

CENG 222
Probability and Statistics
HOMEWORK 4&5
Method of Moments and Maximum Likelihood Estimation

DUE DATE: 21.05.2022 - 23:55

You are expected to implement the following experiments in Python and answer the corresponding questions in a report.

a) MoM and MLE Estimation

- In your **report**, estimate the parameter θ using Method of Moments (MoM) and Maximum Likelihood Estimation (MLE) for the distribution with the following density:

$$f(x) = \begin{cases} \theta x^{\theta-1}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Apply your findings to $X = \{0.3, 0.6, 0.8, 0.9\}$ and calculate the two estimates for this sample set.

- In your **code**, implement the two functions which take a sample set X and return the calculated estimate of the parameter θ using MoM and MLE. Call these two functions to calculate the estimates for the same X given above and print the results.

b) Population Generation

- In your **report**, do the necessary calculations to use the Inverse Transform Method to generate random samples of the given distribution.
- In your **code**, create a population P of size 10 million which has the same distribution as in (a), using the Inverse Transform Method. Set the parameter θ as 2.4.

c) Experiment Simulation

- In your **code**, implement a function that takes the population (P) and sample size (N) as its input, uses 100000 samples of size N from the population P to calculate MoM and MLE estimates of the parameter θ from. Use `np.random.random_integers` to create indices for random sampling. After plotting the histograms of both estimators in a single figure (use 100 bins and $\alpha=0.5$), it returns the mean and the variance of the two estimators.

Call the implemented function for $N = [1, 2, 3, 4, 5, 10, 50, 100, 500, 1000]$. Print the estimator mean and variances for each N .

- In your **report**, comment on the histograms and the findings. How does the numbers change with respect to N ? Which estimator would you prefer? Why?

Submission Rules:

1. You should submit your assignments through TEAMS until due date.
2. You have to submit one zipped file including one file for your code (**py**) and one file for your report (**pdf**).
3. Your homework should be named as **CENG222_HW4-5_studentID.zip**
4. Write your **student ID** both in your code and report files.