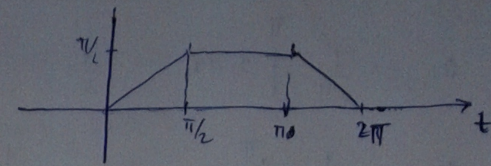
Name – Pratyush kumar Patel

Maths Assignment – II

a1) $f(t) \begin{cases} t & 0 \le t \le \pi/2 \\ \pi/2 & \pi/2 \le t \le \pi \\ \pi - t/2 & \pi \le t \le 2\pi \end{cases}$



$2L = 2\pi$

$$a_0 = \frac{1}{2L}\left[\int_0^{\pi/2} t\,dt + \int_{\pi/2}^{\pi} \pi/2\,dt + \int_{\pi}^{2\pi} \pi - t/2\right]dt$$

$$= \frac{1}{2L}\left[\frac{\pi^2}{8} + \frac{\pi^2}{4} + \frac{2\pi^2}{8}\right] = \frac{1}{2\pi}\cdot\frac{5\pi^2}{8} = \frac{5\pi}{16}$$

$$\cancel{a_n = \frac{1}{\pi}\int_0^{\pi/2} t\cdot\cos t\,dt + \int_{\pi/2}^{\pi}\frac{\pi}{2}\cos t\,dt + \int_{\pi}^{2\pi}\left(\pi - \frac{t}{2}\right)\cos t\,dt}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi/2} t\cdot\cos nt\,dt + \int_{\pi/2}^{\pi}\frac{\pi}{2}\cos nt\,dt + \int_{\pi}^{2\pi}(\pi - t/2)\cos nt\,dt$$

$$= \frac{1}{\pi}\left[\frac{t\sin nt}{n} + \frac{\cos nt}{n^2}\Big|_0^{\pi/2} + \frac{\pi}{2n}\sin nt\Big|_{\pi/2}^{\pi} + \frac{\pi\sin nt}{n}\Big|_{\pi}^{2\pi} - \frac{t\sin nt}{2n} - \frac{\cos nt}{2n^2}\Big|_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi}{2n}\sin(n\pi/2)} - \frac{1+(-1)^{n/2}}{n^2} - \cancel{\frac{\pi}{2n}\sin(n\pi/2)} - \frac{\cos 2n\pi}{2n^2} + \frac{\cos n\pi}{2n^2}\right]$$

$$a_n = n \begin{cases} \text{even} & (-1)^{n/2} - \frac{1}{n^2} - \cancel{\frac{1}{2n^2}} + \cancel{\frac{1}{2n^2}} = \frac{-1+(-1)^{n/2}}{\pi n^2} \\ \text{odd} & -\frac{2}{2n^2} - \frac{1}{2n^2} - \frac{1}{2n^2} = \frac{-4}{\pi n^2} \end{cases}$$

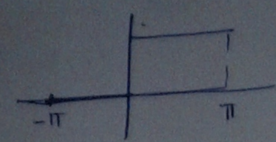
$$b_n = \frac{1}{\pi}\int_0^{\pi/2} t\cdot\sin nt\,dt + \int_{\pi/2}^{\pi}\frac{\pi}{2}\sin nt\,dt + \int_{\pi}^{2\pi}(\pi - t/2)\sin nt\,dt$$

$$= \frac{1}{\pi}\left[-\frac{t\cos nt}{n} + \frac{\sin nt}{n^2}\Big|_0^{\pi/2} - \frac{\pi}{2}\frac{\cos nt}{n}\Big|_{\pi/2}^{\pi} - \frac{\pi\cos nt}{n}\Big|_{\pi}^{2\pi}\right.$$
$$\left. + \frac{t\cos nt}{2n} - \frac{\sin nt}{2n^2}\Big|_{\pi}^{2\pi}\right]$$

$$= -\frac{\pi}{2}\cancel{\frac{\cos n\pi/2}{n}} \quad \frac{1}{\pi}\left\{\frac{\sin n\pi/2}{n^2} - \frac{\pi}{2}\cancel{\frac{\cos n\pi}{n}} + \frac{\pi}{2}\cancel{\frac{\cos n\pi}{n}} - \pi\cancel{\frac{\cos 2n\pi}{n}} + \pi\cancel{\frac{\cos n\pi}{n}} + \cancel{\frac{2\pi\cos 2n\pi}{2n}} - \pi\cancel{\frac{\cos n\pi}{2n}}\right]$$

$$b_n = \begin{cases} \frac{(-1)(-1)^{\frac{n+1}{2}}}{\pi n^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

