

CENG 222 HOMEWORK 4-5 REPORTS

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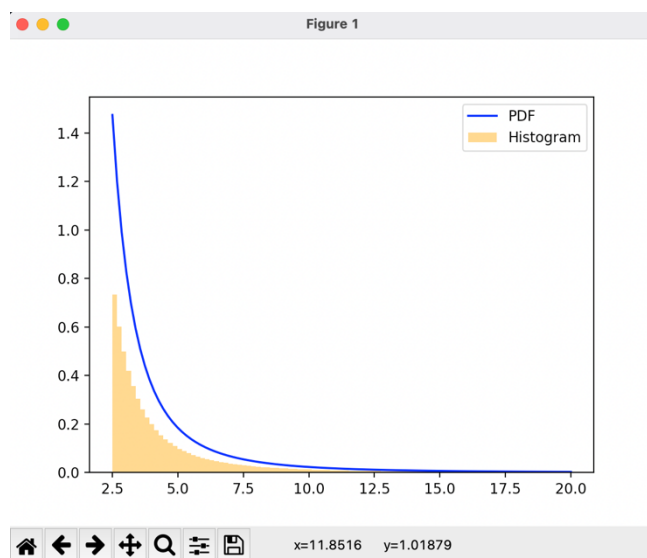
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Outputs:

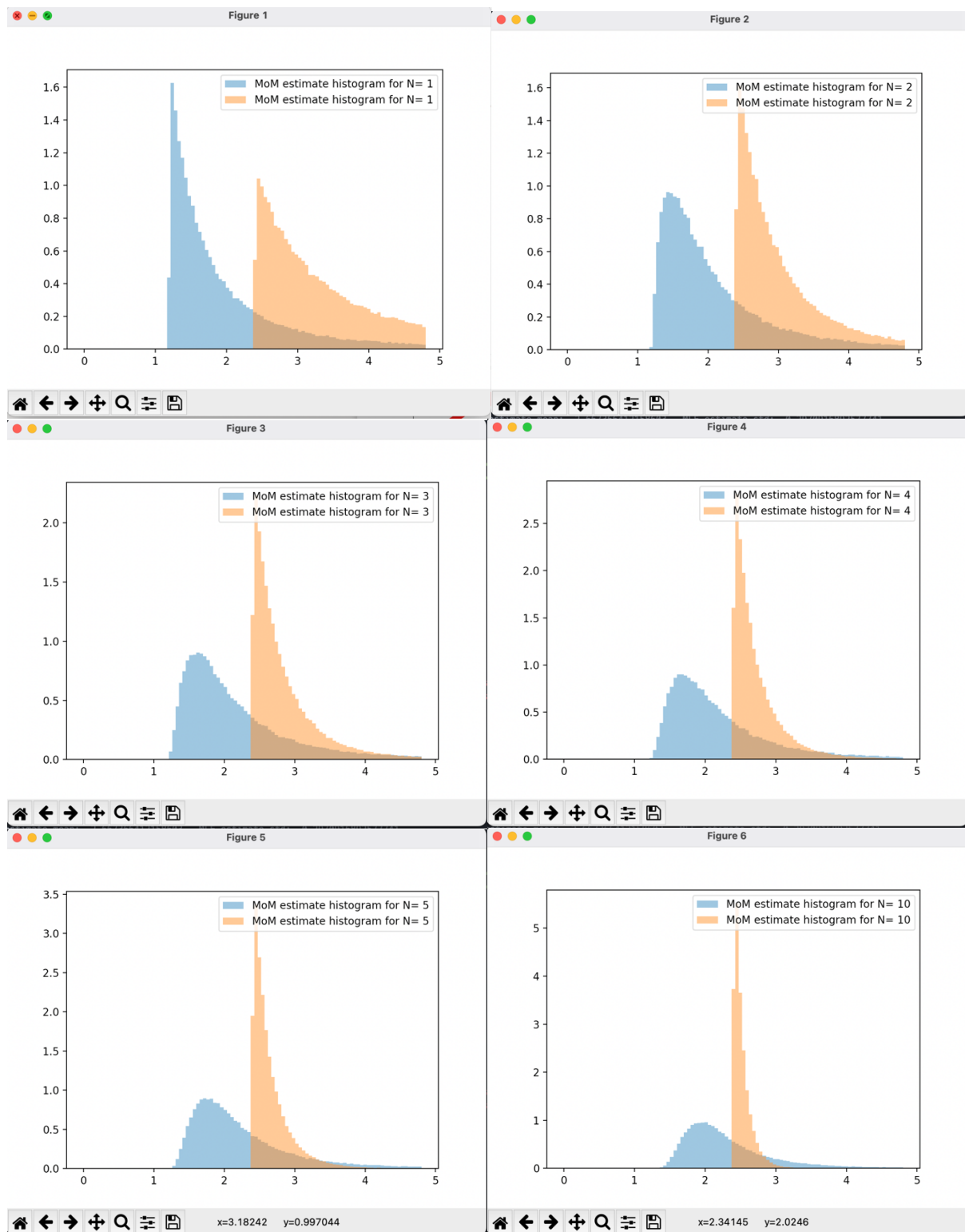
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(base) kaganyalim@Kagan-MBP Desktop % /Users/kaganyalim/opt/anaconda3/bin/python /Users/kaganyalim/Desktop/ceng222hw4.py
MoM estimate for the sample is 0.325
MLE estimate for the sample is 0.3
For N =1:
MoM estimate mean: 2.395487876380141 MoM estimate std: 3.2940683308791643
MLE estimate mean: 4.790975752760282 MLE estimate std: 6.588136661758329
For N =2:
MoM estimate mean: 2.4034021516940163 MoM estimate std: 2.9970035557923724
MLE estimate mean: 3.1954609434346026 MLE estimate std: 1.1222191434778566
For N =3:
MoM estimate mean: 2.3857112615533143 MoM estimate std: 1.9503478548721993
MLE estimate mean: 2.879694427639117 MLE estimate std: 0.5792508412262829
For N =4:
MoM estimate mean: 2.391257308192665 MoM estimate std: 1.69576455276499
MLE estimate mean: 2.74234417464312 MLE estimate std: 0.39666358129280604
For N =5:
MoM estimate mean: 2.4099237970134197 MoM estimate std: 2.4308796345346955
MLE estimate mean: 2.667366412158603 MLE estimate std: 0.2970916892577741
For N =10:
MoM estimate mean: 2.4060098560489442 MoM estimate std: 1.2929105135616026
MLE estimate mean: 2.5264925832816902 MLE estimate std: 0.13325418454761043
For N =50:
MoM estimate mean: 2.399164544785514 MoM estimate std: 0.5721769463296017
MLE estimate mean: 2.424288862687439 MLE estimate std: 0.024330900029065853
For N =100:
MoM estimate mean: 2.400094821570923 MoM estimate std: 0.42694190812395527
MLE estimate mean: 2.412108027178779 MLE estimate std: 0.012187220530715017
For N =500:
MoM estimate mean: 2.3990905415670767 MoM estimate std: 0.18060166580432271
MLE estimate mean: 2.4024083861053565 MLE estimate std: 0.002408612622180012
For N =1000:
MoM estimate mean: 2.400092655573474 MoM estimate std: 0.1296923655052061
MLE estimate mean: 2.401209154645006 MLE estimate std: 0.001209696008710337
(base) kaganyalim@Kagan-MBP Desktop %
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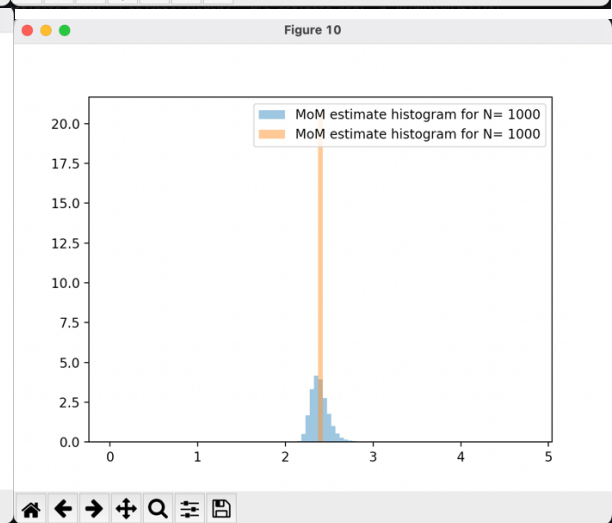
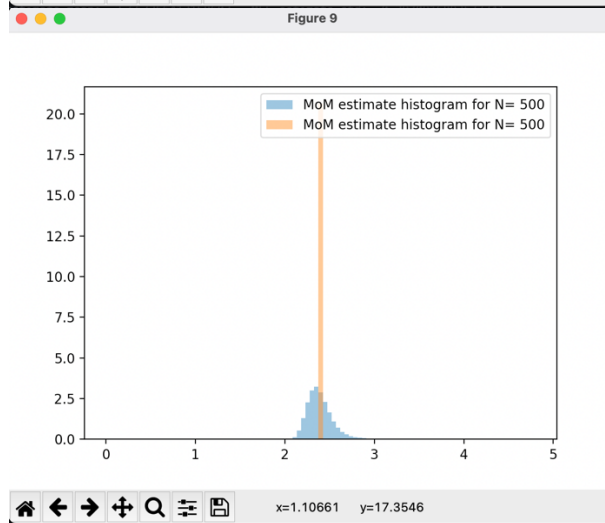
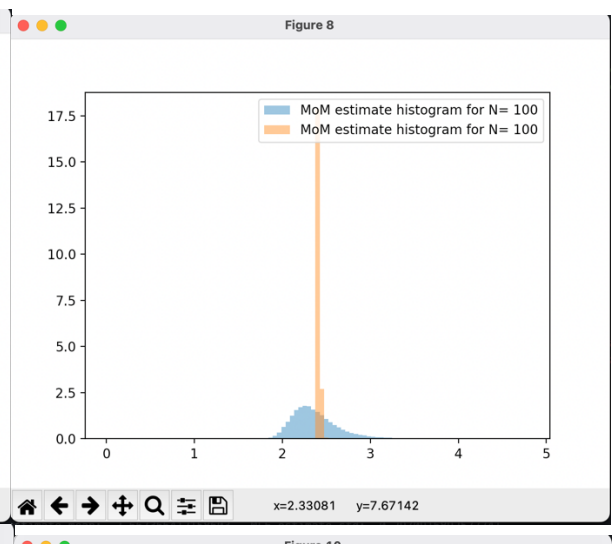
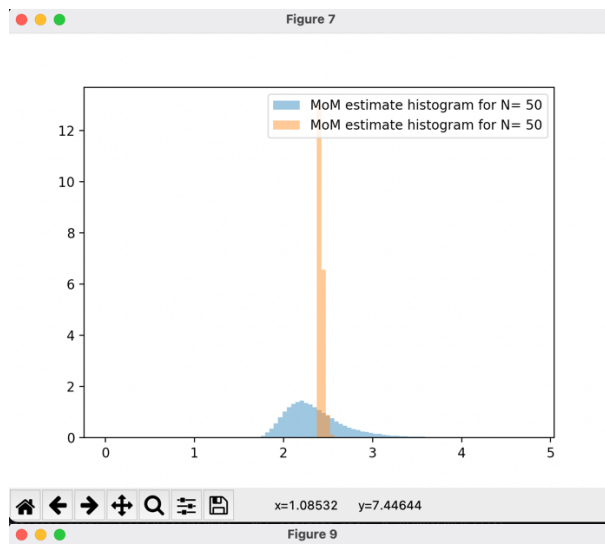
Figures:

Part b figure:



Part c figures:





Mathematical Calculations:

MoM estimation =

$$E(x) = \int_0^{\infty} \frac{2\theta^2}{x^3} dx = \left. -\frac{2\theta^2}{x} \right|_0^{\infty} = 2\theta = \bar{x} \quad \bar{x} = 0,65$$
$$\bar{\theta} = 0,325$$

MLE estimation =

$$\ln(f(x_i)) = \sum_{i=1}^n \ln f(x_i)$$

$$L(\theta) = \sum_{i=1}^n \ln\left(\frac{2\theta^2}{x_i^3}\right) = \sum_{i=1}^n \ln(2) + 2 \sum_{i=1}^n \ln \theta - 3 \sum_{i=1}^n \ln x_i$$

$$= n \ln 2 + 2n \ln \theta - 3 \sum_{i=1}^n \ln x_i$$

$$\frac{dL(\theta)}{d\theta} = \frac{2n}{\theta} = 0 \quad \times \quad (x \text{ is minimum element in array.})$$

Inverse Transform Method =

$$F(x) = \int_0^x f(x) = \int_0^x \frac{2\theta^2}{x^3} dx = -\frac{\theta^2}{x^2} + 1$$

$$1 - \frac{\theta^2}{x^2} = y \quad x = \sqrt{\frac{\theta^2}{1-y}} \quad F^{-1}(x) = \sqrt{\frac{\theta^2}{1-x}}$$