Design and Developments in Underactuated Underwater Vehicles

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***Abstract*—This report describes a general overview of underactuated underwater vehicles. A detailed study on the general procedure of design and development of underactuated underwater vehicles is made using pre-existing designs of AUVs and ROVs. The report also illustrates the kinematic and dynamic modelling of the aforementioned vehicles. Case studies of X4 AUV and RRC ROV have been considered owing to their extensive literature, previously conducted research and practical applications. Their models have been included here to feign relevance to the general models. Recent developments and advancements of UUVs have also been highlighted.**

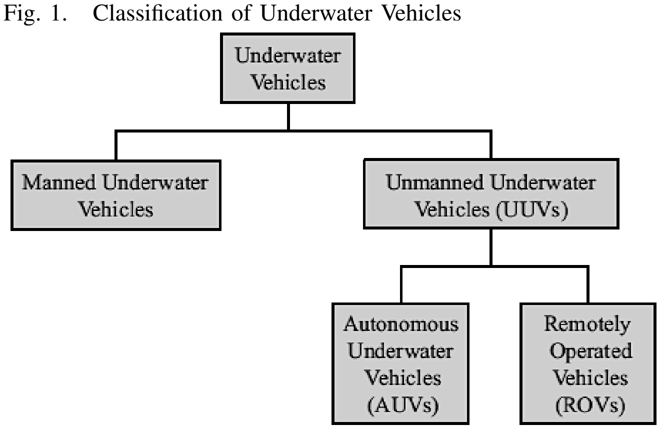
I. Introduction

Underactuated Underwater Vehicles (UUVs) are defined as systems in which the number of independent control actuators is less than the number of independent directions of motion desired.

Generally, underwater vehicles can be divided into three:

- Manned submersibles,
- Remotely Operated Vehicles (ROVs)
- Autonomous Underwater Vehicles (AUVs).

Fig. 1. Classification of Underwater Vehicles



ROVs and AUVs are the main types of the underwater vehicles that received great attention from both the industrial and underwater research community. The difference of AUVs and ROVs is that AUVs are controlled automatically by on-board computers and can work independently without connecting to the surface whereas ROVs are controlled or remotely controlled by human operator via a cable or wireless communication on ship or on the ground.

II. Factors Affecting an UUV

Several forces that require consideration for the design process act on an underwater vehicle. These include buoyancy, hydrodynamic damping, Coriolis and added mass.

- **Buoyancy**: The magnitude of the buoyant force (B) exerted on a body, which is floating or submerged, is equal to the weight of the volume of water displaced by that body. The ability of an object to float depends on the magnitude of the weight of the body (W). Clearly, if B $>$ W, then the body will float, while if B $<$ W it will sink. If $B = W$, then the body remains where it is.
- **Hydrodynamic Damping**: When a body is moving through the water, the main forces acting in the opposite direction to the motion of the body are hydrodynamic damping forces. These damping forces are mainly due to drag and lifting forces, as well as linear skin friction. Damping forces have a significant effect on the dynamics of an underwater vehicle which leads to nonlinearity.
- **Stability**: Assuming no water movement, the stability of a static body underwater is predominantly affected by the position of the center of mass (CM) and the center of buoyancy (CB). The center of buoyancy is the centroid of the volumetric displacement of the body. If CM and CB are not aligned vertically with each other in either the longitudinal or lateral directions, then instability will exist due to the creation of a nonzero moment. If CM and CB coincide in the same position in space, the vehicle will be very susceptible to perturbations. Ideally, the two centroids should be aligned vertically some distance apart from each other with CM below CB. This results in an ideal bottom-heavy configuration with innate stability. In

the case of a dynamic underwater body, stability is affected not only by the centers of mass and buoyancy, but also by factors such as external forces and centers of drag. To increase dynamic stability, the centers of drag, determined by the centroids of the effective surface areas of the vehicle, should be aligned with the centers of the externally applied forces. In this manner, the vehicle will not tend to exhibit undesirable characteristics in its Motion.