Q2)



$\cancel{2}L = \cancel{2}\pi$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} 1\,dx = \frac{1}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi}\cos nx\,dx = \frac{1}{\pi}\frac{\sin nx}{n}\Big|_0^{\pi} \Rightarrow 0$$

$$b_n = \frac{1}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{-1}{\pi}\frac{\cos nx}{n}\Big|_0^{\pi} = \frac{1}{\pi}\left[\frac{1}{n} - \frac{\cos n\pi}{n}\right]$$

n is even $b_n = 0$

n is odd $b_n = \dfrac{2}{n\pi}$

$$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\frac{2}{(2n-1)\pi}\sin(2n-1)x$$

We need to calulate $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$

but $x = \pi/2$

$$f(\pi/2) = \frac{1}{2} + \frac{2}{\pi}\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}\cdots\right]$$

$f(\pi/2) = 1$

$$\frac{1}{2} = \frac{2}{\pi}\left[1 - \frac{1}{3} + \frac{1}{5}\cdots\right]$$

$$= \frac{\pi}{4}$$

Q3) $f(x) = \sin x/2 \quad 0 \le x \le \pi$

$\quad -\sin x/2 \quad \pi < x \le 2\pi$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi}\sin\frac{x}{2}\,dx - \int_{\pi}^{2\pi}\sin\frac{x}{2}\,dx$$

$$= \frac{1}{2\pi}\left[-2\cos\frac{x}{2}\Big|_0^{\pi} + 2\cos\frac{x}{2}\Big|_{\pi}^{2\pi}\right] = \frac{1}{2\pi}\left[2-2\right] = 0$$

$$a_n = \frac{1}{\pi}\int_0^{\pi}\sin\frac{x}{2}\cdot\cos nx + \int_{\pi}^{2\pi}-\sin\frac{x}{2}\cdot\cos nx \qquad 0$$

$$I = \int \sin\frac{x}{2}\cdot\cos nx = -2\cos\frac{x}{2}\cos nx - \int n\sin nx\, 2\cos\frac{x}{2}\,dx$$

$$-2n\int\sin nx\cos\frac{x}{2}\,dx$$

$$\Downarrow$$

$$2n\,2\sin\frac{x}{2}\sin nx + (2n)^2\cos nx\sin\frac{x}{2}$$

$$I = -2\cos\frac{x}{2}\cos nx - 4n\sin\frac{x}{2}\sin nx + (2n)^2\int \cos nx \sin\frac{x}{2}\,dx$$

$$I(1-4n^2) = -2\cos\frac{x}{2}\cos nx - 4n\sin\frac{x}{2}\sin nx$$

$$I = \frac{-1}{(1-4n^2)}\left(2\cos x/_2 \cos nx + 4n\sin x/_2 \sin nx\right)$$

$$a_n = \frac{1}{\pi}\; I\Big]_0^{\pi} + (-I)\Big]_{\pi}^{2\pi}$$

$$= \frac{1}{(1-4n^2)\pi}\left[(+2) + 2\cos\pi\cos 2\pi n\right] = \underline{0}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi}\sin\frac{x}{2}.\sin nx\,dx + \int_{\pi}^{2\pi}-\sin\frac{x}{2}\sin nx\,dx$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi}\frac{\sin(2n-1)\frac{x}{2}}{(n-\frac{1}{2})} - \frac{\sin(2n+1)x/_2}{(n+\frac{1}{2})}\,dx - \int_{\pi}^{2\pi}\frac{\sin(2n+1)\frac{x}{2}}{(n+\frac{1}{2})} - \frac{\sin(2n-1)\frac{x}{2}}{(n-\frac{1}{2})}\right]$$

$$= \frac{2}{2\pi}\left[\frac{2\sin(2n-1)\frac{\pi}{2}}{(2n-1)} - \frac{2\sin(2n+1)\pi/_2}{(2n+1)}\right]$$

$$= \frac{2}{\pi}\;\frac{2}{4n^2-1} = \frac{4}{\pi(4n^2-1)}\begin{cases} n \text{ odd} = \frac{4}{\pi(4n^2-1)} \\ n \text{ even } \frac{-4}{\pi(4n^2-1)}\end{cases} = \frac{(-1)^n(4)}{\pi(1-4n^2)}$$

