Mechanism Design: Beyond Traditional Models, Beyond Traditional Settings

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Mechanism design, also known as "incentive engineering," is concerned with the design of protocols such that rational participants, motivated solely by their self-interest, will end up achieving the designer's goals. The applications of mechanism design in the age of the internet are vast: scheduling tasks in the cloud, routing traffic in a network, buying and selling goods in electronic marketplaces, and assigning ad-slots on search engines, to name a few. These days, many algorithm problems are in fact mechanism design problems. Solving these problems requires merging ideas from computer science, game theory, and economics.

One of the most fundamental goals in mechanism design and a major focus of my own research is profit maximization, e.g., in ad auctions. Indeed, these auctions generate significant revenue for companies like Microsoft, Google, and Facebook. As Hal Varian, Google Chief Economist, has said, "What most people don't realize is that all that money comes in pennies at a time."

Consider, for example, an auctioneer selling m items to n potential buyers, and suppose that the auctioneer has detailed prior knowledge of the distribution from which each buyer's values for items are drawn. What is the revenue-optimal auction that the seller can implement? Unfortunately, despite decades of research on this problem, we are still far from having achieved a full understanding of the answer to this question. In fact, a complete characterization of the optimal auction for selling just two items optimally to a single bidder is still not known!

My work addresses specific yet tractable problems in revenue maximization by studying models that incorporate the uncertainty and specific constraints mechanisms face in practice. These include problems (1) where the seller may not have any prior knowledge about the market; (2) where buyers' attitudes toward risk may vary; (3) where buyers cannot trust the auctioneer to implement a mechanism and wish to verify it; (4) where a buyer's preferences have some strong underlying structure, like a preferred quality-of-service or demand cap, and the seller must reason about the uncertainty of this preference; or (5) where an auction is executed repeatedly, but the auctioneer and auction used may be different in each round.

Beyond revenue maximization, I am also beginning to explore the space of "mechanism design for social good"—real-world problems where current market and government mechanisms are failing and where algorithmic mechanism design can help play a role to improve social welfare, mitigate inequality, or disincentive the waste of resources.

Beyond Traditional Models in Revenue Maximization

Beyond Restricted Distributions: Leveraging Structure

There are two major lines of research directed towards understanding optimal (revenue maximizing) auctions in multi-parameter settings. First, optimal auctions have been characterized for some specific prior distributions of buyer's values. For example, they are known for the following settings: two items each with $U[0,1]$ distributions and quadratic utility [21], six items with uniform distributions [17], and single-crossing distributions [23]. Secondly, some papers have characterized settings in which a specific auction format is optimal, such as selling two items in a bundle to additive bidders [10, 11], selling only the bidders' favorite outcome [19], or deterministically pricing quantities of identical items [13]. Most of these results consider very simple or restricted distributions. There has also been a surge of work recently on designing simple mechanisms that give constant-factor approximations to the optimal revenue in a variety of value settings.

In contrast to work that places restrictions on the distributions or settles for approximation, my work identifies problems that are motivated by real-world settings where buyer preferences have additional structure that, while uncertain to the seller, can be leveraged to exactly characterize the optimal auction without placing any restrictions on the distributions.

In "The FedEx Problem" [16], with Amos Fiat, Anna Karlin, and Elias Koutsoupias, we consider the setting where buyers have a package to ship, a deadline d that they want it received by, and a value v for having the package received by its deadline. This setting also encompasses customers with deadlines for scheduling tasks in the cloud, or app developers buying a certain level of customer support from a cloud service. More generally, this model captures the setting where buyers have a value v for service that meets their quality-of-service demand or time-sensitive deadline d. The seller, e.g., FedEx, must reason about not only the unknown values, but also the unknown deadlines, and must use a mechanism that segments the market into customers with higher or lower quality demands in order to fully extract the optimal revenue.

