$\pi^0 = |1 \quad 0\rangle$,$\pi^- = |1 \quad -1\rangle$
For λ,I=0:
$\lambda = |0 \quad 0\rangle$
For Δ,I=3/2:
$\Delta^{++} = |3/2 \quad 3/2\rangle$,$\Delta^+ = |3/2 \quad 1/2\rangle$,$\Delta^0 = |3/2 \quad -1/2\rangle$,$\Delta^- = |3/2 \quad -3/2\rangle$.

7.2 Discrete symmetry

7.2.1 Parity

Many considered that mirror image of any physical laws are also perfectly possible that is parity invariance of laws of nature.Though parity invariance had evidence in strong and electromagnetic processes there wasn't any for weak

LEPTON CLASSIFICATION

	l	Q	L_e	L_μ	L_τ
First generation	e	-1	1	0	0
	ν_e	0	1	0	0
Second generation	μ	-1	0	1	0
	ν_μ	0	0	1	0
Third generation	τ	-1	0	0	1
	ν_τ	0	0	0	1

FIGURE 8

QUARK CLASSIFICATION

	q	Q	D	U	S	C	B	T
First generation	d	$-\frac{1}{3}$	-1	0	0	0	0	0
	u	$\frac{2}{3}$	0	1	0	0	0	0
Second generation	s	$-\frac{1}{3}$	0	0	-1	0	0	0
	c	$\frac{2}{3}$	0	0	0	1	0	0
Third generation	b	$-\frac{1}{3}$	0	0	0	0	-1	0
	t	$\frac{2}{3}$	0	0	0	0	0	1

FIGURE 9

interactions.Chien-Shiung Wu did experiment in which radioactive cobalt 60 nucleii were carefully aligned so that spin is in one direction,supposing z-direction.She noticed Cobalt 60 underwent beta-decay emitting the electrons in the opposite direction of the nuclear spin that is along negative z-axis.

$$Co^{60} \to Ni^{60} + e + \bar{\nu}_e$$

The same experiment when examined through mirror image we see that still electrons are ejected downwards but the spin is now along negative direction.That is here the spin and electron emission are in same direction.This is called **Parity Violation**

Behaviour of neutrino Particle with helicity +1 is called right-handed and particle with helicity -1 is called left-handed.A right-handed electron moving right when observed by inertial observer moving at speed higher that of the electron he notices electron moves leftwards but still spinning the same way .It became left-handed from right-handed just from changing the frame of reference that is not Lorentz invariant. Neutrino considering massless should travel at speed of light and there is no observer faster than them and impossible to change the direction by moving in faster frame making helicity of neutrino Lorentz invariant.Everyone assumed that neutrinos were half right-handed and half left-handed but which was not the case.

NEUTRINOS ARE LEFT-HANDED

ANTINEUTRINOS ARE RIGHT-HANDED

The mirror image of a neutrino does not exist.

Parity operator

Parity operator(P) when applied to a vector **a** it produces a vector pointing opposite direction($P(a) = -a$).It is multiplicative quantum number.There are some vectors whose sign does not change under parity transformation($P(a) = a$).Former one is polar vectors while the latter one is pseudo vectors.Similary there are two kinds of scalars which does change which we call pseudoscalars and other is ordinary.

7.2.2 Charge conjugation

Charge conjugation operator converts each particle into its anti-particle.This is also multiplicative quantum number.

$$C|p\rangle = \pm|p\rangle = |\overline{p}\rangle$$

$$C|p\rangle = |\overline{p}\rangle$$

Thus only those particles which are their own anti-particles can be eigenstates of C.This inculdes photon and the mesons that lie at the center of their Eightfold-Way diagram.It is conserved in strong and electromagnetic interactions.But when applied to neutrino(left-handed always) gives left-handed anti-neutrino(which does not occur) C operator acting neutral particle alters internal quantum numbers leaving mass,energy,momentum and spin untouched. Hence eigenvalues of C are ± 1

