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Spacetime from Entanglement

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Abstract

This is an idiosyncratic colloquium-style review of the idea that spacetime and gravity can emerge from entanglement. Drawing inspiration from the conjectured duality between quantum gravity in anti de Sitter space and certain conformal field theories, we argue that tensor networks can be used to define a discrete geometry that encodes entanglement geometrically. With the additional assumption that a continuum limit can be taken, the resulting geometry necessarily obeys Einstein's equations. The discussion takes the point of view that the emergence of spacetime and gravity is a mysterious phenomenon of quantum many-body physics that we would like to understand. We also briefly discuss possible experiments to detect emergent gravity in highly entangled quantum systems.

1. THE ORIGIN OF SPACETIME

Our current best description of nature views gravity as arising from the curvature of the geometry of spacetime (1). To quote Wheeler & Fox, "Spacetime tells matter how to move; matter tells spacetime how to curve" (2, p. 235). This beautiful geometric theory gives a good description of gravitational physics across many length scales and makes numerous predictions. Recently, LIGO (Laser Interferometer Gravitational-Wave Observatory) has even directly detected gravitational waves emitted from the merger of a pair of black holes in good accord with theory (3).

Despite the immense success of the geometric theory, this review takes the point of view that spacetime and gravity must ultimately emerge from something else. The main reasons to suspect this have ultimately to do with the problem of combining gravity with quantum physics. Trying to generalize the geometry of spacetime to allow for quantum fluctuations leads to an enormous number of technical and conceptual problems.

In the special case of gravity in anti de Sitter space (AdS), we have a more coherent understanding. This is thanks to a theoretical development known as the anti de Sitter space/conformal field theory (AdS/CFT) duality (4). The duality states that a quantum gravitational system can be equally well understood as an ordinary quantum system (a quantum field theory) without gravity. AdS/CFT is named for the best-understood special case in which the quantum field theory has conformal symmetry (a CFT), but many non-CFT examples are known (5).

The key problem is then to understand the emergence of gravitational dynamics from non-gravitational degrees of freedom. Remarkably, quantum entanglement has been identified as a key component in the emergence of spacetime within AdS/CFT. Roughly speaking, entanglement is the fabric of spacetime (6, 7). We review these developments from the point of view that the emergence of gravity is a mysterious phenomenon of quantum many-body physics, a new phase of matter loosely speaking, that we would like to understand.

Of course, gravity in AdS is not a good model for the physics of our Universe. AdS is a highly symmetric, negatively curved geometry that arises from a negative cosmological constant, but our Universe appears to have a positive cosmological constant and to be approximated by an expanding de Sitter geometry (8, 9). However, quantum gravity is sufficiently rich and confusing that even toy universes can shed enormous light on the physics. In addition to its intrinsic scientific interest in the context of quantum gravity, the emerging relationship between entanglement and geometry has also significantly impacted our understanding of quantum information and quantum phases of matter.

2. WHY DOES SPACETIME NEED TO COME FROM SOMETHING?

For most human-scale experiments, space can be thought of as a fixed background in which dynamical processes take place. Time is similarly fixed, ticking away without reference to the processes it meters. Furthermore, space and time are separate. Dynamics can be viewed as correlations between a system and a reference clock, but there is little harm in assuming that the ticking of a physical clock is well correlated with the flow of some abstract time.

Pushing beyond the human scale to experiments with particles moving near the speed of light reveals new physics. It becomes important to understand how motion affects the behavior of rulers and clocks (10). One output of the improved theory is the unification of space and time (11). The resulting spacetime geometry, as measured by rulers and clocks, is nevertheless fixed and flat.

New physics again arises when the effects of gravity are important. The ticking of a clock is then influenced by its motion and by the presence of other matter. The physics can still be interpreted in terms of a spacetime geometry, but that geometry is no longer flat and fixed (1).

Einstein envisioned the physical meaning of geometry as a set of causal relationships mapped out by a vast network of clocks exchanging light pulses to determine their relative distances (10). In other words, the pattern of interactions should define the geometry. If a system has many nonlocal connections, then it could be that no geometrical interpretation is possible.

This raises important questions: Why does the pattern of interactions have a geometrical interpretation? Assuming there is a geometrical interpretation, why do different probes reveal the same geometry? Could a collection of nongeometrical degrees of freedom organize themselves such that their interactions have a geometrical interpretation?