Canonical attempts at this problem lead to a messy case analysis, even when deadlines are just in 1 or 2 days. Our approach circumvents this. By the revelation principle, without any loss of revenue, we can restrict our attention to "incentive-compatible" mechanisms, where it is in the buyer's best interest to report his true (v, d) pair. In the FedEx setting, we can require that a buyer prefers to report his true deadline (v, d) over any other deadline (v, d') using a single constraint: he should prefer reporting (v, d) over $(v, d-1)$. In our work, we formulate the optimization problem as a infinite linear program. We take the program's dual and use complementary slackness to come up with values for the primal and dual variables that determine the structure that the optimal auction must take. While solving for these variables is still challenging, it is possible due to the limited number of relevant incentive-compatibility constraints where d is varied, making the dimensions somewhat orthogonal. Thus, instead of a case analysis, via the duality approach, we are able to develop a unified solution with a nice inductive structure that is provably optimal.

Another well-motivated problem with specific preferential uncertainty is the Multi-Unit Pricing Problem posed by [13]: "Software companies such as Microsoft sell software subscriptions that can have different levels of service. The levels could be the number of different documents you are allowed to create, or the number of hours you are allowed to use the software." In this setting, buyers have a value v per unit of item, that is, they value each document they create or hour that they use the software. However, buyers also have some demand cap d after which they no longer get additional value from the product. This is a good candidate for our techniques because the incentive-compatibility constraints that ensure a buyer reports his true demand cap reduce to only two constraints per demand cap (plus the usual incentive compatibility constraints for fixed d and varying v). In their paper, Devanur et al. prove that under a distributional assumption known as price-regularity, the optimal mechanism will post increasing prices for bundles of $1, 2, \ldots, n$ items (or documents or hours). When bidders only have demand-caps of 1 or 2 (the $n = 2$ setting), they analyze four cases to break down the options of how the prices for 1 and 2 items will be set.

I am working to characterize the optimal mechanism for the Multi-Unit Pricing Problem without case analysis or distributional assumptions. I have already unified the case analysis for $n = 2$ into a single optimization problem that gives the optimal deterministic pricing under price-regularity. I am in the process of reproving this via the duality framework from the FedEx paper, after which I hope to use the framework to generalize the solution to all unrestricted distributions for $n = 2$, and then to any n.

Future Work: Our work on The FedEx Problem and my progress on the Multi-Unit Pricing Problem suggest that our duality techniques are promising for determining optimal mechanisms in settings with sufficient structure, where we can reduce to a smaller number of relevant constraints. Not only does this help to chip away at the original problem of understanding revenue-optimal auctions, but by choosing well-motivated underlying structure, we are inherently solving fundamental revenue maximization problems and designing mechanisms that actual sellers will be interested in implementing. I propose to continue to identify real-world, important profit maximization problems that have this kind of useful structure, and to leverage these approaches to build up our understanding as we move to more complex settings.

Some specific open questions that intrigue me are the following:

1. The optimal revenue-maximizing auction for the FedEx problem is randomized and could end up involving a lottery over 2^n prices if there are n possible deadlines. On the other hand, we know how to find the optimal deterministic pricing. How big is the revenue gap between the optimal randomized and the optimal deterministic pricing?

2. The formulation of the FedEx problem does not incorporate costs of delivery/service. How does the solution change when these costs are incorporated?

3. Do these techniques extend to a setting where there is a buyer with a value for an item and a budget b that he can spend? In this setting, the seller is uncertain about the value and budget: there are n types of budgets that the buyer might have, and this (value, budget) pair is drawn from a joint distribution.

4. What happens when there are multiple buyers? Can we extend these techniques beyond the single buyer case?

Beyond Complicated Mechanisms: Posted Pricings

Optimal mechanisms, such as those described above, are often very complicated, and involve offering buyers a complex menu of lotteries. A major focus of recent research in the field has been on understanding the efficacy of simple mechanisms that post prices on each service or item that is offered, and then let buyers arrive in any order and take any item they want given the prices. The hope is that posted price mechanisms still guarantee a large fraction of the optimal revenue.

