## Mechanism Design Basics

A mechanism is defined by an *allocation rule* that takes as input the bids (or reports, or actions) of the bidders (or players, agents, buyers) and determines an outcome. It is often accompanied by a payment rule. First, we'll focus on the bidders, and why they are choosing the actions they are choosing.

**Definition 1.** Each bidder i has a private valuation  $v_i$  that is its maximum willingness-to-pay for the item being sold.

Our default assumption is that a bidder's utility is modeled by "quasilinear utility."

**Definition 2.** For a deterministic mechanism with at most one winner, a bidder with *quasilinear* utility has utility

 $u_i(\cdot) = \begin{cases} v_i - p_i & \text{if } i \text{ wins and pays } p_i \\ 0 & \text{otherwise.} \end{cases}$ 

**Definition 3.** A dominant strategy is a strategy (bid) that is guaranteed to maximize a bidder's utility no matter what the other bidders do.

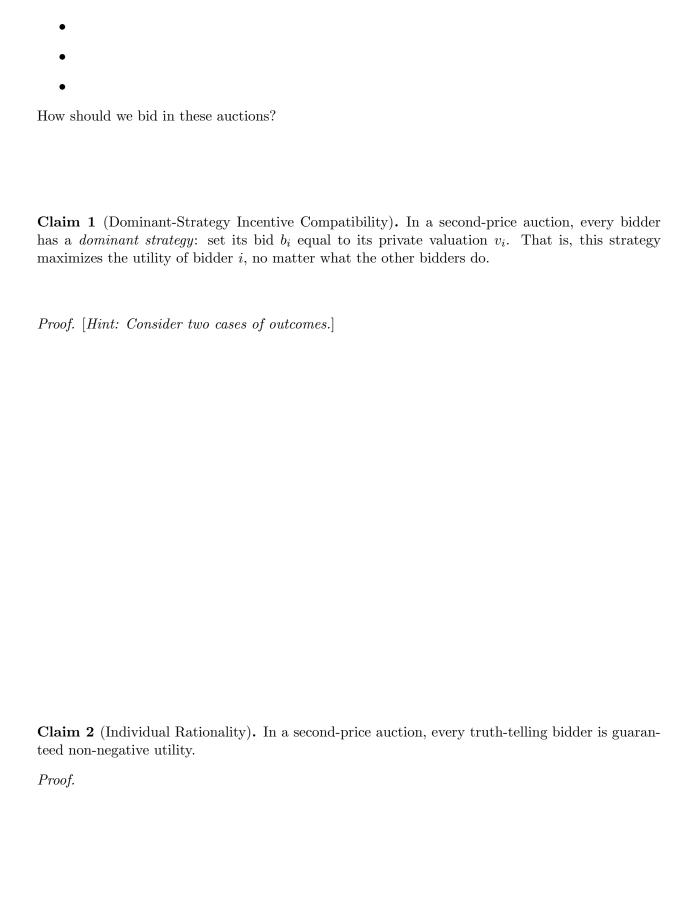
## **Sealed-Bid Auctions:**

- (1) Each bidder i privately communicates a bid  $b_i$  to the auctioneer—in a sealed envelope, if you like.
- (2) The auctioneer decides who gets the item (if anyone).
- (3) The auctioneer decides on a selling price.

How should we do (2) and (3)?

What we'll do for (2):

What about (3)? Some potential auctions:



**Theorem 1** (Vickrey). The Vickrey (second-price) auction satisfies the following three quite different and desirable properties:

- (1) [strong incentive guarantees] It is dominant-strategy incentive-compatible (DSIC) and individually rational (IR), i.e., Claims 1 and 2 hold.
- (2) [strong performance guarantees] If bidders report truthfully, then the auction maximizes the social surplus

$$\sum_{i=1}^{n} v_i x_i,$$

where  $x_i$  is 1 if i wins and 0 if i loses, subject to the obvious feasibility constraint that  $\sum_{i=1}^{n} x_i \leq 1$  (i.e., there is only one item).

(3) [computational efficiency] The auction can be implemented in polynomial time.

In general, as we design mechanisms, we'll take the following design approach:

- Step 1: Assume, without justification, that bidders bid truthfully. Then, how should we assign bidders to slots so that properties (2) strong performance guarantees and (3) computational efficiency hold?
- Step 2: Given our answer to Step 1, how should we set selling prices so that property (1) strong incentive guarantees holds?

## References

[1] Roger B. Myerson. Optimal auction design. *Mathematics of Operations Research*, 6(1):58–73, 1981.