The picture is further complicated by quantum effects. Matter sources curvature and matter can be in a superposition of different positions, so a minimal extension suggests allowing superpositions of different curvatures. In other words, quantum gravity allows superpositions of different geometries. Then one has the distinct feeling of having nowhere left to stand. Of course, the reader is cautioned that the nature of quantum gravity in our own Universe remains mysterious because of lack of experimental input.

Nevertheless, building on insights from string theory, a concrete version of quantum gravity has been put forward that appears to have all the expected features while addressing some of the preceding questions. AdS/CFT duality conjectures that the quantum physics of a metric (and other quantum fields), which asymptotes at large distances to an AdS metric, is equivalent to the quantum physics of certain strongly interacting field theories. A remarkable aspect of the duality is its holographic character (12, 13): The CFT is defined in D spacetime dimensions, but the gravity theory is defined in $D + 1$ dimensions (in the simplest case).

AdS/CFT duality asks us to imagine, beginning with some fixed geometry in which the CFT lives, a geometry that includes both space and time already, and then somehow construct a dynamical extra-dimensional geometry starting from the fixed CFT geometry. Then in the appropriate regime, the dynamics of the CFT is best described in terms of higher-dimensional gravitational variables.

An analogy may be useful. A system of interacting electrons can realize a wide variety of distinct phases of matter. These phases include metals, where the low-energy excitations are electron like, and insulating magnets, where the low-energy excitations are bosonic spin waves that look nothing like electrons. In fractional quantum Hall systems (14), there is good theoretical evidence for the proposition that the electron fractionalizes into excitations carrying a fraction of the electron charge (15). The moral is that the low-energy degrees of freedom of a system need look nothing like the microscopic constituents. The emergence of low-energy modes described by higher-dimensional gravity is an extreme version of this moral.

How can the low-energy degrees of freedom look so different from the microscopic constituents? The key is quantum entanglement. Entanglement enables a large quantum system to be more than the sum of its parts: By organizing the microscopic particles into a complex entangled state, one can create new dynamical degrees of freedom. For example, the quantum state of fractional quantum Hall systems is qualitatively more entangled than the state of an insulator-like diamond (16, 17). The emergence of gravity also requires a highly entangled quantum state, in a surprisingly literal sense.

3. CLUES TO QUANTUM GRAVITY

The first step is to understand more precisely in what sense gravity is holographic. It is useful to begin by counting degrees of freedom in a gravitational system. This is done by studying the thermal entropy as a function of energy density. Consider a flat spacetime containing a box of size R full of hot photons of total energy E. Introducing Newton's constant G and requiring $GE \ll R$

(the speed of light is unity), one finds that the entropy of the box is proportional to R^3 in three dimensions.

In the opposite extreme, when $GE \gg R$, the effects of gravity are strong. The box of photons has collapsed to form a black hole of radius $R_0 = 2GE$. Bekenstein reasoned that the black hole should have entropy because the original photons had entropy (18). Otherwise the second law of thermodynamics would be violated (19). Remarkably, it is possible to assign an entropy to the black hole, but the result scales as the surface area of the black hole instead of as the volume of the black hole (18, 20),

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{BH}}}{4\hbar G}. \qquad 1.$$

In flat space the area of the black hole is proportional to R_0^2, so the entropy scales with the area instead of with the volume of the black hole.

Viewing the entropy of the black hole as counting the number of microstates of the underlying microscopic degrees of freedom (whatever they may be), we reach the remarkable conclusion that the number of degrees of freedom should scale like volume at low energies but like area at higher energies. This led 't Hooft to speculate that a gravitational system extended in D dimensions can ultimately be described by the degrees of freedom of a $D-1$ dimensional system (12). The analogy is to a hologram of light, where some two-dimensional surface encodes a three-dimensional image.

Given this intuition that the space of states in quantum gravity is holographic, surely the dynamical laws of gravity are also holographic? Recall that in three dimensions the motion of a test particle of mass m_1 around a mass $m_2 \gg m_1$ is determined by a force of magnitude

$$F = \frac{Gm_1 m_2}{r^2}. \qquad 2.$$

This implies that the total mass m_2 can be measured using a distant probe by studying its trajectory.

An analogy with electrostatics may be helpful. It is possible to measure the total charge of a system at infinity by measuring the asymptotic electric field. Thinking of the region infinitely far from the charge distribution as a boundary, the total charge of the system is observable at the boundary. The same phenomenon occurs in gravity, except the analog of the charge is the mass. Because all forms of energy gravitate, the total energy of a localized gravitational system is a boundary observable. More formally, within general relativity one can show that the Hamiltonian is a boundary observable (and if there is no boundary, then the Hamiltonian is zero) (21).