One of the most fundamental open questions this motivates is the following "prophet inequality" problem. Suppose a gambler is playing the following game: he will be shown items in some adversarial order, and as each item appears, its weight is revealed, and the gambler must immediately and irrevocably decide whether to take the item or not. However, he can only take at most k items (or abide by some other given matroid feasibility constraint). His goal is to maximize the weight of the set of items that he takes. The gambler wants to use an algorithm that, knowing the prior distributions of the weights of each item, sets a threshold for each item in advance. Then, upon seeing an item's weight, if it exceeds its threshold and it is feasible to take the item given the items he has already taken, he takes it. Can such an algorithm guarantee a constant-factor of the weight of the set taken by the "prophet," who sees all of the weights in advance and knows the order? If so, known reductions tell us that posted prices yields strong revenue guarantees [7, 8].

In ongoing work with Shuchi Chawla, Anna Karlin, and Ben Miller, we are exploring the existence of non-adaptive constant-factor prophet inequalities. Prior work by Kleinberg and Weinberg [20] uses thresholds that adapt over time; this means that thresholds can be selected based on what items have already been taken, and thus based on an understanding of what taking the next item might exclude in the future. It is much more challenging to prove matroid prophet inequalities with non-adaptive thresholds since the reasoning must be done in expectation, which leaves gaps in the known charging arguments. On the other hand, if we can solve this problem, we will be able to use this to construct truthful mechanisms with very desirable properties.

We already have an algorithm that gives a constant-factor approximation for graphic matroids, as well as a candidate algorithm for general matroids. We have found that while it is difficult to argue that an algorithm is generating enough weight directly, we can use randomization to argue that each item's weight is achieved with constant probability. Solving this problem will yield a greater understanding of simple yet approximately optimal mechanisms for matroid-constrained buyers; it may also have implications for rounding fractional solutions that come from a relaxation of online selection problems.

Beyond Known Distributions

While the bulk of revenue maximization reasons about buyers with unknown values drawn from known distributions, it is not always realistic to assume that the seller has access to this information. When the buyer population is dynamic, a seller is unlikely to have precise information about the prior distributions. An auction tailored to a specific input may perform poorly if the seller's knowledge of this information is noisy or flat-out wrong. In practice, sellers may want an auction format they can use successfully every time, including on different populations.

In joint work with Anna Karlin [18], we construct a *prior-independent* mechanism that, without any knowledge of the buyers' prior distributions, guarantees a constant fraction of the expected profit achieved by the optimal mechanism that is tailored to the particular prior distributions. Let v_{ij} be the value of buyer i for item j. This guarantee holds no matter what the distributions happen to be, as long as the v_{ij}'s are all independent and satisfy the fairly standard condition of regularity. We give a mechanism that requires only a single sample from the distribution of each v_{ij}, and when there are at least two bidders from any prior distribution, we can implement this as a prior-independent mechanism. This result extends prior-independent results from the single-parameter setting [14] to the multi-parameter setting for multiple additive bidders.

A crucial lemma in [14][1] is that, for a single-item single-bidder problem, access to a *single sample* from a regular distribution is sufficient to approximate the optimal revenue, which in this case approximates the revenue from setting the reserve price. We combine this result with the recent breakthrough results by Babaioff, Imorlica, Lucier, and Weinberg [2], Yao [25], and Cai, Devanur, Weinberg [5], which give approximately optimal auctions that all only use second-price auctions and reserve pricing for either single items or bundles of items.

[1] This is a reinterpretation of the Bulow-Klemperer Theorem [4].

We observe that the single sample paradigm nearly suffices to construct a prior-independent version of these auctions. There is only one detail to resolve and that relates to the issue of pricing bundles: the sum of regular random variables is not necessarily regular. However, by delving into the proof from Babaioff et al., we found that the solution to this problem essentially writes itself: in the "bad" case, when bundle pricing is necessary for approximating the optimal revenue, the relevant random variable concentrates so that, in fact, a sample bundle price is sufficient.

Future Work: Huge progress has been made recently on the design of relatively simple approximately optimal mechanisms in settings with multiple buyers who are additive up to a matroid constraint [8], with multiple buyers who have XOS valuations [6], or with a single buyer who has subadditive valuations [24]. A key research question here is whether these approximately optimal mechanisms can be turned into near-optimal prior-independent mechanisms.

