Causal mediation

Defintions and estimates

Causal mediation and direct effects

$$CME_0 = Y_{0M_1} - Y_{0M_0}$$
$$CME_1 = Y_{1M_1} - Y_{1M_0}$$

$$CDE_0 = Y_{1M_0} - Y_{0M_0}$$
$$CDE_1 = Y_{1M_1} - Y_{0M_1}.$$

Total causal effects

$$
\begin{aligned}
TCE &= Y_{1M_1} - Y_{0M_0} \\
&= CME_1 + CDE_0 \\
&= CME_0 + CDE_1
\end{aligned}
$$

Estimating causal mediation effects

$$\hat{CME}_0 = \sum_{m \in 0,1} E[Y|A=0, M=m][P(M=m|A=1) - P(M=m|A=0)]$$

$$\hat{CME}_1 = \sum_{m \in 0,1} E[Y|A=1, M=m][P(M=m|A=1) - P(M=m|A=0)]$$

Estimating causal direct effects

$$\hat{CDE}_0 = \sum_{m \in 0,1} (E[Y|A=1, M=m] - E[Y|A=0, M=m]) P(M=m|A=0)$$

$$\hat{CDE}_1 = \sum_{m \in 0,1} (E[Y|A=1, M=m] - E[Y|A=0, M=m]) P(M=m|A=1)$$