- **Coriolis**: Coriolis is an inertial force that acts perpendicular to the direction of motion of a body. The force is proportional to both the velocity and rotation of the coordinate system. The effect of the Coriolis force then, is that the path of the body is deflected. In reality, however, the path of the body is not actually deflected, but only appears to be. This is due to the motion of the bodys coordinate system. Since the coordinate system of an AUV rotates with respect to another reference frame, the effect of the Coriolis force is usually taken into account and included in the equations of motion.
- **Added Mass**: Another phenomenon that affects underwater vehicles is added mass. When a body moves underwater, the immediate surrounding fluid is accelerated along with the body. This affects the dynamics of the vehicle in such a way that the force required to accelerate the water can be modelled as an added mass. Added mass is a fairly significant effect and is related to the mass and inertial values of the vehicle. It is greatly influenced by the shape of the vehicle. Added mass coefficient is extractable using empirical formulas, the simple analytical relations and numerical methods such as Strip theory and lab tests.
- **Environmental Forces**: Environmental disturbances can affect the motion and stability of a vehicle. This is particularly true for an underwater vehicle if waves and currents can perturb the vehicle. When the vehicle is submerged, the effect of wind and waves can be largely ignored. The most significant disturbances then for underwater vehicles are currents. In a controlled environment such as a pool, the effect of these environmental forces is minimal.
- **Pressure**: As with air, underwater pressure is caused by the weight of the medium, in this case water, acting upon a surface. Pressure is usually measured as an absolute or ambient pressure; absolute denoting the total pressure and ambient being of a relativistic nature. At sea level, pressure due to air is 14.7 psi or 1 atm. For every 10 m of depth, pressure increases by about 1 atm and hence, the absolute pressure at 10 m underwater is 2 atm. Although it is linear in nature, the increase in pressure as depth increases is significant and underwater vehicles must be structurally capable of withstanding a relatively large amount of pressure if they are to survive.

III. KINEMATIC MODEL

Linear displacement in the three inertial axes defines the position vector of the vehicle:

$$\eta_1 = \begin{bmatrix} \mathrm{x} & \mathrm{y} & \mathrm{z} \end{bmatrix}^T \tag{1}$$

In the same way, the rotations around each axis of the Earth system, represented by the Euler angles, make up the orientation vector:

$$\eta_2 = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^T \tag{2}$$

The position and orientation change with respect to time defines the linear velocity and angular rate expressed in the body-fixed frame through the vectors 1 and 2 respectively:

$$\nu_1 = \begin{bmatrix} \mathrm{u} & \mathrm{v} & \mathrm{w} \end{bmatrix}^T \tag{3}$$

$$\nu_2 = \begin{bmatrix} \mathrm{p} & \mathrm{q} & \mathrm{r} \end{bmatrix}^T \tag{4}$$

Fig. 2. Reference frames of UUV. The origin of the inertial frames is at an arbitrary point on the surface of the Earth and the position and orientation are described in it. The origin of the body-fixed frame coincides with its centre of gravity and the linear velocities and angular rate are related to it.

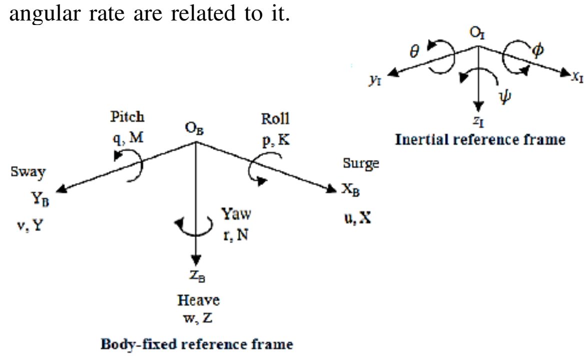


Fig. 2 shows the location of the reference frames and their relation with the robots states. According to the Society of Marine Architects and Naval Engineers (SNAME), the directions of the linear movements are named surge, sway and heave, and the rotational ones roll, pitch, and yaw. The evolution of the position and orientation of the UROV is described by (5)-(6):

$$\dot{\eta}_1 = J_1(\eta_2)\nu_1 \tag{5}$$

$$\dot{\eta}_2 = J_2(\eta_2)\nu_2 \tag{6}$$

where,

$$J_1 = \begin{bmatrix} c\theta c\psi & (\text{-}c\phi s\psi + s\phi s\theta c\phi) & (s\phi s\psi + c\phi s\theta c\psi) \\ c\theta s\psi & (c\phi c\psi + s\phi s\theta s\psi) & (\text{-}s\phi c\psi + c\phi s\theta s\psi) \\ \text{-}s\theta & c\theta s\psi & c\theta c\phi \end{bmatrix} \tag{7}$$

$$J_2 = (1/c\theta)\begin{bmatrix} c\theta & s\phi s\theta & c\phi s\theta \\ 0 & c\phi c\theta & \text{-}s\phi c\theta \\ 0 & s\phi & c\phi \end{bmatrix} \tag{8}$$

Equations (1)-(8) (c: cosine, s: sine) can be described in the matrix form of (9)-(12) and they are used to determine the inertial movements of the robot from its linear velocities and angular rates without regard on the forces and moments that cause them. Nevertheless, to simulate these velocities, it is necessary to know the dynamics of the UUV.

$$\dot{\eta} = J(\eta)\nu \tag{9}$$

$$\eta = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix} \tag{10}$$

$$\nu = \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} \tag{11}$$

$$J(\eta) = \begin{bmatrix} J_1(\eta_2) & 0_{3x3} \\ 0_{3x3} & J_2(\eta_2) \end{bmatrix} \tag{12}$$

IV. Dynamic Model

The dynamic model for the UUV is defined to be able to formulate control algorithms and to perform simulations. The dynamic model

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau \tag{13}$$

is derived from the Newton-Euler equation of a rigid body in fluid, where,

$M = M_{RB} + M_A$ is the inertia matrix for the rigid body and added mass, respectively;

$C(\nu) = C_{RB}(\nu) + C_A(\nu)$ is the Coriolis and centripetal matrix for the rigid body and added mass, respectively;

$D(\nu) = D_q(\nu) + D_t(\nu)$ is the quadratic and linear drag matrix, respectively;

g(η) is the gravitational and buoyancy matrix;

τ is the force/torque vector of the thruster input.

