Advancements and challenges in quantum computers and quantum networks

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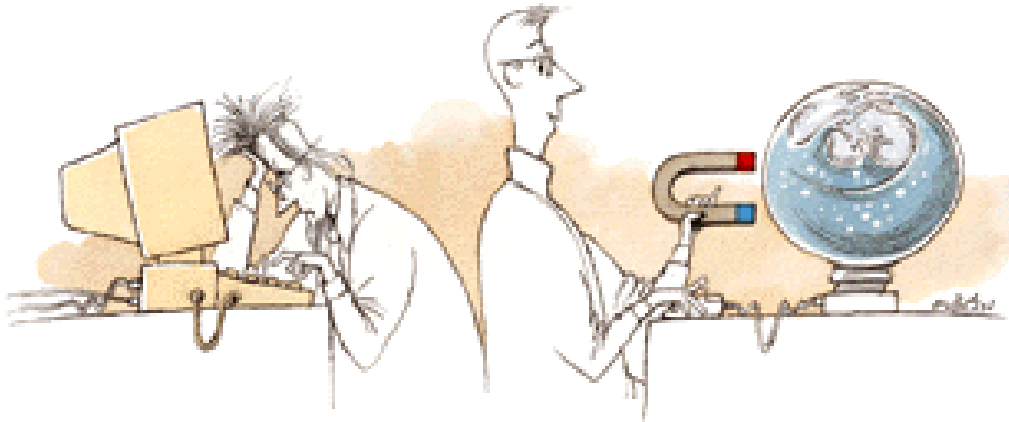
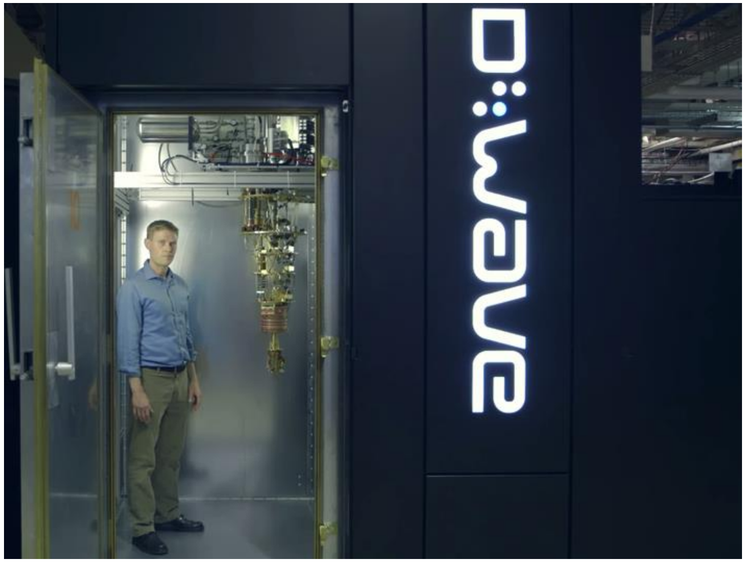
Sharif University, Dec 2016





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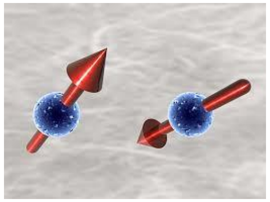
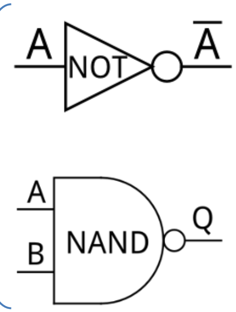


Outlook

- **Reviewing some basics about quantum mechanics**
  - Q-bits Vs. Classical bits
  - Superposition and entanglement
  - Hilbert space and wondering in this big space
- **Quantum gates and quantum processing and act of measurement**
- **A famous example of quantum algorithm : Shor's algorithm**
- **Some examples of real platforms for quantum computers**
  - The role of temperature as a fundamental challenge in quantum computers
  - Role of nano-mechanical oscillators as possible solutions to some challenge

What is a quantum bit?

Classical bit → $\begin{bmatrix}1\\0\end{bmatrix}$ →



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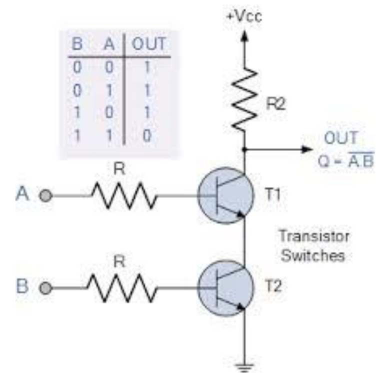
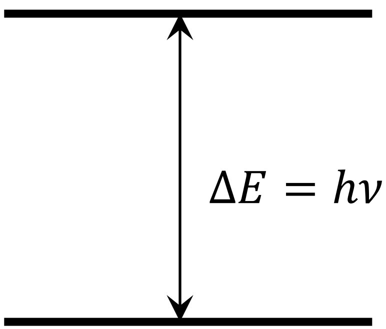


$\begin{bmatrix}5\\0\end{bmatrix}$ volt

Quantum bit (Qubit) → $\begin{bmatrix}|1\rangle\\|0\rangle\end{bmatrix}$ $\begin{bmatrix}|+\rangle\\|-\rangle\end{bmatrix}$ $\begin{bmatrix}|\uparrow\rangle\\|\downarrow\rangle\end{bmatrix}$

Exited State $|1\rangle, |+\rangle, |\uparrow\rangle$



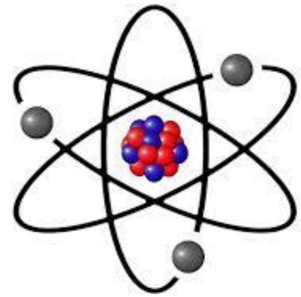
$\Delta E = h\nu$

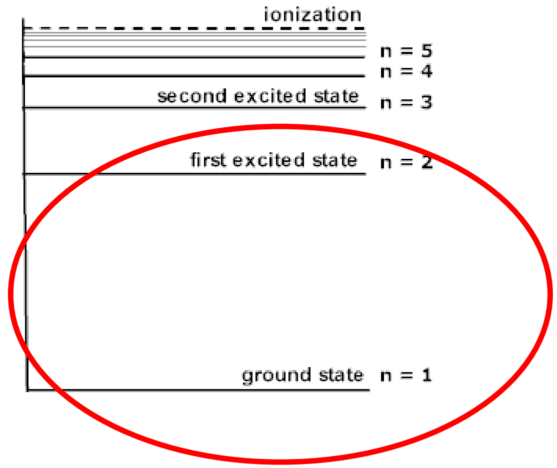
Ground State $|0\rangle, |-\rangle, |\downarrow\rangle$

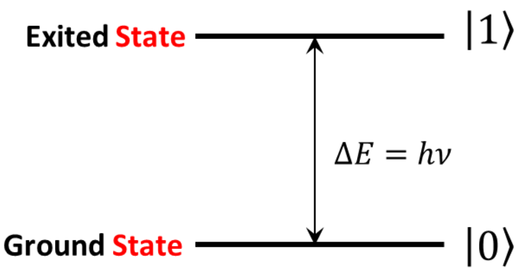
Two level system

What is a two level system?

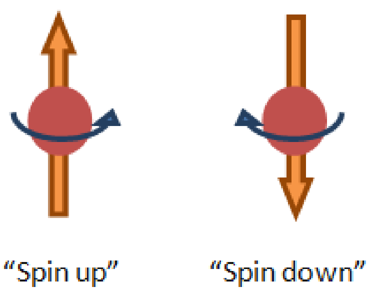
Example of atomic two level systems:

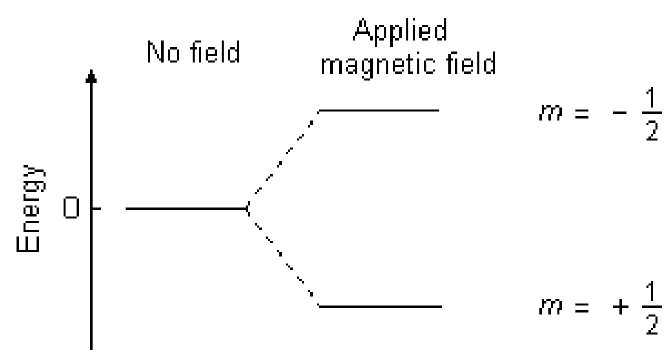






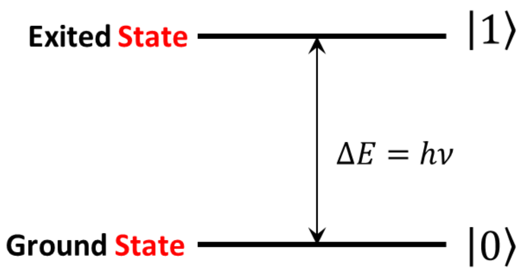
Example of magnetic (spin) two level systems:

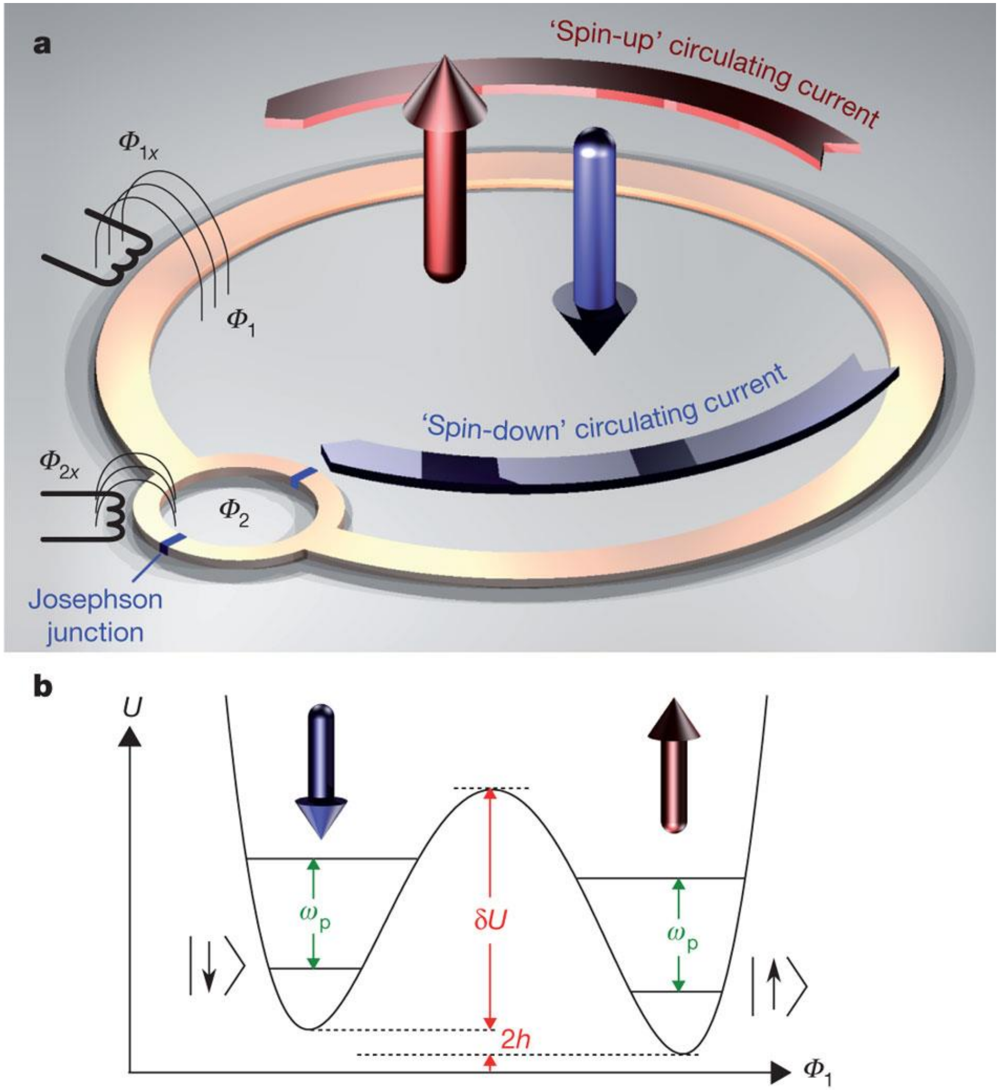




What is a two level system?