Additional On-Going and Future Work

I have begun to investigate a number of other questions related to non-traditional models for revenue maximization with alternative forms of uncertainty. While all of these questions are fundamental from the perspective of mechanisms that are implementable in practice, they have received minimal attention thus far.

Beyond Risk-Neutral: For the most part, single-item revenue maximization focuses on risk-neutral buyers: those who maximize their expected utility. However, many buyers are actually *risk-averse*: they prefer an outcome that is certain over taking their chances on a lottery, even if their payoff in the certain outcome is slightly less. Maskin and Riley [22] studied single-item revenue maximization for risk-averse buyers in 1984; there is less prior work on maximizing revenue in multi-item settings with risk-averse buyers. In collaboration with Shuchi Chawla, Ben Miller, and Manolis Pountourakis, I plan to begin addressing questions in this space. For example, in multi-item settings where we do know the optimal auctions for a single risk-neutral buyer, what is the optimal mechanism for a single risk-averse buyer who is modeled as having a concave utility function? What are the analogues of simple but approximately optimal mechanisms for selling to a single unit-demand risk-averse buyer, or a single additive risk-averse buyer?

I also hope to combine these questions with the approach of studying more structured scenarios like the FedEx setting. One interesting question in this domain is the following: suppose a buyer has a value v and a parameter r indicating how risk-prone or risk-averse the buyer is. For example, r might be the Arrow-Pratt measure of risk-aversion, indicating how concave or convex the buyer's utility function is. Or perhaps the utility function is drawn from a known set of n types. The seller must reason about this uncertainty and design a mechanism that segments her market to leverage the different possible attitudes toward risk and achieve optimal revenue.

Beyond Trusted Sellers: In pursuit of optimal auctions, we seek the revenue-optimal Bayesian Incentive-Compatible mechanism, which we can do without loss of generality due to the revelation principle. However, this reasoning is posited on buyers having full knowledge of the mechanism that the seller is implementing, which relies on trusting the seller. How can a buyer or third party audit the mechanism to determine that the seller is implementing the mechanism he claims? With Shai Vardi, I have begun investigating questions in this space. In particular, we have begun to explore modifications to the second-price auction for a single item that would make it "trustless," or robust to strategizing by the seller. Such strategizing might include the seller lying and using a different allocation rule, or submitting false bids to increase his revenue.

Beyond Centralization: In Bitcoin, a distributed electronic market, individuals called "miners" compete to process a set of transactions in a block, extending the "blockchain" that forms a discretized transactional history. Incentives are necessary to incentivize miners to expend costs on hardware and energy and to follow the prescribed protocol, as otherwise, there will not be universal agreement on the transaction history and thus on who owns which Bitcoins, threatening the security of the currency. As each block is formed, its miner allocates and prices transaction spots in the block, and a customer must acquire a spot to have his transaction processed. Out of the pool of miners, a single miner is chosen by nature as the current block's miner, or auctioneer, with probability proportional to his fraction of the pool's computation power. Under the existing protocol, these payments for spots, or transaction fees, will soon need to be high enough to provide sufficient incentives to the miners. However, there are currently no restrictions on the allocation rules. With Anna Karlin and Matt Weinberg, I am looking into the design of mechanisms that miners might use for allocating transaction spots.

Bitcoin could decide to incorporate a specific mechanism into the protocol, but such a mechanism would need to be verifiable (in the sense of the previous questions) and generate enough revenue to ensure proper incentives. What mechanism would have these desirable properties? A different approach would be to reason about the mechanisms that different miners might use, given that they may be able to generate more revenue by limiting supply, but also given that they only get to be the auctioneer infrequently, and whatever transactions they do not process, a different miner might process, getting the associated payment.