Eqn.(13) does not take into account environmental disturbances, such as underwater currents.

A. Mass and Inertia Matrix

The mass and inertia matrix consists of a rigid body mass and an added mass, respectively M_{RB} and M_A , where

$$M = M_{RB} + M_A \tag{14}$$

The rigid body mass term, $M_{RB}\dot{\nu}$, from eqn.(26) can be written as

$$M_{RB}\dot{\nu} = \begin{bmatrix} m\dot{\upsilon}_B + m\dot{\omega}_B x r_c \\ I_B\dot{\omega}_B + m r_c x \dot{\upsilon}_B \end{bmatrix} \tag{15}$$

where m is the mass of the AUV, I_B is the inertia tensor of the AUV with respect to the B-frame, given by

$$I_B = \begin{bmatrix} I_{xx} & I_{xy} & \text{-}I_{xz} \\ \text{-}I_{yx} & I_{yy} & \text{-}I_{yz} \\ \text{-}I_{zx} & \text{-}I_{zy} & I_{zz} \end{bmatrix} \tag{16}$$

and r_c = $\begin{bmatrix} x_c & y_c & z_c \end{bmatrix}^T$ is the definition of the center of gravity of the AUV with respect to the B-frame. The rigid body mass, M_{RB} is defined by

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & \text{-}my_G \\ 0 & m & 0 & \text{-}mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & \text{-}mx_G & 0 \\ 0 & \text{-}mz_G & my_G & I_{xx} & \text{-}I_{xy} & \text{-}I_{xz} \\ mz_G & 0 & \text{-}mx_G & \text{-}I_{yx} & I_{yy} & \text{-}I_{yz} \\ \text{-}my_G & mx_G & 0 & \text{-}I_{zx} & \text{-}I_{zy} & I_{zz} \end{bmatrix} \tag{17}$$

The AUV is symmetric in the x-z plane and almost symmetric about the y-z plane. Although the AUV is not symmetric about the x-y plane it can be assumed to be symmetric because the vehicle operates at relative low speeds. When the AUV is symmetric in all planes and the origin of the B-frame is positioned at the center of gravity of the AUV, i.e. r_G = $\begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$, then the rigid body mass, M_{RB} , can be simplified into

$$M_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{xx} & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{yy} & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{zz} \end{bmatrix} \tag{18}$$

The effect of the hydrodynamic added mass is modeled with the use of

Since the AUV is assumed to be symmetric in all planes and the origin of the B-frame is positioned at the center of gravity the hydrodynamic added mass, the effect of the hydrodynamic added mass is modeled as

$$M_A = \begin{bmatrix} X_{\dot{u}} & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{\dot{v}} & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\dot{w}} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{\dot{p}} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{\dot{q}} & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{\dot{r}} \end{bmatrix} \tag{19}$$

B. Coriolis and Centripetal Matrix

The Coriolis and Centripetal Matrix consists of a rigid body and an added mass term, respectively $C_{RB}(\nu)$ and $C_A(\nu)$,

$$C(\nu) = C_{RB}(\nu) + C_A(\nu) \tag{20}$$

where,

$$C_{RB}(\nu) = \begin{bmatrix} 0 & 0 & 0 & 0 & m\omega & -m\nu \\ 0 & 0 & 0 & -m\omega & 0 & mu \\ 0 & 0 & 0 & m\nu & -mu & 0 \\ 0 & m\omega & -m\nu & 0 & I_{xx}r & -I_{yy}q \\ -m\omega & 0 & mu & -I_{zz}r & 0 & I_{xx}p \\ m\nu & -mu & 0 & I_{yy}q & -I_{xx}p & 0 \end{bmatrix} \tag{21}$$

$$C_A(\nu) = \begin{bmatrix} 0 & 0 & 0 & 0 & -\alpha_3 & \alpha_2 \\ 0 & 0 & 0 & \alpha_3 & 0 & -\alpha_1\nu \\ 0 & 0 & 0 & -\alpha_2\nu & \alpha_1\nu & 0 \\ 0 & -\alpha_3\nu & \alpha_2 & 0 & -\beta_1 & \beta_2 \\ \alpha_3 & 0 & -\alpha_1 & \beta_3 & 0 & -\beta_1 \\ -\alpha_2 & \alpha_1 & 0 & -\beta_2 & \beta_1 & 0 \end{bmatrix} \tag{22}$$