Example of Superconducting two level systems:



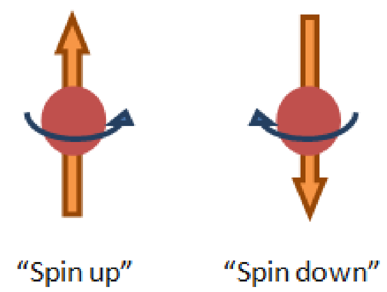


Difference between classical bits and Qubits

A classical bit has only and only two states (0,1)

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

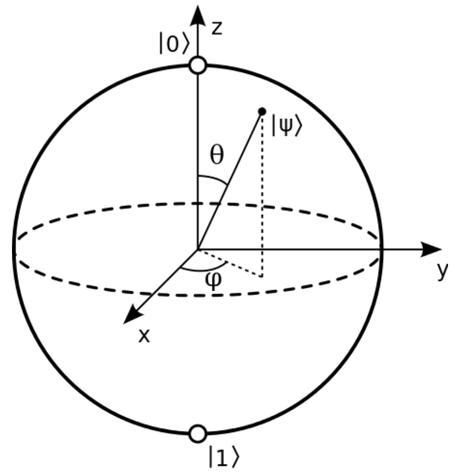
The problem with quantum bits is that the states can live in any possible combination of these two (like vector)



Super position state

$$|a\rangle = \alpha|1\rangle + \beta|0\rangle$$

(Just need to be normalized $\alpha^2 + \beta^2 = 1$)

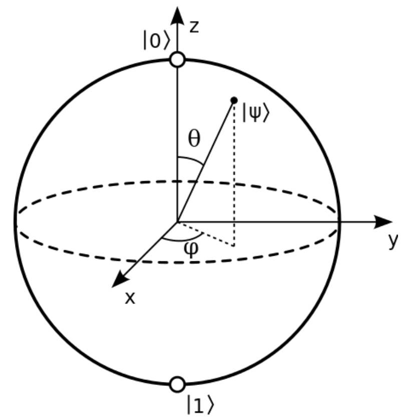


Sphere because α, β can be imaginary

Superposition state for single Qubits

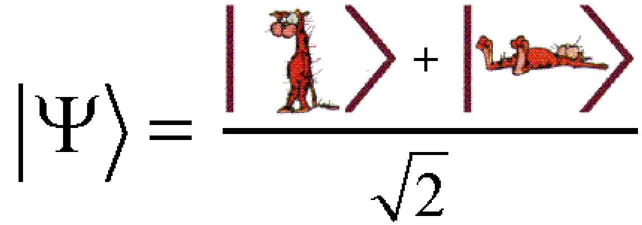
Single qubit in superposition state:

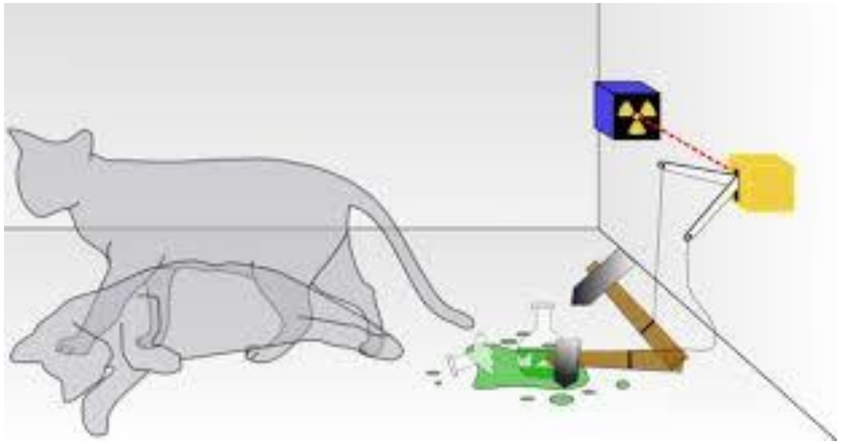
$$|a\rangle = \alpha|1\rangle + \beta|0\rangle$$



Example of a famous states - **Cat state**:

$$|a\rangle = \frac{|1\rangle + |0\rangle}{\sqrt{2}}$$

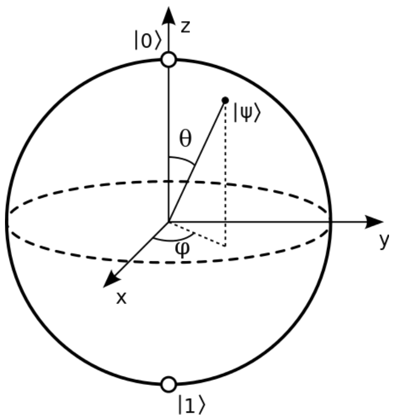




Problem of measurement in quantum mechanics

Single qubit in superposition state:

$$|a\rangle = \alpha|1\rangle + \beta|0\rangle$$

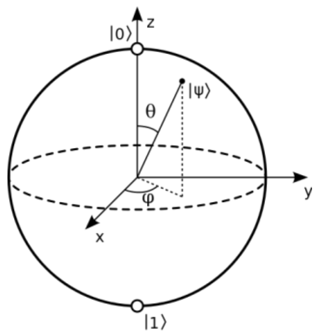


Interesting idea:
a qubit unlike a classical bit does not only have two possibilities but can have infinite states between $|0\rangle$, $|1\rangle$

Question:
What can we do with this which is interesting from the computational point of view

Answer:
Actually unfortunately noting!! and that is because of the problem of measurement in quantum mechanics

Problem of measurement in quantum mechanics



$$|a\rangle = \alpha|1\rangle + \beta|0\rangle$$

Measurement device

0 With probability $\boldsymbol{\beta}$

1 With probability $\boldsymbol{\alpha}$

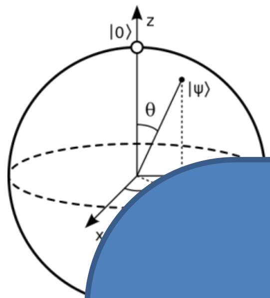
- Although a quantum state can have infinite possibilities, the measurement outcome is **always** is always either zero or one

- This is not because of the way of measurement or the measurement device but it is one of the fundamental laws of quantum mechanics

- The measurement outcome is random with the probability of α, β

- **The order of this randomness in much deeper than statistical randomness and is considered as absolute randomness**

Problem of measurement in quantum mechanics



0 With probability $\boldsymbol{\beta}$

$|a\rangle = a$

ability $\boldsymbol{\alpha}$

Single quantum bits (two level systems)

- Very interesting for us (physicists)
- Boring for you (computer scientists)

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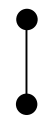
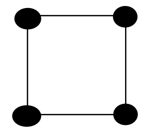
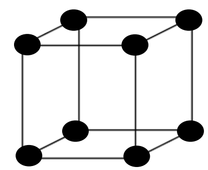
- **The order of this randomness in much deeper than statistical randomness and is considered as absolute randomness**

What happens when we put several bit or Qubits together

Classical case

a_i is either 0 or 1

		Dimension of the space	Number of possibilities
(a_0)		1D	2^1
(a_1, a_0)		2D	2^2
(a_2, a_1, a_0)		3D	2^3
$\vdots$			
$(a_{127}, \cdots, a_1, a_0)$	?	128D	2^{128}

$\underbrace{\text{? \quad 128D}}_{\text{Real space (surface)}}$

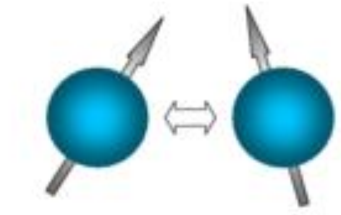
In the classical domain, if I give you any number $X < 2^{128}$:

$$X = x_0 \times 2^0 + x_1 \times 2^1 + \cdots + x_{127} \times 2^{127} \equiv (x_{127}, \cdots, x_1, x_0)$$

What happens when we put several bit or Qubits together

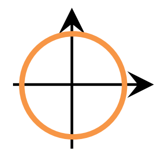
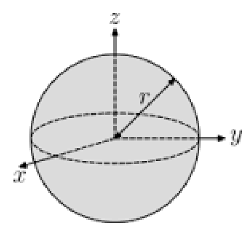
Quantum case:

$|a_1\rangle$ $|a_2\rangle$ $\longrightarrow$ $|a_1\rangle \otimes |a_2\rangle \equiv |a_1, a_2\rangle$



(Tensor product)

With the difference that a $|a_i\rangle$ can have any value between the up and down

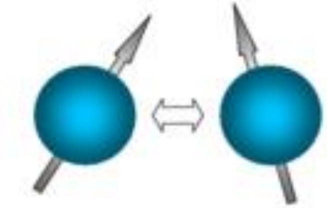
		Dimension of the space	Number of possibilities
$\lvert a_1\rangle$	$\uparrow$	1D	2^1
$\lvert a_1, a_2\rangle$		2D	2^2
$\lvert a_1, a_2, a_3\rangle$		3D	2^3
$\vdots$			
$(a_{127}, \cdots, a_1, a_0)$	?	128D	2^{128}

Right !!?? $\longrightarrow$ **Very wrong!** **Things are interring because of it!**

What happens when we put several bit or Qubits together

Simple example of 2 Qubits:

$|a_1\rangle$ $|a_2\rangle$



$\longrightarrow$ $|a_1\rangle \otimes |a_2\rangle \equiv |a_1, a_2\rangle$

(Tensor product)

If we follow the classical logic:

4 possible basic states:

$|0, 0\rangle$

$|1, 0\rangle$

$|0, 1\rangle$

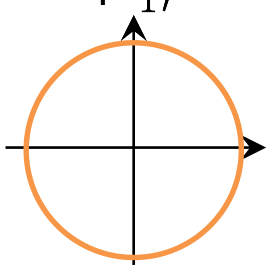
$|1, 1\rangle$

Pulse all the super position states we learnt that exist because of quantum mechanics:

$$|\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle)$$

If we believe the classical picture, this two 2D space forms a complete space that fit any vector consist of 2 Qubits

$|a_1\rangle$



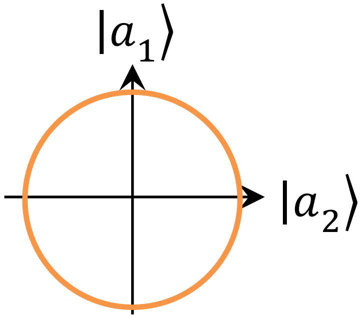
$|a_2\rangle$

What happens when we put several bit or Qubits together

If the classical picture is true and 2 Qubits form a 2D space, any vector in the space of these two Qubits should be factored in this form

$$|\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)\otimes(\alpha_2|0\rangle + \beta_2|1\rangle)$$

$$X = x_0 \times 2^0 + x_1 \times 2^1 + \cdots + x_{127} \times 2^{127} \equiv (x_{127}, \cdots, x_1, x_0)$$



Counter example to above argument:

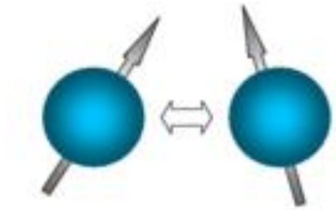
$$\underbrace{|\psi\rangle = \frac{|0,1\rangle + |1,0\rangle}{\sqrt{2}}}_{\textbf{Bell state}} \xrightarrow{?} |\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)\otimes(\alpha_2|0\rangle + \beta_2|1\rangle)$$

This means that all the vectors with this two qubits does not fit in this 2D space

What happens when we put several bit or Qubits together

Quantum case:

$|a_1\rangle \quad |a_2\rangle \longrightarrow |a_1\rangle \otimes |a_2\rangle \equiv |a_1, a_2\rangle$



(Tensor product)

We can show the space that these two qubits are operating is a 4D space with the basics of these vector

$$\begin{cases} |0,0\rangle \\ |1,0\rangle \\ |0,1\rangle \\ |1,1\rangle \end{cases}$$

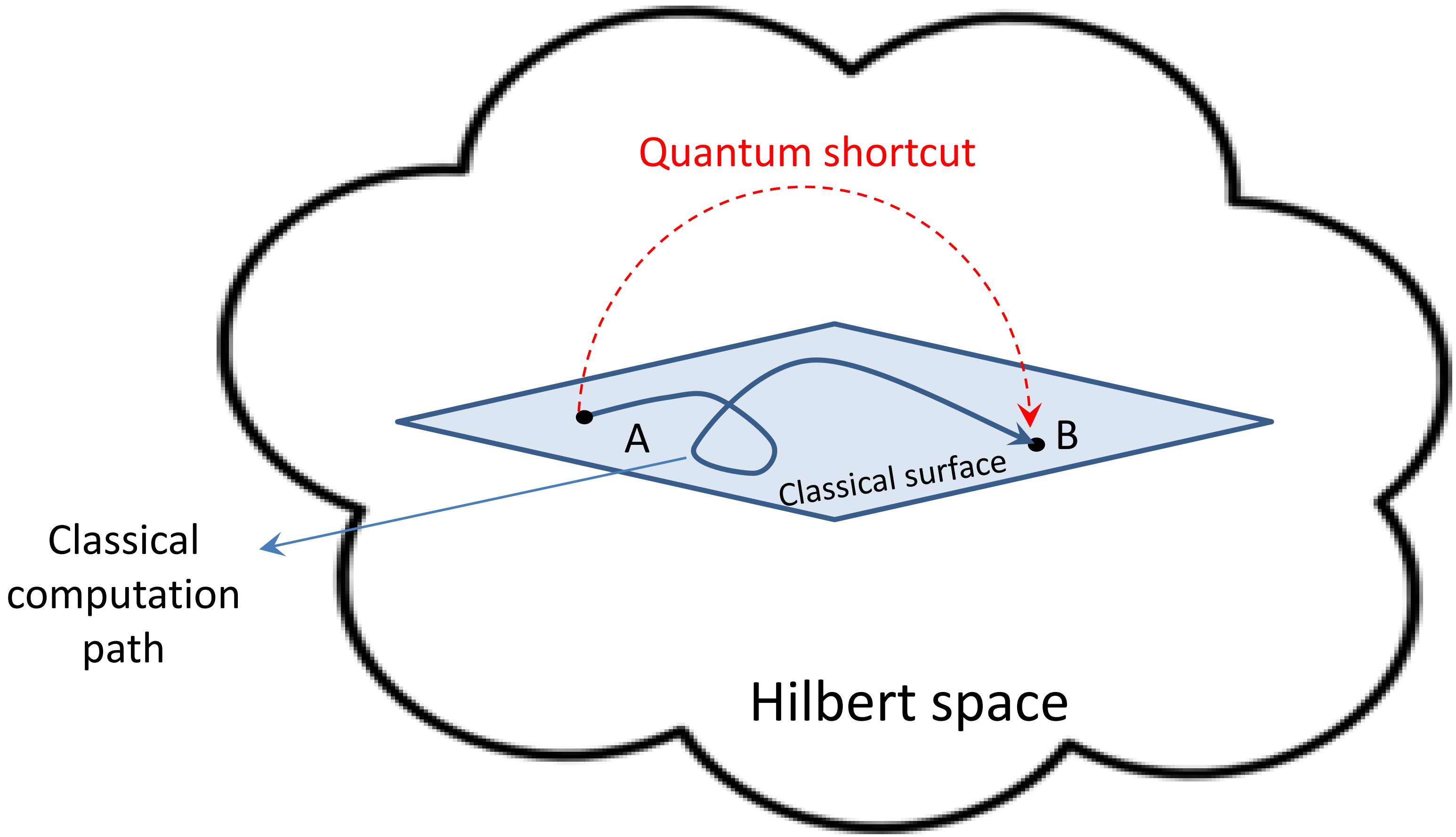
Number of bits	Number of basic vector: Quantum	Number of basic vector: Classical
1	2^0	1
2	2^2	2
3	2^3	3
⋮	⋮	⋮
128	$\underbrace{2^{128}}_{\text{Hilbert Space}}$	128

