

## Kidney Exchange

- There are more than 92,000 on the waitlist for a kidney transplant in the US; this makes up 87% of the organ transplant list [1].
- Healthy people have two kidneys and can survive fine with only one.
- A donor and recipient must be “compatible” (blood and tissue types).
- Two incompatible patient-donor pairs can agree to a kidney exchange. This is legal. (Compensation for kidneys is not, except in Iran.)

**Question:** How would one design a centralized mechanism for kidney exchange, where incompatible patient-donor pairs can register and be matched with others?

## Idea #1: Use the Top Trading Cycle Algorithm

### Vanilla Top Trading Cycles

Consider the housing allocation problem defined by Shapley and Scarf [5]: There are  $n$  agents, and each initially owns one house. Each agent has a total ordering over the  $n$  houses, and need not prefer their own over the others. How can we reallocate the houses to make the agents better off?

**The Top Trading Cycle Algorithm [Gale [5]].**

While agents remain:

- Each remaining agent points to its favorite remaining house. This induces a directed graph  $G$  on the remaining agents in which every vertex has out-degree 1 (Figure 1).
- The graph  $G$  has at least one directed cycle. Self-loops count as directed cycles.
- Reallocate as suggested by the directed cycles, with each agent on a directed cycle  $C$  giving its house to the agent that points to it, that is, to its predecessor on  $C$ .
- Delete the agents and the houses that were reallocated in the previous step.

Observations:

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**Theorem 1.** *The TTCA induces a DSIC mechanism.*

[*Hint:* Divide agents into those allocated to in the  $j$ th iteration.]

*Proof.*

**Definition 1.** A *core allocation* is an allocation such that no coalition of agents can make all of its members better off via internal reallocations.

**Theorem 2.** *For every house allocation problem, the allocation computed by the TTCA is the unique core allocation.*

## Modifications for Kidney Exchange

The first attempt was via the TTCA by Roth, Sönmez, and Ünver [3] before the authors talked extensively to doctors. People’s “preferences” over kidneys would just be via decreasing probability of success of the transplant.

But kidney exchange is more complicated:

- (1) There are patients without living donors, and deceased donors.
- (2) The cycles along which reallocations are made can be arbitrarily long.
- (3) Modeling preferences as a total ordering over the set of living donors is overkill: empirically, patients don't really care which kidney they get as long as it is compatible with them.

Instead:

## Idea #2: Use a Matching Algorithm

(2) Short reallocation cycles and (3) binary preferences motivate looking for *matchings*, as done in [4].

What's the relevant graph for kidney exchange? Describe the vertices, edges, and what a matching would look like.

How do incentives work here? What should the mechanism look like?

- (1)
- (2)
- (3)

But how do we tie-break between maximum-cardinality matchings?

**Theorem 3.** *For every collection  $\{E_i\}_{i=1}^n$  of edge sets and every ordering of the vertices, the priority matching mechanism above is DSIC: no agent can go from unmatched to matched by reporting a strict subset  $F_i$  of  $E_i$  rather than  $E_i$ .*

# Hospital Incentives

Current research is focused on incentive problems at the *hospital* level, rather than at the level of individual patient-donor pairs. Hospitals are the ones who actually report the pairs to the national kidney exchange, but the objectives of a hospital (to match as many of its patients as possible) and of society (to match as many patients overall as possible) are not perfectly aligned.

**The Need for Full Reporting.** Only reporting pairs who the hospital can't match internally can result in fewer exchanges.

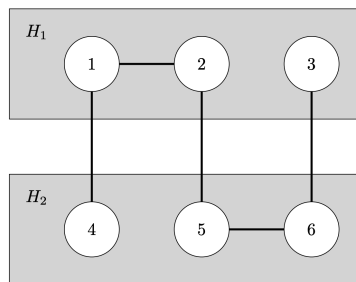


Figure 1: Full reporting by hospitals leads to more matches than with only internal matches.

**Hiding patients.** If  $H_1$  hides patients 2 and 3 from the exchange (while  $H_2$  reports truthfully), then  $H_1$  guarantees that all of its patients are matched. The unique maximum matching in the report graph matches patient 6 with 7 (and 4 with 5), and  $H_1$  can match 2 and 3 internally. On the other hand, if  $H_2$  hides patients 5 and 6 while  $H_1$  reports truthfully, then all of  $H_2$ 's patients are matched. In this case, the unique maximum matching in the graph of report matches patient 1 with 2 and 4 with 3, while  $H_2$  can match patients 5 and 6 internally.

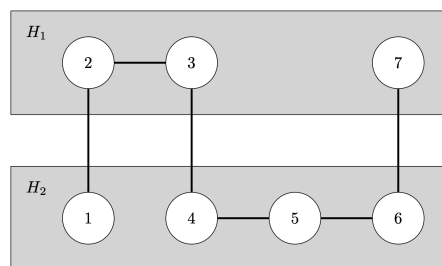


Figure 2: Hospitals can have an incentive to hide patient-donor pairs.

It turns out there cannot be a DSIC mechanism that always computes a maximum-cardinality matching in the full graph.

In light of this example, the revised goal should be to compute an approximately maximum-cardinality matching so that, for each participating hospital, the number of its patients that get matched is approximately as large as in any matching, maximum-cardinality or otherwise. Understanding the extent to which this is possible, in both theory and practice, is an active research topic [2, 6].

## References

- [1] American Kidney Fund, Jun 2022.
- [2] Itai Ashlagi, Felix Fischer, Ian A Kash, and Ariel D Procaccia. Mix and match: A strategyproof mechanism for multi-hospital kidney exchange. *Games and Economic Behavior*, 91:284–296, 2015.
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- [5] Lloyd Shapley and Herbert Scarf. On cores and indivisibility. *Journal of mathematical economics*, 1(1):23–37, 1974.
- [6] Panagiotis Toulis and David C Parkes. A random graph model of kidney exchanges: efficiency, individual-rationality and incentives. In *Proceedings of the 12th ACM conference on Electronic commerce*, pages 323–332, 2011.