Where,

$\alpha_1(\nu)$ = $X_{\dot{u}}$u + $Y_{\dot{v}}$v + $X_{\dot{w}}$w + $X_{\dot{p}}$p + $X_{\dot{q}}$q + $X_{\dot{r}}$r;

$\alpha_2(\nu)$ = $X_{\dot{v}}$u + $Y_{\dot{v}}$v + $Y_{\dot{w}}$w + $Y_{\dot{p}}$p + $Y_{\dot{q}}$q + $Y_{\dot{r}}$r;

$\alpha_3(\nu)$ = $X_{\dot{w}}$u + $Y_{\dot{w}}$v + $Z_{\dot{w}}$w + $Z_{\dot{p}}$p + $Z_{\dot{q}}$q + $Z_{\dot{r}}$r;

$\beta_1(\nu)$ = $X_{\dot{p}}$u + $Y_{\dot{p}}$v + $Z_{\dot{p}}$w + $K_{\dot{p}}$p + $K_{\dot{q}}$q + $K_{\dot{r}}$r;

$\beta_2(\nu)$ = $X_{\dot{q}}$u + $Y_{\dot{q}}$v + $Z_{\dot{q}}$w + $K_{\dot{q}}$p + $M_{\dot{q}}$q + $M_{\dot{r}}$r;

$\beta_3(\nu)$ = $X_{\dot{r}}$u + $Y_{\dot{r}}$v + $Z_{\dot{r}}$w + $K_{\dot{r}}$p + $M_{\dot{r}}$q + $N_{\dot{r}}$r;

C. Hydrodynamic Damping Matrix

The hydrodynamic damping of underwater vehicles normally contains the drag and lift forces. However, the AUV only operates at a low speed which makes the lift forces negligible compared to the drag forces. The drag forces can be separated into a linear and a quadratic term, $D(\nu) = D_q(\nu) + D_t(\nu)$, where $D_q(\nu)$ and $D_t(\nu)$ are the quadratic and the linear drag term, respectively. If the assumption of the vehicle symmetry about all planes holds then

$$D_t(\nu) = \begin{bmatrix} X_u & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_w & 0 & 0 & 0 \\ 0 & 0 & 0 & K_p & 0 & 0 \\ 0 & 0 & 0 & 0 & K_q & 0 \\ 0 & 0 & 0 & 0 & 0 & N_r \end{bmatrix} \tag{23}$$

describes the linear drag of the AUV.

The axial quadratic drag force of the AUV can be modeled with X = -$(1/2\rho C_d A_f)u|u$ =$X_{u|u|}u|u|$ where $X_{u|u|}$ = $\partial X/\partial$(u—u) $= -1/2\rho C_d A_f$. The matrix notation of (2.21) which gives the quadratic drag matrix, is given by $D_q(\nu)$ = diag

$$\{X_{u|u|}|u|, Y_{v|v|}|v|, Z_{w|w|}|w|, K_{p|p|}|p|, K_{q|q|}|q|, N_{r|r|}|r|\} \tag{24}$$

D. Gravitational and Buoyancy Matrix

The gravitational and buoyancy vector, g(h) , can be denoted in matrix form by

$$D_t(\nu) = \begin{bmatrix} f_B + f_C \\ r_B x f_B + r_G x f_G \end{bmatrix} \tag{25}$$

Where the gravitational force vector, f_G , is caused by the weight of the AUV,

$$f_G = R^{BW^{-1}}(\nu_w)\begin{bmatrix} 0 & 0 & W \end{bmatrix}^T \text{ , with W = mg}$$

and the buoyancy force vector, f_B , caused by the buoyancy,

$$f_B = R^{BW^{-1}}(\nu_w)\begin{bmatrix} 0 & 0 & -B \end{bmatrix}^T \text{ , with B = } \rho g \nabla$$

where g is the gravity constant 9.81 m/s^2, ρ is the fluid density, ∇ is the volume of fluid displaced by the AUV [m^3], r_G = $\begin{bmatrix} x_G & y_G & z_G \end{bmatrix}^T$ the center of gravity of the AUV, r_B = $\begin{bmatrix} x_B & y_B & z_B \end{bmatrix}^T$ the center of buoyancy of the AUV.