$$f(x) = \sum_{n=1}^{\infty}\frac{4(-1)^n \sin nx}{\pi(1-4n^2)}$$

Q4) $f(x) = 1-x^2$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(1-x^2)\,dx = \frac{1}{2\pi}\cdot x - \frac{x^3}{3}\Big|_{-\pi}^{\pi} = \frac{1}{2\pi}\left[\left(\pi - \frac{\pi^3}{3}\right) - \left(-\pi + \frac{\pi^3}{3}\right)\right]$$

$$= \frac{1}{2\pi}\left[\pi - \frac{\pi^3}{3} + \pi - \frac{\pi^3}{3}\right] = \frac{1}{2\pi}\left[2\pi - \frac{2\pi^3}{3}\right.$$

$$= 1 - \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1-x^2)\cos nx\,dx \quad \text{as } f(-x) = f(x)$$

$$= \frac{2}{\pi}\int_0^{\pi}(1-x^2)\cos nx\,dx = \frac{2\sin nx}{\pi\, n}\Big]_0^{\pi} - \frac{2}{\pi}\int_0^{\pi}x^2\cos nx\,dx$$

$0 < x < 10$ period $f = 10$

(3)

$$a_n = \frac{2}{\lambda\pi} \sin x \Big]_0^{\pi} - \frac{2}{\pi}\left[x^2 \frac{\sin nx}{n} + \frac{1}{n}\int 2x \sin nx \, dx\right.$$

$$= \frac{2}{n\pi} \sin nx \Big|_0^{\pi} - \frac{2x^2}{\pi n} \sin nx \Big|_{\pi}^{2\pi} + \frac{4x \cos nx}{\pi n^2}\Big|_{\pi}^{2\pi} - \frac{4 \sin nx}{\pi n^3}\Big|_{\pi}^{2\pi}$$

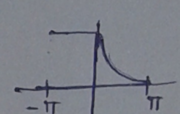
$$= \frac{4\, 2\cancel{\pi} \cos 2\pi x}{\cancel{\pi} n^2} - \frac{4\cancel{\pi} \cos n\pi}{\cancel{\pi} n^2} - \cancel{4 \sin 2n\pi}$$

$$= \frac{8 \cos 2\pi x}{n^2} - \frac{4 \cos n\pi}{n^2} = 4\left[\frac{2}{n^2} - \frac{(-1)^n}{n^2}\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} (1-x^2) \sin nx \, dx \quad f(-x) = -f(x) = 0$$

$$1 - x^2 = 1 - \frac{\pi^2}{3} + \sum_{n=1}^{\infty} 4\left[\frac{2}{n^2} - \frac{(-1)^n}{n^2}\right] \cos nx$$

5) $f(t) \begin{cases} \pi^2 & -\pi < t < 0 \\ (t-\pi)^2 & 0 \le t \le \pi \end{cases}$



$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} \pi^2 dt + \int_0^{\pi} (t-\pi)^2 dt\right] = \frac{1}{2\pi}\left[\pi^3 + \frac{t^3}{3} + \pi^2 t - t^2\pi\right]_0^{\pi}$$

$$= \frac{1}{2\pi}\left[\pi^3 + \frac{\pi^3}{3}\right] + \cancel{\pi^3} - \cancel{\pi^3}$$

$$= \frac{2\, 4\pi^{3\,2}}{3\, 6\pi} = \frac{2\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} \pi^2 \cos nt + \int_0^{\pi} (t-\pi)^2 \cos nt \, dt$$

$$= \frac{1}{\pi}\left(\frac{\pi^2 \sin nt}{n}\right]_{-\pi}^{0} + \int_0^{\pi} t^2 \cos (n\pi - nt)\, dt\Bigg)$$

$$\Downarrow$$

$$\frac{t^2 \sin(n\pi - nt)}{-n}\Big|_0^{\pi} + \int \frac{2t \sin(n\pi - nt)}{n} dt$$

$$\Downarrow$$

$$a_n = \frac{\pi \sin nt}{n}\Big|_{-\pi}^{0} + -\frac{t^2}{n}\sin(n\pi - nt)\Big|_0^{\pi} + \frac{2}{n}\left[\frac{t \cos(n\pi - nt)}{n} + \frac{\sin(n\pi - nt)}{n^2}\right]_0^{\pi}$$

$$= \frac{2\pi}{n^2}$$

$$a_n = \frac{2\pi}{n^2}$$

$0<x<10$ period $f = 10$

7) $f(x) = \sin x$ $0<x<\pi$ fourier cosine series

$$b_n = \int_{-\pi}^{0} \pi^2 \sin nt \, dt + \int_{0}^{\pi} (t-\pi)^2 \sin nt \, dt$$

