DS 574 Algorithmic Mechanism Design	**Lecture #9 Worksheet**
Fall 2025	Prof. Kira Goldner

Time-Inconsistent Planning: Present Bias

Story about Nobel Laureate George Akerlof who needed to mail a package to his friend, Joseph Stiglitz.

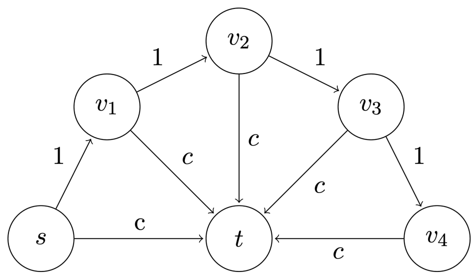

Figure 1: The fan graph for Akerlof's story.

Formally:

- Sending the package has a fixed cost c.
- There is a loss of use cost 1 for each day in which the package cannot be used.
- Total cost for sending on day t is: $c + t$.

The rational behavior is to send the package on the first day to minimize the total cost. *Present bias* [Akerlof] indicates that you perceive the cost of doing something today as inflated by some bias factor b. Thus:

More generally, we define the model as follows:

1. There is a directed acyclic graph G with a source s and a target t.
2. Each edge e corresponds to some task and has a cost which captures the effort required for completing the task.
3. The agent needs to take a path from s to t. At each node v it will choose the $v - t$ path which is the shortest path in a graph in which the costs of all outgoing edges from v are multiplied by a factor of b.

This simple model is based on more elaborate model (quasi-hyperbolic discounting). Formally:

Definition 1 (traversal)**.** An agent currently at v_i will continue to a node $v_{i+1} \in \arg\min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$. We refer to $C(v_i) = \min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$ as the perceived cost of agent i at v_i.

Let's see another example:

Question: Consider an agent with present bias $b = 2$. Which path will he traverse in the graph in Figure 2?

Choice Reduction and Its Benefits

Experiment in a course at MIT: Students need to submit 3 assignments throughout the semester. In the beginning of the semester, each student was asked to set a deadline for each assignment. What is the rational behavior?

What would you do?

In the experiment:

What does this tell us?

Example: 3 week course, 2 task. The cost of completing a single task in a week is 4. The cost for completing both in the same week is 9. The cost of a week of studying without doing any tasks is 1. The task graph in Figure 2 models this scenario. In the graph, node $v_{i,j}$ corresponds to completing j tasks by week i.

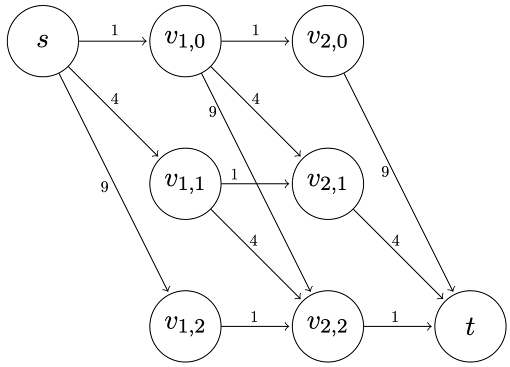

Figure 2: An example featuring the benefits of setting deadlines. Horizontally: weeks. Vertically: tasks.

Now, assume that there is a reward $R = 17$ for completing the course (reaching t) and the agent will traverse the graph as long as its perceived cost is less than R. How will an agent with present bias $b = 2$ traverse the graph?

How can we help the student complete the course? Consider setting a deadline for the first assignment: the first assignment should be submitted by the second week. This means that in graph we delete the node $v_{2,0}$. What will the agent do now?

This leads to the following algorithmic question: given a graph in which the agent does not reach t can we delete nodes and edges such that agent will reach t?

One way for approaching this question is hoping that if there is a traversable subgraph then there is always a traversable subgraph which is just a path. Is this true?

Research Directions:

- Cost ratio: quantifying how much present-biased agents lose due to their bias.
- Characterizing graph structures that lend themselves to bounded or exponential cost ratios.
- Sophisticated agents aware of their present bias.

Obviously Strategy Proof

We need a few more standard game-theoretic definitions before we can understand this concept.

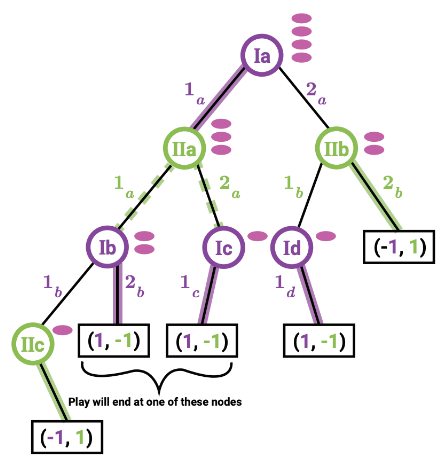

Figure 3: The Subtraction Game: Starting with a pile of four chips, two players alternate taking one or two chips. Player I goes first. The player who removes the last chip wins.

