DS 574 Algorithmic Mechanism Design	**Lecture #17 Worksheet**
Fall 2023	Prof. Kira Goldner

Interdependent Values II

Submodularity over Signals [1]

Definition 1. Valuation $v_i(\cdot)$ is *submodular over signals* if, for all j, when $\mathbf{s}_{-j}$ is lower, $v_i(\cdot)$ is more sensitive to s_j. For all j, and for any $\mathbf{s}_{-j} \leq \mathbf{s}'_{-j}$:

$$\frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}_{-j}) \geq \frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}'_{-j})$$

Random-Sampling Vickrey Auction.

- Elicit s_i from each bidder i.
- Assign each bidder into set A or set B w.p. 1/2 independently.
- For each bidder $i \in A$, calculate and use proxy value $\hat{v}_i = v_i(s_i, \mathbf{0}_{A\setminus i}, \mathbf{s}_B)$.
- Allocate to the potential winner in A with the highest proxy value.

Theorem 1. *The RS Vickrey Auction is EPIC and achieves a prior-free $\frac{1}{4}$-approximation to the optimal welfare.*

To prove this theorem, we need to address (1) truthfulness and (2) the approximation guarantee.

Truthfulness. Is this allocation EPIC?

Approximation. Is $v_i(s_i, \mathbf{0}_{A\setminus i}, \mathbf{s}_B)$ a good way to choose a winner?

Lemma 1 (Key Lemma)**.** *Let v_i be a submodular over signals valuation. Partition all agents other than i uniformly at random into sets A and B. Then*

$$\mathbb{E}_{A,B}[v_i(s_i, \mathbf{0}_A, \mathbf{s}_B)] \geq \frac{1}{2} v_i(\mathbf{s}).$$

Proof.

Proof of Theorem 1.

References

[1] Alon Eden, Michal Feldman, Amos Fiat, Kira Goldner, and Anna R. Karlin. Combinatorial auctions with interdependent valuations: Sos to the rescue. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, EC '19, Phoenix, AZ, USA, 2019. ACM.