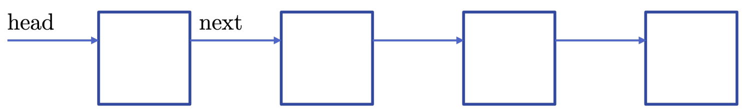
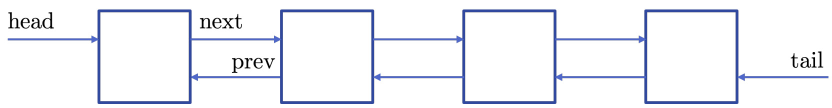
DS 320 Algorithms for Data Science	**Lecture #3 Worksheet**
Spring 2023	Prof. Kira Goldner

Abstract Data Types and Depth-First Search

Linked Lists

Consider a list $L = [x_1, x_2, \ldots, x_n]$ where each x_i is an element in the list. We keep a pointer to the head (and the tail) of the list. Each element x_i has a pointer "next" (and "previous").



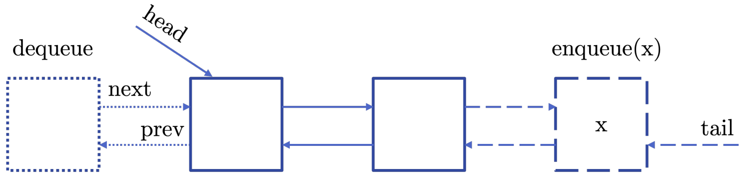


- What is the (worst-case) runtime to find an element?
- What is the (worst-case) runtime to insert or delete an element (once it's found)?

Queues

Queues are First-In, First-Out (FIFO) linked lists. They support the operations:

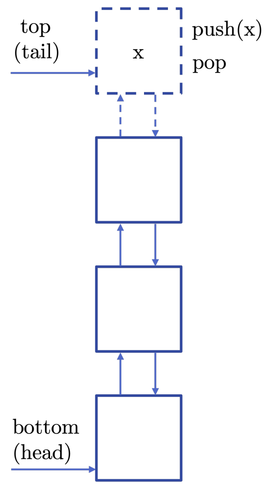
- enqueue(q, x): insert element x to the back of the queue q. Formally, $q = q \circ x$.
- dequeue(q): delete the element at the front of the queue q and return it. Formally, $q = [x_2, \ldots, x_n]$, return x_1.



Stacks

Stacks are what's known as Last-In, First-Out (LIFO) linked lists. They support the operations:

- push(s, x): insert element x to the top (back) of the stack s. Formally, $s = s \circ x$.
- pop(s): delete the element at the top (back) of the stack s and return it. Formally, $s = [x_1, \ldots, x_{n-1}]$, return x_n.



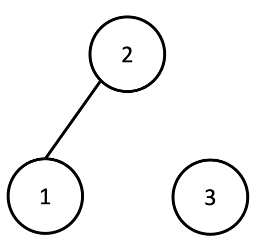
Graphs

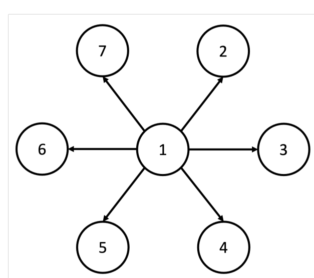
Definition 1. A (directed) *graph* $G = (V, E)$ is defined by a set of vertices V and a set of (ordered) edges $E \subseteq V \times V$.

Definition 2. A *directed edge* is an ordered pair of vertices (u, v) and is usually indicated by drawing a line between u and v, with an arrow pointing towards v.

Definition 3. An *undirected edge* is an unordered pair of vertices $\{u, v\}$ and is usually indicated by drawing a line between u and v. It indicates the existence of ordered edges (u, v) and (v, u).

Typically undirected edges will also be notated (u, v) out of sloppiness.



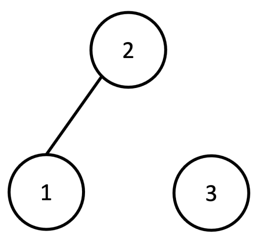


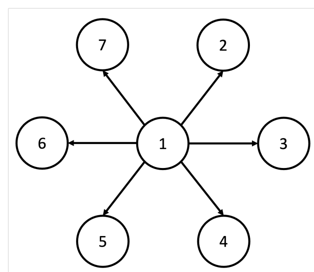
Some conventions:

- We will refer to the number of vertices (or the *size* of the vertex set $|V|$) as n.
- We will refer to the number of edges (or the *size* of the edge set $|E|$) as m.
- Often we will simply name the vertices $V = \{1, \ldots, n\}$ so an edge (i, j) is an edge from the i^{th} vertex to the j^{th} vertex.
- You may also hear vertices referred to as "nodes" or edges referred to as "arcs."