Number of possibilities : insane value of $2^{2^{128}}$

Hilbert space Vs classical space

Hilbert Space >>>>>>>> classical space

$2^{128} \gg 128$



The whole idea of quantum computers: To find some shortcut in Hilbert space for some computational tasks

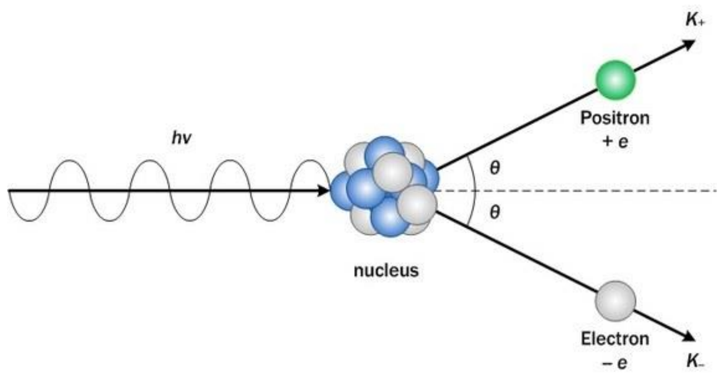
Bell state, entanglement and the problem of local realism

Some physical examples of how to generate entangled states such as Bell state:

$$|\psi\rangle = \frac{|0,1\rangle \; + |1,0\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$$

Output sum of spins = 0 because of the conservation of the spin

Input spin = 0



$$\begin{bmatrix} |\uparrow\rangle \\ |\downarrow\rangle \end{bmatrix} = \begin{bmatrix} |1\rangle \\ |0\rangle \end{bmatrix}$$

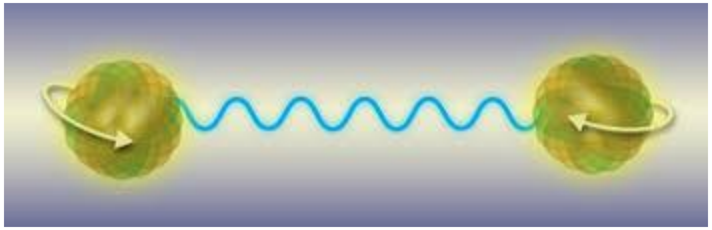
$$|\psi\rangle = \frac{|\uparrow,\downarrow\rangle \; + |\uparrow,\downarrow\rangle}{\sqrt{2}}$$

Entangled particles

Bell state, entanglement and the problem of local realism

Entangled particles

≠ correlated particles



$$|\psi\rangle = \frac{|\uparrow,\downarrow\rangle \; + |\uparrow,\downarrow\rangle}{\sqrt{2}}$$

A two particle system that the two together has a well deign state but individually have no well deign state

Not that we do not know the spin of each individual particle but such a thing doesn't exist

Einstein Paradox of spooky action in distance

↓

It is wrong because it is spooky but there is no interaction in distance and one can not transfer energy/information with it

↓

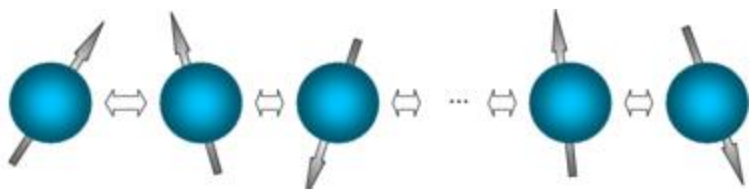
One need to abandon the idea of **Local Realism** ≡ accepting the absolute randomness

Why Hilbert space is so large?

Array of classical bits: $(0,1,1,0,1,0,\cdots,1)$

These bits individually exist and are independent of each other

Array of qubits:



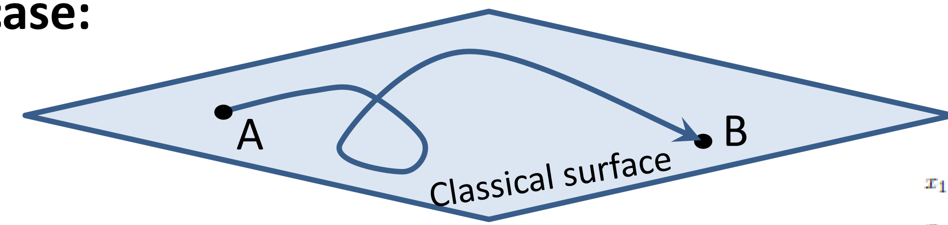
Not only individual states matter but also correlation (entanglement) between particles matter!

- Between each two particle
- Between groups of three
- And etc.

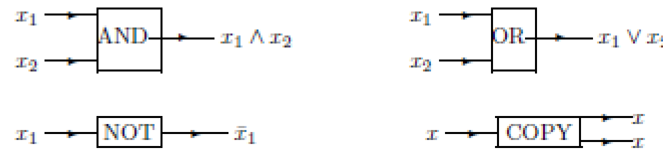
Quantum computers exactly try to take advantage/harvest this parallel integration between qubits for more efficient parallel processing

Quantum gate and moving in Hilbert space

Classical case:

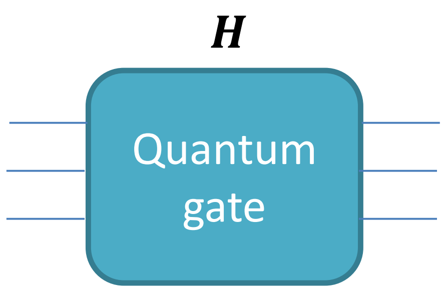


Moving though the classical surface using logical gates



Quantum computers: Quantum gates

$$|\psi_{in}\rangle = |a_0, a_1, \cdots, a_n\rangle$$



$$|\psi_{out}\rangle = H|\psi_{in}\rangle = |b_0, b_1, \cdots, b_n\rangle$$

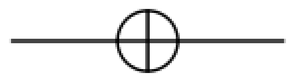
The challenges of the quantum gates:

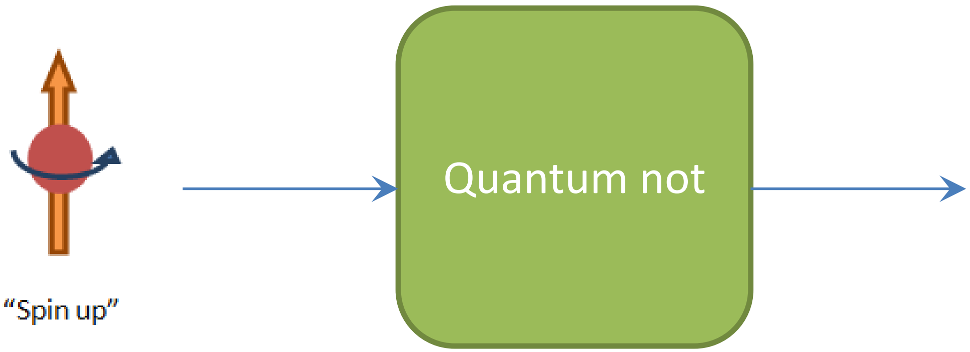
- Same number of inputs and output (unlike the classical gates)
- Gates should be reciprocal ($HH^\dagger = 1$). The transfer matrix is Hermitian. $H^\dagger = (H^T)^*$
- We can not look at the state at any point to do this operation (unlike classical computers)

Quantum gates

Gate quantum **not** (single bit operation):

$$H = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$





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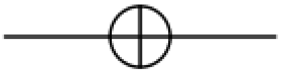

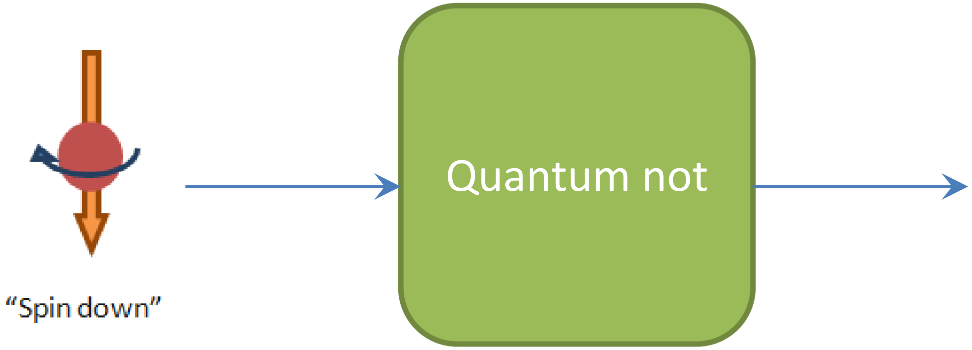

Quantum gates

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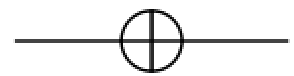