$$= -\frac{\pi^2 \cos nt}{n} \Big]_{-\pi}^{0} + \int_0^{\pi} t^2 \sin(n\pi - nt)\, dt$$

$$\Downarrow$$

$$\frac{t^2 \cos(n\pi - nt)}{n} - \frac{2}{n}\int t \cos(n\pi - nt)\, dt$$

$$\Downarrow$$

$$\frac{t \sin(n\pi - nt)}{-n} + \frac{\cos(n\pi - nt)}{n^2}$$

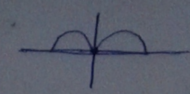
$$= -\frac{\pi^2 \cos nt}{n} \Big|_{-\pi}^{0} + \frac{t^2 \cos(n\pi - nt)}{n} \Big|_0^{\pi} - \frac{2}{n}\left[\frac{t \sin(n\pi - nt)}{-n} + \frac{\cos(n\pi - nt)}{n^2}\right]_0^{\pi}$$

$$= \frac{\pi^2}{n}\cos n\pi - \frac{\pi^2}{n} + \frac{\pi^2}{n} - \frac{2}{n}\left(\frac{1}{n^2} - \frac{\cos n\pi}{n^2}\right)$$

7) $f(x) = \sin x \quad 0 < x < \pi$ fourier cosine series

even expansion

$$f(x) = \begin{cases} \sin x & 0 < x < \pi \\ -\sin x & 0 < x < -\pi \end{cases}$$



(4)

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} -\sin x\,dx + \int_0^{\pi} \sin x\,dx\right] = \frac{1}{2\pi}\left[\left[\cos x\right]_{-\pi}^{0} + \cos x\Big|_{\pi}^{0}\right]$$

$$= \frac{1}{2\pi}(2+2) = \frac{\cancel{4}2}{\cancel{2}\pi} = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -\sin x \cdot \cos nx + \int_0^{\pi} \sin x \cos nx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{1}{2}\left[\int_{-\pi}^{0} \sin(n+1)x - \sin(n-1)x\,dx - \int_0^{\pi} \sin(n+1)x - \sin(n-1)x\,dx\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{\cos(n+1)x}{n+1} - \frac{\cos(n-1)x}{(n-1)}\Big|_{-\pi}^{0} + \left(-\frac{\cos(n+1)x}{n+1}\right) + \left(\frac{\cos(n-1)x}{n-1}\right)\Big|_0^{\pi}\right.$$

$$= \frac{1}{2\pi}\left[\frac{1}{n+1} - \frac{1}{n-1} - \frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} + \frac{-\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1}\right.$$

$$+ \frac{1}{n+1} - \frac{1}{n-1}$$

$$= \frac{1}{\cancel{2}\pi}\left[\frac{\cancel{2}1}{n+1} - \frac{\cancel{2}1}{n-1} - \frac{\cancel{2}\cos(n+1)\pi}{n+1} + \frac{\cancel{2}\cos(n-1)\pi}{n-1}\right] \quad \forall\ \underline{\underline{n \neq 1}}$$

$$a_n \begin{cases} n \text{ odd} \Rightarrow 0 \\ n \text{ even} \Rightarrow \frac{1}{\pi}\left(\frac{2}{n+1} - \frac{2}{n-1}\right) = \frac{4}{\pi(1-n^2)} \\ n = 1 \Rightarrow 0 \end{cases} \quad \& \quad b_n = 0$$

cosine series,

$$\sin x = \frac{2}{\pi} + \sum_{n=1}^{\infty} \frac{4 \cos 2nx}{\pi\left(1-(2n)^2\right)}$$

8) a) $f(x) = \cos x$ sine series

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{0} -\cos x\,dx + \int_0^{\pi} \cos x\,dx = \frac{1}{2\pi}\int_0^{-\pi} \cos x\,dx + \int_0^{\pi} \cos x\,dx$$

$$= \frac{1}{2\pi}\left[\left(\sin(-\pi) - \sin 0\right) + \sin(\pi) - \sin 0\right]$$

$$0$$

$a_n = 0$ as sine series

$$b_n = \frac{1}{\pi}\left[\int_0^{-\pi} \cos x \sin nx\,dx + \int_0^{\pi} \cos x \sin nx\,dx\right] \Rightarrow \frac{1}{2\pi}\left[\int_0^{-\pi} \sin(n+1)x + \sin(n-1)x\,dx\right.$$

