Tree-Structured Indexes

Chapter 9

Outline

- Introduction
- Intuition for Tree Indexes
- Indexed sequential access method (IASM)
- B+ Trees: A dynamic Index Structure
- Search
- Insert
- Delete
- Duplicates
- B+ Trees in practice

Introduction

- *As for any index, 3 alternatives for data entries* **k***:
  - Data record with key value **k**
  - <**k**, rid of data record with search key value **k**>
  - <**k**, list of rids of data records with search key **k**>
- Choice is orthogonal to the *indexing technique* used to locate data entries **k***.
- Tree-structured indexing techniques support efficient insertion, deletion, *range searches* and *equality searches*.
- *ISAM (Indexed Sequetial Access Method)*: static structure;
  - Efficient when the file is not frequently updated and unsuitable for files which grow and shrink.
- *B+ tree*: dynamic, adjusts gracefully under inserts and deletes.
- In both structures, leaf pages contain data entries.

Intuition

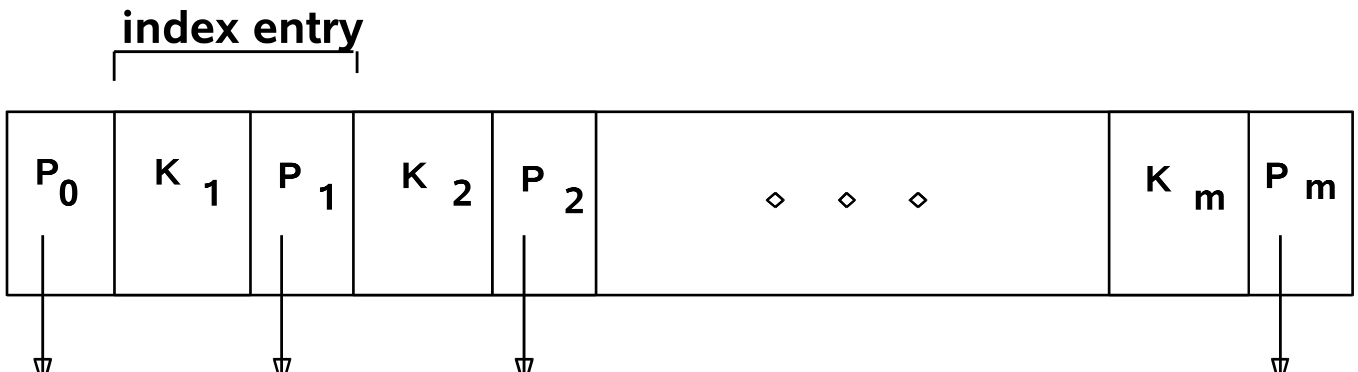
- *``Find all students with gpa > 3.0''*
  - If data is in sorted file, do binary search to find first such student, then scan to find others.
  - Cost of binary search can be quite high.
  - Cost is proportional to number of pages fetched.
- Simple idea: Create an `index' file with one record per page in the original data file, of the form <first key on the page, pointer to page)

☛ *Can do binary search on (smaller) index file!*

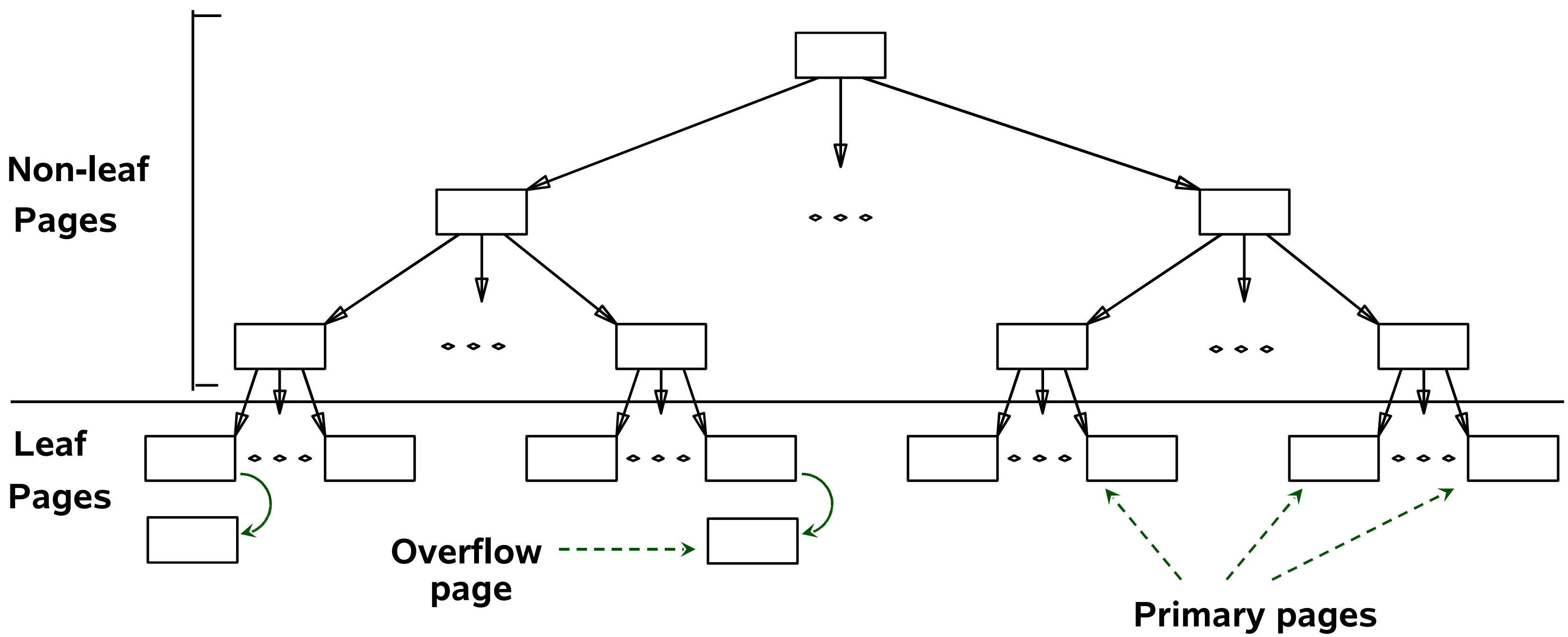
Intuition…

- Index file is much smaller than the data file
- Binary search of index file is much faster than the binary search of data file.
- However, binary search of index file is still expensive.
- Large index file motivates the tree index idea.
  - Applying the idea of auxiliary structure on the collection of index records, until smallest structure is in one page.
  - It results into a tree!
- Locating a record involves a traversal from the root to leaf, with one I/O per level.
- Root will be in the buffer pool
- Given a typical fan-out value (over 100), trees rarely have more than 3-4 levels.