Of course, for most purposes the Hilbert space and dynamics of the Universe may be approximated by those of an effective local quantum field theory, e.g., the Standard Model. One of the great mysteries of quantum gravity is how the effective local bulk emerges holographically from the underlying boundary degrees of freedom.

Some intuition for the meaning of the emergent direction can be gained by thinking about a massive spherical object. Suppose an excited atom emits light of energy E_0 when far removed from any gravitational field. After placing this excited atom around the massive object, the energy of the emitted light, as measured at infinity, will be smaller than E_0 because of gravitational redshift (1). This redshift increases as the atom gets closer to the object and further from the boundary, hence the radial direction is associated with energy scale. We see below that this is precisely the case in AdS gravity (4).

The preceding clues mostly referred to the physics of gravity in asymptotically flat spacetimes, but this is not quite the setting that we will obtain. The holographic principle takes its sharpest form in asymptotically AdS spacetimes, and it is an AdS-like setting that arises from entanglement. The clues above are still relevant, but there are some new twists due to the hyperbolic geometry of AdS; e.g., volumes and areas scale the same way.

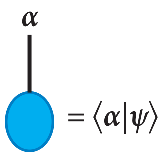

Figure 1

Graphical representation of a one-qubit state.

4. ENTANGLEMENT AND TENSOR NETWORKS

We now turn to quantum entanglement, starting from a single qubit. Tools known as tensor networks turn out to be crucial in the entanglement approach because they give a way to geometrize entanglement.

Consider one qubit. Introduce a set of basis states $\{|0\rangle, |1\rangle\}$ that are eigenstates of the z-component of the spin, $\sigma^z|0\rangle = |0\rangle$ and $\sigma^z|1\rangle = -|1\rangle$. The superposition principle gives the general one-qubit state as a linear combination of basis states,

$$|\psi\rangle = a|0\rangle + b|1\rangle. \qquad 3.$$

A convenient graphical representation of the state is shown in **Figure 1**. The object in **Figure 1** is called a tensor, and the line coming out is called a leg. Tensors are defined mathematically as multilinear operators; here one should think of the tensor as a machine into which we feed an index ($\alpha = 0, 1$) and out of which we get out an amplitude $\langle\alpha|\psi\rangle$ to be in state $|\alpha\rangle$.

Now consider a composite system of two qubits. A complete basis is given by the eigenstates of σ_1^z and σ_2^z, $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. For simplicity, we suppress the tensor product symbol when defining composite states and operators. Again the general state is a linear combination of basis states,

$$|\psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle. \qquad 4.$$

There is a special subclass of states where qubit 1 and qubit 2 are uncorrelated,

$$|\psi'\rangle_{12} = (a'|0\rangle + b'|1\rangle)_1(c'|0\rangle + d'|1\rangle)_2 = |\phi\rangle_1|\phi'\rangle_2. \qquad 5.$$

Such states are called unentangled. States that cannot be written as a product are called entangled.

A graphical representation of the quantum state is shown in **Figure 2*a***. The tensor is drawn with two legs, one for each qubit. A generic quantum state in the Hilbert space has no special

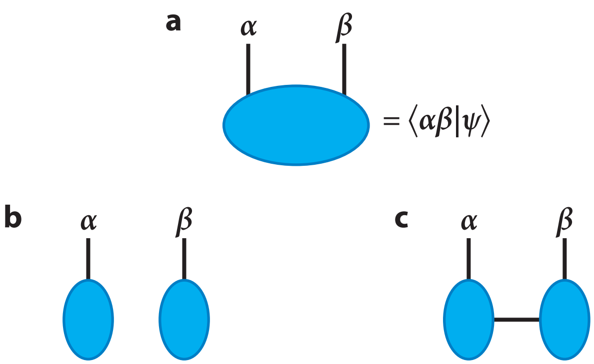

Figure 2

(*a*) Graphical representation of a two-qubit state. (*b*) Unentangled. (*c*) Entangled.