$$+ \int_0^{\pi} \sin(n+1)x + \sin(n-1)x\,dx$$

$$= \frac{-1}{2\pi}\left[\frac{\cos(n+1)x}{n+1} + \frac{\cos(n-1)x}{n-1} \Big|_0^{-\pi} + \frac{\cos(n+1)x}{n+1} + \cos\frac{(n-1)x}{n-1} \Big|_0^{\pi} \right]$$

$$= \frac{-1}{2\pi}\left[\frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{n-1} - \frac{2}{n+1} - \frac{2}{n-1} + \frac{\cos(n+1)\pi}{n+1} + \frac{\cos(n-1)\pi}{(n-1)} \right]$$

$$= \frac{-1}{\cancel{2}\pi}\left[\frac{\cancel{2}\cos(n+1)\pi}{n+1} + \frac{\cancel{2}\cos(n-1)\pi}{(n-1)} - \frac{\cancel{2}}{n+1} - \frac{\cancel{2}}{n-1} \right]$$

$$n \begin{cases} \text{odd} & 0 \\ \text{even} & \frac{-1}{\pi}\left[-\frac{2}{n+1} - \frac{2}{n-1} \right] = \frac{4n}{\pi(1-n^2)} \\ 1 & 0 \end{cases}$$

$$\cos x = 4 \sum_{n=1}^{\infty} \left(\frac{-2n}{\pi(1-(2n)^2)} \right)$$

B) $f(x) = x^3$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} x^3 \, dx = 0$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \cos nx \, dx = 0$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} x^3 \sin nx \, dx = \frac{1}{\pi} \left[-\frac{x^3 \cos nx}{n} + \frac{3x^2 \sin nx}{n^2} + \frac{6x \cos nx}{n^3} + \frac{6 \sin nx}{n^4} \right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{\pi^3 \cos n\pi}{n} - \frac{\pi^3 \cos n\pi}{n} + \frac{6\pi \cos n\pi}{n^3} + \frac{6\pi \cos n\pi}{n^3} \right]$$

$$= \frac{1}{\pi} \cdot \frac{-2\pi^3 \cos n\pi}{n} + \frac{1}{\pi} \cdot \frac{12\pi \cos n\pi}{n^3}$$

$$= \cancel{\frac{2\pi^2(-1)(-1)^n}{n}} = \frac{(12 - 2n^2\pi^2)(-1)^n}{n^3}$$

$$f(x) \approx \sum_{n=1}^{\infty} (-1)^n \left(\frac{12 - 2n^2\pi^2}{n^3} \right) (-1)^n \sin nx$$

(4)

a) $f(x) = \begin{cases} 8 & 0 < x < 2 \\ -8 & 2 < x < 4 \end{cases}$ Periodicity is 4

$$4a_0 = \int_0^2 8\,dx + \int_2^4 (-8)\,dx$$

$$a_0 = \frac{1}{4}\left[16 - 16\right] = 0$$

$$a_n = \frac{1}{2}\int_0^2 8\cos\frac{n\pi}{2}x\,dx \Rightarrow \frac{1}{2}\left[\frac{2\cdot 8}{n\pi}\sin\frac{n\pi}{2}x\right]_0^2 = \frac{8}{n\pi}(\sin n\pi) = 0$$

$$\cancel{a_n} = 0 + \frac{1}{2}\int_2^4 (-8)\cos\frac{n\pi}{2}x\,dx = -\frac{1}{2}\left[\frac{2(8)}{n\pi}\cdot\sin\frac{n\pi}{2}x\right]_2^4 = 0$$

$$b_n = \frac{1}{2}\left[\int_0^2 8\sin\frac{n\pi}{2}x\,dx + \int_2^4 -8\sin\frac{n\pi}{2}x\,dx\right]$$

$$= 4\left[\frac{2}{n\pi}\left[-\cos\frac{n\pi}{2}x\right]_0^2 + \cos\frac{n\pi}{2}x\Big|_2^4\right]$$

$$\cancel{= \frac{8}{n\pi}\left[-\cos n\pi + 1 + \cos 2n\pi - \cos n\pi\right] = \frac{8}{n\pi}\left[1 - \cos 2n\pi\right]}$$