ISAM

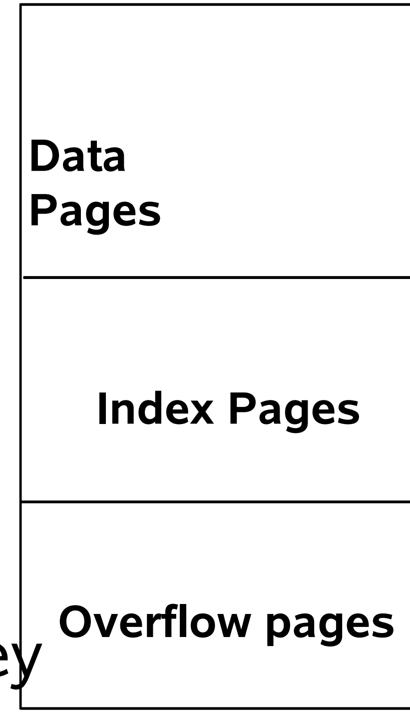


- Index file may still be quite large. But we can apply the idea repeatedly!



☛ *Leaf pages contain data entries.*

Comments on ISAM

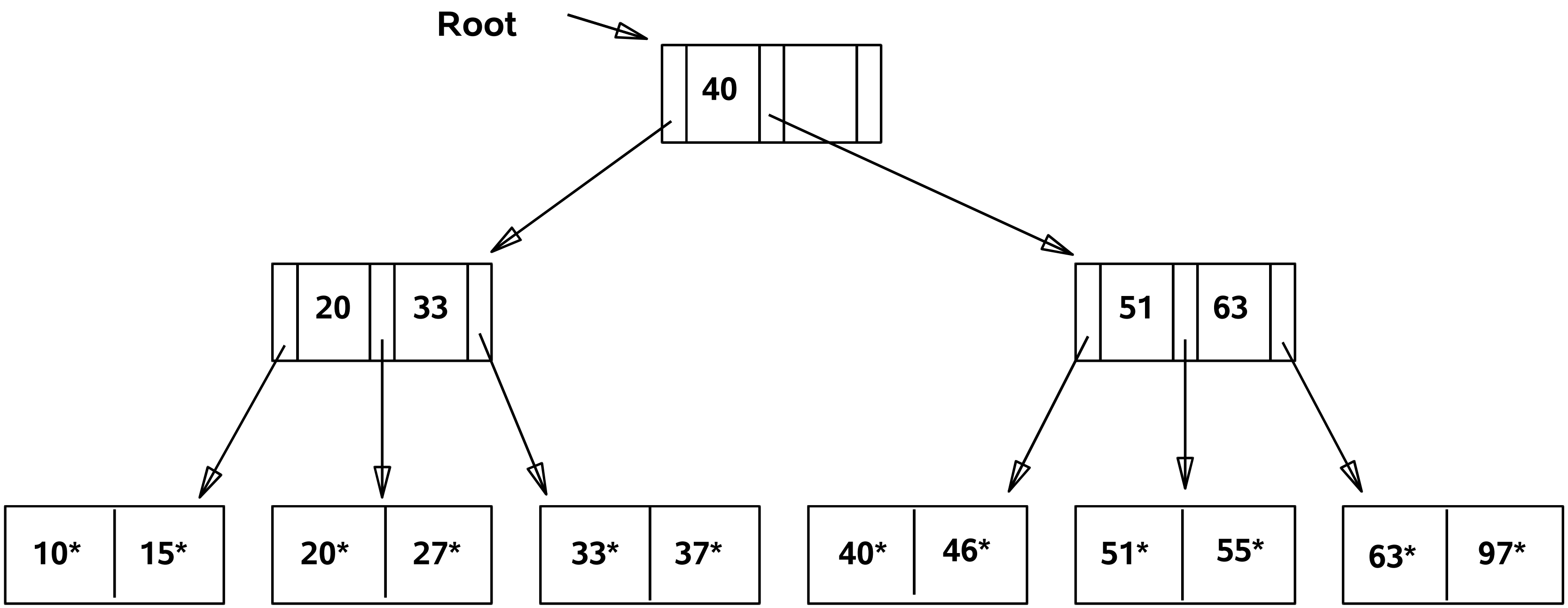


- *File creation*: Leaf (data) pages allocated sequentially, sorted by search key; then index pages allocated, then space for overflow pages.
- *Index entries*: <search key value, page id>; they \`direct' search for *data entries*, which are in leaf pages.
- *Search*: Start at root; use key comparisons to go to leaf. Cost $\propto \log_F N$; F = # entries/index pg, N = # leaf pgs
- *Insert*: Find leaf data entry belongs to, and put it there.
- *Delete*: Find and remove from leaf; if empty overflow page, de-allocate.

