DS 574 Algorithmic Mechanism Design	**Lecture #1 Worksheet**
Fall 2025	Prof. Kira Goldner

Mechanism Design Basics

A mechanism is defined by an *allocation rule* that takes as input the bids (or reports, or actions) of the bidders (or players, agents, buyers) and determines an outcome. It is often accompanied by a payment rule. First, we'll focus on the bidders, and why they are choosing the actions they are choosing.

Definition 1. Each bidder i has a private *valuation* v_i that is its maximum willingness-to-pay for the item being sold.

Our default assumption is that a bidder's utility is modeled by "quasilinear utility."

Definition 2. For a deterministic mechanism with at most one winner, a bidder with *quasilinear utility* has utility

$$u_i(\cdot) = \begin{cases} v_i - p_i & \text{if } i \text{ wins and pays } p_i \\ 0 & \text{otherwise.} \end{cases}$$

Definition 3. A *dominant strategy* is a strategy (bid) that is guaranteed to maximize a bidder's utility *no matter what* the other bidders do.

Sealed-Bid Auctions:

(1) Each bidder i privately communicates a bid b_i to the auctioneer—in a sealed envelope, if you like.

(2) The auctioneer decides who gets the item (if anyone).

(3) The auctioneer decides on a selling price.

How should we do (2) and (3)?

What we'll do for (2):

What about (3)? Some potential auctions:

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How should we bid in these auctions?

Claim 1 (Dominant-Strategy Incentive Compatibility)**.** In a second-price auction, every bidder has a *dominant strategy*: set its bid b_i equal to its private valuation v_i. That is, this strategy maximizes the utility of bidder i, no matter what the other bidders do.

Proof. [*Hint: Consider two cases of outcomes.*]

Claim 2 (Individual Rationality)**.** In a second-price auction, every truth-telling bidder is guaranteed non-negative utility.

Proof.

Theorem 1 (Vickrey)**.** *The Vickrey (second-price) auction satisfies the following three quite different and desirable properties:*

(1) ***[strong incentive guarantees]*** *It is dominant-strategy incentive-compatible (DSIC) and individually rational (IR), i.e., Claims 1 and 2 hold.*

(2) ***[strong performance guarantees]*** *If bidders report truthfully, then the auction maximizes the social surplus*

$$\sum_{i=1}^{n} v_i x_i,$$

where x_i is 1 if i wins and 0 if i loses, subject to the obvious feasibility constraint that $\sum_{i=1}^{n} x_i \leq 1$ (i.e., there is only one item).

(3) ***[computational efficiency]*** *The auction can be implemented in polynomial time.*

In general, as we design mechanisms, we'll take the following design approach:

Step 1: Assume, without justification, that bidders bid truthfully. Then, how should we assign bidders to slots so that properties (2) strong performance guarantees and (3) computational efficiency hold?

Step 2: Given our answer to Step 1, how should we set selling prices so that property (1) strong incentive guarantees holds?

References

[1] Roger B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73, 1981.