7.2.3 CP

CP turns the left-handed antimuon into a right handed muon,which is exactly what we observe in nature

$$CP|K^0\rangle = -|\overline{K^0}\rangle$$

$$CP|\overline{K}^0\rangle = -|K^0\rangle$$

and the normalised eigenstates of CP are $|K_1\rangle = (1/\sqrt{2})(|K^0\rangle - |\overline{K}^0\rangle)$ and $|K_2\rangle = (1/\sqrt{2})(|K^0\rangle + |\overline{K}^0\rangle)$ with $CP|K_1\rangle = |K_1\rangle$ and $CP|K_2\rangle = -|K_2\rangle$ We know kaons decay into 2 or 3 pions and we based on CP it concludes K_1 decays into two pions and K_2 decays into three pions.If we start with beam of K^0's K_1 decays faster than K_2 thus we see should see 2π events near the source and 3 π events farther.K_1 lifetime is less than that of K_2.

7.2.4 CP Violation

By using long enough beam we can produce pure sample of long-lived samples.An experiment was reported by James Cronin and Val Logsdon Fitch in 1964 in which at the end of a beam 57 feet long ,they counted 45 two pion events in a total of 22,700 decays.This shows us the violation of CP.

7.2.5 Time reversal and the TCP Theorem

Time reversal is difficult to test .The direct test like taking particular reaction and run it in reverse.For corresponding conditions of momentum,energy,spin and reaction rate should be same in either direction.In weak interaction inverse-reaction experiments are tough making it difficult for us to observe failure of time reversal as expected. TCP theorem states that the combined operation of time reversal,charge conjugation,and parity is an exact symmetry of any interaction.If CP is violated as in Fitch-Cronin ther must be corresponding violation of T.

8. Light Quark Mesons

The up and down quarks constitute an isospin douplet: u=$|1/2 \quad 1/2\rangle$,d=$|1/2 \quad -1/2\rangle$

When we combine two particle with I=1/2,we obtain an isotriplet

$$|1 \quad 1\rangle = -u\bar{d}$$

$$|1 \quad 0\rangle = (u\bar{u} - d\bar{d})/\sqrt{2}$$

$$|1 \quad -1\rangle = -d\bar{u}$$

and an isosinglet

$$|0 \quad 0\rangle = (u\bar{u} + d\bar{d})/\sqrt{2}$$

In pseudoscalar mesons the triplet is π and for vector mesons it is ρ

$$\pi^0, \rho^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$$

The left η and η' or $(\omega and \phi)$ must be represented by leftover isosinglet combination and $s\bar{s}$. Pseudoscalar mesons appear to be

$$\eta = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$$

$$\eta' = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$$

and vector mesons

$$\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$$

$$\phi = s\bar{s}$$

Since η' which treats u,d and s symmetrically , is uneffected by SU(3) transformations,it is a singlet under SU(3).The η transforms as part of an SU(3) octet,whose other members are three pions and four K's which totally makes up the previous pseudoscalar octet.By contrast ,neither ϕ nor the ω is an SU(3) singlet.They are mixed since the strange quark is isolated from the other two. Incidentally,the other mesons nonetes seem to follow the $\phi - \omega$ mixing pattern.

$$K^+ = u\bar{s}, K^0 = d\bar{s}, \overline{K}^0 = -s\bar{d}, K^- = s\bar{u}$$

In the language of group theory,the three light quarks belong to the fundamental representation of SU(3),whereas the antiquarks belong to conjugate representation.What we have done is combine these representations to obtain an octet and a singlet:

$$3 \otimes \bar{3} = 8 \oplus 1$$

9. Baryons

The quarks carry spin 1/2,so each can occupy either of two states:spin up(↑) or spin down(↓).Thus we have eight possible states for the three quarks :(↑↑↑),(↑↑↓),(↑↓↑),(↑↓↓),(↓↑↑),(↓↑↓),(↓↓↑),(↓↓↓)
The quarks spin can combine to give a total of 3/2 or 1/2 which are :
For spin=3/2 (ψ_s)
$|3/2 \quad 3/2\rangle = (\uparrow\uparrow\uparrow)$
$|3/2 \quad 1/2\rangle = (\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow)/\sqrt{3}$
$|3/2 \quad -1/2\rangle = (\downarrow\downarrow\uparrow + \downarrow\uparrow\downarrow + \uparrow\downarrow\downarrow)/\sqrt{3}$
$|3/2 \quad -3/2\rangle = (\downarrow\downarrow\downarrow)$
For spin=1/2 (ψ_{12})
$|1/2 \quad 1/2\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\uparrow/\sqrt{2}$
$|1/2 \quad -1/2\rangle_{12} = (\uparrow\downarrow - \downarrow\uparrow)\downarrow/\sqrt{2}$
For spin=1/2 (ψ_{23})
$|1/2 \quad 1/2\rangle_{23} = \uparrow(\uparrow\downarrow - \downarrow\uparrow)\sqrt{2}$
$|1/2 \quad -1/2\rangle_{23} = \downarrow(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$
The spin 3/2 combinations are completely symmetric that is interchanging any two particles leaves the state untouched.The spin 1/2 is combinations are partially antisymmetric that is interchanging two particles changes the sign.
In the language of group theory ,the direct product of three fundamental representations of SU(2) decomposes into the direct sum of a four-dimensional representation and two two-dimensional representations.

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

The wave functions of a baryon consists of several pieces;there is the spatial part,describing the locations of the three quarks;there is the spin part,representing their spins ;there is a flavour component,indicating what combination of u,d and s is involved;and there is a color term,specifying the color of three quarks:

$$\psi = \psi(space)\psi(spin)\psi(flavor)\psi(color)$$

This whole must be anti-symmetric under the interchange of any two quarks.Spatial state is symmetric as as there is no angular momentum since l=l'=0.The spin state can be either completely symmetric (j=3/2) or of mixed symmetry (j=1/2).As for flavour there are 3^3=27 possibilites:uuu,uud,udu,udd,...,sss,which we reshuffle intro symmetric,antisymmetric and mixed combinations;they form irreducible representations of SU(3).These are convinently displayed in Eightfold-Way patterns: Thus the combination of three light-quark flavours gives us a decuplet,a singlet,and two octets that is language of group theory , the direct product of three fundamental representations of SU(3) decomposes according to the rule