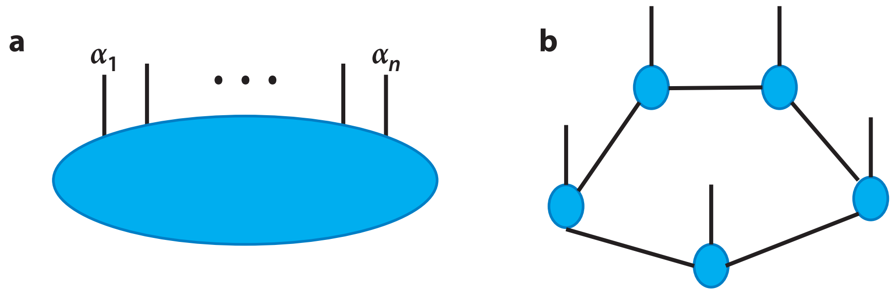

Figure 3

(*a*) Graphical representation of an n-qubit state. (*b*) Simplifications from locality.

structure in the tensor product basis, but unentangled states are special. They are represented graphically by breaking the two qubit tensor into two disconnected tensors as in **Figure 2*b***.

If the qubits are entangled, then the two-qubit tensor can still be broken into pieces, but those pieces must be connected. The new element in **Figure 2*c*** is the internal leg that mediates entanglement between the qubits. The entangled two-qubit state is

$$\langle \alpha\beta|\psi\rangle = \sum_{j=1}^{\chi} X_j^{\alpha} Y_j^{\beta}, \tag{6}$$

where the range of the internal leg is χ, the bond dimension. This illustrates the general calculational rule: The amplitude associated with any tensor network is obtained from the contraction of all internal legs.

Finally, consider the general case of n qubits. The Hilbert space of n qubits is 2^n-dimensional, and the most general n-qubit state can be represented graphically as a tensor with n legs as in **Figure 3*a***. However, states encountered in nature are not typically generic. Instead, they often inherit the locality properties of the interactions that generate entanglement among the qubits. **Figure 3*b*** shows one example in which qubits are arranged in a one-dimensional array and are only locally entangled. The tensor network in **Figure 3*b*** is known as a matrix product state (22, 23); it underlies the functioning of the powerful density matrix renormalization group (24).

Tensor networks are completely general: If the bond dimension of the internal legs of the tensors in **Figure 3*b*** is large enough (of order 2^n), then any n-qubit state can be represented. However, tensor networks are most useful when the bond dimension is small. In this case they provide an efficient compression of the quantum state. For example, suppose each external leg in the tensors in **Figure 3*b*** takes values in $\{0, 1\}$, and suppose each internal leg takes χ values (the bond dimension); then the total number of parameters describing the n-qubit quantum state is $2\chi^2 n$. Provided χ is a polynomial in n, the tensor network compressed the 2^n amplitudes defining the state into polynomial in n many parameters.

For our purposes, tensor networks provide a convenient parameterization of the space of states that are relevant for the emergence of spacetime. To see this, let us understand quantitatively how entanglement is distributed in a tensor network. Separate the physical qubits into two sets, A and B, such that the combined set AB includes everything. Given a tensor network for the state of AB, if internal legs connect A and B, then generically there is entanglement between A and B.

To quantify entanglement, take a pure quantum state $|\psi\rangle_{AB}$ and define the reduced density matrix of A by

$$\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|_{AB}), \tag{7}$$

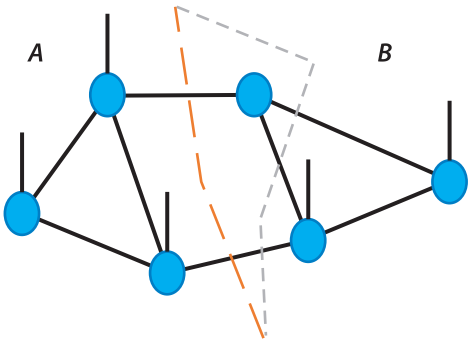

Figure 4

A is the left three qubits (*vertical lines*); B is the right two qubits. The orange dashed line shows the minimal cut; the gray dotted line shows a nonminimal cut. Note that not all tensors need to have physical legs.

where Tr_B denotes the partial trace. The trace throws away all information about the qubits in B. If A and B are unentangled, then ρ_A will also be a pure state,

$$\rho_A = |\phi\rangle\langle\phi|. \tag{8}$$

However, if A and B are entangled, then ρ_A will generically be mixed and it will have entropy. The von Neumann entropy or entanglement entropy is defined as

$$S(A) = -\mathrm{Tr}_A(\rho_A \ln \rho_A). \tag{9}$$

It provides a quantitative measure of the entanglement between A and B. Returning to tensor networks, the fundamental entanglement property of these networks is that the entropy $S(A)$ of a region A is bounded by a constant times the minimum number of internal legs that must be cut to isolate A from B (see **Figure 4**). More precisely, in a tensor network state $|\mathrm{TN}\rangle_{AB}$ of bond dimension χ, the entropy is bounded by

$$S(A, |\mathrm{TN}\rangle_{AB}) \leq (\text{minimum cut}) \ln \chi. \tag{10}$$

If the tensor network is well adapted to the state of interest, so that tensors and their connectivity are not wasted, then the above bound can be expected to give the qualitative scaling of $S(A)$ as a function of A.

