

Statistical Analysis on the Time Complexity of Quick Sort

Background:

Sorting is one of the most fundamental problems in computer science. When given a set of items, what is the most efficient way to sort them? Many different sorting algorithms currently exist, and they all have the individual advantages and disadvantages. One way to classify these algorithms is by using time complexity. Time complexity describes how the time of algorithm changes as the number of items increases. Time complexity is denoted with big-Oh notation, $O(n)$, where n represents the number of items. For example, $O(n)$ represents an algorithm that increases linearly as the number of items increases. $O(n^2)$ would represent a quadratic algorithm. With this big-Oh notation, algorithms can be defined in a quantitative manner.

A popular sorting algorithm is called Quick Sort. Quick Sort has $O(n \log n)$ time complexity, meaning that Quick Sort increases log-linearly as the number of items increases (less than a linear function, but more than a logarithmic function). Although the time complexity of Quick Sort is theoretically $O(n \log n)$, does it change when run on an actual data set? Could there be other factors that change affect the time complexity? Using hypothesis testing, we can determine the answers to these questions.

Experimental Design:

To determine if the time complexity of Quick Sort is actually $O(n \log n)$, I decided to evaluate the time difference between the Quick Sort of 200 items and 400 items. Using the equation below ($n = 200$ items), the ratio between 400 items and 200 items should be around 2.26164.

$$\frac{t_{2n}}{t_n} = \frac{2n * \log(2n)}{n * \log(n)}$$

To set up this experiment, I used a Python implementation of Quick Sort online. I wrote a Python script that generated a random list of numbers (200 or 400 items), measured the time it took to Quick Sort them, and then created 500 samples from the results. Each sample contained 800 different Quick Sort times. The data was written into two separate files, one containing the 200-item times and the other containing the 400-item times. All the times are reported in milliseconds.

*The Python script and the raw data are included at the Appendix

Data:

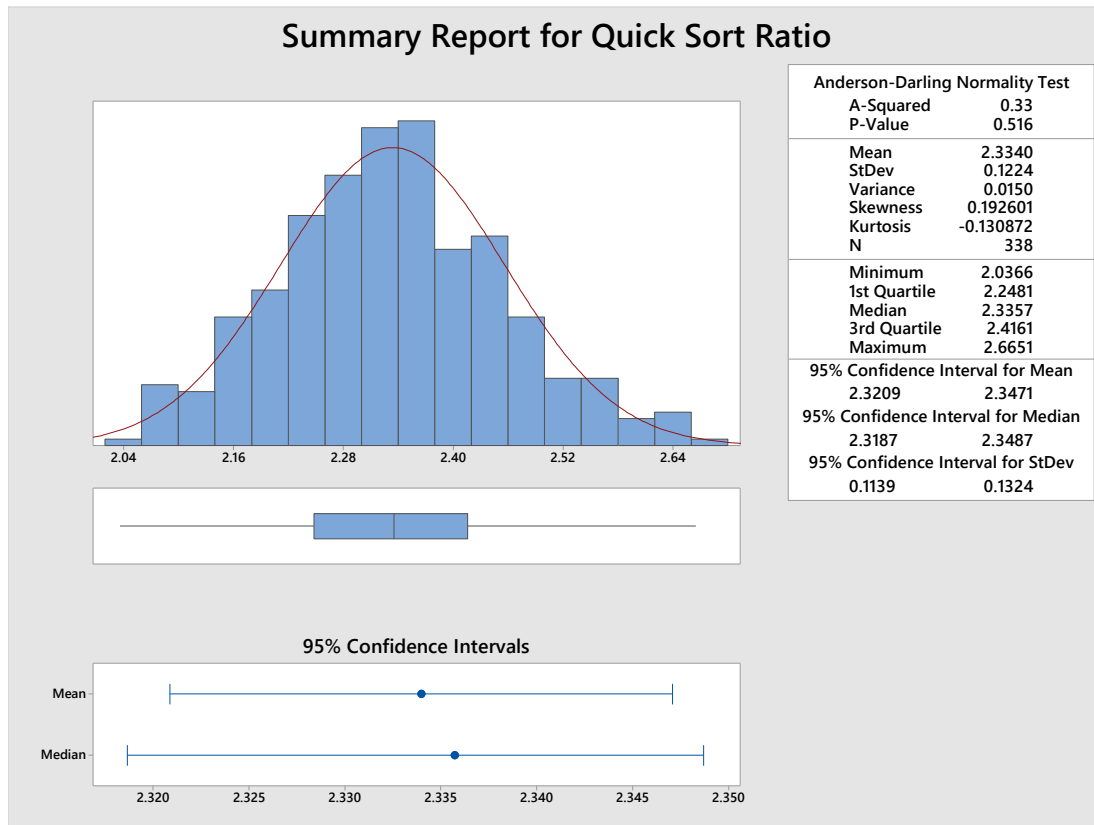


Figure 1: Minitab Graphical Summary of the ratio of times between the Quick Sort of 400 items and 200 items. The graphical summary tells us that this data set is normally distributed (Anderson-Darling test). There is also minimal skew, and the mean of this set lies around 2.33.

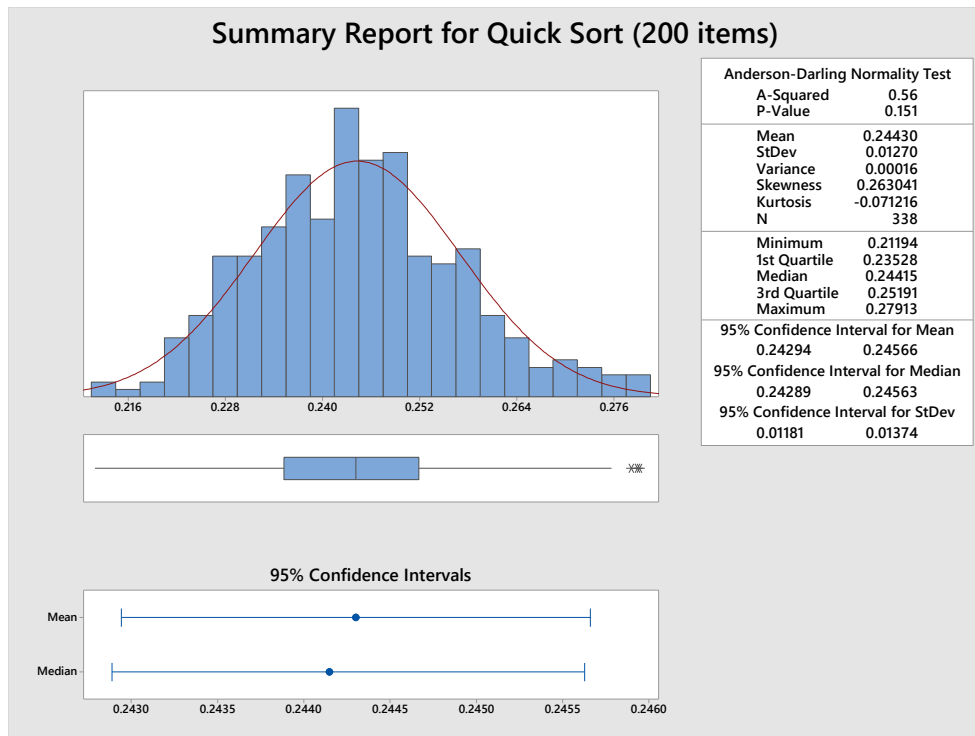


Figure 2: Minitab Graphical Summary of the average time of 500 samples of Quick Sorting 200 items. The graphical summary tells us that this data set is normally distributed (Anderson-Darling test). The mean of this set lies around 0.244ms.