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

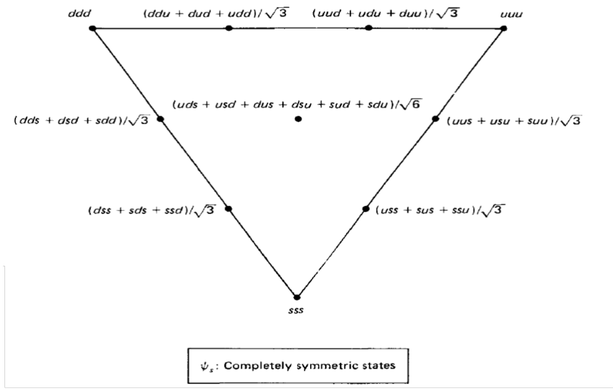

FIGURE 10

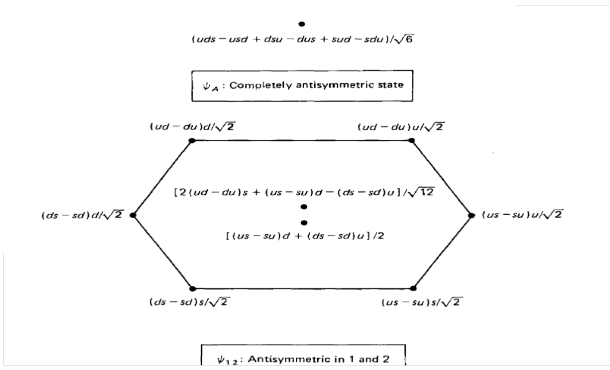

FIGURE 11

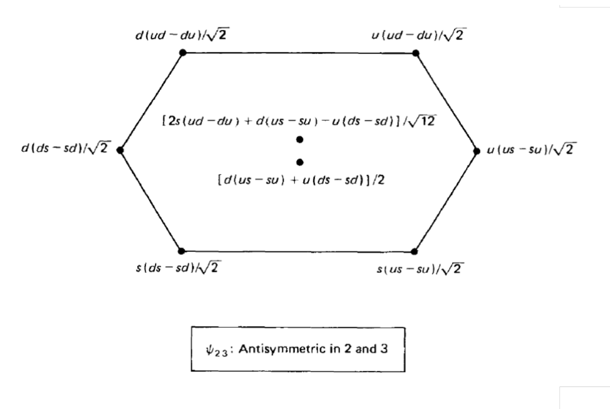

FIGURE 12

10. Timeline for various discoveries

TABLE 1. Timeline

Event	Year	Discovery
Discovery of X-rays	1895	Wilhelm Conrad Roentgen
Discovery of Radioactivity	1896	Antoine Henri Becquerel
Discovery of electron	1897	Joseph John Thompson
Discovery of proton	1909	Ernest Rutherford
Dirac's equation	1927	Paul Adrien Maurice Dirac
Discovery of Positron	1932	Carl Anderson
Discovery of neutron	1932	James Chadwick
Strong Force Theory	1934	Hideki Yukawa
Discovery of Kaons	1947	Clifford Butler and George Rochester
Eightfold way	1961	Murray Gell-Mann
The Quark Model	1964	Murray Gell-Mann and George Zweig

[a]Example footnote.

11. Plot between I_3 and Hyperchage

Hypercharge(H) is given baryon number(B) plus strangeness number(S): Y=B+S
We see that when we plot values of I3 vs H ,the resultant resembles the original Eightfold Way structures.