Definition 2. A k-player finite *extensive-form* game is defined by a finite, rooted tree T . Each node in T represents a possible state in the game, with leaves representing terminal states. Each internal (nonleaf) node v in T is associated with one of the players, indicating that it is his turn to play if/when v is reached. The edges from an internal node to its children are labeled with actions, the possible moves the corresponding player can choose from when the game reaches that state. Each leaf/terminal state results in a certain payoff for each player. A pure strategy for a player in an extensive-form game specifies an action to be taken at each of that player's nodes. A mixed strategy is a probability distribution over pure strategies.

Definition 3. Given an extensive-form game, the *normal form* of the game is the matrix of possible pure strategies and their resulting payoffs.

Sealed-bid second-price auction and ascending English auction have the same normal form, but not the same extensive form. In practice, people play them quite differently.

Earliest Point of Departure: Nodes I_i are in the *information set* $\alpha(S_i^1, S_i^2)$ if and only if

1. $S_i^1 \neq S_i^2$ at I_i and
2. I_i could have been reached by playing either S_i^1 or S_i^2.

Let $u_i^G(h, S_i, S_{-i}, v_i)$ be the utility to agent i in game G as a function of starting from history h with play proceeding according to S_i, S_{-i} and the resulting outcome evaluated according to preferences v_i.

Definition 4. A strategy S_i is *weakly dominant* if for all deviating strategies S_i' and other bidder strategies S_{-i}, $u_i^G(h_0, S_i, S_{-i}, v_i) \geq u_i^G(h_0, S_i', S_{-i}, v_i)$

Definition 5. A strategy S_i is *obviously dominant* if for all deviating strategies S_i' and nodes in the earliest point of departure $I_i \in \alpha(S_i, S_i')$: $\inf_{h \in I_i, S_{-i}} u_i^G(h, S_i, S_{-i}, v_i) \geq \sup_{h \in I_i, S_{-i}} u_i^G(h, S_i', S_{-i}, v_i)$

Definition 6. A mechanism is *obviously strategyproof* if truth-telling is an obviously dominant strategy.

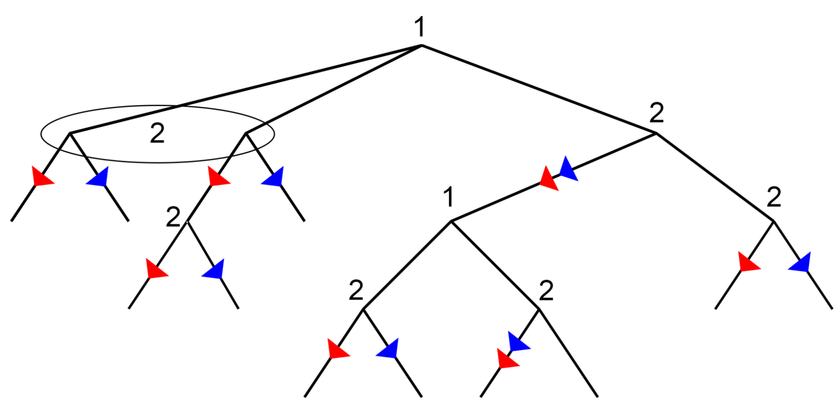

Figure 4

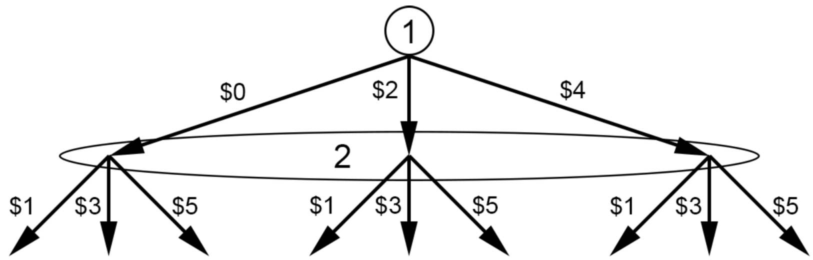

Figure 5

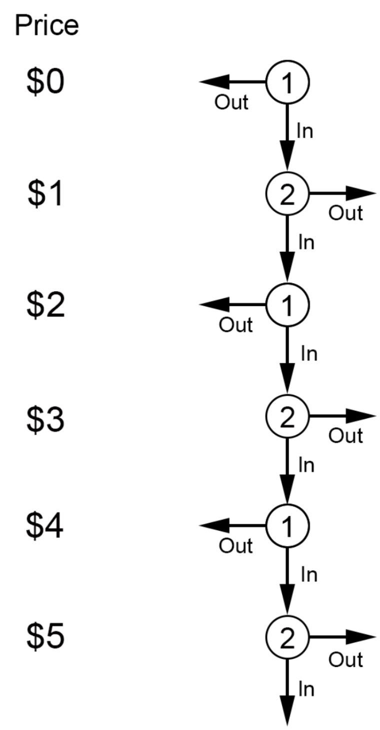

Figure 6

Acknowledgements

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