Mechanism Design for a Complex World: Rethinking Standard Assumptions

Overview. This thesis addresses barriers to progress in three fundamental directions in mechanism design by rethinking standard models and assumptions and provides positive results in all three cases. First, we design revenue-optimal mechanisms in "interdimensional" settings—highly structured correlated settings that sit in between the assumed dichotomy of single-dimensional and multi-dimensional settings. Second, we propose a new model of proportional complementarities and construct an intuitive, simple mechanism that guarantees near-optimal revenue. Third, we study welfare maximization in the interdependent values setting without the single-crossing condition, and guarantee strong approximations for the most general setting of combinatorial auctions.

Interdimensional Mechanism Design. The problem of designing *optimal* (or revenue-maximizing) auctions has been the subject of intense study for decades. The problem is completely solved when the auctioneer has a single item to sell [11]; however, even in the case where a seller has only two items to sell, we fail to fully understand optimal auctions. The optimal auction in this setting can be incredibly complex—highly randomized and intractable to compute—and thus characterizations remain elusive.

Our work introduces a new subclass of natural and ubiquitous multi-dimensional settings that we call *interdimensional* settings. The complexity of maximizing revenue in these settings lies strictly between the "easy" case of "single-parameter" settings (where the buyer's private information is characterized by a single number) and the fully general "multi-dimensional" setting.

Our main result is a complete characterization of the optimal solution in the interdimensional setting we call "The Fedex Problem," where a buyer has a value and a deadline, and a shipping service is worth his value to him only if his package is received on or before his deadline. This is one of the first multi-dimensional settings where a closed-form solution is obtained without any restriction on the form of the mechanism and without any assumption about the prior distributions from which buyer values are drawn. Our approach is via duality inspired by [8]. We also present multi-dimensional analogues of Myerson's theory: revenue curves, ironing (needed even in the single-bidder case!), and virtual values. This work originally appeared in EC 2016 [7].

We then characterize the optimal mechanism for selling to a single-minded buyer: a set of items is worth his value only if it contains his desired bundle. While this is only a slight generalization of the FedEx setting, we find that the *menu complexity*, or the number of distinct options offered to the buyer, jumps from exponential in the FedEx setting (specifically, 2^{m-1} for m services) to *unbounded but finite* (even for $m = 3$ services) in the single-minded setting. This sharply separates the single-minded setting from both the FedEx setting and the "multi-dimensional" two-item additive setting, which is known to have uncountably infinite menu complexity [2]. This work is in submission [3]; both papers were presented as part of an EC 2018 tutorial on Menu Complexity [9].

Mechanism Design with Complements. Items have *complements* when a buyer derives some additional value from receiving a set of items beyond just the sum of his values for the individual items. For example, a pairs of shoes is worth much more than the sum of values for a lone left shoe and a lone right shoe.

Positive results for maximizing revenue in settings with complements have been few and far between, and either preclude decent approximation factors or rely on the often unrealistic assumption of independence between valuations for every different bundles of items.

We introduce a new way to model complementarities that replaces the independence assumptions of previous models with known proportionalities observed from data. Our model is motivated by the Microsoft Pricing Engine, a product that was developed to take in a company's sales history dataset and output a suggestion for what prices to sell that company's products at, whether to sell them individually or bundle them together, and how to capitalize on any complementarities that may exist among the items. For example, if it is known that users derive some extra value from using Microsoft Word with Excel, say, by creating charts in Excel and dragging them into Word, it may increase the seller's revenue to sell Word and Excel together as a bundle.

Our main result is a new, simple, and intuitive mechanism that capitalizes on these proportionalities. The mechanism first gives a number of items away for free, which the buyer will, of course, happily take. Then, capitalizing on the fact that there are complementarities among these free items and the items remaining for sale, the latter (which are now more valuable to the buyer) can be sold separately at inflated prices. We see this mechanism often in practice, such as Android giving the operating system away for free to sell ads across it, or Microsoft giving OneNote away for free with compatibility to Office. Using the better of this mechanism and selling the grand bundle, we are able to give approximations to revenue that are linear in the smaller of (1) the maximum-degree and (2) the largest-hyperedge for a hypergraphic model of complementarities. This contrasts sharply with prior work [4] which gives an approximation that is linear in the maximum-degree but exponential in the largest-hyperedge, and also requires a strong independence assumption. This work originally appeared at EC 2019 [1] and is under revision at Operations Research.

Mechanism Design with Interdependent Valuations. In the standard model of *independent private values*, a buyer's value for an item is unaffected by other buyers' values for the item. This is often unrealistic. Buyers may not know their value, may only have partial information regarding an item, and may be impacted by the resale value of an item. For example, if two firms are bidding for oil drilling rights, and one firm learns the amount of oil available, that information directly impacts the value that the other firm has for the drilling rights. The *interdependent values model* [10] captures the idea that a buyer's information and value may impact the values of other buyers.

Unfortunately, for interdependent valuations, there are strong impossibility results that preclude the existence of mechanisms for attaining optimal social welfare beyond very restricted settings (such as single-item auctions when buyer valuations satisfy the "single-crossing" condition).

Our goal is thus to *approximate* social welfare—but without the single-crossing condition, without dependence on buyers' prior distributions, and in more complex settings such as those with multi-dimensional signals or multi-item environments. In an EC 2018 paper [5], Eden, Feldman, Fiat, and Goldner show that without *any* condition on the valuations, no approximation guarantee is possible; the question then is: *what is the right condition?*

In follow-up work, we introduce a new assumption on interdependent valuations that we call "submodularity over signals." This assumption is natural and benign; in particular, it is satisfied by the typical examples of interdependent valuations in the literature.

Submodularity allows us to use a simple random-sampling idea to help unentangle some of the incentive problems. We split the buyers randomly into "potential winners" and "definite losers" and then use proxy valuations for the potential winners that depend only on their own signals and those of the definite losers. Then, we maximize welfare with respect to the proxy valuations; the key observation is that submodularity ensures that our proxy valuations are close enough in expectation to the true values such that they give a good approximation for welfare maximization.

This technique, combined with a new way to set payments, allows us to move beyond the realm of single-parameter settings, and to obtain strong approximately-optimal welfare guarantees in more general interdependent settings such as unit-demand settings and combinatorial auctions—the first positive results beyond effectively single-dimensional environments. Our approach also improves the approximation factor in single-dimensional settings without single-crossing from $O(\sqrt{n}c^{3/2})$ under c-single-crossing to 4 under submodularity. This work originally appeared at EC 2019 [6] and received the award for Best Paper with a Student Lead Author.

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