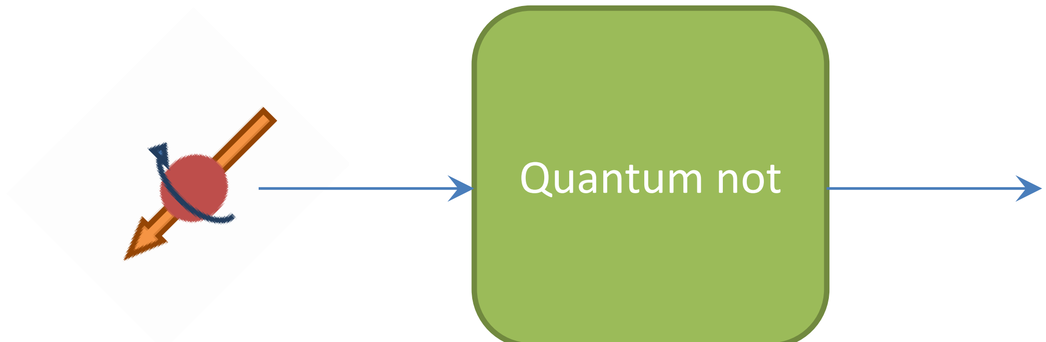

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle$$

Quantum gates

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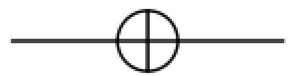
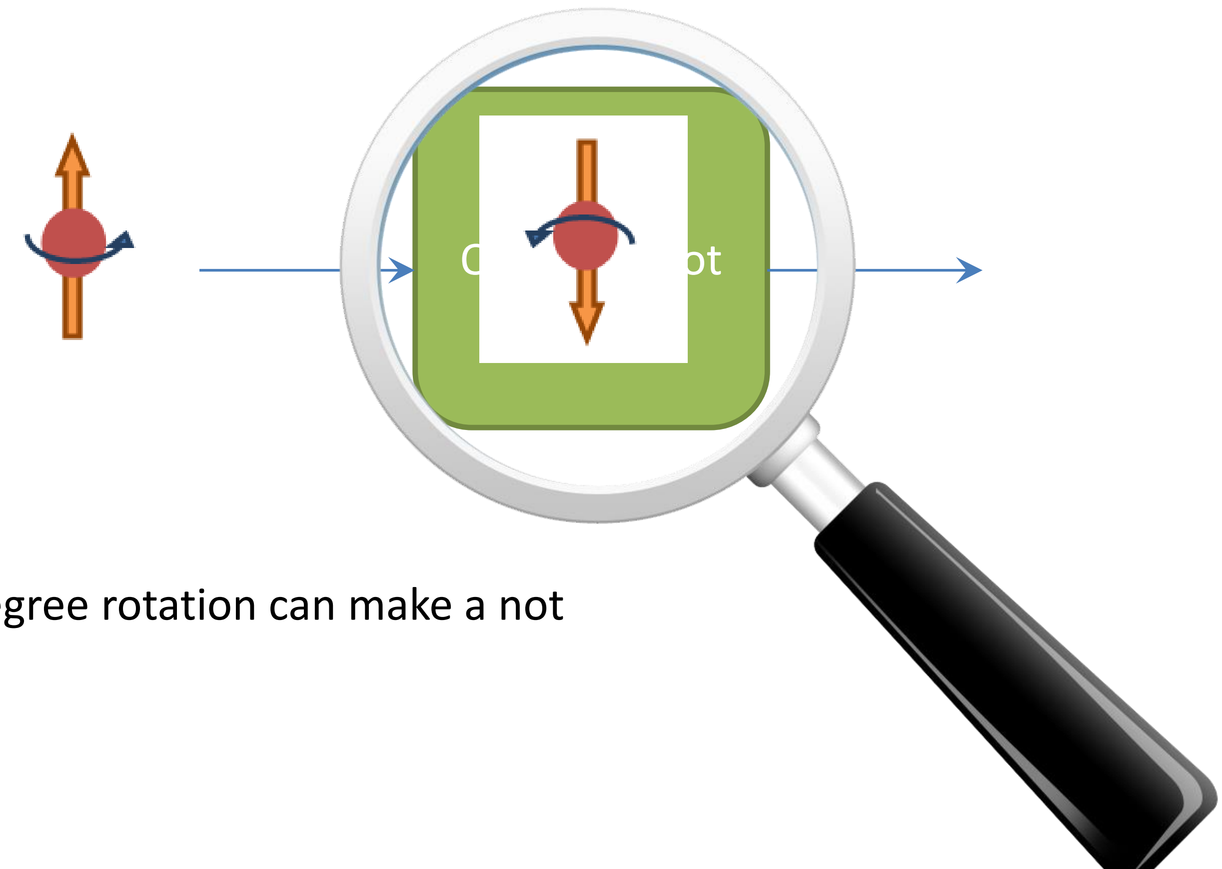
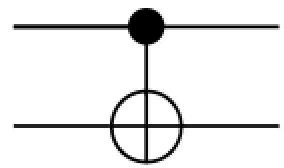




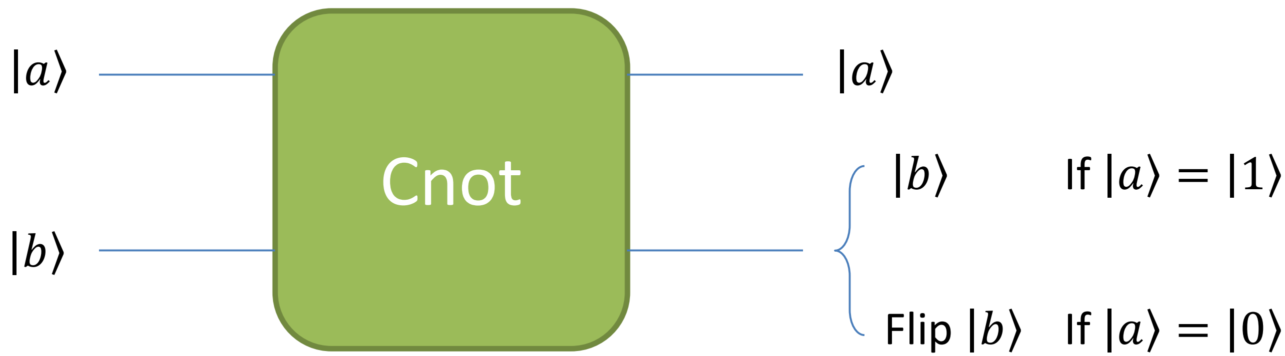
A 180 degree rotation can make a not gate

Quantum gates

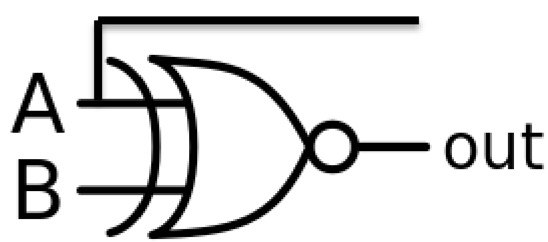
Gate quantum CNot (2 bit operator):



Controlled not

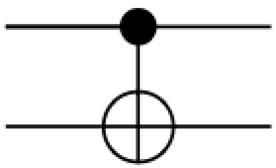


Very similar to classical Xnor

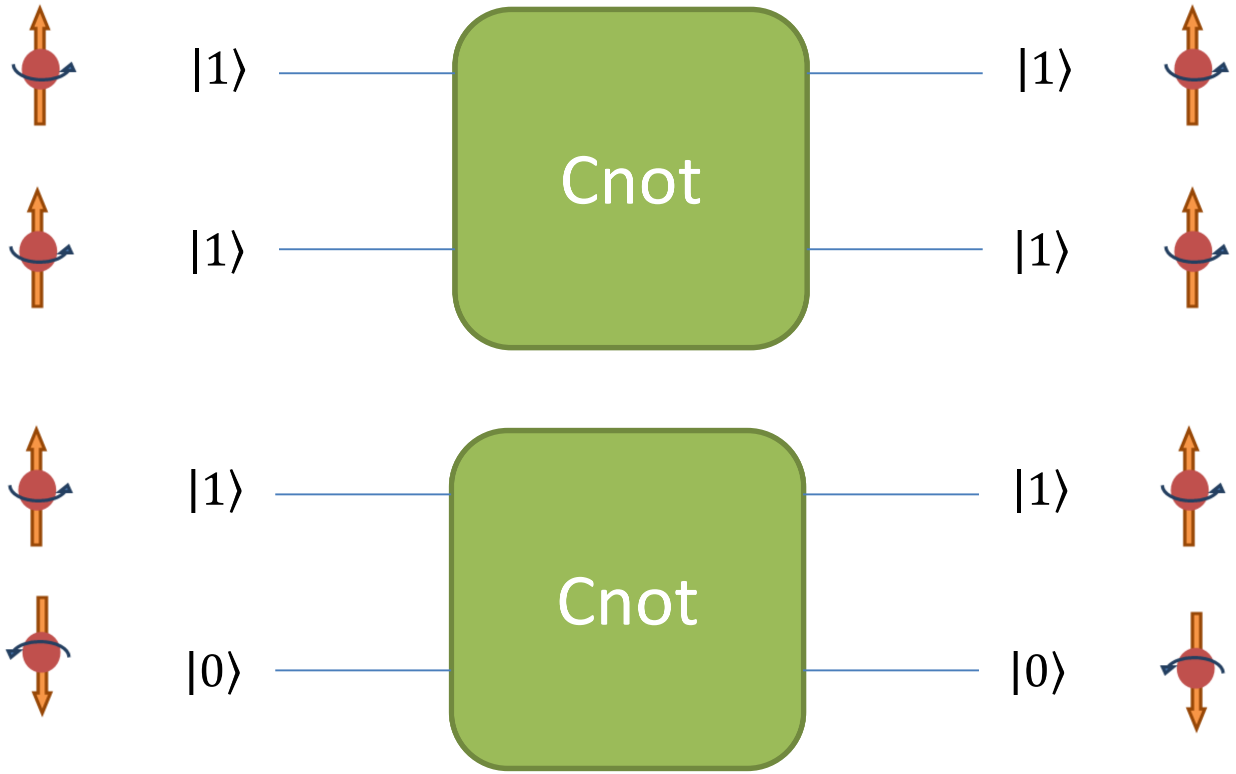


Quantum gates

Gate quantum CNot (2 bit operator):



Controlled not



Quantum gates

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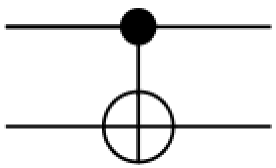

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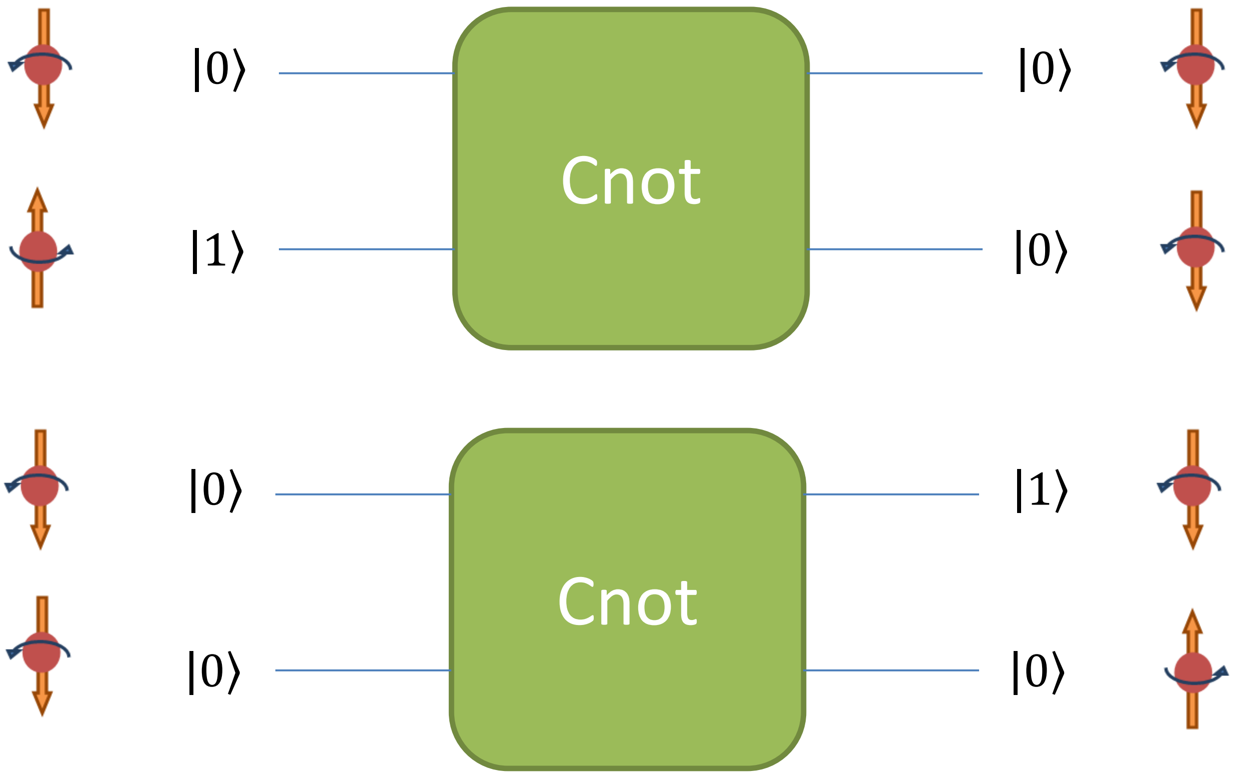

Quantum gates

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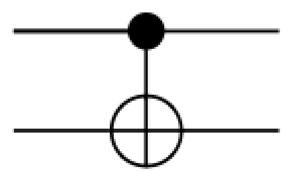