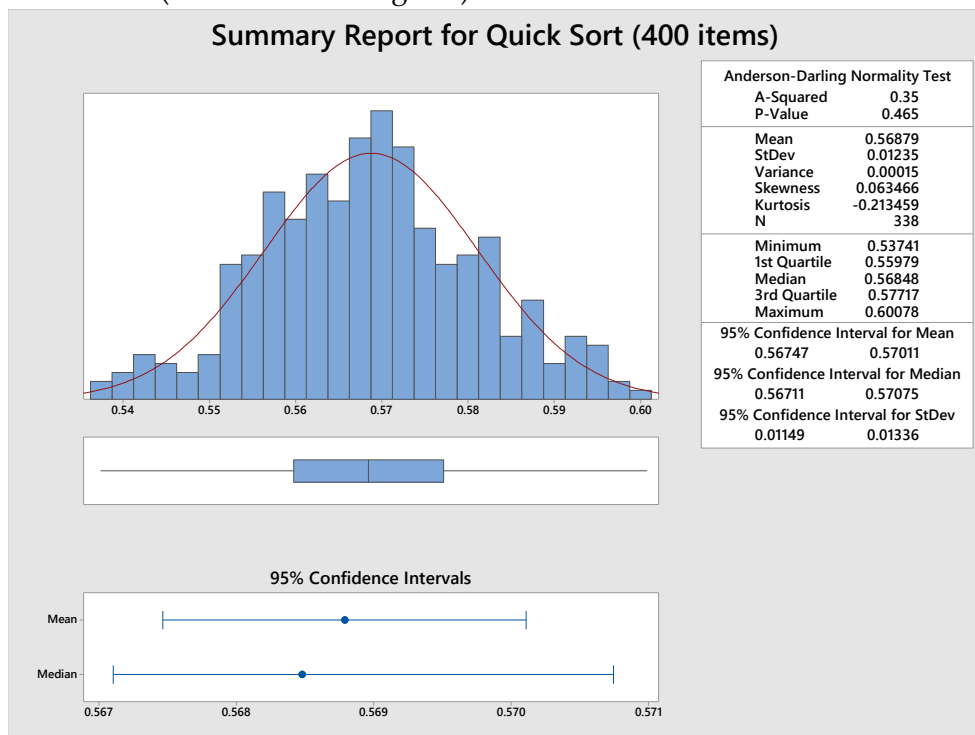


Figure 3: Minitab Graphical summary of the average time of 500 samples of Quick Sorting 400 items. The graphical summary tells us that this data set is normally distributed (Anderson-Darling test). The mean of this set lies around 0.568ms.

Hypothesis Testing:

Paired t-test: Quick Sort times of 200 items and Quick Sort times of 400 items

In Minitab: 1-Sample t-test: Ratio of times between 400 items and 200 items

1. $H_0: \mu = 2.26164$
2. $H_A: \mu > 2.26164$
3. $\alpha = 0.05$
4. Data: Shown above
5. T-Value: 10.86 p-Value = 0

Test

Null hypothesis	$H_0: \mu = 2.26164$
Alternative hypothesis	$H_1: \mu > 2.26164$
T-Value	P-Value
10.86	0.000

6. Since $p < \alpha$, we reject the null hypothesis.
7. Both data sets (200 items and 400 items) meet 1/25/250 since 500 samples of each set were collected. All conditions are met for this test.
8. Result: We are 95% confident that the average ratio of time between the Quick Sort of 400 items and 200 items is greater than 2.26164.

Conclusion:

After conducting the hypothesis test, we are 95% confident that the average ratio of time between the Quick Sort of 400 items and 200 items is greater than 2.26164. The data shows that ratio between two times is slightly greater than $\frac{2n \cdot \log(2n)}{n \cdot \log(n)}$. This increase could have been due to two things, the Quick Sort of 200 items is faster than expected or the Quick Sort of 400 items is longer than expected. The latter option is more probable as some of the computer's energy is devoted to the overhead required to keep the computer functional. Thus, the time of the Quick Sort algorithm may have taken longer than it should ideally. In addition, the times may not be as accurate because each Quick Sort takes under 1ms. More time and computational power would be necessary to run a bigger data set, which would create samples that better represent the actual population. Further experimentation could be conducted to determine how other factors affect the time an algorithm takes to run.