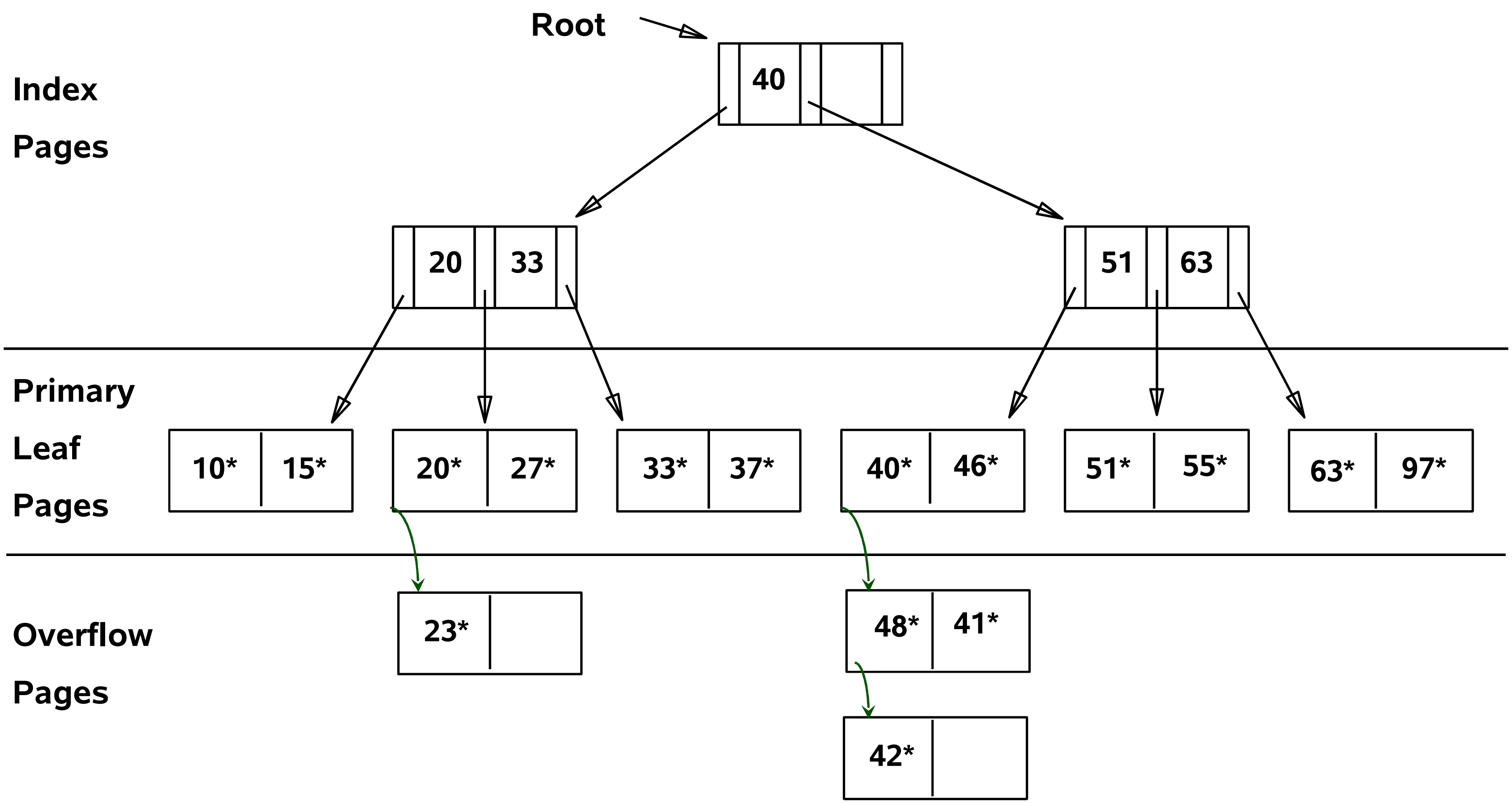
Static tree structure: *inserts/deletes affect only leaf pages.*

Example ISAM Tree

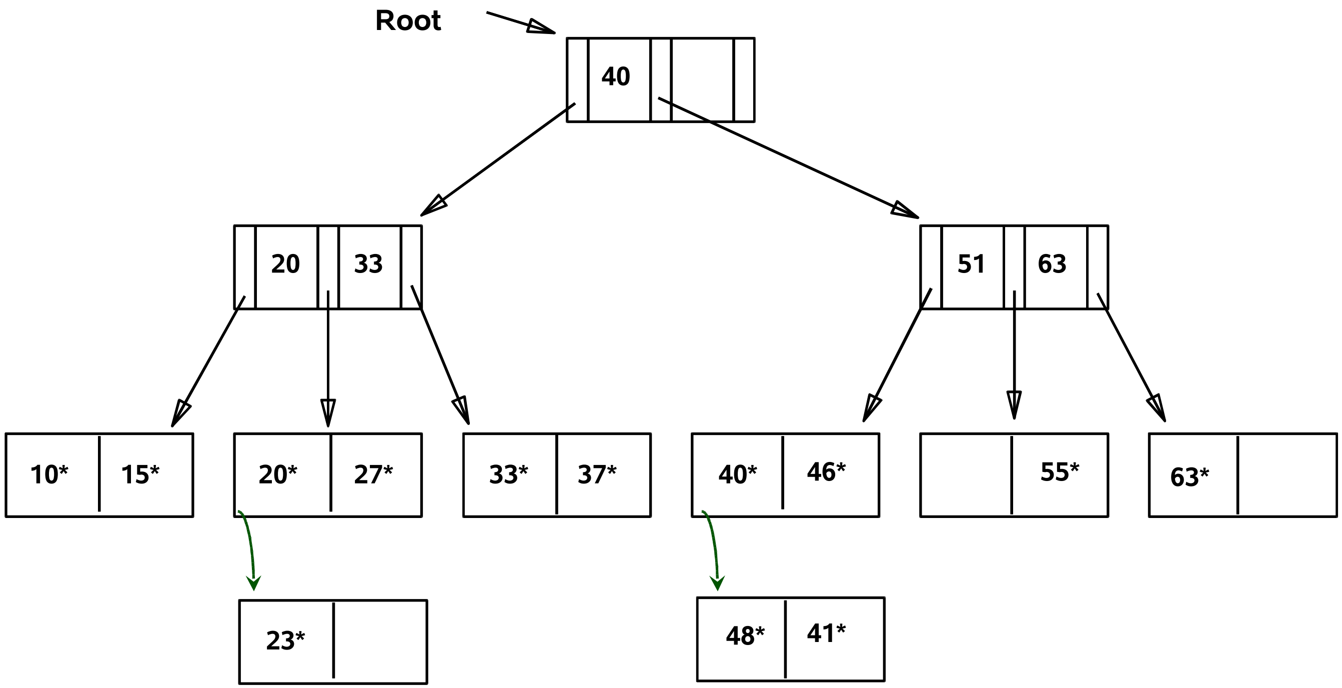
- Each node can hold 2 entries; no need for `next-leaf-page' pointers. (Why?)



After Inserting 23*, 48*, 41*, 42* ...



