Schema Refinement and Normal Forms

References: (i) Chapter 19 of Ramakrishnan Book

(ii) Chapter 7 of Silberschatz book

Outline

- First Normal Form
- Evils of Redundancy
- Functional dependencies
  - Definition of BCNF and 3NF
- Functional Dependency theory
  - Algorithms for BCNF and 3NF
- Multi-valued dependencies and 4 NF
- Further normal forms (5NF and Domain-Key Normal form
- Definition of 2NF
- Summary

First Normal Form

- Domain is atomic if its elements are considered to be indivisible units
  - Examples of non-atomic domains:
    - Set of names, composite attributes
    - Identification numbers like CS101 that can be broken up into parts
- A relational schema R is in first normal form if the domains of all attributes of R are atomic
- Non-atomic values complicate storage and encourage redundant (repeated) storage of data
  - E.g. Set of accounts stored with each customer, and set of owners stored with each account
  - We assume all relations are in first normal form

First Normal Form (Contd.)

- Atomicity is actually a property of how the elements of the domain are used.
  - E.g. Strings would normally be considered indivisible
  - Suppose that students are given roll numbers which are strings of the form *CS0012* or *EE1127*
  - If the first two characters are extracted to find the department, the domain of roll numbers is not atomic.
  - Doing so is a bad idea: leads to encoding of information in application program rather than in the database.

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The Evils of Redundancy

- *Redundancy* is at the root of several problems associated with relational schemas:
  - redundant storage, insert/delete/update anomalies
- Integrity constraints, in particular *functional dependencies*, can be used to identify schemas with such problems and to suggest refinements.
- Main refinement technique: *decomposition* (replacing ABCD with, say, AB and BCD, or ACD and ABD).
- Decomposition should be used judiciously:
  - Is there reason to decompose a relation?
  - What problems (if any) does the decomposition cause?

Problems of first normal form

- INSERT anomalies: We can not insert the fact that a particular supplier is located in a particular city until that supplier supplies at least one part.
  - It does not allow to represent the fact that S5 is located in Paris unless S5 supplies some part.
- DELETE anomalies: If we delete the only FIRST tuple for a particular supplier , we destroy
  - Shipment connecting that supplier to part
  - Information that the supplier is located in a particular city.
  - Example: If we delete the fact that S3 supplies P2, we loose the information that S3 is located in Paris.
- UPDATE anomalies: Redundancy creates update problems.
  - If supplier changes city either we have to update every tuple or produce inconsistent result.

S#	STATUS	CITY	P#	QTY
S1	20	London	P1	300
S1	20	London	P2	200
S1	20	London	P3	400
S1	20	London	P4	200
S!	20	London	P5	100
S1	20	London	P6	100
S2	10	Paris	P1	300
S2	10	Paris	P2	400
S3	10	Paris	P2	200
S4	20	London	P2	200
S4	20	London	P4	300
S4	20	London	P5	400

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Goal — Devise a Theory for the Following

- Decide whether a particular relation R is in "good" form.
- In the case that a relation R is not in "good" form, decompose it into a set of relations $\{R_1, R_2, ..., R_n\}$ such that
  - each relation is in good form
  - the decomposition is a lossless-join decomposition
- Our theory is based on:
  - functional dependencies
  - multivalued dependencies

Functional Dependencies

- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.
- A functional dependency is a generalization of the notion of a *key.*

Functional Dependencies (Cont.)

- Let R be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The functional dependency

$$\alpha \rightarrow \beta$$

holds on R if and only if for any legal relations r(R), whenever any two tuples t_1 and t_2 of r agree on the attributes α, they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

- Example: Consider $r(A, B)$ with the following instance of r.

1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold.

Functional Dependencies (Cont.)

- K is a superkey for relation schema R if and only if $K \rightarrow R$
- K is a candidate key for R if and only if
  - $K \rightarrow R$, and
  - for no $\alpha \subset K$, $\alpha \rightarrow R$
- Functional dependencies allow us to express constraints that cannot be expressed using superkeys. Consider the schema:

  bor_loan = (*<u>customer_id, loan_number</u>, amount*).

  We expect this functional dependency to hold:

  loan_number → *amount*

  but would not expect the following to hold:

  amount → *customer_name*

Use of Functional Dependencies

- We use functional dependencies to:
  - test relations to see if they are legal under a given set of functional dependencies.
    - If a relation *r* is legal under a set *F* of functional dependencies, we say that *r* satisfies *F.*
  - specify constraints on the set of legal relations
    - We say that *F* holds on *R* if all legal relations on *R* satisfy the set of functional dependencies *F.*
- Note: A specific instance of a relation schema may satisfy a functional dependency even if the functional dependency does not hold on all legal instances.
  - For example, a specific instance of *loan* may, by chance, satisfy
    $amount \rightarrow customer\_name.$

Functional Dependencies (Cont.)

- *A* functional dependency is trivial if it is satisfied by all instances of a relation
  - Example*:*
    - *customer_name, loan_number* $\rightarrow$ *customer_name*
    - *customer_name* $\rightarrow$ *customer_name*
  - In general, $\alpha \rightarrow \beta$ is trivial if $\beta \subseteq \alpha$

Closure of a Set of Functional Dependencies

- Given a set *F* of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of all functional dependencies logically implied by *F* is the *closure* of *F*.
- We denote the *closure* of *F* by F^+.
- F^+ is a superset of *F*.

Boyce-Codd Normal Form

A relation schema R is in BCNF with respect to a set F of functional dependencies if for all functional dependencies in F^+ of the form

$$\alpha \rightarrow \beta$$

where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following holds:

- $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$)
- α is a superkey for R

Example schema *not* in BCNF:

bor_loan = (*customer_id, loan_number, amount*)

because *loan_number* $\rightarrow$ *amount* holds on *bor_loan* but *loan_number* is not a superkey

Decomposing a Schema into BCNF

- Suppose we have a schema *R* and a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF.

  We decompose *R* into:

  - $(\alpha \cup \beta)$
  - $(R - (\beta - \alpha))$
- In our example,
  - α = *loan_number*
  - β = *amount*

  and *bor_loan* is replaced by

  - $(\alpha \cup \beta)$ = (*loan_number, amount*)
  - $(R - (\beta - \alpha))$ = (*customer_id, loan_number*)

BCNF and Dependency Preservation

- Constraints, including functional dependencies, are costly to check in practice unless they pertain to only one relation
- If it is sufficient to test only those dependencies on each individual relation of a decomposition in order to ensure that *all* functional dependencies hold, then that decomposition is **dependency preserving**.
- Because it is not always possible to achieve both BCNF and dependency preservation, we consider a weaker normal form, known as *third normal form.*

Third Normal Form

- A relation schema *R* is in third normal form (3NF) if for all:

$$\alpha \rightarrow \beta \text{ in } F^+$$

at least one of the following holds:

  - $\alpha \rightarrow \beta$ is trivial (i.e., $\beta \in \alpha$)
  - α is a superkey for *R*
  - Each attribute *A* in $\beta - \alpha$ is contained in a candidate key for *R.*

  (**NOTE***:* each attribute may be in a different candidate key)

- If a relation is in BCNF it is in 3NF (since in BCNF one of the first two conditions above must hold).
- Third condition is a minimal relaxation of BCNF to ensure **dependency preservation** (will see why later).

Goals of Normalization

- Let R be a relation scheme with a set F of functional dependencies.
- Decide whether a relation scheme R is in "good" form.
- In the case that a relation scheme R is not in "good" form, decompose it into a set of relation scheme $\{R_1, R_2, \ldots, R_n\}$ such that
  - each relation scheme is in good form
  - the decomposition is a lossless-join decomposition
  - Preferably, the decomposition should be dependency preserving.

How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a database

  classes (*course, teacher, book*)

  such that (c, t, b) $\in$ *classes* means that t is qualified to teach c, and b is a required textbook for c
- The database is supposed to list for each course the set of teachers any one of which can be the course's instructor, and the set of books, all of which are required for the course (no matter who teaches it).

How good is BCNF? (Cont.)

course	*teacher*	*book*
database	Avi	DB Concepts
database	Avi	Ullman
database	Hank	DB Concepts
database	Hank	Ullman
database	Sudarshan	DB Concepts
database	Sudarshan	Ullman
operating systems	Avi	OS Concepts
operating systems	Avi	Stallings
operating systems	Pete	OS Concepts
operating systems	Pete	Stallings

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if Marilyn is a new teacher that can teach database, two tuples need to be inserted

(database, Marilyn, DB Concepts)
(database, Marilyn, Ullman)

How good is BCNF? (Cont.)

- Therefore, it is better to decompose *classes* into:

course	*teacher*
database	Avi
database	Hank
database	Sudarshan
operating systems	Avi
operating systems	Jim

teaches

course	*book*
database	DB Concepts
database	Ullman
operating systems	OS Concepts
operating systems	Shaw

text

This suggests the need for higher normal forms, such as Fourth Normal Form (4NF), which we shall see later.

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Functional-Dependency Theory

- We now consider the formal theory that tells us which functional dependencies are implied logically by a given set of functional dependencies.
- We then develop algorithms to generate lossless decompositions into BCNF and 3NF
- We then develop algorithms to test if a decomposition is dependency-preserving

Closure of a Set of Functional Dependencies

- Given a set *F* set of functional dependencies, there are certain other functional dependencies that are logically implied by *F*.
  - For example: If $A \rightarrow B$ and $B \rightarrow C$, then we can infer that $A \rightarrow C$
- The set of all functional dependencies logically implied by *F* is the *closure* of *F*.
- We denote the *closure* of *F* by F^+.
- We can find all of F^+ by applying Armstrong's Axioms:
  - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$ **(reflexivity)**
  - if $\alpha \rightarrow \beta$, then $\gamma \alpha \rightarrow \gamma \beta$ **(augmentation)**
  - if $\alpha \rightarrow \beta$, and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$ **(transitivity)**
- These rules are
  - sound (generate only functional dependencies that actually hold) and
  - complete (generate all functional dependencies that hold).

Example

- $R = (A, B, C, G, H, I)$
  $F = \{ A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- some members of F^+
  - $A \rightarrow H$
    - by transitivity from $A \rightarrow B$ *and* $B \rightarrow H$
  - $AG \rightarrow I$
    - by augmenting $A \rightarrow C$ with G, to get $AG \rightarrow CG$
      and then transitivity with $CG \rightarrow I$
  - $CG \rightarrow HI$
    - by augmenting $CG \rightarrow I$ to infer $CG \rightarrow CGI$,
      and augmenting of $CG \rightarrow H$ to infer $CGI \rightarrow HI$,
      and then transitivity

Procedure for Computing F^+

- To compute the closure of a set of functional dependencies F:

$F^+ = F$
repeat
    for each functional dependency f in F^+
        apply reflexivity and augmentation rules on f
        add the resulting functional dependencies to F^+
    for each pair of functional dependencies f_1 and f_2 in F^+
        if f_1 and f_2 can be combined using transitivity
            then add the resulting functional dependency to F^+
until F^+ does not change any further

Closure of Functional Dependencies (Cont.)

- We can further simplify manual computation of F^+ by using the following additional rules.
  - If $\alpha \rightarrow \beta$ holds *a*nd $\alpha \rightarrow \gamma$ holds, then $\alpha \rightarrow \beta\gamma$ holds **(union)**
  - If $\alpha \rightarrow \beta\gamma$ holds, then $\alpha \rightarrow \beta$ holds and $\alpha \rightarrow \gamma$ holds **(decomposition)**
  - If $\alpha \rightarrow \beta$ holds *a*nd $\gamma\beta \rightarrow \delta$ holds, then $\alpha\gamma \rightarrow \delta$ holds **(pseudotransitivity)**

  The above rules can be inferred from Armstrong's axioms.

Closure of Attribute Sets

- Given a set of attributes α, define the *closure* of α under F (denoted by α^+) as the set of attributes that are functionally determined by α under F

- Algorithm to compute α^+, the closure of α under F

result := α;
while (changes to *result*) **do**
    for each $\beta \rightarrow \gamma$ **in** *F* **do**
      begin
        if $\beta \subseteq$ *result* **then** *result* := *result* $\cup\ \gamma$
      end

Example of Attribute Set Closure

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B$
  $A \rightarrow C$
  $CG \rightarrow H$
  $CG \rightarrow I$
  $B \rightarrow H\}$
- $(AG)^+$
  1. $result = AG$
  2. $result = ABCG$ $(A \rightarrow C$ and $A \rightarrow B)$
  3. $result = ABCGH$ $(CG \rightarrow H$ and $CG \subseteq AGBC)$
  4. $result = ABCGHI$ $(CG \rightarrow I$ and $CG \subseteq AGBCH)$
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$

Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- Testing for superkey:
  - To test if α is a superkey, we compute $\alpha^{+,}$ and check if α^{+} contains all attributes of *R*.
- Testing functional dependencies
  - To check if a functional dependency $\alpha \rightarrow \beta$ holds (or, in other words, is in F^{+}), just check if $\beta \subseteq \alpha^{+}$.
  - That is, we compute α^{+} by using attribute closure, and then check if it contains β.
  - Is a simple and cheap test, and very useful
- Computing closure of F
  - For each $\gamma \subseteq R$, we find the closure γ^{+}, and for each $S \subseteq \gamma^{+}$, we output a functional dependency $\gamma \rightarrow S$.

Canonical Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - For example: $A \rightarrow C$ is redundant in: $\{A \rightarrow B, \quad B \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g.: on RHS: $\{A \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow CD\}$ can be simplified to
      $$\{A \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow D\}$$
    - E.g.: on LHS: $\{\mathrm{A} \rightarrow B, \quad B \rightarrow C, \quad AC \rightarrow D\}$ can be simplified to
      $$\{\mathrm{A} \rightarrow B, \quad B \rightarrow C, \quad A \rightarrow D\}$$
- **Intuitively, a canonical cover of F is a "minimal" set of functional dependencies equivalent to F, having no redundant dependencies or redundant parts of dependencies**

Extraneous Attributes

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
  - Attribute A is extraneous in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is extraneous in β if $A \in \beta$ and the set of functional dependencies $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (I.e. the result of dropping B from $AB \rightarrow C$).
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C

Testing if an Attribute is Extraneous

- Consider a set F of functional dependencies and the functional dependency $\alpha \rightarrow \beta$ in F.
- To test if attribute $A \in \alpha$ is extraneous in α
  1. compute $(\{\alpha\} - A)^+$ using the dependencies in F
  2. check that $(\{\alpha\} - A)^+$ contains β; if it does, A is extraneous in α
- To test if attribute $A \in \beta$ is extraneous in β
  1. compute α^+ using only the dependencies in
     $F' = (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$,
  2. check that α^+ contains A; if it does, A is extraneous in β

Canonical Cover

- A *canonical cover* for F is a set of dependencies F_c such that
  - F logically implies all dependencies in $F_{c,}$ and
  - F_c logically implies all dependencies in F, and
  - No functional dependency in F_c contains an extraneous attribute, and
  - Each left side of functional dependency in F_c is unique.
- To compute a canonical cover for F:
  repeat
  Use the union rule to replace any dependencies in F
  $\alpha_1 \rightarrow \beta_1$ and $\alpha_1 \rightarrow \beta_2$ with $\alpha_1 \rightarrow \beta_1 \beta_2$
  Find a functional dependency $\alpha \rightarrow \beta$ with an
  extraneous attribute either in α or in β
  If an extraneous attribute is found, delete it from $\alpha \rightarrow \beta$
  until F does not change
- Note: Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied

Computing a Canonical Cover

- $R = (A, B, C)$
  $F = \{A \rightarrow BC$
  $B \rightarrow C$
  $A \rightarrow B$
  $AB \rightarrow C\}$
- Combine $A \rightarrow BC$ and $A \rightarrow B$ into $A \rightarrow BC$
  - Set is now $\{A \rightarrow BC, B \rightarrow C, AB \rightarrow C\}$
- A is extraneous in $AB \rightarrow C$
  - Check if the result of deleting A from $AB \rightarrow C$ is implied by the other dependencies
    - Yes: in fact, $B \rightarrow C$ is already present!
  - Set is now $\{A \rightarrow BC, B \rightarrow C\}$
- C is extraneous in $A \rightarrow BC$
  - Check if $A \rightarrow C$ is logically implied by $A \rightarrow B$ and the other dependencies
    - Yes: using transitivity on $A \rightarrow B$ *and* $B \rightarrow$ C.
      - Can use attribute closure of A in more complex cases
- The canonical cover is: $A \rightarrow B$
  $B \rightarrow C$

Lossless-join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \Pi_{R1}(r) \bowtie \Pi_{R2}(r)$$

- A decomposition of R into R_1 and R_2 is lossless join if and only if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B, B \rightarrow C)$
  - Can be decomposed in two different ways
- $R_1 = (A, B), \quad R_2 = (B, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{B\} \text{ and } B \rightarrow BC$$

  - Dependency preserving
- $R_1 = (A, B), \quad R_2 = (A, C)$
  - Lossless-join decomposition:

    $$R_1 \cap R_2 = \{A\} \text{ and } A \rightarrow AB$$

  - Not dependency preserving
    (cannot check $B \rightarrow C$ without computing $R_1 \bowtie R_2$)

Dependency Preservation

- Let F_i be the set of dependencies F^+ that include only attributes in R_i.
  - A decomposition is dependency preserving, if

$$(F_1 \cup F_2 \cup \ldots \cup F_n)^+ = F^+$$

  - If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive.

Testing for Dependency Preservation

- To check if a dependency $\alpha \rightarrow \beta$ is preserved in a decomposition of R into $R_1, R_2, \ldots, R_n$ we apply the following test (with attribute closure done with respect to F)
  - *result* = α
    while (changes to *result*) do
      for each R_i in the decomposition
        $t = (result \cap R_i)^+ \cap R_i$
        $result = result \cup t$
  - If *result* contains all attributes in β, then the functional dependency $\alpha \rightarrow \beta$ is preserved.
- We apply the test on all dependencies in F to check if a decomposition is dependency preserving
- This procedure takes polynomial time, instead of the exponential time required to compute F^+ and $(F_1 \cup F_2 \cup \ldots \cup F_n)^+$

Example

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $\quad B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF
- Decomposition $R_1 = (A, B)$, $R_2 = (B, C)$
  - R_1 and R_2 in BCNF
  - Lossless-join decomposition
  - Dependency preserving

Testing for BCNF

- To check if a non-trivial dependency $\alpha \rightarrow \beta$ causes a violation of BCNF
  1. compute α^+ (the attribute closure of α), and
  2. verify that it includes all attributes of *R*, that is, it is a superkey of *R*.
- Simplified test: To check if a relation schema *R* is in BCNF, it suffices to check only the dependencies in the given set *F* for violation of BCNF, rather than checking all dependencies in F^+.
  - If none of the dependencies in *F* causes a violation of BCNF, then none of the dependencies in F^+ will cause a violation of BCNF either.
- However, using only *F* is incorrect when testing a relation in a decomposition of R
  - Consider $R = (A, B, C, D, E)$, with $F = \{ A \rightarrow B, BC \rightarrow D\}$
    - Decompose *R* into $R_1 = (A,B)$ and $R_2 = (A,C,D, E)$
    - Neither of the dependencies in *F* contain only attributes from (A,C,D,E) so we might be mislead into thinking R_2 satisfies BCNF.
    - In fact, dependency $AC \rightarrow D$ in F^+ shows R_2 is not in BCNF.

Testing Decomposition for BCNF

- To check if a relation R_i in a decomposition of R is in BCNF,
  - Either test R_i for BCNF with respect to the restriction of F to R_i (that is, all FDs in F^+ that contain only attributes from R_i)
  - or use the original set of dependencies F that hold on R, but with the following test:
    - for every set of attributes $\alpha \subseteq R_i$, check that α^+ (the attribute closure of α) either includes no attribute of $R_i - \alpha$, or includes all attributes of R_i.
  - If the condition is violated by some $\alpha \rightarrow \beta$ in F, the dependency

    $$\alpha \rightarrow (\alpha^+ - \alpha) \cap R_i$$

    can be shown to hold on R_i, and R_i violates BCNF.
  - We use above dependency to decompose R_i

BCNF Decomposition Algorithm

result := $\{R\}$;
done := false;
compute F^+;
while (not *done)* **do**
  if (there is a schema R_i in *result* that is not in BCNF)
    then begin
      let $\alpha \rightarrow \beta$ be a nontrivial functional dependency that holds on R_i
        such that $\alpha \rightarrow R_i$ is not in F^+,
        and $\alpha \cap \beta = \emptyset$;
      result := (*result* – R_i) $\cup$ (R_i – β) $\cup$ (α, β);
      end
    else *done* := **true;**

Note: each R_i is in BCNF, and decomposition is lossless-join.

Example of BCNF Decomposition

- $R = (A, B, C)$
  $F = \{A \rightarrow B$
  $\quad B \rightarrow C\}$
  Key = $\{A\}$
- R is not in BCNF ($B \rightarrow C$ but B is not superkey)
- Decomposition
  - $R_1 = (B, C)$
  - $R_2 = (A,B)$

Example of BCNF Decomposition

- Original relation R and functional dependency F

  R = (*branch_name, branch_city, assets,*

  customer_name, loan_number, amount)

  F = {*branch_name* $\rightarrow$ *assets branch_city*

  loan_number $\rightarrow$ *amount branch_name* }

  Key = {*loan_number, customer_name*}

- Decomposition
  - R_1 = (*branch_name, branch_city, assets*)
  - R_2 = (*branch_name, customer_name, loan_number, amount*)
  - R_3 = (*branch_name, loan_number, amount*)
  - R_4 = (*customer_name, loan_number*)
- Final decomposition

  R_1, R_3, R_4

BCNF and Dependency Preservation

It is not always possible to get a BCNF decomposition that is dependency preserving

- $R = (J, K, L)$
  $F = \{JK \rightarrow L$
  $\quad L \rightarrow K\}$
  Two candidate keys = JK and JL
- R is not in BCNF
- Any decomposition of R will fail to preserve

$$JK \rightarrow L$$

This implies that testing for $JK \rightarrow L$ requires a join

Third Normal Form: Motivation

- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important
- Solution: define a weaker normal form, called Third Normal Form (3NF)
  - Allows some redundancy (with resultant problems; we will see examples later)
  - But functional dependencies can be checked on individual relations without computing a join.
  - There is always a lossless-join, dependency-preserving decomposition into 3NF.

3NF Example

- Relation R:
  - $R = (J, K, L)$
    $F = \{JK \rightarrow L, L \rightarrow K\}$
  - Two candidate keys: *JK* and *JL*
  - *R* is in 3NF

$JK \rightarrow L$	*JK* is a superkey
$L \rightarrow K$	*K* is contained in a candidate key

Redundancy in 3NF

- There is some redundancy in this schema
- Example of problems due to redundancy in 3NF
  - $R = (J, K, L)$
    $F = \{JK \rightarrow L, L \rightarrow K\}$

J	L	K
j_1	l_1	k_1
j_2	l_1	k_1
j_3	l_1	k_1
null	l_2	k_2

- repetition of information (e.g., the relationship l_1, k_1)
- need to use null values (e.g., to represent the relationship l_2, k_2 where there is no corresponding value for J).

Testing for 3NF

- Optimization: Need to check only FDs in F, need not check all FDs in F^+.
- Use attribute closure to check for each dependency $\alpha \rightarrow \beta$, if α is a superkey.
- If α is not a superkey, we have to verify if each attribute in β is contained in a candidate key of R
  - this test is rather more expensive, since it involve finding candidate keys
  - testing for 3NF has been shown to be NP-hard
  - Interestingly, decomposition into third normal form (described shortly) can be done in polynomial time

3NF Decomposition Algorithm

```
Let Fc be a canonical cover for F;
i := 0;
for each functional dependency α → β in Fc do
  if none of the schemas Rj, 1 ≤ j ≤ i contains α β
        then begin
                i := i + 1;
                Ri := α β
          end
if none of the schemas Rj, 1 ≤ j ≤ i contains a candidate key for R
  then begin
          i := i + 1;
          Ri := any candidate key for R;
        end
return (R1, R2, ..., Ri)
```

3NF Decomposition Algorithm (Cont.)

- Above algorithm ensures:
  - each relation schema R_i is in 3NF
  - decomposition is dependency preserving and lossless-join

3NF Decomposition: An Example

- Relation schema:

  cust_banker_branch = (*customer_id, employee_id*, *branch_name, type*)
- The functional dependencies for this relation schema are:
  1. *customer_id, employee_id* $\rightarrow$ *branch_name, type*
  2. *employee_id* $\rightarrow$ *branch_name*
  3. *customer_id, branch_name* $\rightarrow$ *employee_id*
- We first compute a canonical cover
  - *branch_name* is extraneous in the r.h.s. of the 1st dependency
  - No other attribute is extraneous, so we get F_C =

    customer_id, employee_id $\rightarrow$ *type*
    employee_id $\rightarrow$ *branch_name*
    customer_id, branch_name $\rightarrow$ *employee_id*

3NF Decompsition Example (Cont.)

- The **for** loop generates following 3NF schema:

  (*customer_id, employee_id, type*)

  (*employee_id, branch_name*)

  (*customer_id, branch_name, employee_id)*

  - Observe that (*customer_id, employee_id, type*) contains a candidate key of the original schema, so no further relation schema needs be added

- If the FDs were considered in a different order, with the 2nd one considered after the 3rd,

  (*employee_id, branch_name*)

  would not be included in the decomposition because it is a subset of

  (*customer_id, branch_name, employee_id)*

- Minor extension of the 3NF decomposition algorithm: at end of for loop, detect and delete schemas, such as (*employee_id, branch_name*), which are subsets of other schemas

  - result will not depend on the order in which FDs are considered

- The resultant simplified 3NF schema is:

  (*customer_id, employee_id, type*)

  (*customer_id, branch_name, employee_id)*

Comparison of BCNF and 3NF

- It is always possible to decompose a relation into a set of relations that are in 3NF such that:
  - the decomposition is lossless
  - the dependencies are preserved
- It is always possible to decompose a relation into a set of relations that are in BCNF such that:
  - the decomposition is lossless
  - it may not be possible to preserve dependencies.

Design Goals

- Goal for a relational database design is:
  - BCNF.
  - Lossless join.
  - Dependency preservation.
- If we cannot achieve this, we accept one of
  - Lack of dependency preservation
  - Redundancy due to use of 3NF
- Interestingly, SQL does not provide a direct way of specifying functional dependencies other than superkeys.

  Can specify FDs using assertions, but they are expensive to test
- Even if we had a dependency preserving decomposition, using SQL we would not be able to efficiently test a functional dependency whose left hand side is not a key.

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How good is BCNF?

- There are database schemas in BCNF that do not seem to be sufficiently normalized
- Consider a database

  classes (*course, teacher, book*)

  such that $(c, t, b) \in$ *classes* means that t is qualified to teach c, and b is a required textbook for c
- The database is supposed to list for each course the set of teachers any one of which can be the course's instructor, and the set of books, all of which are required for the course (no matter who teaches it).

How good is BCNF? (Cont.)

course	*teacher*	*book*
database	Avi	DB Concepts
database	Avi	Ullman
database	Hank	DB Concepts
database	Hank	Ullman
database	Sudarshan	DB Concepts
database	Sudarshan	Ullman
operating systems	Avi	OS Concepts
operating systems	Avi	Stallings
operating systems	Pete	OS Concepts
operating systems	Pete	Stallings

- There are no non-trivial functional dependencies and therefore the relation is in BCNF
- Insertion anomalies – i.e., if Marilyn is a new teacher that can teach database, two tuples need to be inserted

  (database, Marilyn, DB Concepts)
  (database, Marilyn, Ullman)

How good is BCNF? (Cont.)

- Therefore, it is better to decompose *classes* into:

course	*teacher*
database	Avi
database	Hank
database	Sudarshan
operating systems	Avi
operating systems	Jim

teaches

course	*book*
database	DB Concepts
database	Ullman
operating systems	OS Concepts
operating systems	Shaw

text

This suggests the need for higher normal forms, such as Fourth Normal Form (4NF), which we shall see later.

Multivalued Dependencies (MVDs)

- Let R be a relation schema and let $\alpha \subseteq R$ and $\beta \subseteq R$. The *multivalued dependency*

$$\alpha \rightarrow\rightarrow \beta$$

holds on R if in any legal relation $r(R)$, for all pairs for tuples t_1 and t_2 in r such that $t_1[\alpha] = t_2[\alpha]$, there exist tuples t_3 and t_4 in r such that:

$$\begin{aligned}
t_1[\alpha] = t_2[\alpha] = t_3[\alpha] &= t_4[\alpha] \\
t_3[\beta] &= t_1[\beta] \\
t_3[R - \beta] &= t_2[R - \beta] \\
t_4[\beta] &= t_2[\beta] \\
t_4[R - \beta] &= t_1[R - \beta]
\end{aligned}$$

MVD (Cont.)

- Tabular representation of $\alpha \rightarrow\rightarrow \beta$

	α	β	$R - \alpha - \beta$
t_1	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$a_{j+1} \ldots a_n$
t_2	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$b_{j+1} \ldots b_n$
t_3	$a_1 \ldots a_i$	$a_{i+1} \ldots a_j$	$b_{j+1} \ldots b_n$
t_4	$a_1 \ldots a_i$	$b_{i+1} \ldots b_j$	$a_{j+1} \ldots a_n$

Example

- Let R be a relation schema with a set of attributes that are partitioned into 3 nonempty subsets.

$$Y, Z, W$$

- We say that $Y \rightarrow\rightarrow Z$ (Y multidetermines Z) if and only if for all possible relations r (R)

$$< y_1, z_1, w_1 > \in r \text{ and } < y_1, z_2, w_2 > \in r$$

then

$$< y_1, z_1, w_2 > \in r \text{ and } < y_1, z_2, w_1 > \in r$$

- Note that since the behavior of Z and W are identical it follows that

$Y \rightarrow\rightarrow Z$ if $Y \rightarrow\rightarrow W$

Example (Cont.)

- In our example:

$$\textit{course} \rightarrow\rightarrow \text{teacher}$$
$$\textit{course} \rightarrow\rightarrow \textit{book}$$

- The above formal definition is supposed to formalize the notion that given a particular value of Y (*course*) it has associated with it a set of values of Z *(teacher)* and a set of values of W *(book)*, and these two sets are in some sense independent of each other.
- Note:
  - If $Y \rightarrow Z$ then $Y \rightarrow\rightarrow Z$
  - Indeed we have (in above notation) $Z_1 = Z_2$
    The claim follows.

Use of Multivalued Dependencies

- We use multivalued dependencies in two ways:
  1. To test relations to determine whether they are legal under a given set of functional and multivalued dependencies
  2. To specify constraints on the set of legal relations. We shall thus concern ourselves *only* with relations that satisfy a given set of functional and multivalued dependencies.
- If a relation *r* fails to satisfy a given multivalued dependency, we can construct a relations *r′* that does satisfy the multivalued dependency by adding tuples to *r.*

Theory of MVDs

- From the definition of multivalued dependency, we can derive the following rule:
  - If $\alpha \rightarrow \beta$, then $\alpha \rightarrow\rightarrow \beta$

  That is, every functional dependency is also a multivalued dependency
- The **closure** D^+ of *D* is the set of all functional and multivalued dependencies logically implied by *D*.
  - We can compute D^+ from *D*, using the formal definitions of functional dependencies and multivalued dependencies.
  - We can manage with such reasoning for very simple multivalued dependencies, which seem to be most common in practice

Fourth Normal Form

- A relation schema *R* is in 4NF with respect to a set *D* of functional and multivalued dependencies if for all multivalued dependencies in D^+ of the form $\alpha \rightarrow\rightarrow \beta$, where $\alpha \subseteq R$ and $\beta \subseteq R$, at least one of the following hold:
  - $\alpha \rightarrow\rightarrow \beta$ is trivial (i.e., $\beta \subseteq \alpha$ or $\alpha \cup \beta = R$)
  - α is a superkey for schema *R*
- If a relation is in 4NF it is in BCNF

Restriction of Multivalued Dependencies

- The restriction of D to R_i is the set D_i consisting of
  - All functional dependencies in D^+ that include only attributes of R_i
  - All multivalued dependencies of the form

$$\alpha \rightarrow\rightarrow (\beta \cap R_i)$$

where $\alpha \subseteq R_i$ and $\alpha \rightarrow\rightarrow \beta$ is in D^+

4NF Decomposition Algorithm

result: = {*R*};
done := false;
compute D^+;
Let D_i denote the restriction of D^+ to R_i

while (**not** *done*)
  if (there is a schema $\mathbf{R}_i$ in *result* that is not in 4NF) **then**
    begin
      let $\alpha \rightarrow\rightarrow \beta$ be a nontrivial multivalued dependency that holds on R_i such that $\alpha \rightarrow R_i$ is not in D_i, and $\alpha \cap \beta = \phi$;
      result := $(result - R_i) \cup (R_i - \beta) \cup (\alpha, \beta)$;
    end
  else *done*:= true;

Note: each R_i is in 4NF, and decomposition is lossless-join

Example

- $R = (A, B, C, G, H, I)$

  $F = \{ A \rightarrow\rightarrow B$

  $B \rightarrow\rightarrow HI$

  $CG \rightarrow\rightarrow H \}$
- R is not in 4NF since $A \rightarrow\rightarrow B$ and A is not a superkey for R
- Decomposition

  a) $R_1 = (A, B)$ (R_1 is in 4NF)

  b) $R_2 = (A, C, G, H, I)$ (R_2 is not in 4NF)

  c) $R_3 = (C, G, H)$ (R_3 is in 4NF)

  d) $R_4 = (A, C, G, I)$ (R_4 is not in 4NF)
- Since $A \rightarrow\rightarrow B$ and $B \rightarrow\rightarrow HI$, $A \rightarrow\rightarrow HI$, $A \rightarrow\rightarrow I$

  e) $R_5 = (A, I)$ (R_5 is in 4NF)

  f) $R_6 = (A, C, G)$ (R_6 is in 4NF)

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Further Normal Forms

- **Join dependencies** generalize multivalued dependencies
  - lead to **project-join normal form (**PJNF) (also called **fifth normal form**)
- A class of even more general constraints, leads to a normal form called **domain-key normal form**.
- Problem with these generalized constraints: are hard to reason with, and no set of sound and complete set of inference rules exists.
- Hence rarely used

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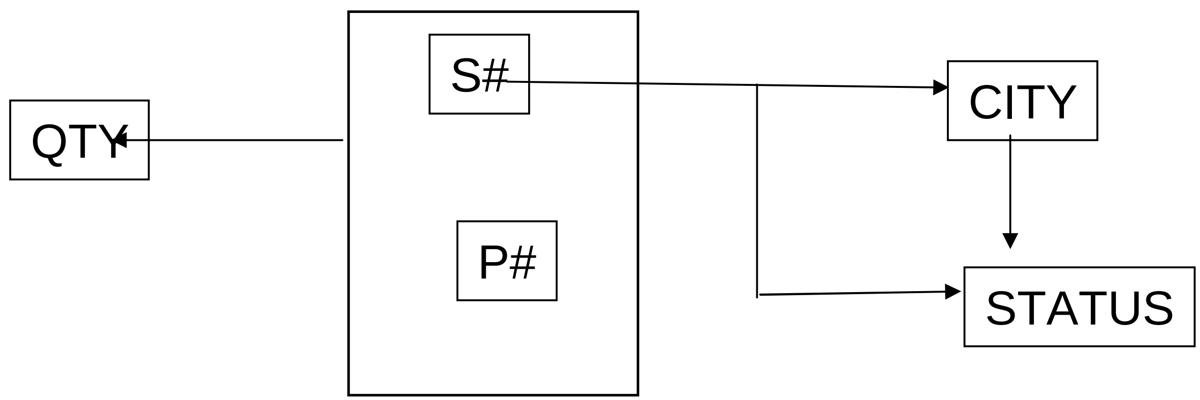
Second normal form

- **Definition: (definition assuming only one candidate key which is also primary key)** A relation is in 2NF if and only if it is in 1NF and every non key attribute is irreducibly dependent on the primary key.
- FDs of 1NF relation: (S#, STATUS, CITY, P#, QTY)
  - PRIMARY KEY (S#, P#)



- Replace the relation with SECOND(S#, STATUS, CITY) and SP(S#, P#, QTY)

SECOND

S#	STATUS	CITY
S1	20	London
S2	10	Paris
S3	10	Paris
S4	20	London
S5	30	Athens

SP

S#	**P#**	**QTY**
S1	P1	300
S1	P2	200
S1	P3	400
S1	P4	200
S!	P5	100
S1	P6	100
S2	P1	300
S2	P2	400
S3	P2	200
S4	P2	200
S4	P4	300
S4	P5	400

REVISED STRUCTURE

SECOND

S#	STATUS	CITY
S1	20	London
S2	10	Paris
S3	10	Paris
S4	20	London
S5	30	Athens

SP

S#	P#	QTY
S1	P1	300
S1	P2	200
S1	P3	400
S1	P4	200
S!	P5	100
S1	P6	100
S2	P1	300
S2	P2	400
S3	P2	200
S4	P2	200
S4	P4	300
S4	P5	400

- **INSERT:** We can insert the information that S5 is located in Paris, even though S5 does not supply any parts.
- **DELETE:** We can delete information connecting S3 and P2 without loosing information that S3 is located in Paris.
- **UPDATE:** The city for a supplier appears once, not many times.

Problems of second normal form

- **Relation SP is in second normal form.**
- **However relation SECOND suffers from the following problems.**
- **INSERT anomalies**: We can not insert the fact that a particular city has a particular status., until we have some supplier located in that city.
- **DELETE anomalies:** If we delete the second tuple for a particular city, we destroy the information not only the information for the supplier concerned but also information that the city has that particular status.
- **UPDATE anomalies**: The status for the city appears many times.

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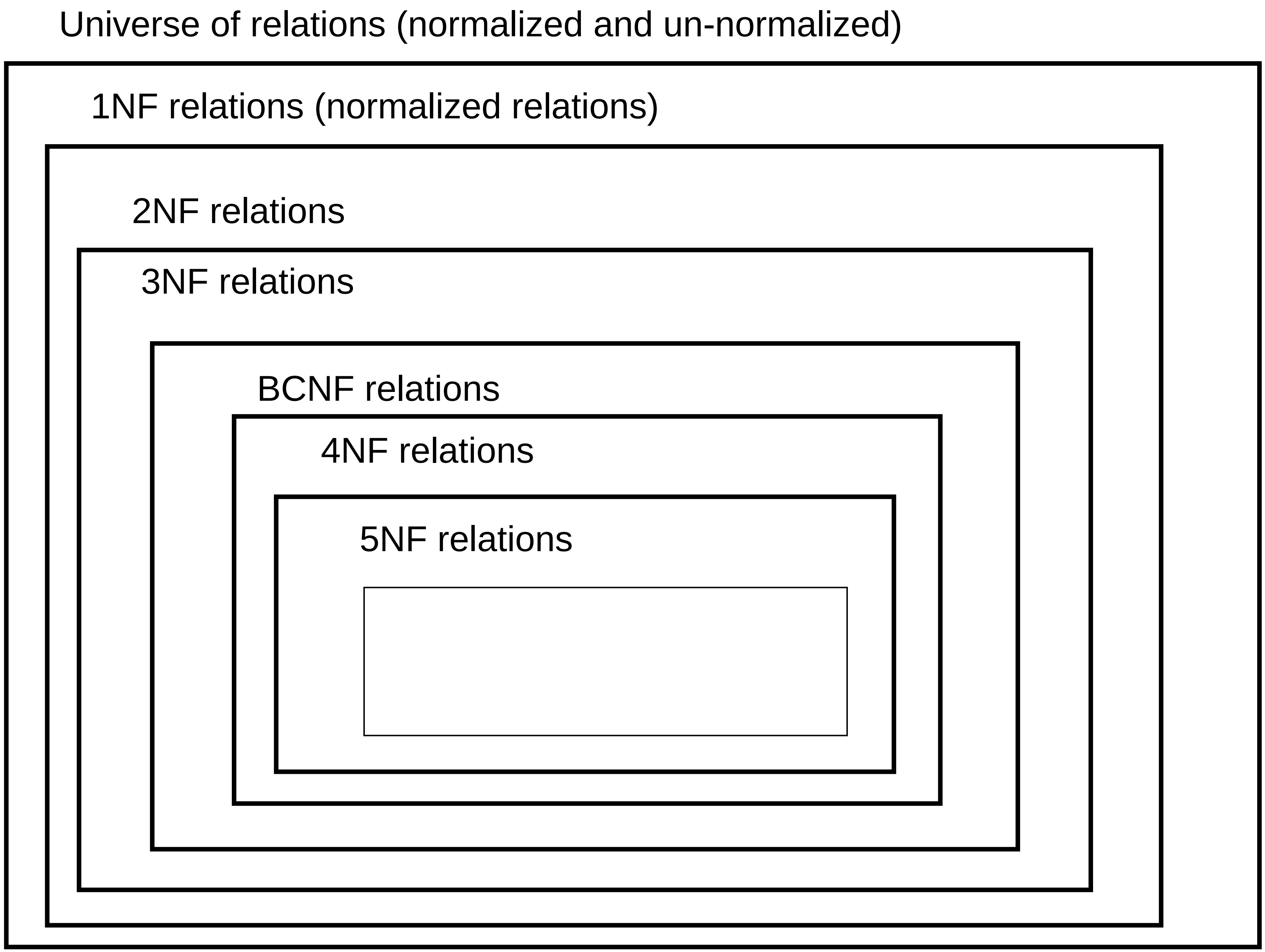
Universe of relations (normalized and un-normalized)
1NF relations (normalized relations)
2NF relations
3NF relations
BCNF relations
4NF relations
5NF relations

Overall Database Design Process

- We have assumed schema *R* is given
  - *R* could have been generated when converting E-R diagram to a set of tables.
  - *R* could have been a single relation containing *all* attributes that are of interest (called **universal relation**).
  - Normalization breaks *R* into smaller relations.
  - *R* could have been the result of some ad hoc design of relations, which we then test/convert to normal form.

ER Model and Normalization

- When an E-R diagram is carefully designed, identifying all entities correctly, the tables generated from the E-R diagram should not need further normalization.
- However, in a real (imperfect) design, there can be functional dependencies from non-key attributes of an entity to other attributes of the entity
  - Example: an *employee* entity with attributes *department_number* and *department_address*, and a functional dependency *department_number* $\rightarrow$ *department_address*
  - Good design would have made department an entity
- Functional dependencies from non-key attributes of a relationship set possible, but rare --- most relationships are binary

Denormalization for Performance

- May want to use non-normalized schema for performance
- For example, displaying *customer_name* along with *account_number* and *balance* requires join of *account* with *depositor*
- Alternative 1: Use denormalized relation containing attributes of *account* as well as *depositor* with all above attributes
  - faster lookup
  - extra space and extra execution time for updates
  - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a materialized view defined as
  account $\bowtie$ depositor
  - Benefits and drawbacks same as above, except no extra coding work for programmer and avoids possible errors

Summary of Schema Refinement

- If a relation is in BCNF, it is free of redundancies that can be detected using FDs. Thus, trying to ensure that all relations are in BCNF is a good heuristic.
- If a relation is not in BCNF, we can try to decompose it into a collection of BCNF relations.
  - Must consider whether all FDs are preserved. If a lossless-join, dependency preserving decomposition into BCNF is not possible (or unsuitable, given typical queries), should consider decomposition into 3NF.
  - Decompositions should be carried out and/or re-examined while keeping *performance requirements* in mind.