Appendix:

Python Script Used to Generate Data:

```
import random
import time
import statistics

def quick_sort(alist):
    quickSortHelper(alist,0,len(alist)-1)
    return alist

def quickSortHelper(alist,first,last):
    if first<last:

        splitpoint = partition(alist,first,last)

        quickSortHelper(alist,first,splitpoint-1)
        quickSortHelper(alist,splitpoint+1,last)

def partition(alist,first,last):
    pivotvalue = alist[first]

    leftmark = first+1
    rightmark = last

    done = False
    while not done:
        while leftmark <= rightmark and alist[leftmark] <= pivotvalue:
            leftmark = leftmark + 1

        while alist[rightmark] >= pivotvalue and rightmark >= leftmark:
            rightmark = rightmark -1

        if rightmark < leftmark:
            done = True
        else:
            temp = alist[leftmark]
            alist[leftmark] = alist[rightmark]
            alist[rightmark] = temp

    temp = alist[first]
    alist[first] = alist[rightmark]
    alist[rightmark] = temp

    return rightmark
```

```

def generate_list(n): # generates a list of size n
    lst = []
    for _ in range(n): #
        lst.append(random.randint(0, 10000000))
    return lst

def main():
    # records samples times for merge sort and quick sort
    quick_file = "quick_times.txt"
    quick_double_file = "quick_double_times.txt"
    quick = open(quick_file, "w")
    quick_double = open(quick_double_file, "w")
    quick_sample_times = []
    quick_double_sample_times = []
    for _ in range(550): # generates 550 samples
        for _ in range(800): # each sample contains 800 averages
            quick_lst = generate_list(200) # each list contains 200 items

            start = time.time()
            quick_sort(quick_lst) # quick sort on list
            stop = time.time()
            elapsed_time = stop - start
            quick_sample_times.append(elapsed_time * 1000) # append the time(ms)
            # taken to quick sort samples

            quick_lst = generate_list(400) # each list contains 400 items
            start = time.time()
            quick_sort(quick_lst) # quick sort on list
            stop = time.time()
            elapsed_time = stop - start
            quick_double_sample_times.append(elapsed_time * 1000) # append the
            # time(ms) taken to quick sort samples

            quick.write(str(statistics.mean(quick_sample_times)) + '\n')
            quick_double.write(str(statistics.mean(quick_double_sample_times)) +
            '\n')

            quick_sample_times = []
            quick_double_sample_times = []
        quick.close()
        quick_double.close()

if __name__ == '__main__':
    main()

```

Quick Sort Times (ms) - 200 items:

0.25303810834884644	0.31056761741638184	0.2405300736427307	0.25551557540893555
0.23543298244476318	0.26026278734207153	0.24315893650054932	0.22725403308868408
0.228215754032135	0.24240642786026	0.2465084195137024	0.2344679832458496
0.2679350972175598	0.26703566312789917	0.27551352977752686	0.2146393060684204
0.2545657753944397	0.2469545602798462	0.233040452003479	0.2593386173248291
0.2620294690132141	0.23167133331298828	0.23062795400619507	0.2418091893196106
0.252993106842041	0.25037050247192383	0.25590717792510986	0.24926602840423584
0.23798435926437378	0.2544763684272766	0.2570018172264099	0.26549458503723145
0.23214906454086304	0.24917930364608765	0.24214982986450195	0.25470972061157227
0.24215281009674072	0.26062965393066406	0.2579832077026367	0.22822469472885132
0.23443102836608887	0.25450944900512695	0.24563908576965332	0.24288564920425415
0.22563576698303223	0.24449646472930908	0.255565345287323	0.2470150589942932
0.23792237043380737	0.2219870686531067	0.2618032693862915	0.2582356333732605
0.22193193435668945	0.22187650203704834	0.2206510305404663	0.25560617446899414
0.24931639432907104	0.2281409502029419	0.24801969528198242	0.23444831371307373
0.2530527114868164	0.23339956998825073	0.24696111679077148	0.23451447486877441
0.23192346096038818	0.25797873735427856	0.23483484983444214	0.2391698956489563
0.2343517541885376	0.24783194065093994	0.27178674936294556	0.2644196152687073
0.2418622374534607	0.2467745542526245	0.2408677339553833	0.6404614448547363
0.24311572313308716	0.264207124710083	0.25807708501815796	0.6989806890487671
0.24559468030929565	0.21815240383148193	0.23568570613861084	0.5856204032897949
0.23809999227523804	0.23810714483261108	0.23562192916870117	0.6487271189689636
0.23564070463180542	0.23925483226776123	0.24315416812896729	0.5822637677192688
0.2566900849342346	0.2555796504020691	0.24069994688034058	0.6570547819137573
0.24453550577163696	0.2410075068473816	0.2741265296936035	0.47897040843963623
0.24308443069458008	0.23833096027374268	0.22794842720031738	0.544809103012085
0.2356073260307312	0.24669915437698364	0.23057013750076294	0.5254602432250977
0.24935513734817505	0.25688886642456055	0.24558722972869873	0.465659499168396
0.2319207787513733	0.28143495321273804	0.23065060377120972	0.49261122941970825
0.23819297552108765	0.2815958857536316	0.25443851947784424	0.45606791973114014
0.22833555936813354	0.23438215255737305	0.25538861751556396	0.4440408945083618
0.2443888783454895	0.26073694229125977	0.24942278861999512	0.3597685694694519
0.2389243245124817	0.25929808616638184	0.25792956352233887	0.3913119435310364
0.24185597896575928	0.2456161379814148	0.2543279528617859	0.3248029947280884
0.26914745569229126	0.23060113191604614	0.2607494592666626	0.31994372606277466
0.24675577878952026	0.24008750915527344	0.23923516273498535	0.36869943141937256
0.2691170573234558	0.2505779266357422	0.2531009912490845	0.3489851951599121
0.23562699556350708	0.24795055389404297	0.2394309639930725	0.26614516973495483
0.24299174547195435	0.24548888206481934	0.26900798082351685	0.2332514524459839
0.248071551322937	0.24795830249786377	0.2268904447555542	0.27812033891677856
0.2231636643409729	0.25941669940948486	0.2616792917251587	0.21270990371704102
0.23936063051223755	0.22423714399337769	0.2517509460449219	0.2364635467529297
0.24436742067337036	0.24184972047805786	0.24925380945205688	0.2105712890625
0.24188220500946045	0.23936450481414795	0.23672670125961304	0.23348838090896606
0.24680167436599731	0.22573590278625488	0.2793887257575989	0.2931573987007141
0.2765420079231262	0.25453686714172363	0.275704562664032	0.22794485092163086
0.25566548109054565	0.24056166410446167	0.271778404712677	0.25862276554107666
0.24826794862747192	0.2630993723869324	0.22159308195114136	0.23797988891601562