... Then Deleting 42*, 51*, 97*



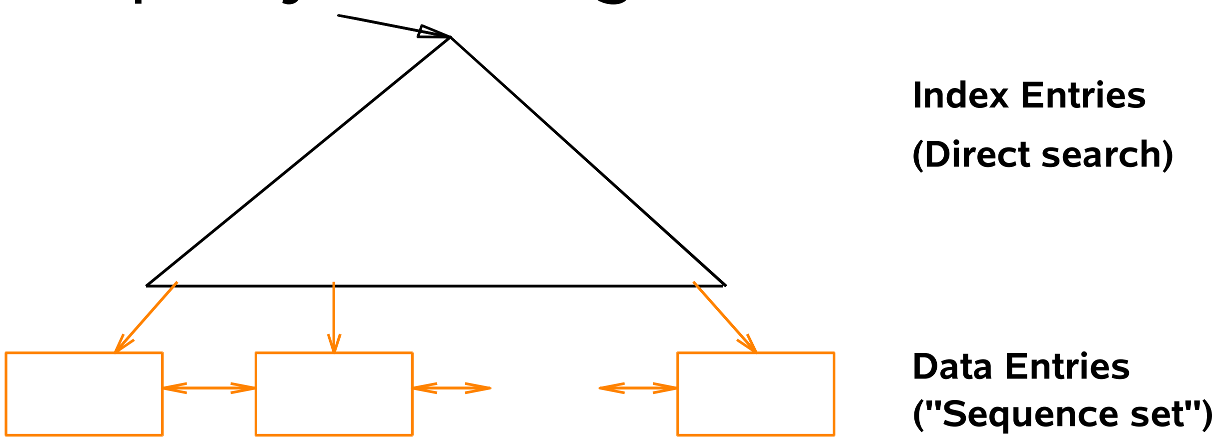
☛ *Note that 51* appears in index levels, but not in leaf!*

ISAM--Discussion

- More inserts and deletes results into long overflow chains, which increases the retrieval time because overflow chains have to searched.
- In case of concurrent searches, it offers significant advantages.
  - Need not loch index pages as they are not modified.
  - If the data distribution size is static with no overflow chains, IASM may be preferable to B+ trees.

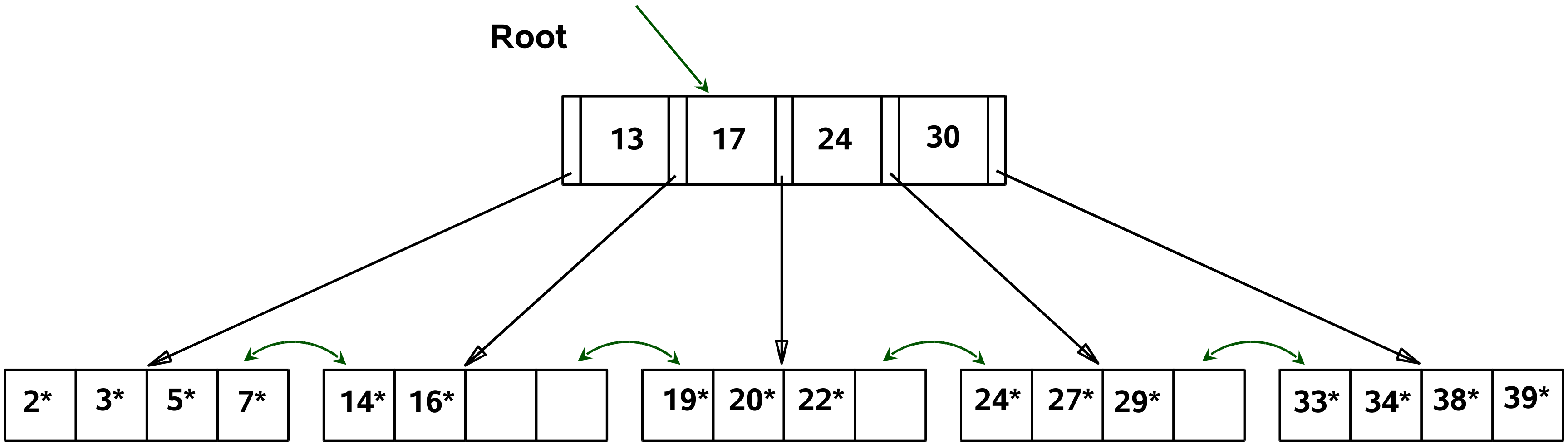
B+ Tree: Most Widely Used Index

- Insert/delete at $\log_F N$ cost; keep tree *height-balanced*. (F = fanout, N = # leaf pages)
- Minimum 50% occupancy (except for root). Each node contains **d** <= *m* <= 2**d** entries. The parameter **d** is called the *order* of the tree.
- Supports equality and range-searches efficiently.



Example B+ Tree

- Search begins at root, and key comparisons direct it to a leaf (as in ISAM).
- Search for 5*, 15*, all data entries >= 24* ...



☛ *Based on the search for 15*, we <u>know</u> it is not in the tree!*

B+ Trees in Practice

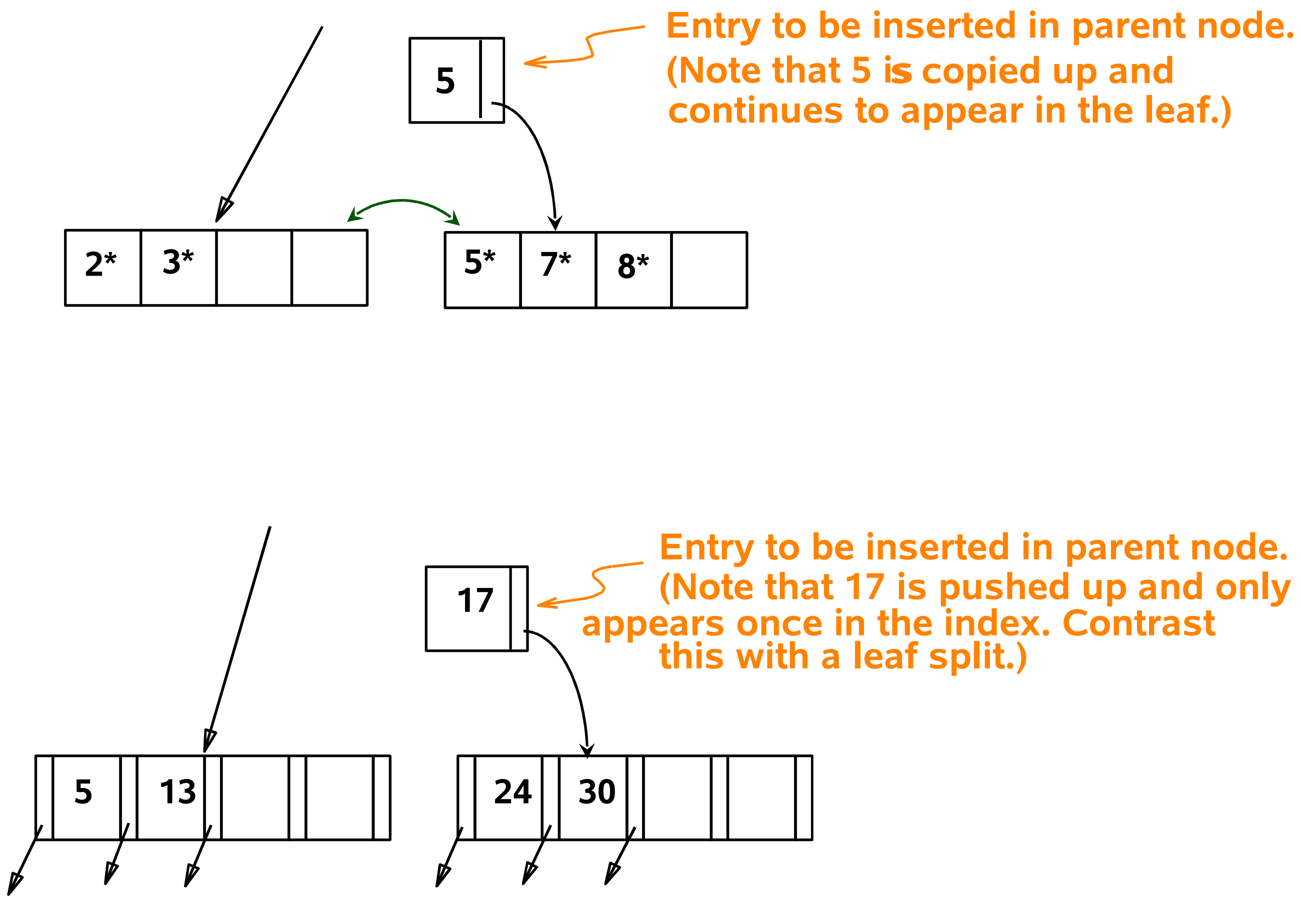
- Typical order: 100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Can often hold top levels in buffer pool:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