Interesting mechanism design questions extend to other blockchain-based systems as well. NameCoin is a blockchain-based system where miners run a mechanism at each time step to allocate, price, and resolve contention among requested domain names. Similarly, the blockchain could be used to implement order books and prediction markets. We plan to study a variety of interesting questions related to revenue maximization for different blockchain-based markets, each with its own specific structure and challenges.

Beyond Traditional Settings: Mechanism Design for Social Good

In addition to the practical and mathematically fascinating objective of revenue maximization, in May I was inspired by a talk given by Cynthia Dwork entitled "Theory for Society" [15]. Since then, I have been working to identify applications that give rise to interesting theoretical mechanism design questions and where the objective of such mechanisms is to mitigate inequality, disincentivize the waste of resources, or work for some other social good. I believe that algorithmic tools and ideas from economics can be merged to come up with actual solutions to societal issues and the current inefficient or broken systems that are in place. As with my agenda for revenue-maximization, the goal is to ground research questions in the constraints that accurately represent reality. The mechanism design community has succeeded before at identifying applications for social good and actually contributing to the mechanisms used in practice, as with kidney exchange and school choice. I aim to expand our community's focus to mechanism design questions in low-income housing, healthcare, income inequality, the online labor market, transportation, and climate change/resource conservation. I am currently running a remote reading group with Rediet Abebe to learn about these topics and identify research questions in this area [1].

One example involves low-income housing. The Move-to-Opportunity study subsidized rent costs for families that moved to lower poverty neighborhoods, and research has demonstrated that children benefited substantially, including earning higher income in the long-run and thus paying income taxes that fully account for the cost of the study [9]. In addition, studies on eviction show that eviction has long-term negative effects on the financial, mental, and physical well-being of families, and that a primary cause of eviction is sudden shock to income or wealth, as opposed to simply a budgetary issue [12]. How can we allocate public housing funds efficiently given that households suffer a very negative outcome when they experience a shock to their wealth without aid? What mechanisms can both minimize eviction and give guarantees on the expected future increase in income taxes so that they essentially fund themselves?

Another example pertains to healthcare. Imagine there are n sick individuals and m hospitals. Individuals have valuations for their treatment, hospitals incur a cost for each treatment specific to that hospital, and some third-party "payer" (such as the government or insurance company) pays for the costs of all of the hospitals, subject to some budget constraint. Wait times are introduced for over-saturated hospitals. Braverman et al. show in [3] that maximizing social welfare is NP-hard, but can be computed efficiently if the budget can be relaxed by a factor of $1 + \varepsilon$. These results, however, rely on the mechanism designer having full knowledge of the individuals' valuations. They also assume that wait times burn welfare equally for each individual, which is not an accurate assumption when some medical conditions are more dire. I would like to extend these results to account for individual sensitivities to wait times and to incorporate private valuations into the model.

My goal is not only to work on problems in this domain myself, but by formulating interesting theoretical questions on fundamental societal problems, I hope to create and kickstart a research community focused on algorithmic and mechanism design problems where the objective is social good. To this end, while membership to the reading group is limited due to logistical constraints, I am maintaining a website [1] where I am posting our reading list, as well as notes generated from our discussion. I am excited by how well the idea has been received by the experts we have asked for suggestions and the various researchers who are following along using the website.

References

[1] Mechanism design for social good reading group 2016-17. `http://homes.cs.washington.edu/~kgoldner/MD4SG.html`.

[2] Moshe Babaioff, Nicole Immorlica, Brendan Lucier, and S. Matthew Weinberg. A simple and approximately optimal mechanism for an additive buyer. In *Proceedings of the 55th Annual Symposium on Foundations of Computer Science*, FOCS '14, 2014.

[3] Mark Braverman, Jing Chen, and Sampath Kannan. Optimal provision-after-wait in healthcare. *Mathematics of Operations Research*, 41(1):352–376, 2016.

[4] Jeremy Bulow and Paul Klemperer. Auctions vs. negotiations. Technical report, National Bureau of Economic Research, 1994.

[5] Yang Cai, Nikhil R. Devanur, and S. Matthew Weinberg. A duality based unified approach to bayesian mechanism design. In *Proceedings of the forty-eighth annual ACM symposium on Theory of computing*. ACM, 2016.