Let us call the physical legs the boundary and the internal legs the bulk. Using this language, the entanglement of a boundary region is upper bounded by the size of the minimal cut in the bulk that separates the boundary region from the rest. In a cubic lattice that approximates a three-dimensional world, such a minimal cut would correspond to a minimal area surface. In an abuse of notation, it is common to use the nomenclature of minimal area surfaces even when the graph geometry is not three dimensional.

The tensor network shown in **Figure 3*b*** describes well the lowest-energy state of a wide variety of one-dimensional systems, namely those with an energy gap to the first excited state (25, 26). Furthermore, the requisite bond dimension is not large (23). The resulting tensor network geometry (the geometry of the bulk) is essentially the same as physical one-dimensional geometry of the spins (the geometry of the boundary). Applying the entanglement bound Equation 10 to a block of spins of length ℓ shows that the entropy is bounded by a constant independent of ℓ.

But nature provides us with more than just gapped phases of matter. One of the most familiar phases of matter, the metal, has an excitation spectrum that is gapless in the thermodynamic limit. In a one-dimensional metal, e.g., a quantum wire, the ground-state entanglement of a region of

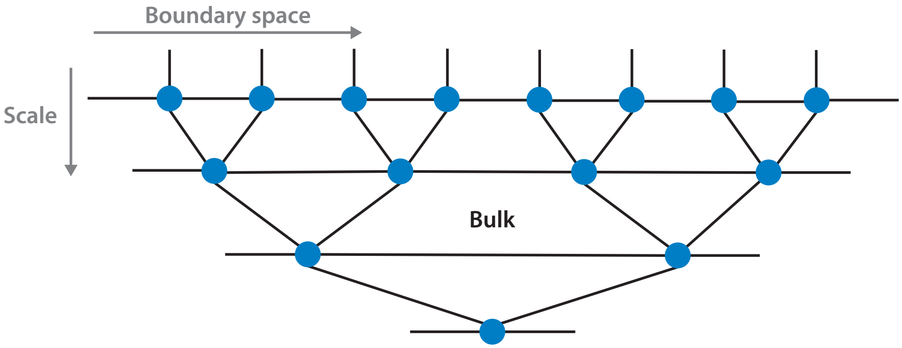

Figure 5

Scale-invariant network describing entanglement in a gapless system.

length ℓ scales like $\ln \ell$ (27, 28). For large ℓ, this is inconsistent with any constant bond dimension in a one-dimensional tensor network.

Although all correlations and entanglement in a one-dimensional gapped state are short ranged, a gapless state can support power-law correlations and scale-invariant physics. Such a state evidently also has longer-range entanglement, so we need a network capable of generating more entanglement.

Figure 5 shows a scale-invariant tensor network where the physical legs reside at the top, and the bulk of the network has a self-similar structure. Vidal introduced a network of this type called the multiscale entanglement renormalization ansatz (MERA) (29). Let us call a general network of the type shown in **Figure 5** a renormalization group (RG) network.

How does the entanglement of a block of length ℓ scale in an RG network? Assuming the bound in Equation 10 gives the qualitative scaling, we find an entanglement entropy proportional to $\ln \ell$. The physics is that every scale contributes a fixed amount of entanglement and the number of scales that contribute is proportional to $\ln \ell$. Thus the entanglement of the gapless state can be captured using the RG network, but the resulting network geometry is richer than the original one-dimensional geometry of the boundary.

These networks can be generalized to higher dimensions and other kinds of states. Given some lattice system of qubits whose low-energy physics is described by a CFT, a plausible conjecture is that there exists an RG network that captures the ground state using only a modest bond dimension (30). More precisely, with a bond dimension polynomial in total system size, $\chi \sim n^a$, the overlap of the tensor network state with the true ground state is greater than $1 - n^{-b}$ and approaches one in the thermodynamic limit.

There is numerical evidence for this conjecture for simple CFTs in one dimension (31, 32). In higher dimensions, there are rigorous constructions of RG networks for long-range entangled topological states like the fractional quantum Hall systems (30). There are also constructions of RG networks for some gapless models with scale invariance but not full conformal invariance (33). Work on CFTs has not yet yielded a rigorous result.