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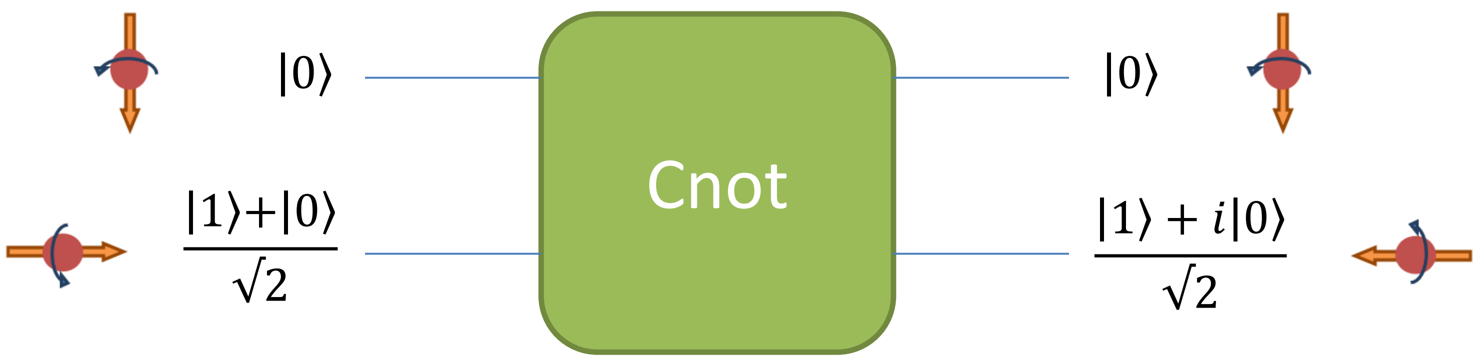
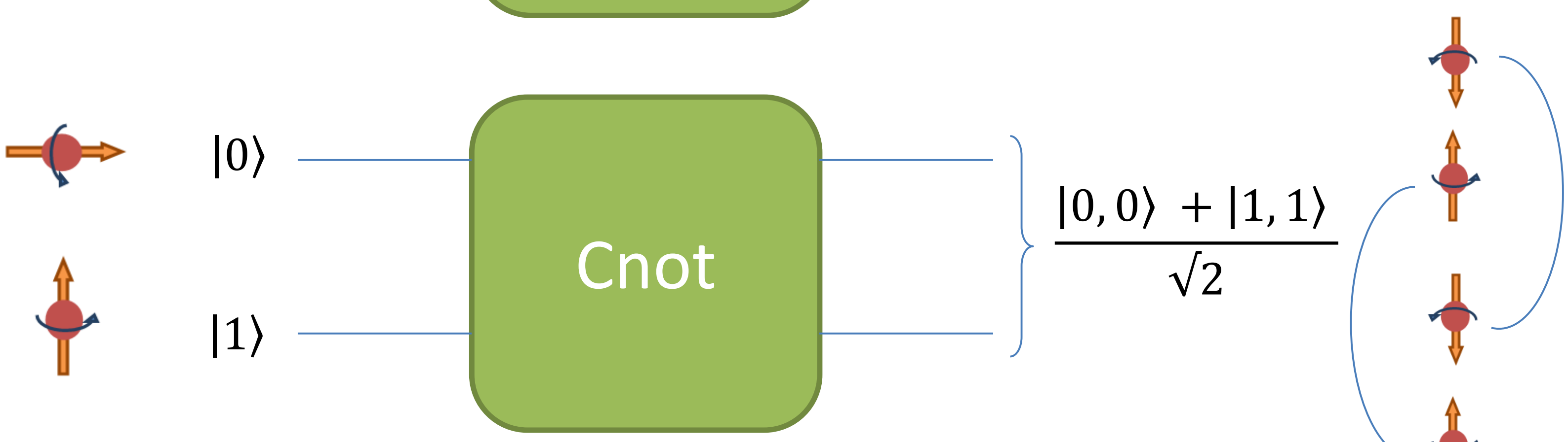


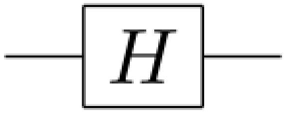


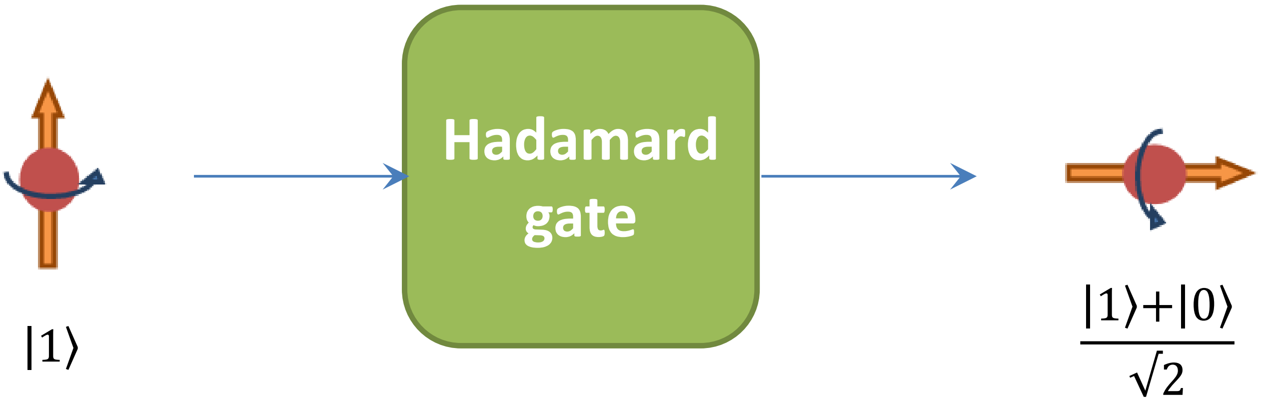
Out put is entangled state

Quantum gates

Gate quantum **Hadamard gate** (single bit operation):

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$



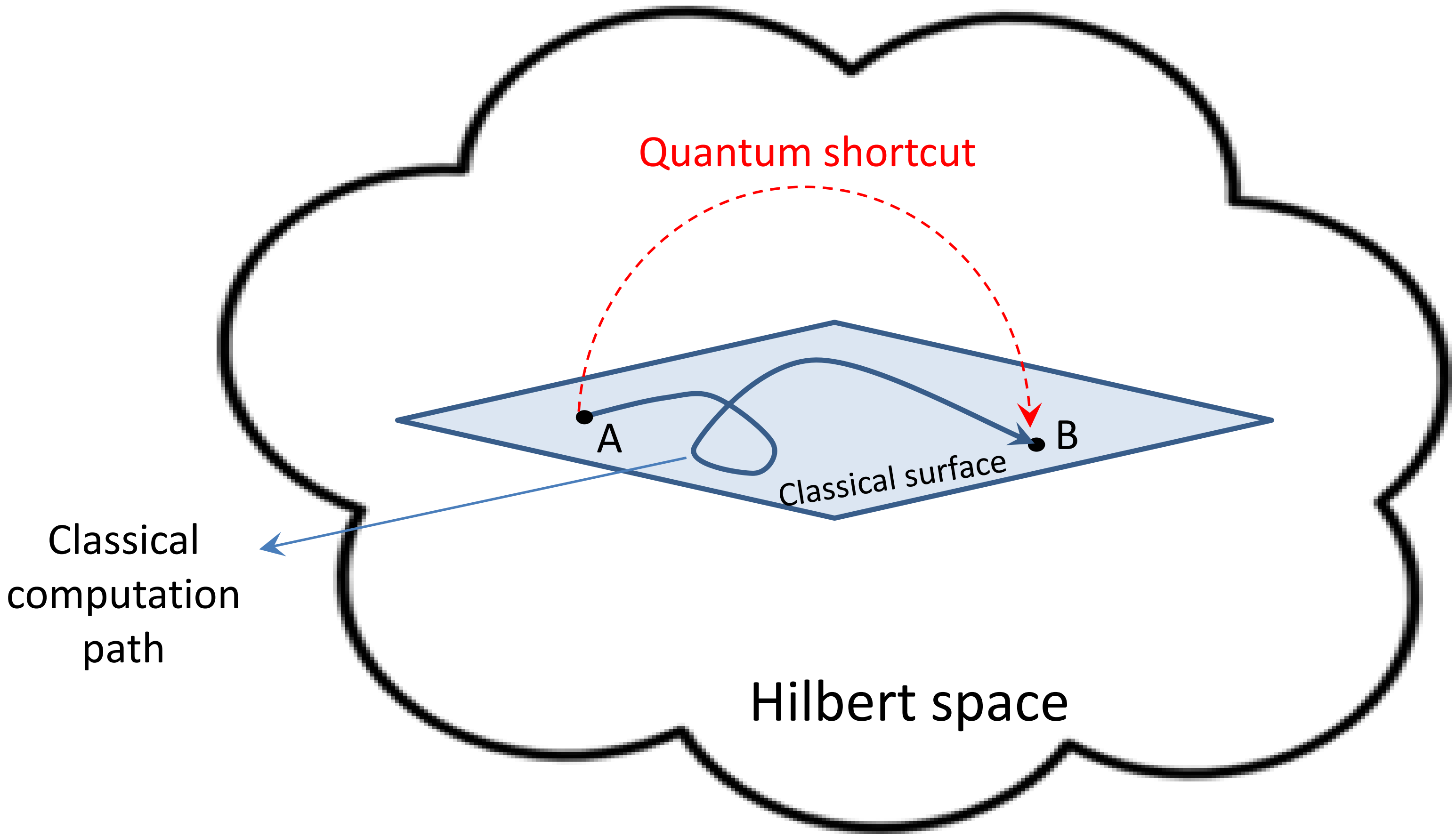


Equivalent of 90 degree rotation

Hilbert space Vs classical space

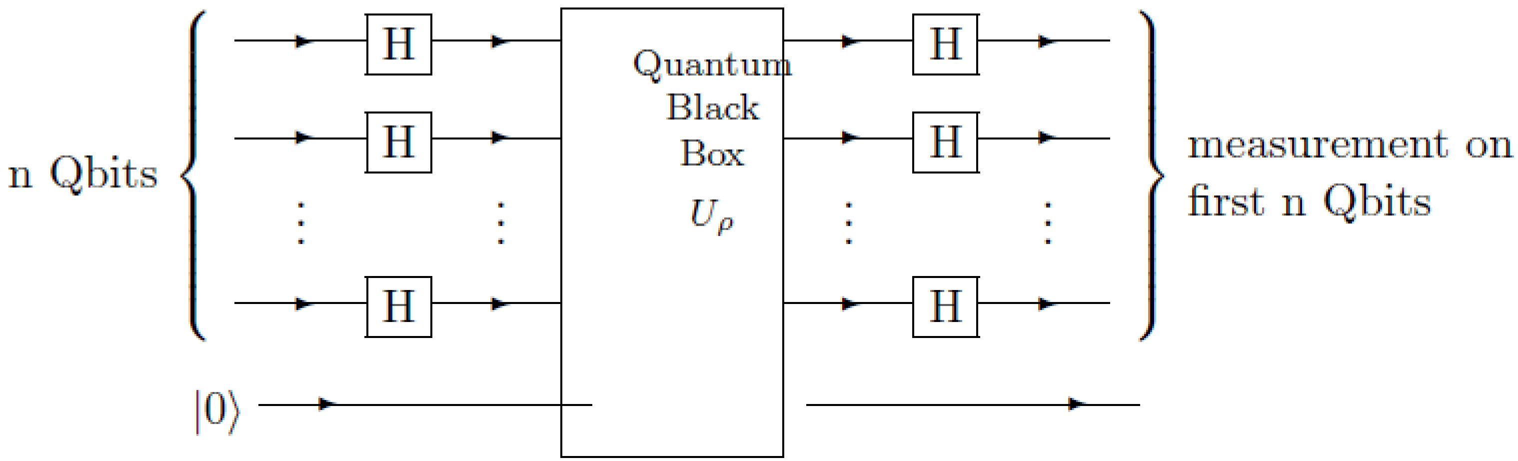
Hilbert Space >>>>>>>> classical space

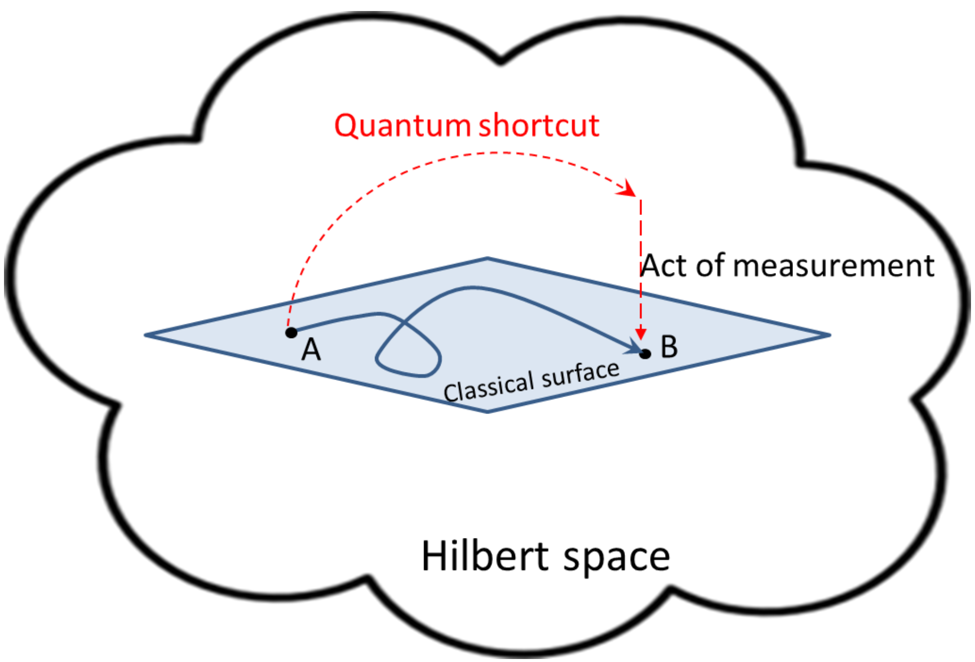
$$2^{128} \gg 128$$



The whole idea of quantum computers: To find some shortcut in Hilbert space for some computational tasks