Definition 4. We call vertices i and j *adjacent* or *neighbors* if there is an edge $(i, j) \in E$. In directed graphs, we may explicitly refer to *out-neighbors* ($\{j : (i, j) \in E\}$) or *in-neighbors* ($\{j : (j, i) \in E\}$).

Definition 5. The *degree* of a vertex v is the number of neighbors it has. That is, $d_v = |\{u : (v, u) \in E\}|$. For directed graphs, we may refer to a vertex's *in-degree* or *out-degree*, and its *degree* is the sum of these.





Definition 6. A *path* from u to w is a sequence of edges $e_1, e_2, \ldots, e_k$ such that $e_1 = (u, v_1), e_i = (v_{i-1}, v_{i+1})$, and $e_k = (v_{i-1}, w)$. That is, the first edge starts at u, the last edge ends at w, and each proceeding edge ends where the previous edge starts.

Definition 7. We say that a pair of vertices are *connected* if there exists a path between them.

We see graphs all over; networks are an entire field of study! What can you represent with graphs?

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What graph problems do you know?

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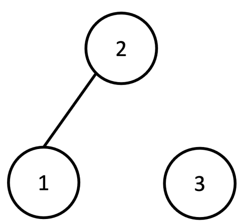
Abstract Data Types for Graphs

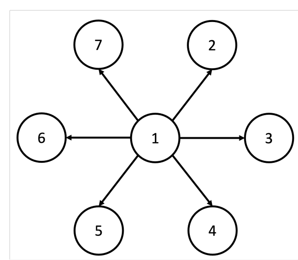
There are two primary ways that we represent graphs in the computer.

Definition 8. An *adjacency matrix* for $G = (V, E)$ is an $n \times n$ binary matrix A where $A_{ij} = 1$ if and only if $(i, j) \in E$.

Pros of using an adjacency matrix:

Cons of using an adjacency matrix:





Definition 9. An *adjacency list* for $G = (V, E)$ is an array A of length n where the i^{th} entry contains a linked list of i's neighbors. That is, j is in the list $A[i]$ if and only if $(i, j) \in E$.

Pros of using an adjacency list:

Cons of using an adjacency list:

Exercise: Ask yourself the following questions for both adjency matrices and adjency lists to fill out the pros and cons (above) for each graph ADT above:

- What is the worst-case runtime to look up a specific edge (i, j)?
- What is the worst-case space needed to store the graph?
- What is the runtime to list all edges adjacent to i? On average, per edge adjacent to i?

Depth-First Search

Graph search algorithms: good for exploring a graph, determining whether two nodes are connected, determining some properties regarding the ordered structure of a directed graph.

Depth-First Search: shoots as far away from a node possible to see if it results in a successful path, and only turns around if it dead ends.

Algorithm 1 search(v, G)

```
Input: Graph G = (V, E) and vertex v.
mark v as explored
for (v, w) ∈ E do
    if w is unexplored then
        search(w, G)
    end if
end for
```

Algorithm 2 DFS(s, G)

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Input: Graph G = (V, E) and vertex v.
for each v ∈ V do
    v is unexplored
end for
search(s, G)
```

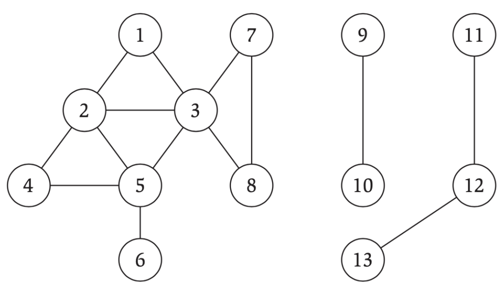

Figure 1: Example graph G. From Kleinberg Tardos.

Exercise.

1. Consider the pseudocode when called on the above example, that is, what happens when we run DFS(1, G) where G is the graph above? Draw the DFS tree as the graph is explored.

2. DFS implicitly uses a stack. Draw the stages of the stack as it runs on the example graph.