5. EINSTEIN'S EQUATIONS FROM ENTANGLEMENT

The previous section argued that certain kinds of gapless states of matter have a pattern of entanglement that is best described by a scale-invariant tensor network. The resulting bulk geometry has an extra dimension relative to the space in which the physics qubits naïvely reside.

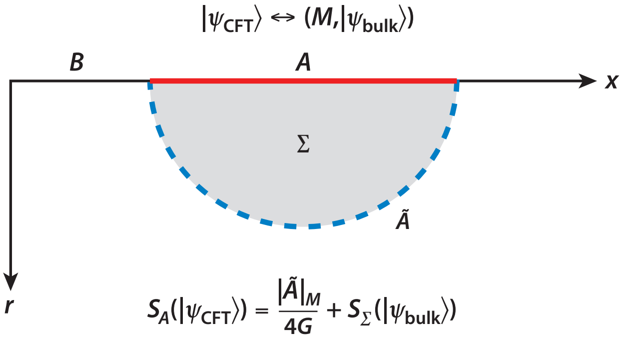

Figure 6

Schematic of the Ryu–Takayanagi formula for entanglement entropy.

Furthermore, the entanglement of a set A of the microscopic qubits was proportional to the boundary of A as measured in the emergent tensor network geometry.

The encoding of entanglement in terms of minimal areas is strongly reminiscent of AdS/CFT duality. For example, black holes have entropy proportional to their area. AdS/CFT associates to certain CFT states $|\psi_{\text{CFT}}\rangle$ a bulk geometry M and state of bulk quantum fields $|\psi_{\text{bulk}}\rangle$. The CFT ground state corresponds to empty AdS, whereas a thermal state of the CFT corresponds to a black hole in AdS. The Ryu–Takayanagi (RT) formula (34) states that the entropy $S(A)$ of any region A in the boundary is given by the area $|\tilde{A}|_M$ in Planck units of a bulk minimal surface $\tilde{A}$ that hangs from the boundaries of A (see **Figure 6**),

$$S(A, |\psi_{\text{CFT}}\rangle) = \frac{|\tilde{A}|_M}{4\hbar G} + S(\Sigma, |\psi_{\text{bulk}}\rangle). \qquad 11.$$

This is strongly reminiscent of Equation 10, where the minimum cut is identified with the minimal area surface. The second term in Equation 11 encodes the entanglement of the bulk quantum fields (35).

To cement the analogy, consider the ground state that corresponds to $M = \text{AdS}_3$. In convenient coordinates, the metric is

$$ds^2 = \frac{L^2}{r^2}(-dt^2 + dx^2 + dr^2), \qquad 12.$$

with L being the AdS radius; x and t are field theory coordinates, whereas the r coordinate describes the emergent direction. This smooth geometry is similar to the RG network geometry provided that we identify $\ln r$ with the number of coarse-graining steps (6). For example, the length of a minimal curve of boundary separation ℓ is proportional to $\ln \ell$. Also, the time component of the metric can be extracted from the renormalization of the Hamiltonian by the tensor network (6).

But is this only an analogy or something more? Suppose it were possible to take a continuum limit of the discrete tensor network geometry such that the resulting smooth geometry computed entanglement entropies via the RT formula. Within AdS/CFT duality the geometry would obey Einstein's equations, but it is far from obvious that this is the only possibility.

Remarkably, assuming that we have a CFT in which states correspond to geometries and in which entanglement is computed with the RT formula (as suggested by the tensor network), it is possible to show that small perturbations to the AdS geometry must obey the source-free Einstein equations linearized about AdS. Apparently, a geometry with the right properties built from entanglement has to obey the gravitational equations of motion.

The argument proceeds in several steps. First, the ground state is identified with AdS because this is the only spacetime with the same conformal symmetry. Second, one relates small changes in the entropy of a ball-shaped region to small changes in the modular energy of the ball (this energy is given by the integral over the ball of the CFT energy density times a weighting function) (36). This relationship between entropy change and energy change is called the entanglement first law by analogy with the first law of thermodynamics. Third, one translates the entanglement first law into geometric quantities using the RT formula. Then one finds that the resulting constraints on the bulk geometry are the source-free Einstein equations linearized about AdS (37, 38).