Inserting a Data Entry into a B+ Tree

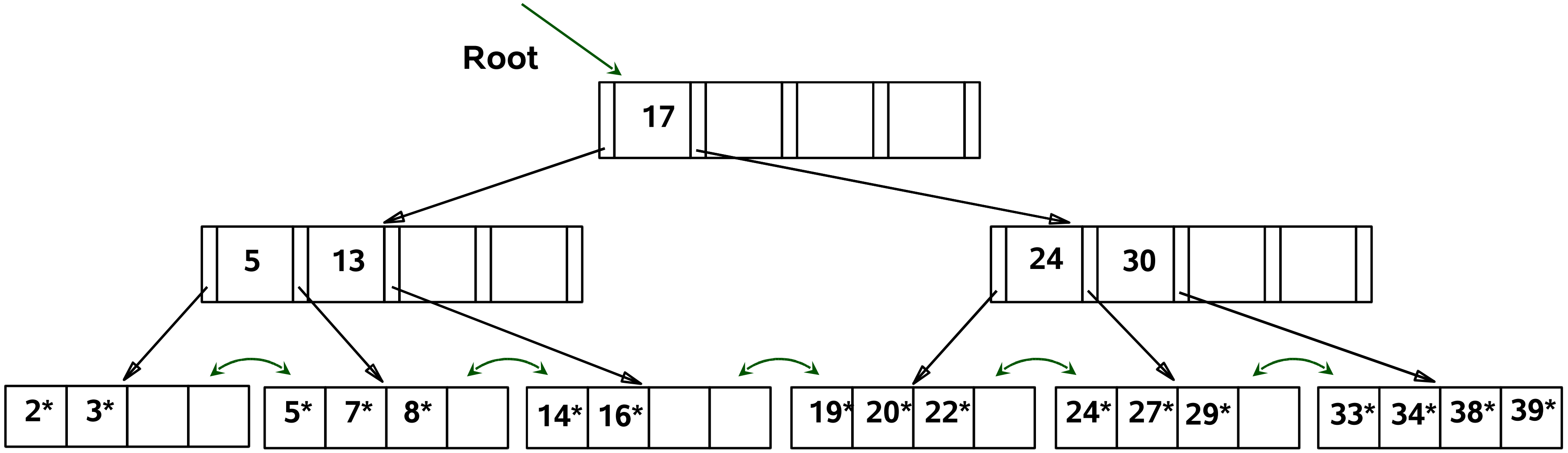
- Find correct leaf *L*.
- Put data entry onto *L*.
  - If *L* has enough space, *done*!
  - Else, must *split* *L (into L and a new node L2)*
    - Redistribute entries evenly, **copy up** middle key.
    - Insert index entry pointing to *L2* into parent of *L*.
- This can happen recursively
  - To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)
- Splits "grow" tree; root split increases height.
  - Tree growth: gets *wider* or *one level taller at top.*

Inserting 8* into Example B+ Tree

- Observe how minimum occupancy is guaranteed in both leaf and index pg splits.
- Note difference between *copy-up* and *push-up*; be sure you understand the reasons for this.



Example B+ Tree After Inserting 8*



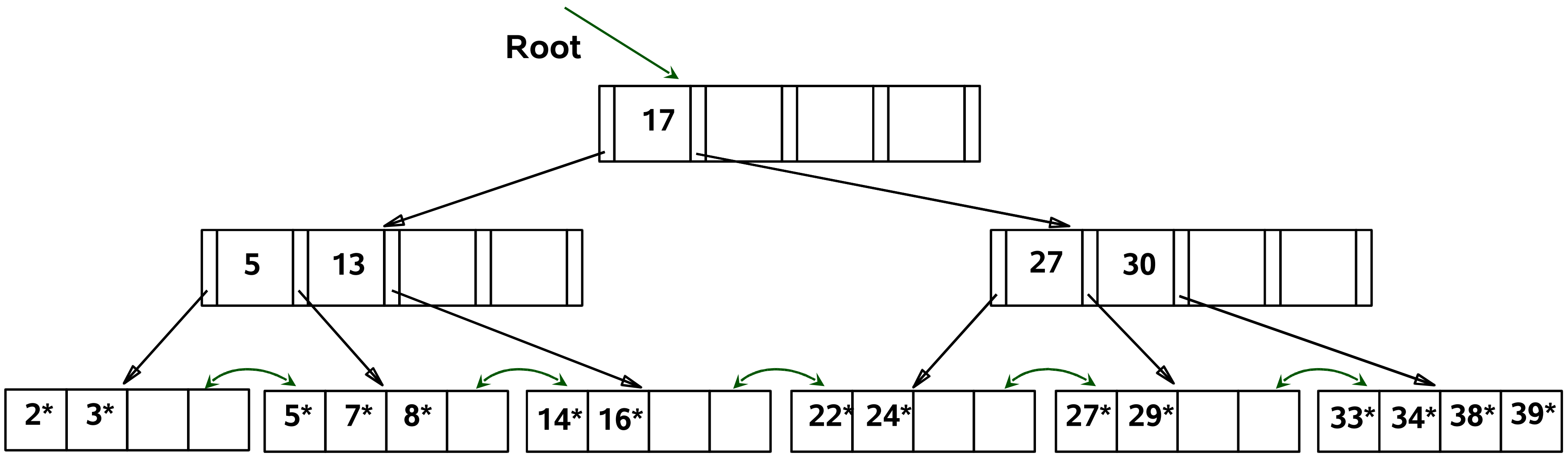
Notice that root was split, leading to increase in height.

