DS 574 Algorithmic Mechanism Design	**Lecture #9 Worksheet**
Fall 2025	Prof. Kira Goldner

Time-Inconsistent Planning: Present Bias

First, some stories:

Suppose people have not drank in 4 hours, eat salty potato chips, and then are given the options: 1 sip now 2 sips in 5 minutes

What about, choose now: 1 sip in 20 minutes 2 sips in 25 minutes

Participants were given the choice of their snack in a week: fruit chocolate

Then, after the week, they were allowed to choose their snack: fruit chocolate

Let's model exercising (-6) and its benefits (+8 in the future):

- today
- tomorrow

Nobel Laureate George Akerlof who needed to mail a package to his friend, Joseph Stiglitz.

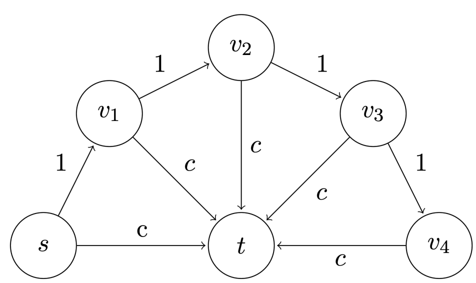

Figure 1: The fan graph for Akerlof's story.

Formally:

- Sending the package has a fixed cost c.
- There is a loss of use cost 1 for each day in which the package cannot be used.
- Total cost for sending on day t is: $c + t$.

The rational behavior is to send the package on the first day to minimize the total cost. *Present bias* [Akerlof] indicates that you perceive the cost of doing something today as inflated by some bias factor b. Thus:

More generally, we define the model as follows:

1. There is a directed acyclic graph G with a source s and a target t.
2. Each edge e corresponds to some task and has a cost which captures the effort required for completing the task.
3. The agent needs to take a path from s to t. At each node v it will choose the $v - t$ path which is the shortest path in a graph in which the costs of all outgoing edges from v are multiplied by a factor of b.

This simple model is based on more elaborate model (quasi-hyperbolic discounting). Formally:

Definition 1 (traversal)**.** An agent currently at v_i will continue to a node $v_{i+1} \in \arg\min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$. We refer to $C(v_i) = \min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$ as the perceived cost of agent i at v_i.

Let's see another example:

Question: Consider an agent with present bias $b = 2$. Which path will he traverse in the graph in Figure 2?

Choice Reduction and Its Benefits

Experiment in a course at MIT: Students need to submit 3 assignments throughout the semester. In the beginning of the semester, each student was asked to set a deadline for each assignment. What is the rational behavior?

What would you do?

In the experiment:

What does this tell us?

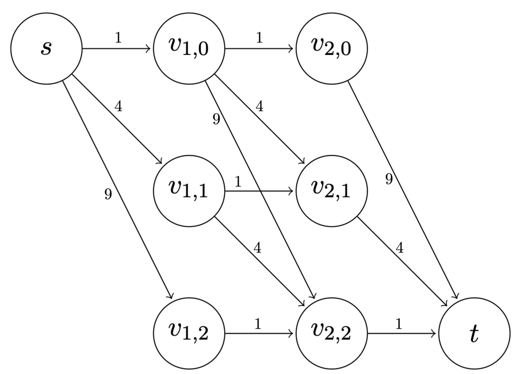

Figure 2: An example featuring the benefits of setting deadlines. Horizontally: weeks. Vertically: tasks.

Example: 3 week course, 2 task. The cost of completing a single task in a week is 4. The cost for completing both in the same week is 9. The cost of a week of studying without doing any tasks is 1. The task graph in Figure 2 models this scenario. In the graph, node $v_{i,j}$ corresponds to completing j tasks by week i.

Now, assume that there is a reward $R = 17$ for completing the course (reaching t) and the agent will traverse the graph as long as its perceived cost is less than R. How will an agent with present bias $b = 2$ traverse the graph?

How can we help the student complete the course? Consider setting a deadline for the first assignment: the first assignment should be submitted by the second week. This means that in graph we delete the node $v_{2,0}$. What will the agent do now?

This leads to the following algorithmic question: given a graph in which the agent does not reach t can we delete nodes and edges such that agent will reach t?

One way for approaching this question is hoping that if there is a traversable subgraph then there is always a traversable subgraph which is just a path. Is this true?

Research Directions:

- Cost ratio: quantifying how much present-biased agents lose due to their bias.
- Characterizing graph structures that lend themselves to bounded or exponential cost ratios.
- Sophisticated agents aware of their present bias.

Obviously Strategy Proof

We need a few more standard game-theoretic definitions before we can understand this concept.

Definition 2. A k-player finite *extensive-form* game is defined by a finite, rooted tree T. Each node in T represents a possible state in the game, with leaves representing terminal states. Each internal (nonleaf) node v in T is associated with one of the players, indicating that it is his turn to play if/when v is reached. The edges from an internal node to its children are labeled with actions, the possible moves the corresponding player can choose from when the game reaches that state. Each leaf/terminal state results in a certain payoff for each player. A pure strategy for a player in an extensive-form game specifies an action to be taken at each of that player's nodes. A mixed strategy is a probability distribution over pure strategies.

Definition 3. Given an extensive-form game, the *normal form* of the game is the matrix of possible pure strategies and their resulting payoffs.