Usual architecture of the quantum computer



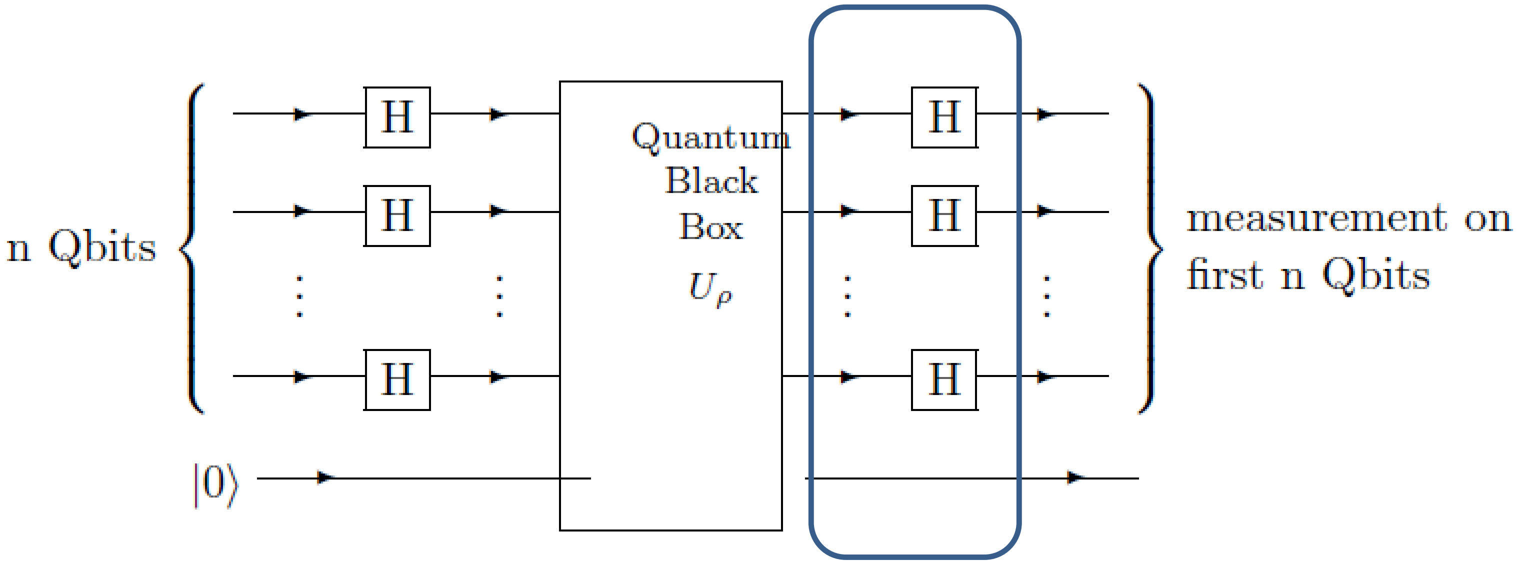


Quantum computer works on the basis that we start with an initial condition (normally $|0,0,\cdots,0\rangle$) and prepare a highly hyper entangled state which has the maximize probability on the classical results.

The results is revealed in the act of measurement (with some statistical error)

Usual architecture of the quantum computer

Or a Quantum Fourier Transform (QFT)



$$(H|0\rangle \otimes \ldots \otimes H|0\rangle) \otimes |0\rangle = \frac{1}{2^{n/2}}(|0\rangle + |1\rangle) \otimes \ldots \otimes (|0\rangle + |1\rangle) \otimes |0\rangle$$

$$= \frac{1}{2^{n/2}} \sum_{\underline{b}} |\underline{b}\rangle \otimes |0\rangle \quad ; \quad \underline{b} = (b_1 \ldots b_n) \quad ; \quad b_i \in \{0, 1\}$$

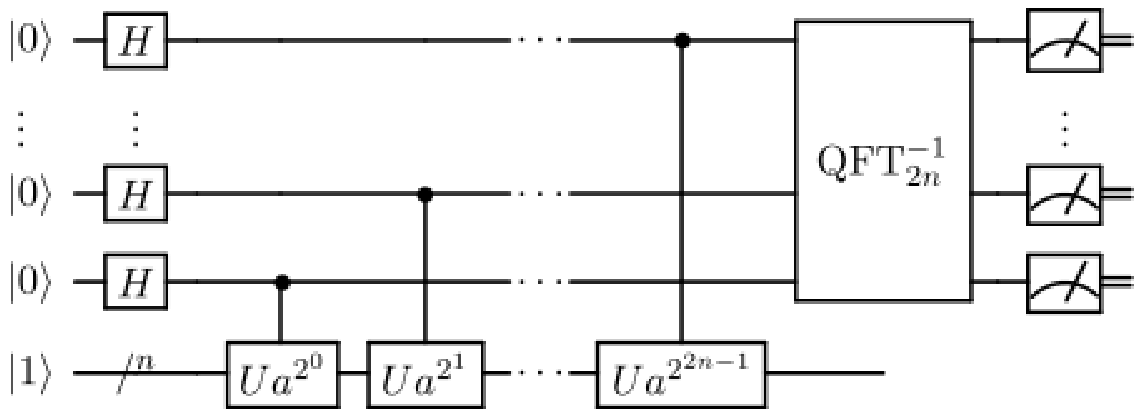
Input with all possible combination

↓

A lot more mathematics needed to go any further

Shor's algorithm

A famous example of quantum algorithm which is faster than classical computer:



Speed using the most efficient know classical algorithm (General number field sieve)

- It is sub-exponential time – about $O(e^{1.9\ (\log N)1/3\ (\log\log N)2/3})$

With Shor's algorithm it is **polynomial time**:

- About $O((\log N)^2(\log\log N)(\log\log\log N))$

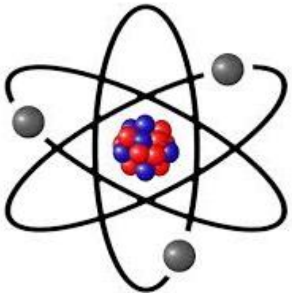
Different platforms to implement qubits

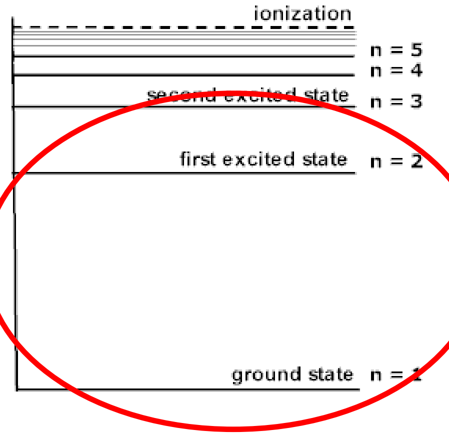
Natural ones

- **Electronic transition in atoms or molecules**
- Two level systems via electron spin and magnetic field
- Two level systems via nuclear spins state (NMR)

Pros:

- Naturally exist in the nature (Cs or Rb atom in NIR regime)
- $\Delta E = h\nu \gg K_B T$ this means that they are safe from the environment (thermal fluctuation)





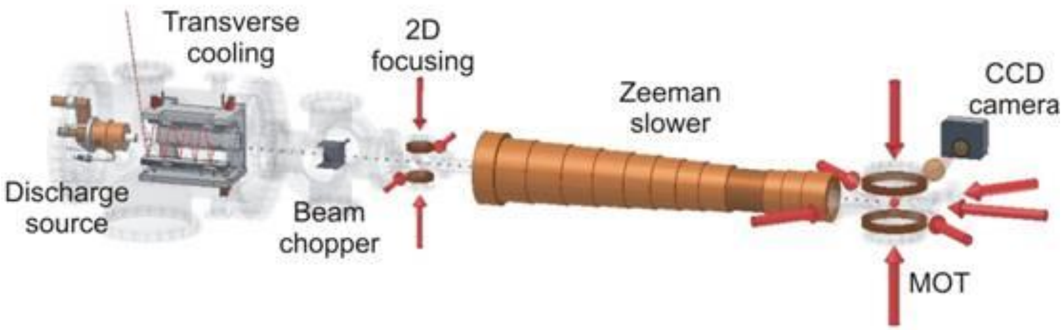
Cons:

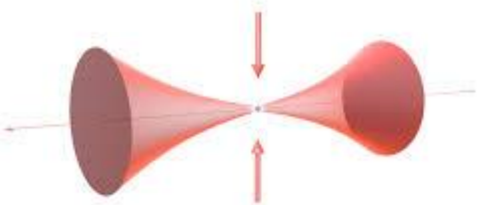
- One need to trap a single atom or ion in space (very complex and bulky)
- Can not be integrated and is not scalable

Atomic traps

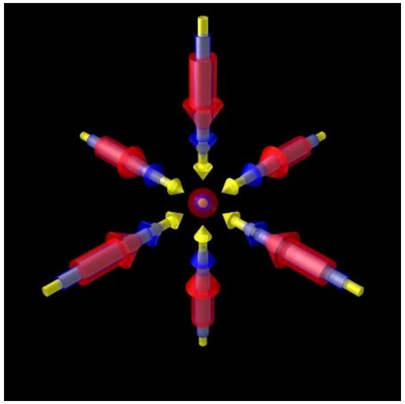
Laser trapping (optical lattice)

Atoms are pulled toward the maximum of intensity (optical tweezer)

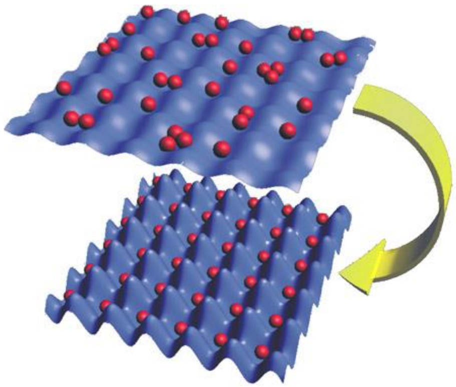




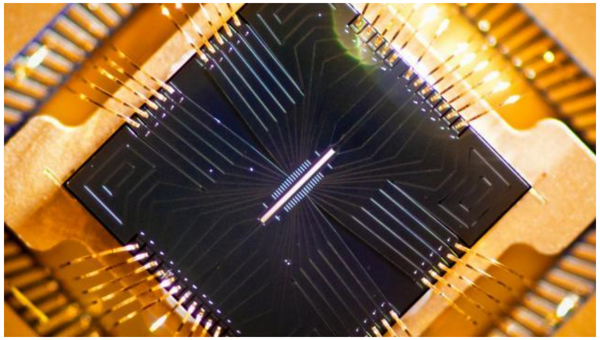
3D confinement

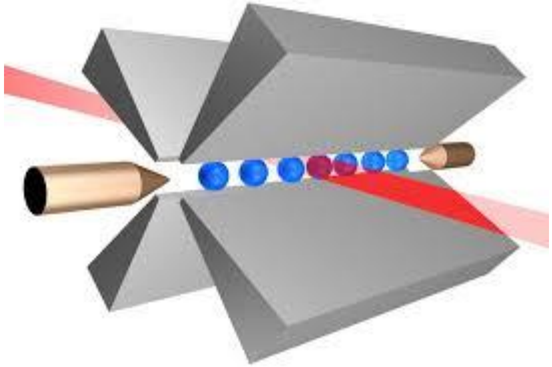


Optical lattices



Ion trapping:



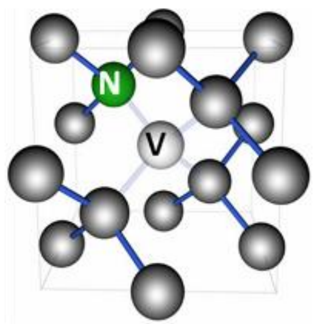


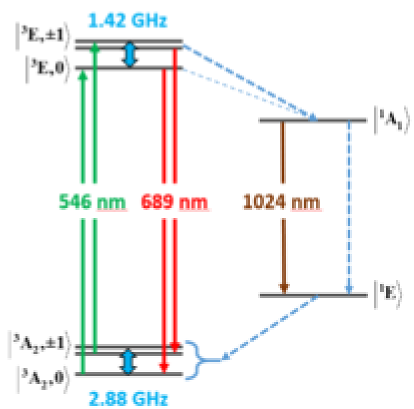
NV color centers

Atoms trap in the solid state crystal:

A very famous is NV centers in diamond crystal: a diamond which is doped with nitrogen atoms

Nitrogen vacancy color centers



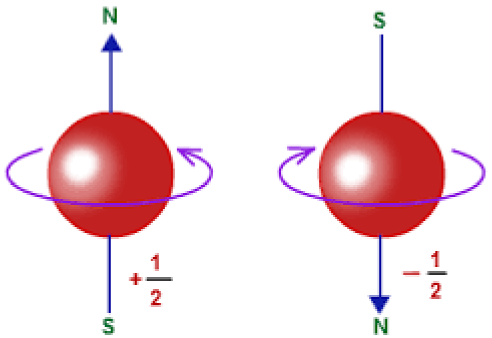


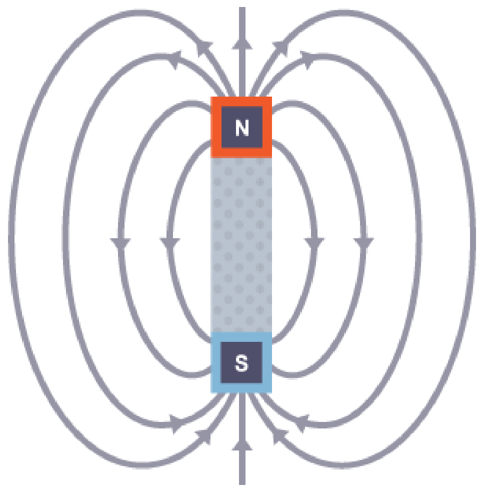
Very interesting system (much simpler than trapping)