0.25054484605789185	0.2515745162963867	0.25935858488082886	0.2530065178871155
0.21942675113677979	0.24077415466308594	0.2455475926399231	0.24935811758041382
0.21193891763687134	0.2445685863494873	0.2428758144378662	0.25186002254486084
0.2060145139694214	0.2503138780593872	0.22675514221191406	0.23805618286132812
0.30612409114837646	0.23927241563796997	0.22920608520507812	0.24431735277175903
0.2379673719406128	0.252135694026947	0.24427920579910278	0.2520400285720825
0.27126312255859375	0.2396625280380249	0.233117938041687	0.25173187255859375
0.24769365787506104	0.2543172240257263	0.24804413318634033	0.2329719066619873
0.33544600009918213	0.2468070387840271	0.2256762981414795	0.2362227439880371
0.255606472492218	0.23933768272399902	0.24324476718902588	0.26526421308517456
0.22343814373016357	0.246792733669281	0.22682517766952515	0.25412052869796753
0.24365752935409546	0.2345523238182068	0.24810343980789185	0.2418091893196106
0.27887046337127686	0.24567663669586182	0.2404242753982544	0.3088781237602234
0.25880903005599976	0.24289458990097046	0.2581319212913513	0.29569536447525024
0.26759475469589233	0.2343478798866272	0.24313300848007202	0.2666664123535156
0.2791261672973633	0.24398267269134521	0.2693396806716919	0.259246826171875
0.25180429220199585	0.219479501247406	0.22939354181289673	0.25274962186813354
0.24896055459976196	0.24319350719451904	0.24693071842193604	0.25807738304138184
0.2297496795654297	0.23429900407791138	0.23432523012161255	0.2455306053161621
0.27537792921066284	0.2231106162071228	0.2429857850074768	0.23047834634780884
0.28159916400909424	0.25303661823272705	0.24406492710113525	0.24948924779891968
0.27340978384017944	0.22957831621170044	0.24801671504974365	0.24703919887542725
0.22570550441741943	0.2457648515701294	0.2429121732711792	0.2593681216239929
0.24933665990829468	0.22432535886764526	0.22062689065933228	0.27081936597824097
0.27183204889297485	0.250527560710907	0.22808313369750977	0.2499234676361084
0.25175660848617554	0.2491489052772522	0.23575693368911743	0.24686425924301147
0.2511635422706604	0.24940669536590576	0.2394390106201172	0.24545341730117798
0.23037970066070557	0.2392229437828064	0.23818165063858032	0.2457141876220703
0.27905821800231934	0.23663997650146484	0.24818986654281616	0.24905681610107422
0.2315455675125122	0.23191243410110474	0.2803066372871399	0.25684237480163574
0.6346529722213745	0.24568110704421997	0.2605125308036804	0.2495160698890686
0.27155280113220215	0.23437172174453735	0.2555385231971741	0.25802820920944214
0.23695677518844604	0.22928714752197266	0.24425536394119263	0.22904425859451294
0.2472686767578125	0.26522397994995117	0.24324148893356323	0.2406635880470276
0.23711085319519043	0.22407114505767822	0.2618005871772766	0.24780452251434326
0.23326009511947632	0.24208664894104004	0.25057166814804077	0.23334771394729614
0.25433868169784546	0.22811055183410645	0.2442324161529541	0.2403894066810608
0.22686690092086792	0.2368423342704773	0.24800211191177368	0.2356222719192505
0.24434059858322144	0.26547104120254517	0.24292826652526855	0.2518153190612793
0.26178866624832153	0.2516961097717285	0.2304437756538391	0.2231389284133911
0.2367684245109558	0.22571980953216553	0.26014596223831177	0.23195713758468628
0.23572593927383423	0.24814307689666748	0.2293461561203003	0.26146262884140015
0.2671054005622864	0.26462316513061523	0.22711753845214844	0.27480393648147583
0.26010215282440186	0.25693565607070923	0.24322420358657837	0.2333703637123108
0.2241736650466919	0.2367207407951355	0.23159503936767578	0.2631351351737976
0.23184865713119507	0.2466145157814026	0.2318677306175232	0.2495601773262024
0.2344098687171936	0.2480316162109375	0.25025516748428345	0.24706900119781494
0.2256685495376587	0.2583104372024536	0.2393379807472229	0.23192882537841797
0.24446189403533936	0.24576663970947266	0.25058120489120483	0.25809794664382935

0.23685574531555176
0.23437172174453735
0.2431708574295044
0.23191511631011963
0.24059385061264038
0.24198144674301147
0.237007737159729
0.2256295084953308
0.24688869714736938
0.24917840957641602
0.23551791906356812