E. Force and Torque Vector

The thrust force and torque of the AUV are provided by six trolling motors. The trolling motors can deliver a force or torque in the surge, heave, roll, pitch and yaw directions. The force and torque vector is defined by

τ = LU ,

where L is the mapping matrix

$$L = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \text{-l}_1 & \text{l}_2 & \text{-l}_1 & \text{l}_2 \\ \text{l}_3 & \text{l}_3 & \text{-l}_6 & \text{-l}_6 & \text{l}_5 & \text{l}_5 \\ \text{-l}_1 & \text{l}_2 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{26}$$

and U the thrust vector

$$U = \begin{bmatrix} \text{T}_1 & \text{T}_2 & \text{T}_3 & \text{T}_4 & \text{T}_5 & \text{T}_6 \end{bmatrix}^T \tag{27}$$

The mapping matrix indicates the direction of the force in the surge and heave degrees of freedom and the direction and arm of the torque for the roll, pitch and yaw degrees of freedom. The thrust vector indicates the thrust that every trolling motor delivers, where T_1 to T_6 represent trolling motor 1 to 6, indicated in Figure The figure also shows the trolling motor numbering of the AUV and distances l_i which the trolling motors are separated from the center of gravity.

V. Tracking Control of AUV

The control laws for the surge and yaw degrees of freedom need to control three degrees of freedom, which means that one is dealing with an under-actuated problem. Motion in heave is neglected in this situation, since the under-actuated problem only concerns the surge, sway and yaw degrees of freedom. The idea is to derive control laws is such way that the controlled system is able to follow a path in the x-y plane and an error in sway is corrected by controlling the system in the surge and yaw direction. Note that when the AUV is making a curve and is able to move in sway the control laws will need to take in consideration the Coriolis and Centripetal forces will try to push the vehicle out of track.

The dynamics of the AUV in the surge, sway and yaw degrees of freedom are defined in matrix form by

$$\dot{\eta} = J(\eta)\dot{\nu} \tag{28}$$

$$M\dot{\nu} = -C(\nu)\nu - D(\nu)\nu + \tau \tag{29}$$

which follow directly from the dynamic model (13) for 6 degrees of freedom. Under the modeling assumptions, the matrix form of the AUV dynamics is rewritten into

$$\dot{x} = uC(\psi) - \nu S(\psi) \tag{30}$$

$$\dot{y} = uS(\psi) - \nu C(\psi) \tag{31}$$

$$\dot{\psi} = r \tag{32}$$

$$m_{11}\dot{u} = m_{22}\nu r - d_{11}u + \tau_u \tag{33}$$

$$m_{22}\dot{\nu} = m_{11}ur - d_{22}\nu \tag{34}$$

$$m_{33}\dot{r} = (m11 - m_{22})u\nu r - d_{33}u + \tau_r \tag{35}$$

where $\nu = \begin{bmatrix} \text{u} & \nu & \text{r} \end{bmatrix}^T$, $\eta = \begin{bmatrix} \text{x} & \text{y} & \psi \end{bmatrix}^T$,

$$M = diag\{m_{11}, m_{22}, m_3\} \tag{36}$$

$$D(\nu) = diag\{d_{11}, d_{22}, d_3\} \tag{37}$$

$\tau = \nu = \begin{bmatrix} \tau_u & 0 & \tau_r \end{bmatrix}^T$,

$$J(\eta) = diag \begin{bmatrix} \text{C}\phi & \text{-S}\phi & 0 \\ \text{S}\phi & \text{C}\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{38}$$

$$C(\nu) = diag \begin{bmatrix} 0 & 0 & \text{-m}_{22}\nu \\ 0 & 0 & \text{m}_{11}u \\ \text{m}_{22}\nu & \text{-m}_{11}u & 0 \end{bmatrix}, \tag{39}$$

Where x, y and y represent the position and orientation of the AUV in the earth-fixed frame and u, v and r are the velocities in surge, sway and yaw respectively. The parameters m_{11} ,m_{22} and m_{33} represent the rigid body and added mass of the inertia and mass of the AUV and d_{11},d_{22} and d_{33} represent the linear damping terms of the AUV.

The position error between the current position of the AUV and a reference position of a virtual AUV is given by $x - x_r$, $y - y_r$, $\psi - \psi_r$ which represents the position errors in the earth fixed frame. Where x , y and ψ represent the current position of the AUV and x_r, y_r and ψ_r the reference position of a virtual AUV.

This problem is sorted by a change of coordinates, which led to the natural error coordinates given by

$$\begin{bmatrix} x_e \\ y_e \\ \psi_e \end{bmatrix} = \begin{bmatrix} C\phi & -S\phi & 0 \\ S\phi & C\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_r \\ y - y_r \\ \psi - \psi_r \end{bmatrix} \quad (40)$$