By including the physics of bulk excitations in Equation 11, one can derive Einstein's equations for gravity coupled to matter, again to leading order (39). In particular, the equivalence principle and Newton's law of gravity (in AdS) arise from the dynamics of entanglement. This result further justifies the claim that spacetime arises from entanglement. Spacetime quite literally describes the structure and dynamics of entanglement in the CFT.

5.1. Assumptions in the Argument

The above derivation of Einstein's equations rested on the assumption that a continuum limit with the right properties could be obtained, but this is highly nontrivial because most physical systems behave nothing like gravity in AdS. The existence of an RG-like network seems quite generic, but the detailed dynamical properties of the network hinge on additional physics. One could speculate that every physical system is dual to some kind of theory that includes gravity, but most such theories have so many other moving parts that they are not recognizable as Einstein gravity. For example, the resulting spacetime will be weakly curved and semiclassical only if the cosmological constant is small compared with certain microscopic energy scales; the cosmological constant problem is still a problem in AdS gravity.

We do not know in general what kind of quantum dynamics can give rise to emergent spacetime. We do understand some special cases, for example, maximally supersymmetric $SU(N)$ Yang–Mills theory (4). In the limit of large N and strong coupling, this CFT is well described by supergravity in AdS. Are there any lessons we can draw from this example? Imagine a lattice regulated version of this theory. What might the tensors of such a system look like? Perhaps they are quasi-random because a strongly interacting system will rapidly mix information among its degrees of freedom (40).

Remarkably, the physics of random tensor networks does share similarities with the expected properties of AdS/CFT duality (41). For example, the network version of the RT formula is approximately obeyed (the bound in Equation 10 is tight), and the entanglement of bulk excitations is correctly included. Similar features arise from perfect tensor models (42). Of course, many features of the CFT are not captured correctly, especially conformal symmetry. Furthermore, these random tensor networks are not the ground state of a local Hamiltonian, so the dynamics is still missing.

5.2. Why Entanglement?

Let us conclude this section by asking, Why entanglement? One clue has emerged recently. The bulk geometry is encoded in the entanglement structure of the CFT, but it is hidden from local probes of the CFT. Small boundary regions are associated with small parts of the bulk geometry near the boundary. The deep interior apparently describes very nonlocal entanglement among the CFT degrees of freedom.

How is the bulk quantum information encoded into the nonlocal entanglement of the boundary CFT in such a way that it is hidden from local probes? The answer is quantum error correction

(43). By encoding the bulk information into the boundary using a quantum error correcting code, it is possible to hide the bulk in the boundary in a way that circumvents our naïve expectations about the geometry of the system (42). Because nonlocal entanglement is required to have a quantum error correcting code, the need to robustly encode the bulk into the boundary shows that entanglement is crucial for constructing the bulk.

6. DETECTING EMERGENT GEOMETRY

There are at least two parameters, the number of degrees of freedom and the coupling, that are relevant for the emergence of spacetime (5). To see the physics of classical gravity, the corresponding quantum theory must also have a kind of classical limit. This is usually accomplished by considering a CFT with many local degrees of freedom. A strong coupling limit is also important for the emergence of gravity. AdS/CFT duality ultimately involves a string theory on the gravity side, but the role of strong coupling in the CFT is to push the string states up to high energy where they are hard to detect.

As mentioned above, perhaps one can think of any quantum system as being dual to a kind of quantum string theory that includes gravity. However, there is little independent understanding of how quantum string theory should behave. Certainly, it need not look like a weakly curved classical geometry obeying the equations of Einstein gravity. Hence, we should formulate conditions under which we can say spacetime has emerged.

The simplest kind of probe is ordinary correlation functions of boundary observables. In AdS/CFT at large N and strong coupling, these correlation functions are obtained from classical wave equations in the bulk geometry (5). The mass of the bulk field is related to the scaling dimension of the boundary operator, but the geometry underlying each wave equation is the same. In other words, all boundary correlators probe the same bulk geometry. One requirement might thus be the compatibility of boundary correlations with a bulk single geometry.

Boundary correlation functions should also have special features that encode bulk locality. In a Lorentz invariant quantum field theory, the two-point correlation function $\langle \phi(x, t)\phi(0, 0)\rangle$ of a field ϕ has singularities on the light cone $|x| = |t|$. However, for field theories with a gravity dual, correlators must have additional approximate singularities that arise from the bulk light cone (44). These unexpected singularities are also an important feature of emergent geometry.