Quick Sort Times (ms) - 400 items

0.5856546759605408	0.6614884734153748	0.5867370963096619	0.5810496211051941
0.595778226852417	0.6134042143821716	0.5785909295082092	0.5806711316108704
0.5871176719665527	0.6410232186317444	0.5649900436401367	0.5700331926345825
0.5623263120651245	0.5788016319274902	0.5708089470863342	0.570594072341919
0.5834001302719116	0.569722056388855	0.5873608589172363	0.5611363053321838
0.5496343970298767	0.5623149871826172	0.6046938896179199	0.5934825539588928
0.5773058533668518	0.5646565556526184	0.5667009949684143	0.5822905898094177
0.5885019898414612	0.5870741605758667	0.5918154120445251	0.5823346972465515
0.5732977390289307	0.5898872017860413	0.5731773376464844	0.5704590678215027
0.569816529750824	0.5683755874633789	0.5573248863220215	0.5750441551208496
0.5669951438903809	0.581144392490387	0.5684882402420044	0.5825629830360413
0.5534419417381287	0.5720165371894836	0.5559566617012024	0.5811440944671631
0.5798158049583435	0.5583140254020691	0.5585065484046936	0.5771949887275696
0.5698341131210327	0.5536317825317383	0.5622711777687073	0.5572500824928284
0.569705069065094	0.552261471748352	0.5535167455673218	0.5535906553268433
0.5535605549812317	0.570797324180603	0.5748143792152405	0.5770677328109741
0.5397149920463562	0.584648847579956	0.5725613236427307	0.5787327885627747
0.5747175216674805	0.5739352107048035	0.5811172723770142	0.5761563777923584
0.5734413862228394	0.5623871088027954	0.572267472743988	1.6275927424430847
0.5597439408302307	0.5709549784660339	0.5773758888244629	1.3196980953216553
0.5584582686424255	0.564788281917572	0.5622181296348572	1.4153888821601868
0.5622488260269165	0.5810749530792236	0.5720207095146179	1.5374305844306946
0.5597478151321411	0.5658045411109924	0.5745702981948853	1.468941867351532
0.5687263607978821	0.5570900440216064	0.5666917562484741	1.4655432105064392
0.5710187554359436	0.5633634328842163	0.5582904815673828	1.4312288165092468
0.5723190307617188	0.5609714984893799	0.5584162473678589	1.3302633166313171
0.5684813857078552	0.5761590600013733	0.5460178852081299	1.162165105342865
0.5559968948364258	0.6306436657905579	0.5535686016082764	1.0114750266075134
0.5584922432899475	0.7005524635314941	0.5671900510787964	1.0756593942642212
0.5720552802085876	0.6230899691581726	0.5709615349769592	0.9781444072723389
0.5669811367988586	0.5755993723869324	0.5788758397102356	0.9888032078742981
0.5698889493942261	0.5710822343826294	0.5834782123565674	0.9446218609809875
0.5662590265274048	0.5818834900856018	0.5797222256660461	0.8646905422210693
0.5745178461074829	0.5736339092254639	0.5917865037918091	0.8106672763824463
0.5709266662597656	0.5771580338478088	0.5657455325126648	0.7533538341522217
0.567186176776886	0.5711549520492554	0.5844709277153015	0.7667171955108643
0.5596563220024109	0.603296160697937	0.5643957853317261	0.6474661827087402
0.5671325325965881	0.5772089958190918	0.5899316072463989	0.5844810605049133
0.5788189172744751	0.5672025680541992	0.5758768320083618	0.5566757917404175
0.5547261238098145	0.5623167753219604	0.5836844444274902	0.5823120474815369
0.5560484528541565	0.5856698751449585	0.5962377786636353	0.5668929219245911
0.5585071444511414	0.5873090028762817	0.5919620394706726	0.5631232261657715
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