In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

Deleting a Data Entry from a B+ Tree

- Start at root, find leaf *L* where entry belongs.
- Remove the entry.
  - If L is at least half-full, *done!*
  - If L has only **d-1** entries,
    - Try to re-distribute, borrowing from *sibling* *(adjacent node with same parent as L)*.
    - If re-distribution fails, *merge* *L* and sibling.
- If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.
- Merge could propagate to root, decreasing height.

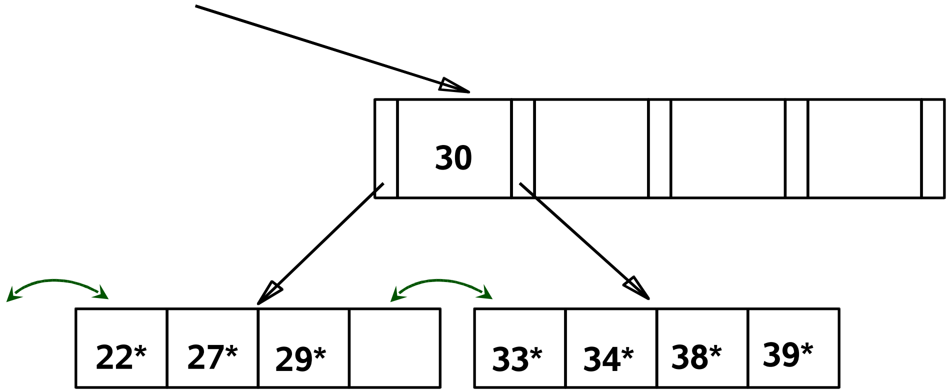
Example Tree After (Inserting 8*, Then) Deleting 19* and 20* ...

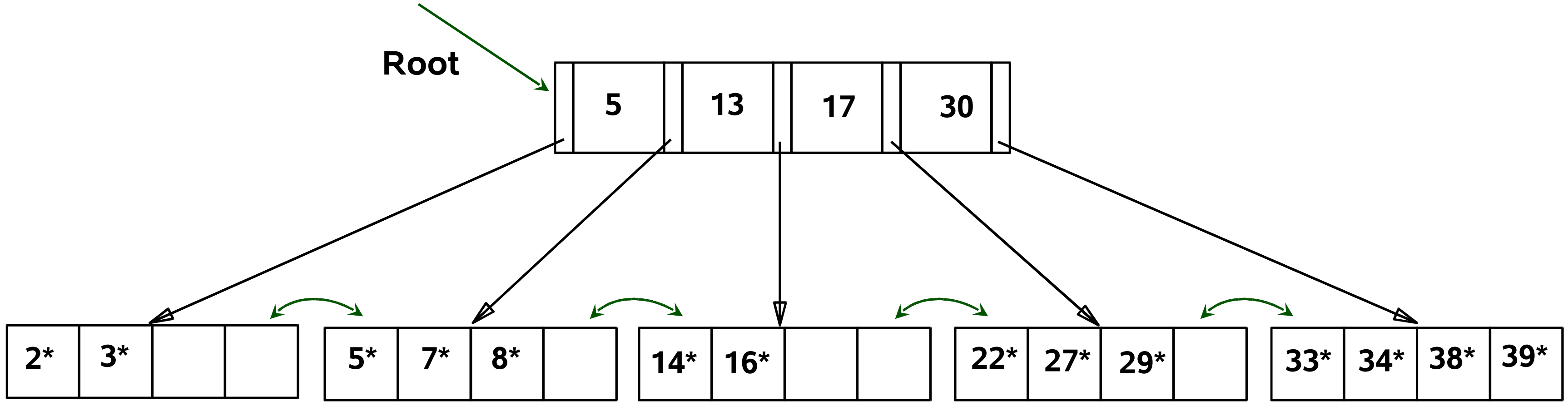


- Deleting 19* is easy.
- Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.

... And Then Deleting 24*

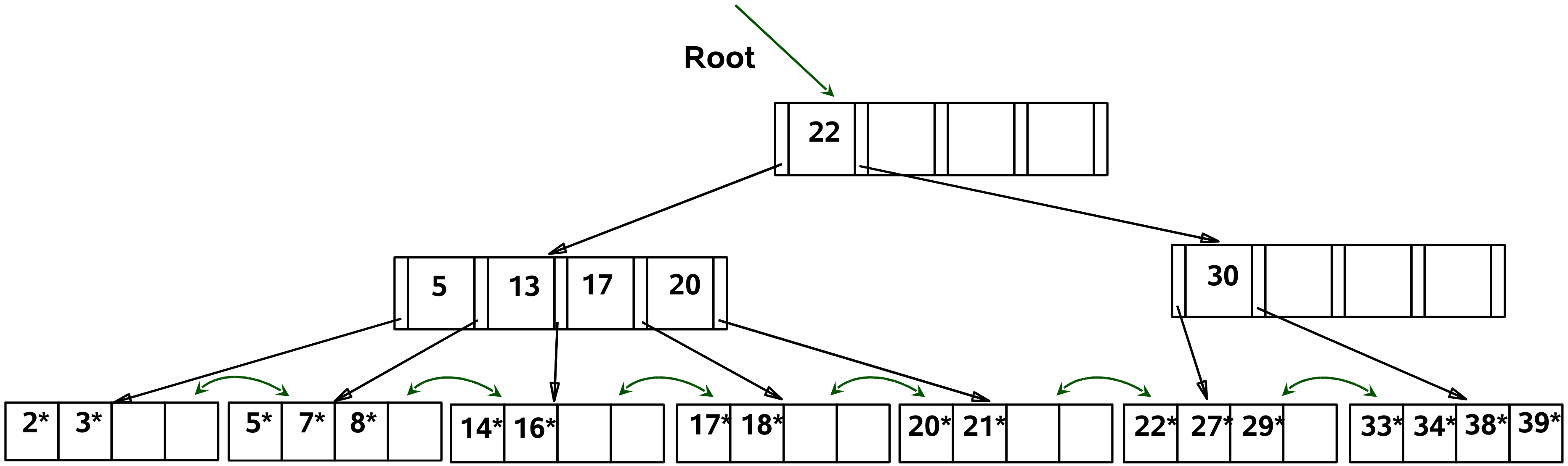
- Must merge.
- Observe *`toss'* of index entry (on right), and *`pull down'* of index entry (below).