[6] Yang Cai and Mingfei Zhao. Simple mechanisms for subadditive buyers via duality. 2016.

[7] S Chawla, JD Hartline, DL Malec, and B Sivan. Multi-parameter mechanism design and sequential posted pricing. *Proceedings of the forty- . . .*, 2010.

[8] Shuchi Chawla and J. Benjamin Miller. Mechanism design for subadditive agents via an ex-ante relaxation. *CoRR*, abs/1603.03806, 2016.

[9] Raj Chetty, Nathaniel Hendren, and Lawrence F Katz. The effects of exposure to better neighborhoods on children: New evidence from the moving to opportunity experiment. *The American Economic Review*, 106(4):855–902, 2016.

[10] Constantinos Daskalakis, Alan Deckelbaum, and Christos Tzamos. Mechanism design via optimal transport. In *Proceedings of the 14th ACM Conference on Electronic Commerce*, EC '13, pages 269–286, 2013.

[11] Constantinos Daskalakis, Alan Deckelbaum, and Christos Tzamos. Strong duality for a multiple-good monopolist. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation*, pages 449–450. ACM, 2015.

[12] Matthew Desmond. Unaffordable america: Poverty, housing, and eviction. *Fast Focus, Institute for Research on Poverty*, 22, 2015.

[13] Nikhil Devanur, Nima Haghpanah, and Christos Alexandros Psomas. Optimal multi-unit pricing is deterministic under price regularity. 2016.

[14] Peerapong Dhangwatnotai, Tim Roughgarden, and Qiqi Yan. Revenue maximization with a single sample. In *Proceedings of the 11th ACM Conference on Electronic Commerce*, EC '10, pages 129–138, New York, NY, USA, 2010. ACM.

[15] Cynthia Dwork. Theory for society. Women in Theory, 2016.

[16] Amos Fiat, Kira Goldner, Anna R. Karlin, and Elias Koutsoupias. The fedex problem. In *Proceedings of the 2016 ACM Conference on Economics and Computation*, pages 21–22. ACM, 2016.

[17] Yiannis Giannakopoulos and Elias Koutsoupias. Duality and optimality of auctions for uniform distributions. In *Proceedings of the 15th ACM Conference on Economics and Computation*, EC '14, pages 259–276, 2014.

[18] Kira Goldner and Anna R. Karlin. A prior-independent revenue-maximizing auction for multiple additive bidders. In *International Workshop on Internet and Network Economics*, page to appear. Springer, 2016.

[19] Nima Haghpanah and Jason Hartline. Reverse mechanism design. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation*, pages 757–758. ACM, 2015. Updated version: http://arxiv.org/abs/1404.1341.

[20] Robert Kleinberg and S. Matthew Weinberg. Matroid prophet inequalities. In *Proceedings of the forty-fourth annual ACM symposium on Theory of computing*, pages 123–136. ACM, 2012.

[21] Jean-Jacques Laffont, Eric Maskin, and Jean-Charles Rochet. Optimal nonlinear pricing with two-dimensional characteristics. *Information, Incentives and Economic Mechanisms*, pages 256–266, 1987.

[22] Eric Maskin and John Riley. Optimal auctions with risk averse buyers. *Econometrica: Journal of the Econometric Society*, pages 1473–1518, 1984.

[23] R. Preston McAfee and John McMillan. Multidimensional incentive compatibility and mechanism design. *Journal of Economic Theory*, 46(2):335 – 354, 1988.

[24] Aviad Rubinstein and S Matthew Weinberg. Simple mechanisms for a subadditive buyer and applications to revenue monotonicity. In *Proceedings of the Sixteenth ACM Conference on Economics and Computation*, pages 377–394. ACM, 2015.

[25] Andrew Chi-Chih Yao. An n-to-1 bidder reduction for multi-item auctions and its applications. In *Proceedings of the 26th Annual ACM-SIAM Symposium on Discrete Algorithms*, pages 92–109, 2015.