The position errors are now considered in the body fixed frame of the AUV, so the tracking errors of the AUV can now be defined by

$$x_e = C(\psi)(x - x_r) + S(\psi)(y - y_r) \quad (41)$$

$$y_e = -S(\psi)(x - x_r) + C(\psi)(y - y_r) \quad (42)$$

$$\psi_e = \psi - \psi_r \quad (43)$$

$$u_e = u - u_r \quad (44)$$

$$\nu_e = \nu - \nu_r \quad (45)$$

$$r_e = r - r_r \quad (46)$$

Where u_r , v_r , r_r , x_r , y_r , ψ_r , $\tau_{u,r}$ and $\tau_{r,r}$ represent the reference coordinates which have to satisfy

$$\dot{x_r} = u_r C(\psi) - \nu_r S(\psi) \quad (47)$$

$$\dot{y_r} = u_r S(\psi) - \nu_r C(\psi) \quad (48)$$

$$\dot{\psi}_r = r_r \quad (49)$$

$$m_{11}\dot{u_r} = m_{22}\nu_r r_r - d_{11}u_r + \tau_{u,r} \quad (50)$$

$$m_{22}\dot{\nu_r} = -m_{11}u_r r_r - d_{22}\nu_r \quad (51)$$

$$m_{33}\dot{r_r} = (m11 - m_{22})u_r\nu_r - d_{33}r_r + \tau_{r,r} \quad (52)$$

Differentiating the tracking errors will lead to the tracking error dynamics, which are given by

$$\dot{x_e} = u - u_r C(\psi_e) - \nu_r S(\psi_e) + r_e y_e + r_r(t) y_e \quad (53)$$

$$\dot{y_e} = \nu - \nu_r C(\psi) + u_r S(\psi_e) - r_e x_e + r_r(t) x_e \quad (54)$$

$$\dot{\psi}_e = r_e \quad (55)$$

$$\dot{u_e} = (m_{22}/m_{11})(\nu_e r_e + \nu_e r_r(t) + \nu_r r_e) - d_{11}u_e/m_{11} + (1/m_{11})(\tau_u \quad (56)$$

$$\dot{\nu_e} = -(m_{11}/m_{22})(u_e r_e + u_e r_r(t) + u_r r_e) - d_{22}u_e/m_{22} \quad (57)$$

$\dot{r_r} = (m_{11} - m_{22})/m_{33})(\nu_e u_e + \nu_e u_r(t) + \nu_r u_e) -$

$d_{33}u_e/m_{33} + (1/m_{33})(\tau_r)$ (58)

The goal now is to stabilize the tracking error dynamics with a suitable choice of control laws for τ_u in surge and τ_r in yaw in order to establish tracking control for the AUV.

It is experimentally proven that the control law has a certain robustness against modeling errors and disturbances due to currents and wave drift forces. The control law can easily be transformed to a control law suitable for the AUV, since the dynamical model of a ship and AUV are similar in the x-y plane. A feedback control law for τ_r , given by

$$\tau_r = \tau_{rr} - (m11 - m_{22})(u\nu - u_r\nu_r) - d_{33}r_e + m_{33}\nu \quad (59)$$

is proposed, which will change the tracking dynamics into the linear system given by

$$\dot{\psi}_e = r_e \quad (60)$$

$$\dot{r_e} = \nu \quad (61)$$

respectively. The linear system can be stabilized by the linear control law given by

$$\nu = -k_1 r_e - k_2\dot{\psi}_e, k_1 > 0, k_2 > 0 \quad (62)$$

which will change the control law for τ_r into

$\tau_r = \tau_{rr} - (m_{11} - m_{22})(u\nu - u_r\nu_r) - k_1 r_e + k_2\dot{\psi}_e,$

$k_1 > -d_{33}, k_2 > 0$(63)
and the tracking dynamics into

$$\begin{bmatrix} \dot{\psi}_e \\ \dot{r_e} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -(d_{33} + k_1)/m33 & -k_2/m33 \end{bmatrix} \begin{bmatrix} \psi_e \\ r_e \end{bmatrix} \quad (64)$$

Under the assumption that the above equation is stabilized and that r_e and ψ_e go to zero, one is able to rewrite the remaining error tracking dynamics into

$$\dot{x_e} = u_e + r_r(t)y_e \quad (65)$$

$$\dot{y_e} = \nu_e - r_r(t)x_e \quad (66)$$

$$\dot{u_e} = (m_{22}/m_{11})\nu_e r_r(t) - d_{11}u_e/m_{11} + (1/m_{11})(\tau_u - \tau_{u,r}) \quad (67)$$

$$\dot{\nu_e} = -(m_{11}/m_{22})u_e r_r(t) - d_{22}u\nu_e/m_{22} \quad (68)$$

$r_e = 0$ and $\psi_e = 0$ are substituted and the error tracking dynamics denoted in matrix form is given by

$$\begin{bmatrix} \dot{x_e} \\ \dot{y_e} \\ \dot{u_e} \\ \dot{\nu_e} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \mathrm{r}_r(t) & 0 & 0 & 1 \\ 0 & 0 & -\mathrm{d}_{11}/m_{11} & \mathrm{m}_{22}r_r(t)/m_{11} \\ 0 & 0 & \mathrm{m}_{11}r_r(t)/m_{22} & \mathrm{d}_{22}/m_{22} \end{bmatrix} \begin{bmatrix} \mathrm{x}_e \\ \mathrm{y}_e \\ \mathrm{u}_e \\ \nu_e \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 1/\mathrm{m}_{11} \\ 0 \end{bmatrix} \begin{bmatrix} \tau_u - \tau_{u,r} \end{bmatrix} \quad (69)$$

The controllability can now be checked with the lemma, which states that the tracking error dynamics are uniformly completely controllable if the reference yaw velocity $r_r(t)$ is persistently exciting [83].

