DS 574 Algorithmic Mechanism Design	**Lecture #16**
Fall 2025	Prof. Kira Goldner

Ascending Auctions

In *ascending auctions*, an auctioneer initializes prices for each item, iteratively raises the prices, and bidders decide which items to bid on in each round. Sometimes *activity rules* are enforced, e.g., once you drop out on an item, you can not bid on it again.

The most famous ascending auction is the single-item version, the English Auction.

The English Auction(ε):

a. Initialize the item's price p_0 to 0.

b. The initial set S_0 of "active bidders" (willing to pay p_0 for the item) is all bidders.

c. For iteration $t = 1, 2, \ldots,$:

   (a) Ask the set of active bidders S_{t-1} if they're willing to pay $p_{t-1} + \varepsilon$. Let S_t be the bidders who say yes. (Hopefully, $v_i \geq p_{t-1} + \varepsilon$.)

   (b) If $|S_t| \leq 1$: terminate the auction. Allocate the item to the remaining active bidder at a price of p_{t-1}. If no bidders remain, randomly allocate to a bidder from S_{t-1} at p_{t-1}.

   (c) Otherwise, $p_t = p_{t-1} + \varepsilon$.

Benefits of using ascending auctions:

- Ascending auctions are easier for bidders. It is generally easier to answer simple queries than to report a valuation. This point will become especially relevant in more complex scenarios.
- Less information leakage. The winner of an ascending auction does not reveal its valuation, just the fact that it is at least the second-highest bid.
- Transparency. The cause of a high selling price is generally more obvious in open ascending auctions than in sealed-bid auctions.
- Potentially more seller revenue. For example, ascending auctions encourage "bidding wars." There is also some supporting theoretical work on this point [1].
- When there are multiple items, the opportunity for "price discovery." A bidder has the opportunity for mid-course corrections and to better coordinate with other bidders.

What about k identical items? What should we do here?

The English Auction for k Identical Items:

The same as above, but replace step 3(b) with the following:

(b) If $|S_t| \leq k$: terminate the auction. Allocate the items to the remaining active bidders at a price of p_{t-1}. If there are items leftover (i.e., $k - |S_t| > 0$), randomly allocate them to bidders from $S_{t-1} \setminus S_t$ at p_{t-1}.

Definition 1. In an ascending auction, *sincere bidding* means that a player answers all queries honestly.

Claim 1. In the k identical item setting, in an English auction, sincere bidding is a dominant strategy for every bidder (up to ε).

Claim 2. In the k identical item setting, if all bidders bid sincerely in an English auction, the welfare of the outcome is within $k\varepsilon$ of the maximum possible.

The English auction for k Identical Items terminates in $v_{\max}/\varepsilon$ iterations.

The above claims are left as an exercise.

We can use the following design process for ascending auctions:

a. As a sanity check, design a direct-revelation DSIC welfare-maximizing polytime mechanism.

b. Implement this as an ascending auction.

c. **(Truthfulness)** Check that its EPIC.

d. **(Performance)** Check that it still maximizes welfare under sincere bidding.

e. **(Tractability)** Check that it terminates in a reasonable number of iterations.

Additive Valuations, Parallel Auctions

The Additive Setting: There are m non-identical items and n bidders where each bidder i has private valuation v_{ij} for each item j. Bidder i has an additive valuation for each set S, that is,

$$v_i(S) := \sum_{j \in S} v_{ij}.$$

Step 1: What is the welfare-optimal direct revelation mechanism here? Just handle each item separately—m Vickrey auctions!

What's the analogous ascending implementation?

Parallel English Auctions: Maintain a set of interested bidders for each item, and the auction for item j terminates when there's only one active bidder remaining, breaking ties arbitrarily.

Is this DSIC? No!

Example: Two bidders, two items. $\mathbf{v}_1 = (3, 2)$ and $\mathbf{v}_2 = (2, 1)$.

What happens under sincere bidding? The first bidder wins both items at prices of 2 and 1 respectively.

Alternatively, bidder 2 could threaten the following strategy: if bidder 1 bids on item 1 in the first turn, then bidder 2 will keep bidding on both items forever (or up to a price of 3). If not, they will bid sincerely until the auction terminates.

Then bidder 1 bidding sincerely triggers bidder 2's threat, causing bidder 1 to lose both items, so bidder 1 would prefer to abandon item 1.

Recall that a dominant strategy maximizes a bidder's utility independent of the actions played by any other player. Bidder 2's strategy may not maximize their utility, but it still implies that sincere bidding is not a *dominant* strategy for bidder 1.

Instead, we need a different solution concept.

Definition 2. A strategy profile $(\sigma_1, \ldots, \sigma_n)$ is an *ex post Nash equilibrium (EPNE)* if, for every bidder i and valuation $v_i \in V_i$, the strategy $\sigma_i(v_i)$ is a best-response to every strategy profile $\sigma_{-i}(\mathbf{v}_{-i})$ with $\mathbf{v}_{-i} \in \mathbf{V}_{-i}$.

In comparison, in a dominant-strategy equilibrium (DSE), for every bidder i and valuation v_i, the action $\sigma_i(v_i)$ is a best response to every action profile $\mathbf{a}_{-i}$ of $\mathbf{A}_{-i}$, whether of the form $\sigma_{-i}(\mathbf{v}_{-i})$ or not.

Definition 3. A mechanism is *ex post incentive compatible (EPIC)* if sincere bidding is an ex post Nash equilibrium in which all bidders always receive nonnegative utility.

Claim 3. For n additive bidders with m heterogenous items, in parallel English auctions, sincere bidding by all bidders is an ex post Nash equilibrium (up to $m\varepsilon$).

Acknowledgements

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