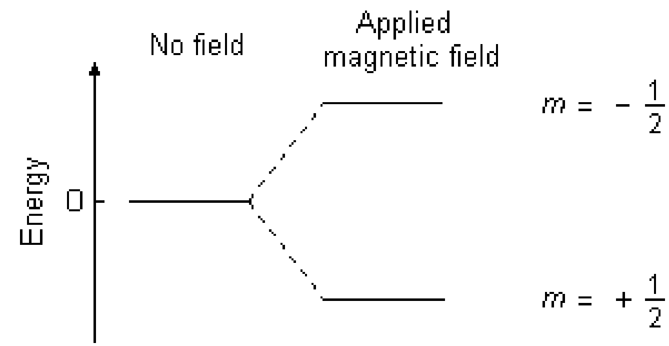
- The location of the color center (NV system) is random

Spin two level system

Energy splitting: $\Delta E = \vec{B} \cdot \vec{\mu} \approx 1 \text{ MHz} - 10 \text{ GHz}$







The most fundamental problem with these low energies is temperature:

Energy of 1 GHz oscillator : $E = h\nu \sim 4 \times 10^{-7}\ eV = 4\ \mu eV$

Energy of thermal fluctuation: $E = k_B T \sim 25\ meV$ at room temperature

Thermal fluctuations >> quantum energy splitting

We need to cool by a factor of above 10^4 $\longrightarrow$ means working at temperature of few mK

Different platforms to implement qubits

Qubits are produced via two level systems in reality:

Artificial ones

- Semiconductor Quantum dots
- **Superconducting Josephson junction qubits**

Pros:

- One can integrate them on a chip
- Scalable to more than one qubits

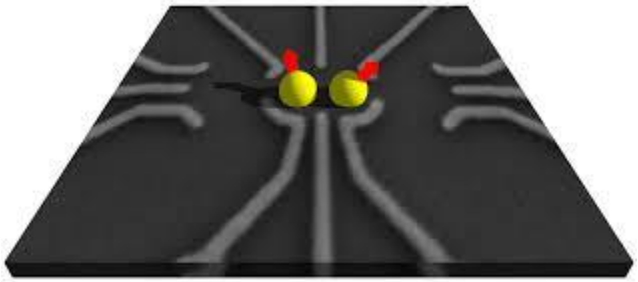
Cons:

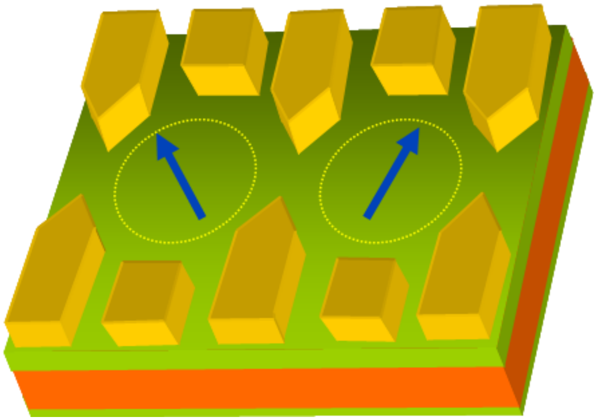
- Based on spin system at microwave frequencies and need operation in mK regime

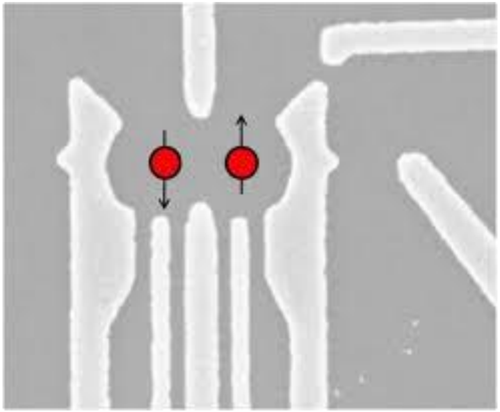
Quantum dots

Semiconductor Quantum dots:

A box that only one electron would fit in it:







Cons:

- Normally are very short live states

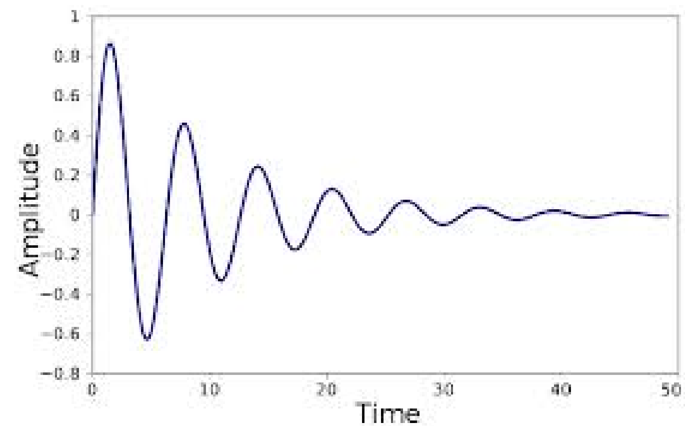
In the business of thermal fluctuation, the energies are not enough/complete

More important parameter is thermal decoherence rate

Thermal decoherence rate

Damped classical harmonic oscillator:

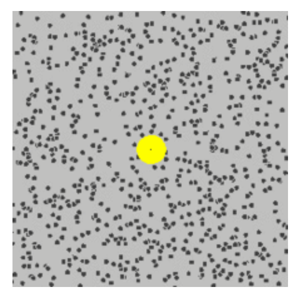
Damping rate = Γ

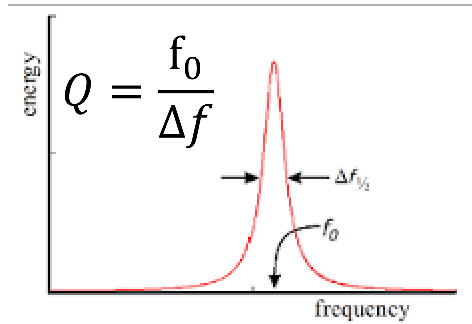


A quantum oscillator (TLS) is much much more fragile:

It would take a lot of gas molecules to stop a classical oscillator (Γ_m)

But only on interaction is enough to destroy the quantum state. This rate is called thermal decoherence rate $\Gamma_{\mathrm{th}} = \mathrm{n_{th}}\ \Gamma$



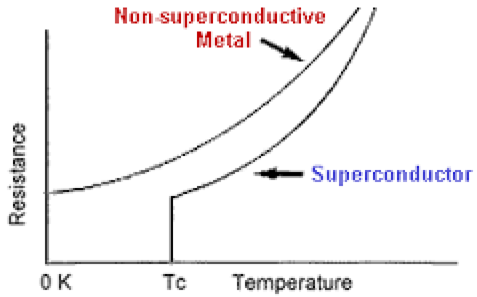


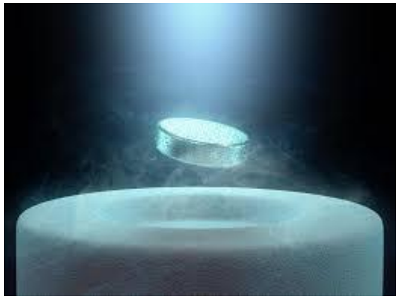
$$n_{th} = \frac{k_B T}{h\nu} \longrightarrow f > \Gamma_{th} \longrightarrow f > \frac{k_B \boldsymbol{T}}{h\ \boldsymbol{Q}}$$

Condition for at least one coherent oscillations

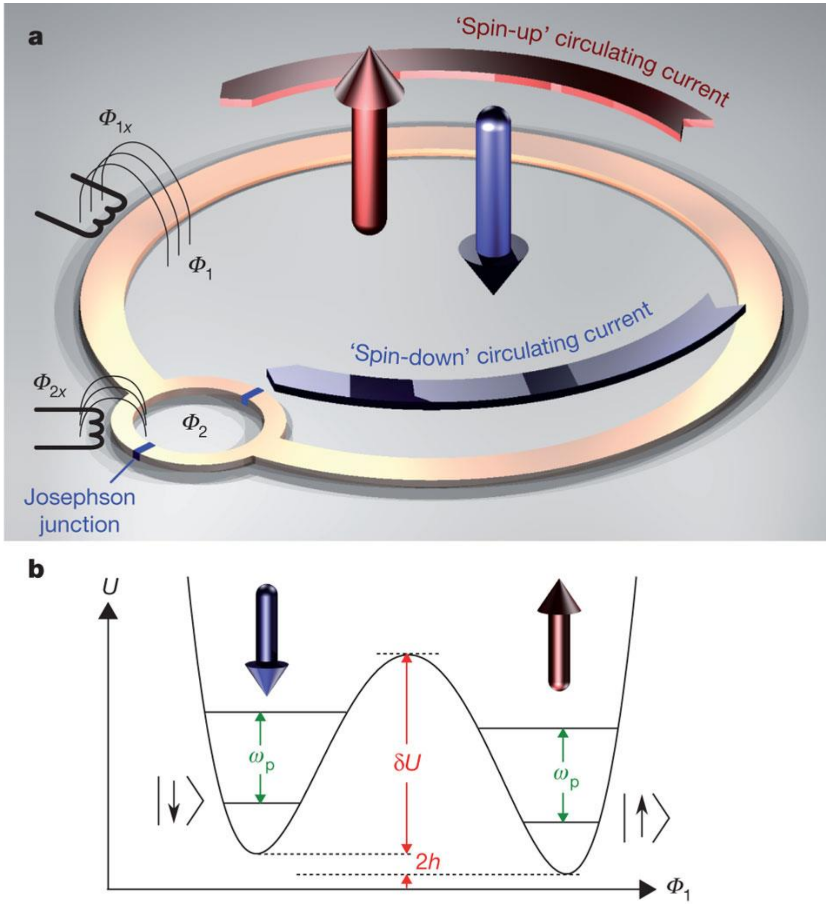
Superconducting Josephson junction qubits

Superconducting Josephson junction qubits



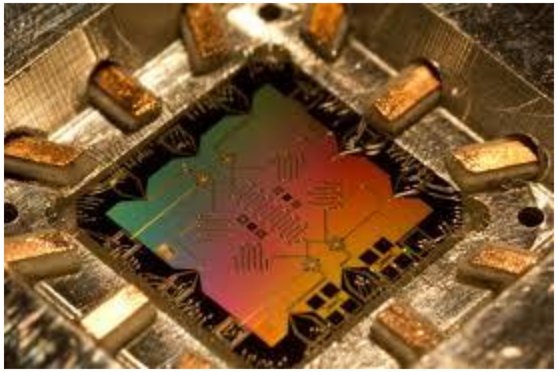


Magnetic flux are quantized in superconducting circuits



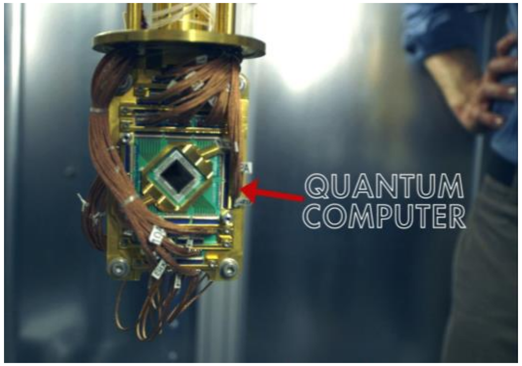
More than 95% present of real demonstrations of the quantum computers are using these qubits

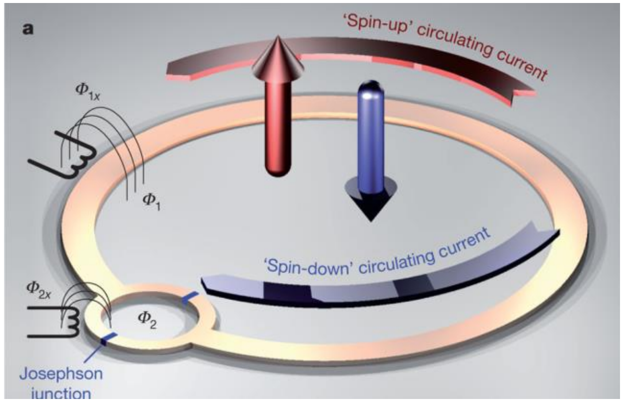
They are relatively high Q and very flexible

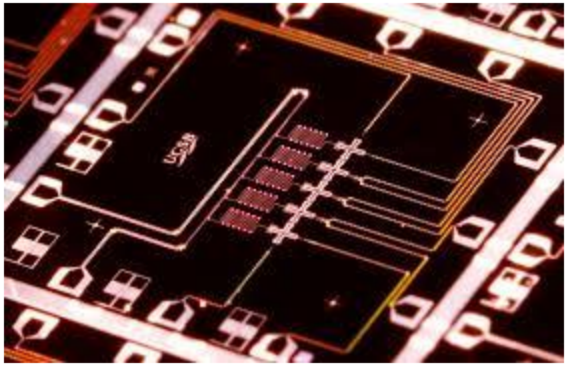


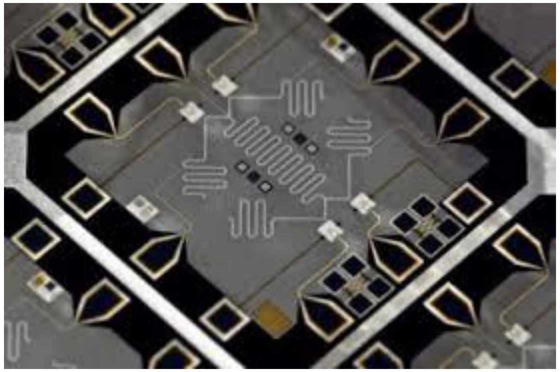
Superconducting Josephson junction qubits