Another probe of emergent geometry is the geometrical structure of entanglement. If the entanglement entropy of different regions can all be computed as minimal area surfaces in the same bulk geometry, then one might say that a classical bulk has emerged. One complication is that higher derivative theories of gravity have entropy given by some more general functional of the geometry. Thus entanglement could still be determined geometrically but by something more complex than the minimal area. One necessary condition for the existence of a geometry obeying the RT formula is that entropies should lie inside the holographic entropy cone (45).

Chaos in the CFT provides another useful probe. The gravity dual of the growth of chaos in the CFT is high-energy scattering near the horizon of a black hole (46). This growth of chaos can be diagnosed by measuring the squares of commutators of local operators W and V, $C(t) = -\langle [W(t), V]^2\rangle$. Recently, it has been shown how to measure such unusual correlation functions (47, 49; N. Y. Yao, F. Grusdt, B. Swingle, M. D. Lukin, D. M. Stamper-Kurn, et al., manuscript submitted, arXiv:1607.01801), but the proposals to date either require a time machine or are associated with an exponentially small signal. Remarkably, black holes in Einstein gravity are maximally chaotic in the sense that $C(t)$ grows exponentially with the largest possible rate (50). Detecting such maximal chaos would be further evidence of emergent geometry.

Unfortunately, none of these probes constitute a smoking gun signature of emergent spacetime. It is not even clear what set of features one should require of emergent gravity. This may be because the emergence of gravity is less like a distinct phase of matter and more like a special limit. However, taken together these probes would paint a compelling picture of emergent spacetime.

What kinds of systems might have these features? One possibility is the Sachdev–Ye–Kitaev model (51, 52). This is a system of interacting Majorana fermions with random all-to-all four fermion interactions. Remarkably, this quantum mechanical system contains a sector described by gravity in AdS_2 (53–55). There have been several proposals to realize this model in cold atoms and solid-state systems (56, 57). More generally, the promise of quantum simulation allows us to fantasize about one day engineering a system whose low-energy dynamics approximates a supersymmetric quantum field theory with a known holographic dual. The experiment could then be tuned away from the classical limit to simulate quantum gravity. Still, it bears repeating that we are largely ignorant of the dynamics of gravity away from the classical limit, so there is plenty of room for surprises.

7. GOING FORWARD

We outlined a systematic framework for understanding the emergence of spacetime from entanglement. The main issue is that we do not understand how to take a continuum limit and show that the smooth version of the RT formula is obtained. If this point could be addressed, then we could fully derive gravity from entanglement, at least for the special case of AdS gravity.

The obvious frontier is to take the lessons learned in AdS and try to apply them to gravity in Minkowski space or de Sitter space. A naïve extrapolation is difficult because of significant differences in the boundary structure of these different spacetimes. The challenges of emerging light-like or time-like dimensions thus remain open, and it seems new ideas are needed. There are also many other promising directions.

Here, we viewed the tensor network as a direct representation of AdS. However, recent progress has been made using an alternative tensor network tailored to describe the physics of AdS via a geometry known as kinematic space (58). In the simplest case, kinematic space is the space of geodesics in AdS, and aspects of AdS/CFT duality can be reformulated using kinematic space. There are intriguing connections between kinematic space and MERA networks. For CFTs in two spacetime dimensions these connections have led to a construction of tensor networks for thermal states from the ground-state tensor network (59).

Another crucial issue is locality at small scales. In AdS/CFT there are multiple scales ranging from the AdS radius, which is essentially the radius of curvature of the Universe, to the string length and Planck length, which both provide short-distance cutoffs to bulk effective field theory. It was understood early that the nodes in the RG tensor network describe AdS radius-sized chunks of the geometry (6). This is how the scale-invariant network can be general: Every theory is local on the AdS scale. However, locality at scales smaller than the AdS radius is mysterious from this point of view. This is closely related to the problem of taking a continuum limit of the tensor network.

The final direction we mention relates to black holes. We focused on the lowest-energy state of the CFT and perturbations thereof. These were dual to empty AdS and gravitational waves on top of it. One can also construct tensor networks to describe thermal states dual to black holes (60). These networks can include the interior of the black hole (61). A remarkable fact about the interior is that it grows with time long after other observables have settled into their equilibrium values. What in the CFT is dual to the persistent growth of the interior? One idea is that the growth of the black hole interior is dual to growth of computational complexity of the CFT state

(62–64). Roughly speaking, the computational complexity is the number of tensors in the minimal tensor network that prepares the state. Interestingly, the interior continues to grow long after all entanglement entropies equilibrate, which is an observation that suggests "entanglement is not enough" (65, p. 1).

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