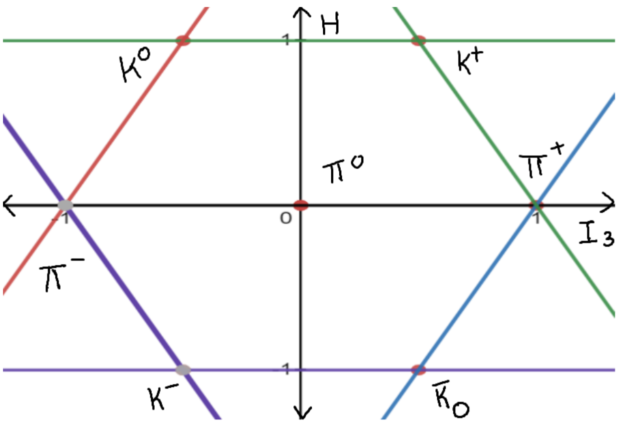

FIGURE 13. Pseudoscalar Mesons

References

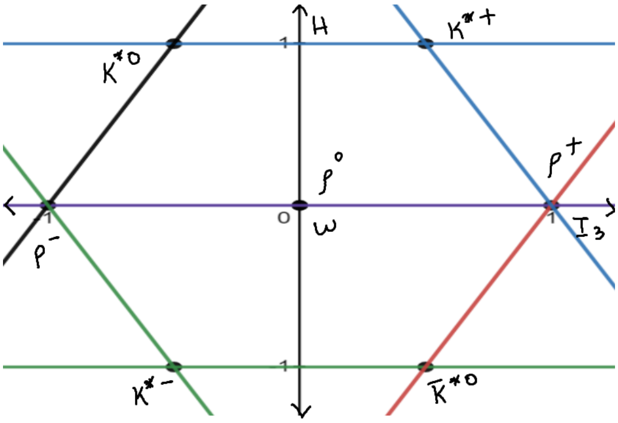

FIGURE 14. Vector mesons

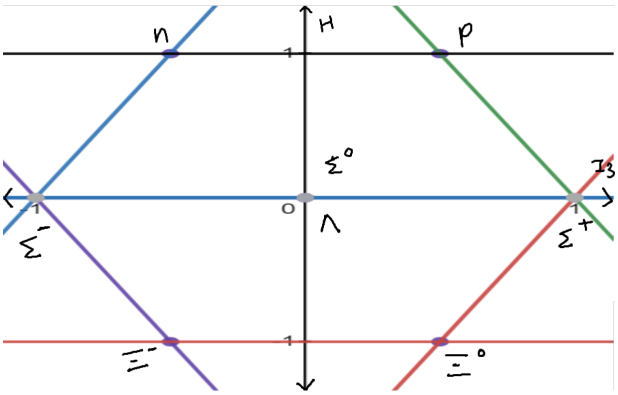

FIGURE 15. Baryons

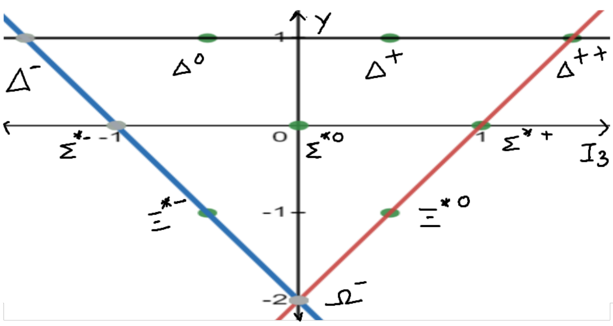

FIGURE 16. Resonance baryons

TABLE 2. Pseudoscalar mesons, $J^P = 0^-$(Spin=0)

Particle	I	I3	B	S	Y
π^+	1	+1	0	0	0
π^0	1	0	0	0	0
π^-	1	-1	0	0	0
K^+	1/2	+1/2	0	1	1
K^0	1/2	-1/2	0	1	1
$\overline{K}^0$	1/2	+1/2	0	-1	-1
K^-	1/2	-1/2	0	-1	-1
n	0	0	0	0	0

TABLE 3. Vector Mesons, $J^P = 1^-$(Spin=1)

Particle	I	I3	B	S	Y
ρ^+	1	+1	0	0	0
ρ^0	1	0	0	0	0
ρ^-	1	-1	0	0	0
K^{+*}	1/2	+1/2	0	1	1
K^{0*}	1/2	-1/2	0	1	1
$\overline{K^{0*}}$	1/2	+1/2	0	-1	-1
K^{-*}	1/2	1/2	0	-1	-1
ω	0	0	0	0	0

TABLE 4. Baryons, $J^P = (1/2)^+$(Spin=1/2)

Particle	I	I3	B	S	Y
p	1/2	+1/2	1	0	1
n	1/2	-1/2	1	0	1
λ^0	0	0	1	-1	0
Σ^+	1	+1	1	-1	0
Σ^0	1	0	1	-1	0
Σ^-	1	-1	1	-1	0
Ξ^0	1/2	1/2	1	-2	-1
Ξ^-	1/2	-1/2	1	-2	-1

TABLE 5. Resonance Baryons, $J^P = (3/2)^+$(Spin=3/2)

Particle	I	I3	B	S	Y
Δ^{++}	3/2	+3/2	1	0	1
Δ^+	3/2	+1/2	1	0	1
Δ^0	3/2	-1/2	1	0	1
Δ^-	3/2	-3/2	1	0	1
$\Sigma*+$	1	+1	1	-1	0
$\Sigma*0$	1	0	1	-1	0
$\Sigma*-$	1	-1	1	-1	0
Ξ^{*0}	1/2	+1/2	1	-2	-1
Ξ^{*-}	1/2	-1/2	1	-2	-1
Ω^-	0	0	1	-3	-2