If the reference yaw velocity $r_r(t)$ is persistently exciting the system can be stabilized by the control law

$$\tau_u = \tau_{u,r} - k_3u_e + k_4r_r(t)\nu_e - k_5x_e + k_6r_r(t)y_e \quad (70)$$

where the parameters k have to following restrictions:
$k_3 > d_{22}d_{11}$,
k_4 = (k_6 + k_3 + d_{11} - d_{22})/(m_{11}/m_{22})($d_{22}k_6$ + $m_{11}k_5$),
$0 < k_5 < (k_3 + d_{11} - d22)d_{22}/m11$,
$k_6 > 0$;

Since this control mechanism uses the surge velocity to correct an error in the sway degree of freedom, the controlled system explained above is only stable if r_r (t) is persistently exciting,

A differential and integral gain are added to the surge control law in order to improve the linear controller into a PID controller. The differential gain is denoted by

$$\dot{u_e}k_7, k_7 > 0 \quad (71)$$

and the integral gain by

$$\int u_e k_8, k_8 > 0 \quad (72)$$

forming the surge control law

$$\tau_u = \tau_{u,r} - k_3u_e - \int u_e k_8 - \dot{u_e}k_7 + k_4r_r(t)\nu_e - k_5x_e +$$

$\mathrm{k}_6 r_r(t)y_e$(73)

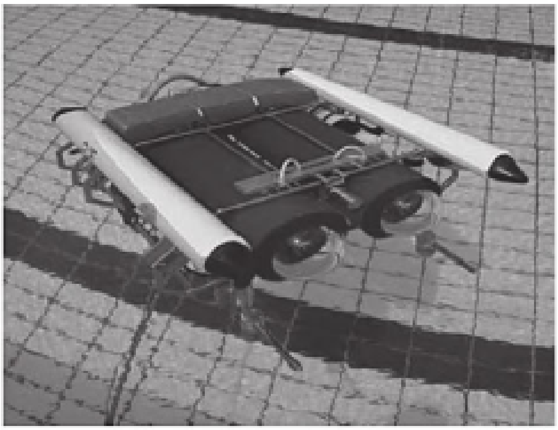
The under-actuated control problem was investigated here assuming sway motion of the AUV is not negligible, so Coriolis and centripetal forces need to be considered. A state feedback control method was explained as a result of which the controlled system is able to follow a reference trajectory in the x-y plane. The only disadvantage is that the yaw degree of freedom of the reference trajectory needs to be persistently exciting, which makes following a straight line impossible. The control law uses the control in the surge degree of freedom to correct a position error in sway. The control law for the yaw degree of freedom was used to minimize the error in yaw. Simulations reveal that the state feedback control law was able to track a circular reference trajectory within a reasonable error and was not affected by measurement noise and perturbed mass and inertia parameters. However perturbed damping parameters seem to affect the control law, which means that damping parameters should be estimated close to the exact plant parameters or an adaption law should be used to tune the damping parameters online.

VI. CASE STUDY: RRC ROV

The ROV designed by Robotics Research Centre (RRC) in Nanyang Technological University (NTU) is known as RRC ROV (see Figure). It is used to perform underwater pipeline inspections such as locating pipe leakages or cracks. This can be done with the operator focusing on inspection while the ROV automatically tracks the pipeline.

The twin eye-ball ROV depicted in Figure has an open-frame structure and is 1 m long, 0.9 m wide, and 0.9 m high. It has a dry weight of 115 kg and a current operating depth of 100 m. Its designed tasks include inspections of underwater pipelines. The RRC ROV is underactuated as it has four thruster inputs for six degrees of freedom (DOFs) (that is, surge, sway, heave, roll, pitch, and yaw velocity) with a high degree of cross coupling between them. The vehicle is equipped with two lateral thrusters mainly for surge, sway and yaw motion control in the horizontal plane, two vertical thrusters for heave in the vertical plane and a suite of sensors for position and velocity measurements. Roll and pitch motions are passive as the metacentric height (that is, the distance between the center of gravity of a ROV and its metacenter) is sufficiently large to provide adequate static stability.