Sealed-bid second-price auction and ascending English auction have the same normal form, but not the same extensive form. In practice, people play them quite differently.

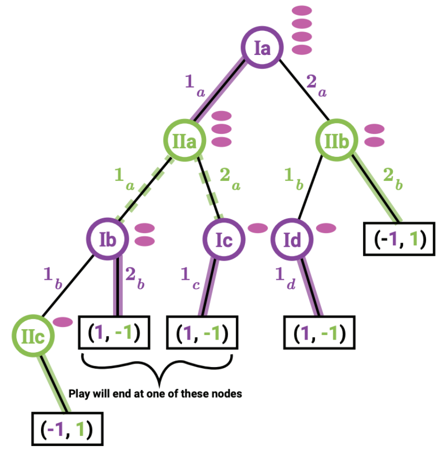

Figure 3: The Subtraction Game: Starting with a pile of four chips, two players alternate taking one or two chips. Player I goes first. The player who removes the last chip wins.

Earliest Point of Departure: Nodes I_i are in the *information set* $\alpha(S_i^1, S_i^2)$ if and only if

1. $S_i^1 \neq S_i^2$ at I_i and
2. I_i could have been reached by playing either S_i^1 or S_i^2.

Let $u_i^G(h, S_i, S_{-i}, v_i)$ be the utility to agent i in game G as a function of starting from history h with play proceeding according to S_i, S_{-i} and the resulting outcome evaluated according to preferences v_i.

Definition 4. A strategy S_i is *weakly dominant* if for all deviating strategies S_i' and other bidder strategies S_{-i}, $u_i^G(h_0, S_i, S_{-i}, v_i) \geq u_i^G(h_0, S_i', S_{-i}, v_i)$

Definition 5. A strategy S_i is *obviously dominant* if for all deviating strategies S_i' and nodes in the earliest point of departure $I_i \in \alpha(S_i, S_i')$: $\inf_{h \in I_i, S_{-i}} u_i^G(h, S_i, S_{-i}, v_i) \geq \sup_{h \in I_i, S_{-i}} u_i^G(h, S_i', S_{-i}, v_i)$

Definition 6. A mechanism is *obviously strategyproof* if truth-telling is an obviously dominant strategy.

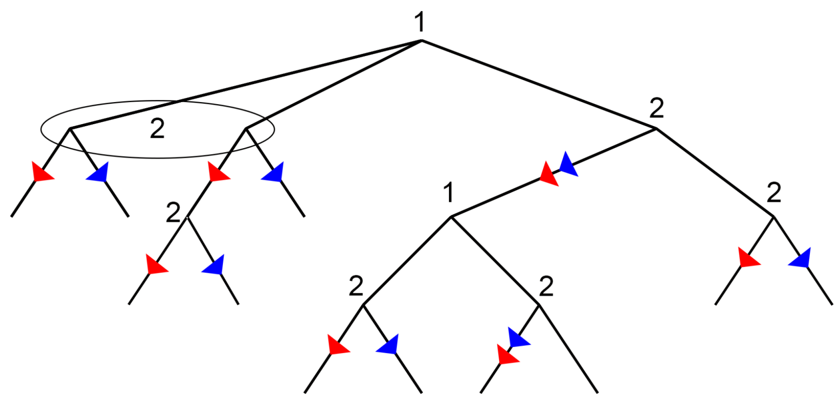

Figure 4

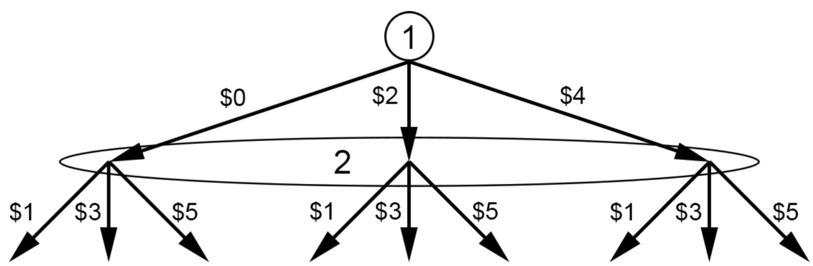

Figure 5

Acknowledgements

This lecture was developed by using materials from Shengwu Li's 2019 tutorial on Behavioral Economics and Imperfect Rationality at ACM EC, Sigal Oren's lecture notes, and David Labison's tutorial at BEACH Day April 2025.

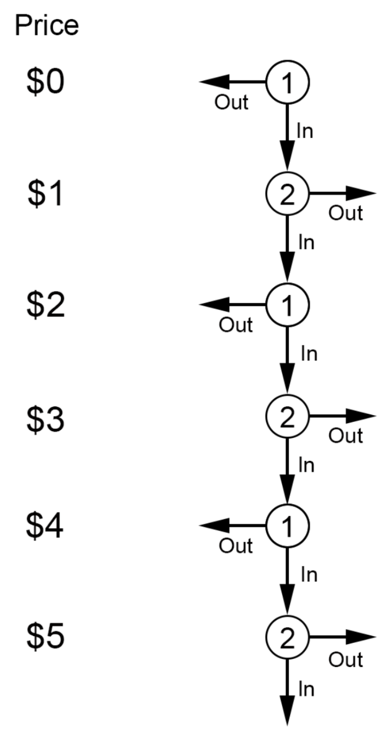

Figure 6