Superconducting Josephson junction qubits



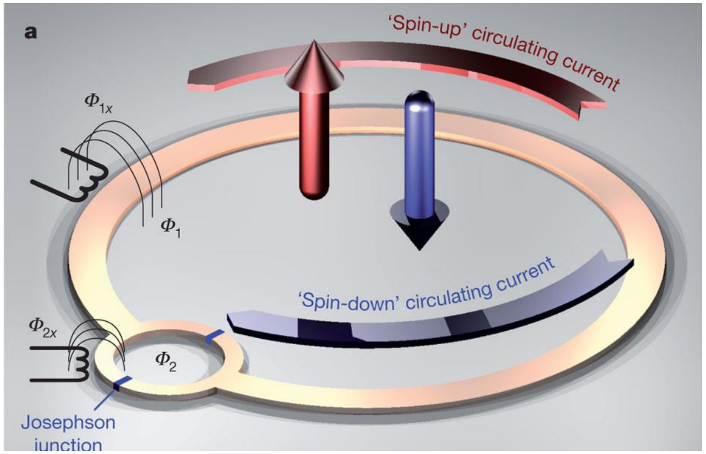






Superconducting Josephson junction qubits

Superconducting Josephson junction qubits



There are three important challenges in these platform and all are related to the temperature:

- Need very low temperature (mK regime) $\longrightarrow$ expensive and bulky

- Limited coherent oscillation because of the limited Q ($f > \frac{k_B \boldsymbol{T}}{h\, \boldsymbol{Q}}$)

- One can not extract the quantum information from the heart of dilution refrigerator for quantum network applications

Quantum networks

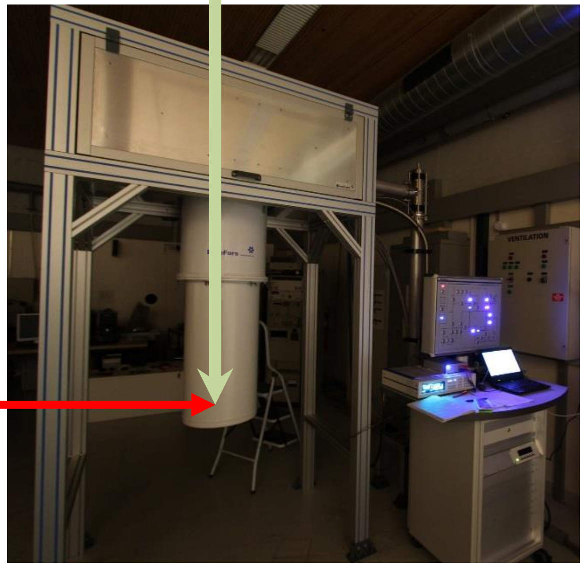
Not possible with GHz microwave photons but is possible to optical photons

$h\nu_{microwave} \ll k_B T$

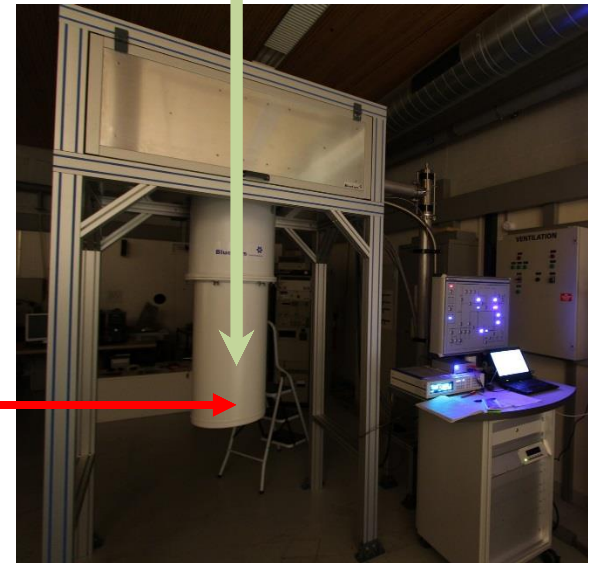
Outside is 300 K

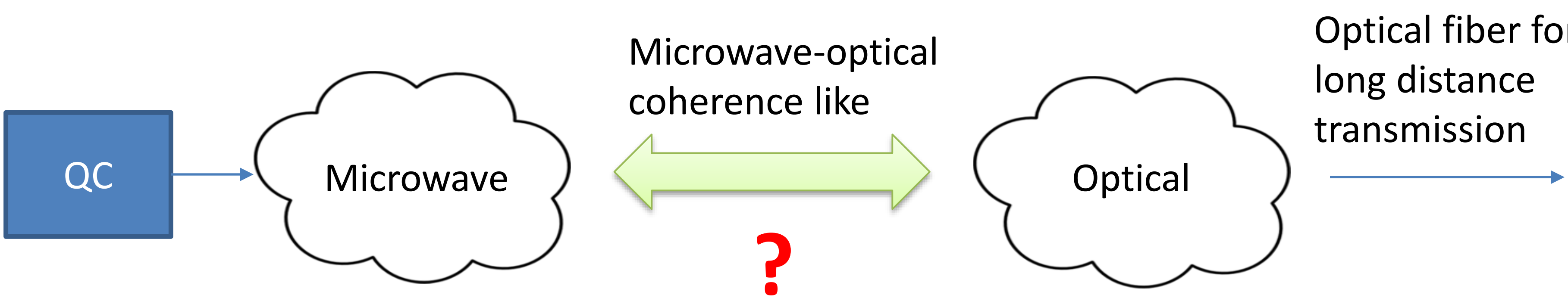
$h\nu_{optical} \gg k_B T$

10 mK



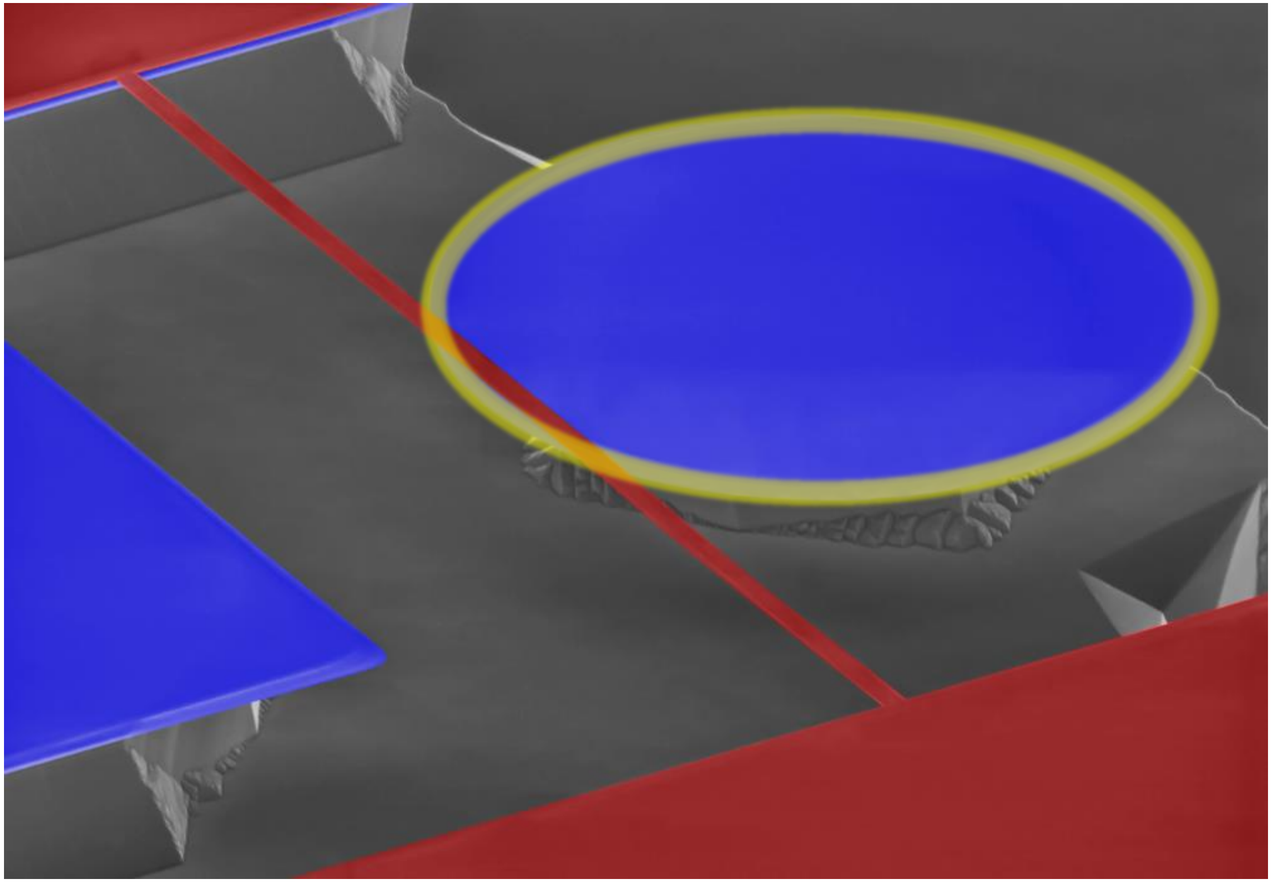
10 mK





Optomechanical systems

A system that mechanical motions are detected and controlled via a quantum limited optical sensors (cavity)



Advantage of the mechanical systems:

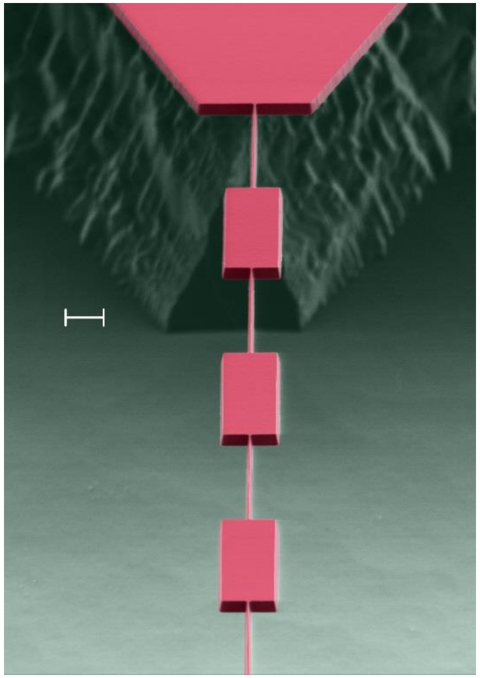
- The usually exhibit very large quality factor
- They couple very easily to different systems

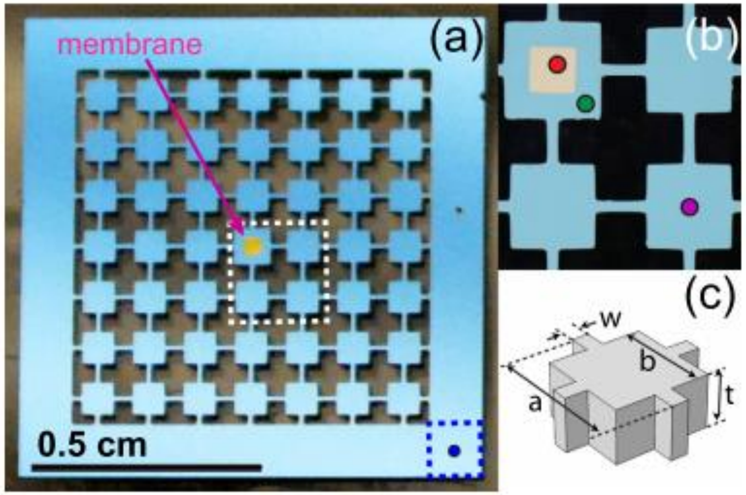
Mechanical resonators are not good candidates for TLS

Mechanical systems for quantum memories

We recently learn to make ultra-high Q mechanical resonator at room temperature or low temperature

- This is achieved by combining several novel techniques such Phononic crystals and stress engineering and etc.
- We have recently observed the mechanical Q of about 1 billion at ~ 3 MHz.
- (if it was a gitar string with the same Q, the ring down would take about a year)
- Several tens of millisecond coherence time even at room temperature (equivalent of the optical fiber with the length of equator of the earth)
- 1000s of coherent oscillation even at room temperature





Question?

- Review some basics in quantum mechanics (TLS, Hilbert space, entanglement)
- Quantum gate and quantum computer architecture (Shor's algorithm)
- Review some physical platform to implement the quantum computers