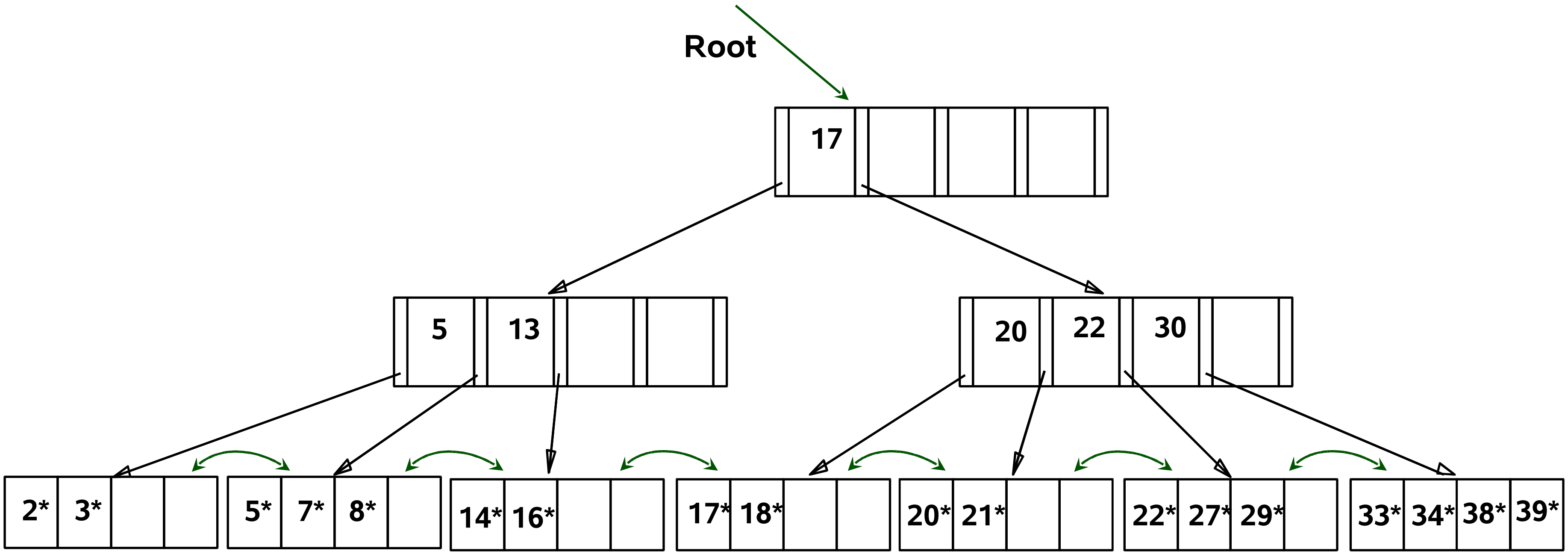
Example of Non-leaf Re-distribution

- Tree is shown below *during deletion* of 24*. (What could be a possible initial tree?)
- In contrast to previous example, can re-distribute entry from left child of root to right child.



After Re-distribution

- Intuitively, entries are re-distributed by *`pushing through'* the splitting entry in the parent node.
- It suffices to re-distribute index entry with key 20; we've re-distributed 17 as well for illustration.

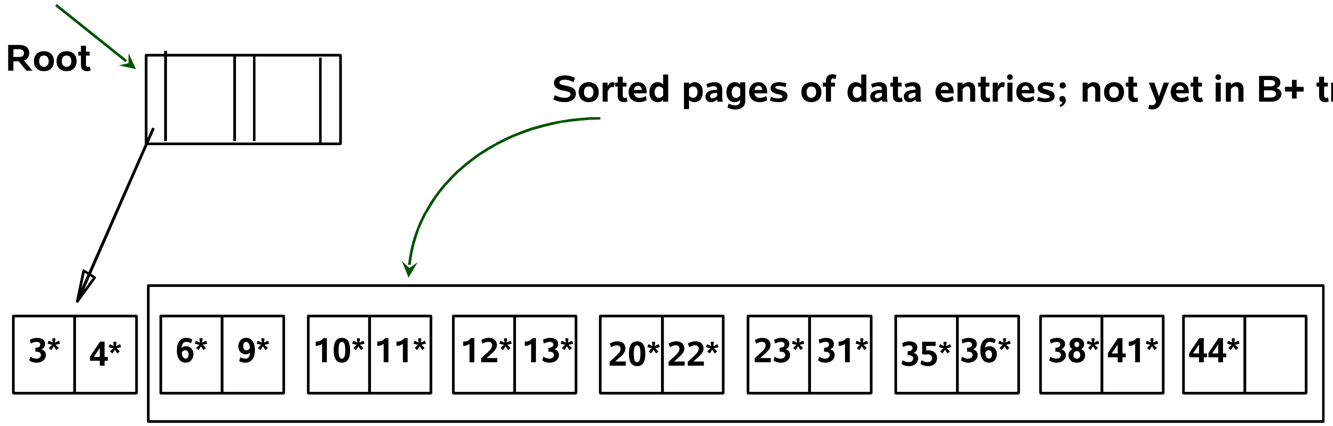


Prefix Key Compression

- Important to increase fan-out. (Why?)
- Key values in index entries only `direct traffic'; can often compress them.
  - E.g., If we have adjacent index entries with search key values *Dannon Yogurt*, *David Smith* and *Devarakonda Murthy*, we can abbreviate *David Smith* to *Dav*. (The other keys can be compressed too ...)
    - Is this correct? Not quite! What if there is a data entry *Davey Jones*? (Can only compress *David Smith* to *Davi*)
    - In general, while compressing, must leave each index entry greater than every key value (in any subtree) to its left.
- Insert/delete must be suitably modified.

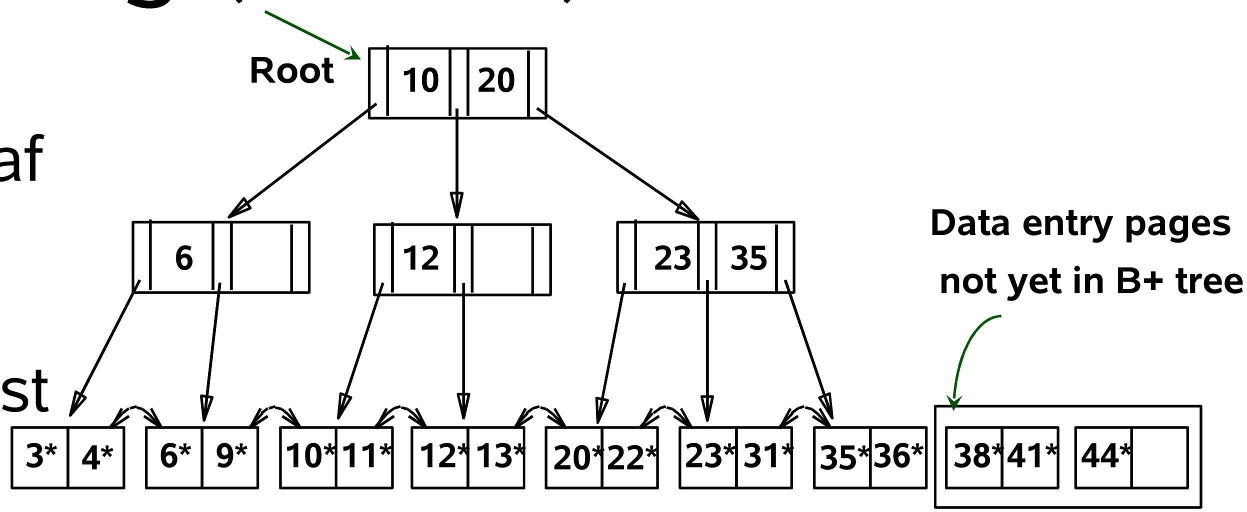
Bulk Loading of a B+ Tree

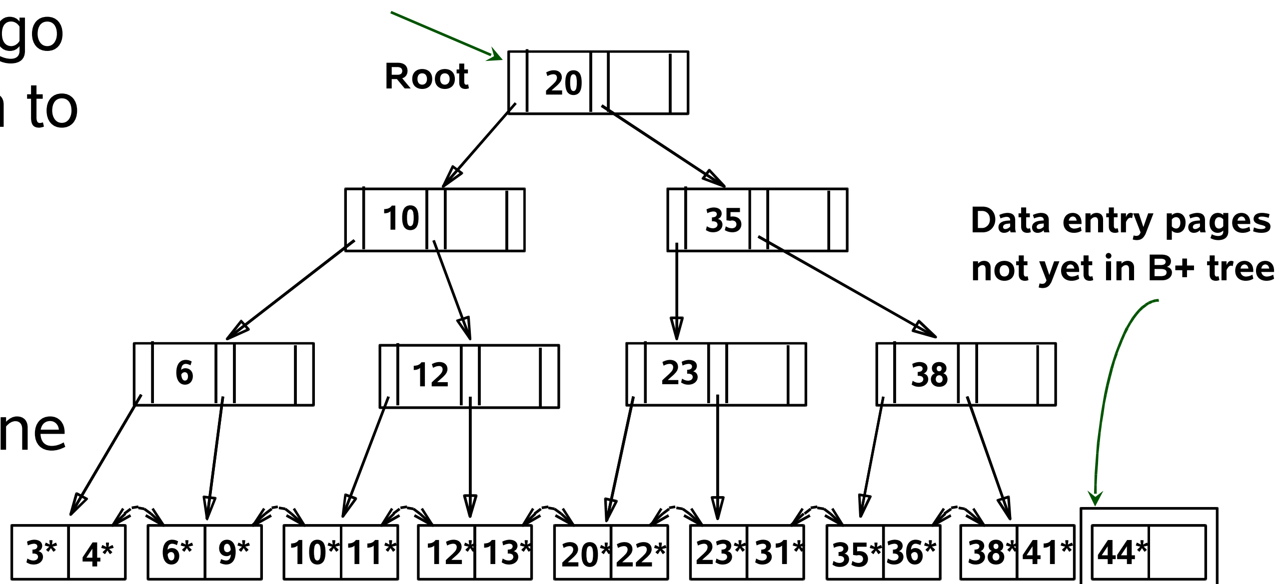
- If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.
- *Bulk Loading* can be done much more efficiently.
- *Initialization*: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk Loading (Contd.)

- Index entries for leaf pages always entered into right-most index page just above leaf level. When this fills up, it splits. (Split may go up right-most path to the root.)
- Much faster than repeated inserts, especially when one considers locking!





Summary of Bulk Loading

- Option 1: multiple inserts.
  - Slow.
  - Does not give sequential storage of leaves.
- Option 2: *Bulk Loading*
  - Has advantages for concurrency control.
  - Fewer I/Os during build.
  - Leaves will be stored sequentially (and linked, of course).
  - Can control "fill factor" on pages.

A Note on `Order'

- *Order* (**d**) concept replaced by physical space criterion in practice (`*at least half-full*').
  - Index pages can typically hold many more entries than leaf pages.
  - Variable sized records and search keys mean different nodes will contain different numbers of entries.
  - Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Summary

- Tree-structured indexes are ideal for range-searches, also good for equality searches.
- ISAM is a static structure.
  - Only leaf pages modified; overflow pages needed.
  - Overflow chains can degrade performance unless size of data set and data distribution stay constant.
- B+ tree is a dynamic structure.
  - Inserts/deletes leave tree height-balanced; $\log_F N$ cost.
  - High fanout (**F**) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.

Summary (Contd.)

  - Typically, 67% occupancy on average.
  - Usually preferable to ISAM, modulo *locking* considerations; adjusts to growth gracefully.
  - If data entries are data records, splits can change rids!

- Key compression increases fanout, reduces height.
- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- Most widely used index in database management systems because of its versatility. One of the most optimized components of a DBMS.