DS 574 Algorithmic Mechanism Design	**Lecture #7 Worksheet**
Fall 2025	Prof. Kira Goldner

Interdependent Values II

Given Myerson's Lemma in the interdependent setting, what allocation will maximize social welfare?

Payments. What are the payments?

Truthfulness. Is this mechanism EPIC? When might it not be?

Assumptions. What assumption could we place on the class of valuations to ensure that the mechanism is always EPIC?

Beyond Single-Crossing [1]

What happens when we don't have single-crossing? Can we at least guarantee some approximation to social welfare?

Example. [Impossibility for deterministic prior-free mechanisms without SC.]

Example. [Impossibility result for randomized mechanisms without SC.]

A Restricted Class. Optimal welfare is not attainable for general valuations. For what *natural* restricted class of valuations can we achieve some α-approximation to optimal social welfare for every profile of signals $\mathbf{s}$ (prior-free) with an EPIC mechanism?

Submodularity over Signals [2]

Definition 1. Valuation $v_i(\cdot)$ is *submodular over signals* if, for all j, when $\mathbf{s}_{-j}$ is lower, $v_i(\cdot)$ is more sensitive to s_j. For all j, and for any $\mathbf{s}_{-j} \leq \mathbf{s}'_{-j}$:

$$\frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}_{-j}) \geq \frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}'_{-j})$$

Random-Sampling Vickrey Auction.

- Elicit s_i from each bidder i.
- Assign each bidder into set A or set B w.p. 1/2 independently.
- For each bidder $i \in A$, calculate and use proxy value $\hat{v}_i = v_i(s_i, \mathbf{0}_{A \setminus i}, \mathbf{s}_B)$.
- Allocate to the potential winner in A with the highest proxy value.

Theorem 1. *The RS Vickrey Auction is EPIC and achieves a prior-free $\frac{1}{4}$-approximation to the optimal welfare.*

To prove this theorem, we need to address (1) truthfulness and (2) the approximation guarantee.

Truthfulness. Is this allocation EPIC?

Approximation. Is $v_i(s_i, \mathbf{0}_{A\setminus i}, \mathbf{s}_B)$ a good way to choose a winner?

Lemma 1 (Key Lemma)**.** *Let v_i be a submodular over signals valuation. Partition all agents other than i uniformly at random into sets A and B. Then*

$$\mathbb{E}_{A,B}[v_i(s_i, \mathbf{0}_A, \mathbf{s}_B)] \geq \frac{1}{2}v_i(\mathbf{s}).$$

Proof.

Proof of Theorem 1.

References

[1] Alon Eden, Michal Feldman, Amos Fiat, and Kira Goldner. Interdependent values without single-crossing. In *Proceedings of the 2018 ACM Conference on Economics and Computation*, EC '18, pages 369–369, New York, NY, USA, 2018. ACM.

[2] Alon Eden, Michal Feldman, Amos Fiat, Kira Goldner, and Anna R. Karlin. Combinatorial auctions with interdependent valuations: Sos to the rescue. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, EC '19, Phoenix, AZ, USA, 2019. ACM.