Fig. 3. RRC ROV

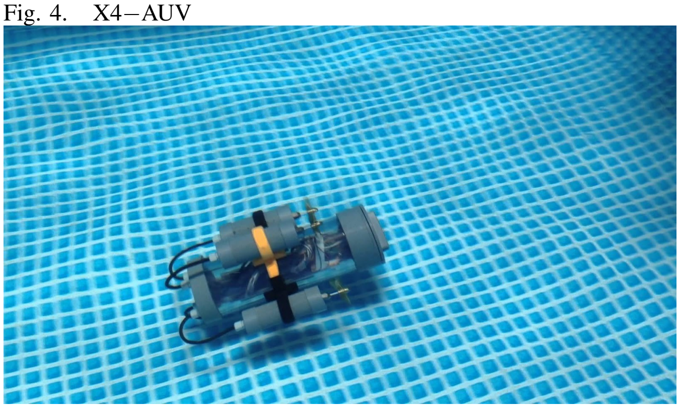


The predefined underwater pipeline in subsea operation is shown in Figure 4.2. In an actual vehicle deployment, the predefined pipeline locations are usually provided by the company and the vehicle uses external cameras onboard the ROV and a navigation system to track it.

VII. Case Study: X−4 AUV

An X4-AUV with a spherical hull shape was studied by Okamura. X4-AUV equipped with four thrusters and 6-DOFs motion capability falls in an underactuated system and also had nonholonomic features. An X4-AUV is modelled as a slender, axisymmetric rigid body whose mass equals the mass of the fluid which it displaces; thus, the vehicle is neutrally buoyant.. When the X4-AUV moves underwater, additional forces and moment coefficients are added to account for the effective mass of the fluid that surrounds the robot, which causes an excessive acceleration of the robot, compared to the case where there is no any added mass and moment of inertia.

Fig. 4. X4−AUV



An X4-AUV with a new type of hull shape was considered with an ellipsoid body to reduce the drag forces against a stream. The X4-AUV posed many challenging control problems because they were underactuated imposing nonintegrable acceleration constraints.The existence of several complex and nonlinear forces such as hydrodynamic drag, damping, lift forces, Coriolis and centrifugal forces, gravity and buoyancy forces, thruster forces, and environmental disturbances acting on an underwater vehicle, makes the control of X4-AUV trickier. It is noted that X4-AUV kinematic and dynamic models were highly nonlinear and coupled, thus X4-AUV control design was complicated. Therefore, nonlinear control methods for an underactuated and nonholonomic system were used for the development of control schemes to steer the dynamical model of an X4-AUV in the horizontal plane to a final target point, with a desired orientation.

Fig. 5. Former X4−AUV: Spherical Hull Design

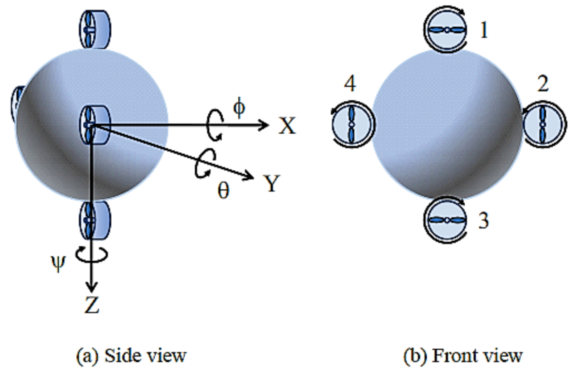
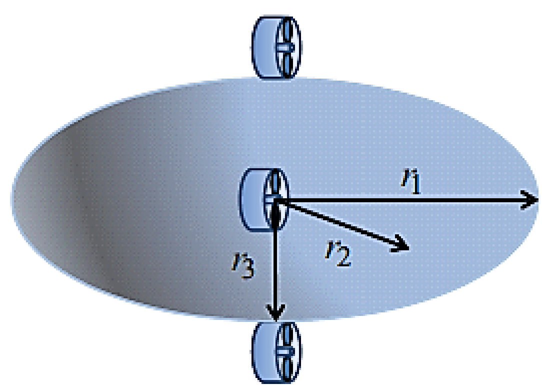


Fig. 6. Modified X4−AUV: Ellipsoidal Hull Design



VIII. Conclusions

A comprehensive review of the two types of Underactuated underwater vehicles is established and their kinematic and dynamic modelling have been summarized. Furthermore, a section on the various factors that are needed to be considered while designing and path programming an UUV is set up referencing the review of pre-existing UUV designs of the RRC ROV and X4-AUV given. Needless to say that there is much scope for development and improvements in the domains of UUVs with the introduction of the RRC ROV II over the RRC ROV I and the modified X4 AUV over the former X4 AUV model incorporating sensor and control improvements and structural advancements respectively in each to cite a few examples. Further research into the domains of new control methods, sensor and navigation technology and weight optimization etc can be carried out for advancing Underactuated Underwater Vehicle technology which can result in vast expansion of its practical applications and its efficiency in performing them.

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