

Greedy Exchange II: Scheduling to Minimize Lateness

In the problem of scheduling to minimize lateness, we have n scheduling requests. Request i has a deadline d_i and requires time t_i to process the job. We'll assign start time s_i and finish time f_i to job i . Let lateness $\ell_i := f_i - d_i$. The goal is to minimize the maximum lateness $\max_i \ell_i$.

Ideas for Greedy Metrics: (and counterexamples)

Our algorithm:

Lemma 1. *There is an optimal schedule with no idle time.*

Proof of correctness. By Greedy Exchange.

Step 1: Label your algorithm's solution ($A = \{a_1, a_2, \dots, a_n\}$) and a general solution ($O = \{o_1, o_2, \dots, o_n\}$).

Step 2: Compare greedy with the other solution. Assume they're not the same and isolate some difference.

Step 3: Exchange. Swap the elements in O without making the solution worse. Argue that swapping a finite number of times will result in A .

Hence, greedy is just as good as *any* optimal or arbitrary solution. □

Runtime: