First-Order Logic – Examples

Translate to 1st Order Logic

1. Snakes are reptiles.
   (S(x): x is a snake, R(x): x is a reptile)
2. None but the brave deserve the fair.
   (B(x): x is brave, D(x): x deserves the fair)
   (B(x): x is brave, F(x): x is fair, D(x,y): x deserves y)
3. No coat is waterproof, unless it has been specially treated.
   (C(x): x is a coat, W(x): x is waterproof, S(x): x has been specially treated)
4. No car that is over 10 years old will be repaired if it is severely damaged.
   (C(x): x is a car, O(x): x is over 10 years old, R(x): x will be repaired, D(x): x is severely damaged)

Solutions

1. Snakes are reptiles.
   $\forall x\,(S(x) \rightarrow R(x))$ Not: $\forall x\,(S(x) \wedge R(x))$
2. None but the brave deserve the fair.
   $\forall x\,(D(x) \rightarrow B(x))$
   $\forall x\,(\forall y\,(F(y) \wedge D(x,y)) \rightarrow B(x))$
3. No coat is waterproof, unless it has been specially treated.
   $\forall x\,(C(x) \rightarrow \neg W(x) \vee S(x))$
4. No car that is over 10 years old will be repaired if it is severely damaged.
   $\forall x\,(C(x) \wedge O(x) \rightarrow (D(x) \rightarrow \neg R(x))$

More Translation

5. If anything is damaged, someone will be blamed.
   (D(x): x is damaged, P(x): x is a person, B(x): x is blamed)
6. If any bananas are yellow, they are ripe.
   (B(x): x is a banana, Y(x): x is yellow, R(x): x is ripe)
7. If any bananas are yellow, then some bananas are ripe.
8. Dead men tell no tales.
   (D(x): x is dead, M(x): x is a man, T(x): x is a tale, T(x,y): x tells y)
9. There is a store from which everyone buys something.
   (P(x): x is a person, S(x): x is a store, B(x,y,z): x buys y from z)

Solutions

5. If anything is damaged, someone will be blamed.
   $(\exists x\ D(x)) \rightarrow \exists y\ (P(y) \wedge B(y))$ or
   $\forall x\ (\ D(x) \rightarrow \exists y\ (P(y) \wedge B(y))\ )$ – notice braces
6. If any bananas are yellow, they are ripe.
   $\forall x\ B(x) \rightarrow (Y(x) \rightarrow R(x))$ or
   $\forall x\ B(x) \wedge Y(x) \rightarrow R(x)$
7. If any bananas are yellow, then some bananas are ripe.
   $\forall x\ B(x) \rightarrow (Y(x) \rightarrow \exists y\ (B(y) \wedge R(y)))$
8. Dead men tell no tales.
   $\forall x\ (D(x) \wedge M(x)) \rightarrow \forall y\ (T(y) \rightarrow \neg T(x,y))$
9. There is a store from which everyone buys something. $\exists x\ (S(x) \wedge \forall y\ (P(y) \rightarrow \exists z\ B(y,z,x)))$

Still More Translation

H(x): x is a horse

C(x): x is gentle

T(x): x has been well trained

10. Any horse that is gentle has been well trained.
11. Gentle horses have all been well trained.
12. If something is a well-trained horse, then it must be gentle.

Prove by Resolution Refutation

1. All athletes are brawny. Shyam is not brawny. Therefore, Shyam is not an athlete. A(x), B(x), s
2. Dates are edible. Only food items are edible. All food items are good. Therefore, all dates are good. D(x), E(x), F(x), G(x)

3. All astronauts are brave. Shyam is brave. Therefore, Shyam is an astronaut. A(x), B(x), s

Translate Peano's Axioms

Based on 3 concepts:

- A constant: *zero*
- A predicate indicating numbers: *N*
- A successor function: *S*

The Axioms

- Zero is a number.
- If *x* is a number, then successor of x is also a number.
- If the successors of two numbers are equal, then those two numbers are equal.
- No successor of any number is equal to zero.
- If φ (a predicate) applies to zero, then if it can be shown that φ applies to the successor of any number given that it applies to that number, then it follows that φ applies to all numbers.

In 1st order logic

- Zero is a number: $N(zero)$
- If x is a number, then successor of x is also a number: $\forall x\ N(x) \rightarrow N(S(x))$
- If the successors of two numbers are equal, then those two numbers are equal: $\forall x,y\ [\ N(x) \wedge N(y) \wedge S(x) = S(y) \rightarrow x = y\ ]$
- No successor of any number is equal to zero: $\neg\ (\exists x\ N(x) \wedge (S(x) = zero))$
- If φ (a predicate) applies to zero, then if it can be shown that φ applies to the successor of any number given that it applies to that number, then it follows that φ applies to all numbers: $\varphi(zero) \wedge \forall x\ (\ N(x) \wedge (\varphi(x) \rightarrow \varphi(S(x)))) \rightarrow \forall x\ (\ N(x) \rightarrow \varphi(x)\ )$

Prove using Resolution Refutation

- Any fish can swim faster than any smaller one. Therefore, if there is a largest fish, then there is a fastest fish.

  (F(x): x is a fish, L(x,y): x is larger= than y, S(x,y): x can swim faster= than y)

Knowledge Engineering

- Choose the objects and relationships that you want to represent.
- Determine axioms.
- Build a knowledge-base.
- Infer new facts.

Symbolize and Translate

- Any single capital letter is a *wff* (well-formed-formula).
- If P is a *wff* then $\neg$P is a *wff*.
- If P and Q are *wff*s, then P $\wedge$ Q and P $\vee$ Q are *wff*s.
- No formula will be regarded as being a *wff* unless its being so follows from this definition.

Symbolize and Translate

Preserve the *intended* meaning.

- One more outburst like that and you will be out of class.
- Spiderman is on TV tonight, if you're interested.
- Well, I like AI and I don't like AI.
- Any politician can fool some people all the time, all people some of the time, but not all people all of the time.

Situation Calculus

- Representing *change* in terms of situations and actions
- Add extra "situation argument" to all predicates and functions that can change over time.
  - E.g. $President(\text{Narayanan}, S_0) \wedge President(\text{Kalam}, S_1)$
- Result situations: From one situation to another.
  - E.g. $Result(StudyHard(\text{Shyam}), S_2) = S_3$
  - Represents actions
  - E.g. $President(\text{Kalam}, Result(Nomination, S_0))$
  - E.g. $\forall x,s\ (Undergraduate(x, s) \rightarrow Graduate(x, Result(StudyHard(x), s)))$

Try out:

- A non-rich person can become rich either by working hard or by stealing from a bank.

(P(x): x is a person, R(x,s): x is rich in situation s, W(x): x worked hard, S(x,y): x stole from y, B(x): x is a bank)