$$= \frac{8}{n\pi}\left[-2\cos n\pi + 1 + \cos 2n\pi\right]$$

$$= \frac{16}{n\pi}\left[1 - (-1)^n\right]$$

$$f(x) \sim \sum_{n=1}^{\infty} \frac{16}{n\pi}\left[1 - (-1)^n\right]\sin nx$$

b) $f(x) \begin{cases} 2x & 0 \le x \le 3 \\ 0 & -3 \le x < 0 \end{cases}$ periodicity 6

$$a_0 = \frac{1}{6}\int_0^3 2x\,dx = \frac{1}{6}\left[\frac{2x^2}{2}\right]_0^3 = \frac{9}{6} = 3/2$$

$$a_n = \frac{1}{3}\int_0^3 2x\cdot\cos\frac{n\pi}{3}x\,dx = \frac{1}{3}\left[\frac{6x}{n\pi}\sin\frac{n\pi}{3}x + \frac{18}{(n\pi)^2}\cos\frac{n\pi x}{3}\right]_0^3$$

$$= \frac{2}{n\pi}\left[3\sin n\pi + \frac{3\cos n\pi - 3}{(n\pi)}\right]$$

$$= \frac{6}{(n\pi)^2}\left[(-1)^n - 1\right]$$

$$b_n = \frac{1}{3}\int_0^3 2x\cdot\sin\frac{n\pi}{3}x\,dx = \frac{1}{3}\left[\frac{-6x}{n\pi}\cos\frac{n\pi}{3}x + 2\left(\frac{3}{n\pi}\right)^2\sin\frac{n\pi x}{3}\right]_0^3$$

$$= \frac{1}{3}\left(\frac{-6(3)\cos n\pi}{n\pi}\right) = \frac{-6(-1)^n}{n\pi}$$

$$f(x) = \frac{3}{2} + \sum_{n=1}^{\infty} \frac{6}{(n\pi)^2}\left((-1)^n - 1\right)\cos nx - \frac{6(-1)^n}{n\pi}\sin nx$$

c) $f(x) = 4x \quad 0<x<10 \qquad$ preriod $T = 10$

$$a_0 = \frac{1}{10}\left[\frac{4x^2}{2}\right]_0^{10} = \frac{200}{10} = 20$$

$$a_n = \frac{1}{5}\int 4x\cos\frac{n\pi x}{5}\,dx = \frac{1}{5}\left(\frac{5}{n\pi}\right)4x\sin\frac{n\pi x}{5} + 4\left(\frac{5}{n\pi}\right)^2\cos\frac{n\pi x}{5}\Bigg|_0^{10}$$

$$= \frac{1}{5}4\left(\frac{5}{n\pi}\right)^2\left[\cos 2n\pi - 1\right] = 0$$

$$b_n = \frac{1}{5}\int 4x\sin\frac{n\pi}{5}x\,dx = \frac{1}{5}\,-4\left(\frac{5}{n\pi}\right)x\cos\frac{n\pi x}{5} + 4\sin\frac{n\pi x}{5}\left(\frac{5}{n\pi}\right)^2\Bigg|_0^{10}$$

$$= -\frac{4}{5}\left(\frac{5}{n\pi}\right)10\cos 2n\pi = -\frac{40}{n\pi}$$

$$f(x) \sim 20 + \sum_{n=1}^{\infty} -\frac{40}{n\pi}\sin nx$$

9 B) $f(x) = e^{-ax} \qquad -\pi < x < \pi$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx = \frac{1}{2\pi}\left[\frac{e^{-ax}}{-a}\right]_{-\pi}^{\pi} = \frac{-e^{-a\pi}+e^{a\pi}}{2\pi a} = \frac{1}{\pi a}\left(\frac{e^{a\pi}-e^{-a\pi}}{2}\right)$$

$$= \frac{1}{\pi a}\sinh a\pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}(-a\cos nx + n\sin nx)\right]_{-\pi}^{\pi}$$

$$= \frac{-a}{\pi}\frac{2}{a^2+n^2}\,\frac{e^{-\pi a}\cos n\pi - e^{\pi a}\cos n\pi}{2}$$

$$= \frac{2a(-1)^n}{\pi(a^2+n^2)}\sinh \pi a$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}(a\sin nx - n\cos nx)\right]_{-\pi}^{\pi}$$

$$= \frac{2\,n}{\pi(a^2+n^2)}\left(\frac{e^{a\pi}\cos\pi n - e^{-\pi a}\cos\pi n}{2}\right)$$

$$= \frac{2n(-1)^n}{\pi(a^2+n^2)}\sinh\pi a$$

Σ

10) $f(x) = x^2 \qquad -\pi < x < \pi$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x^2\,dx = \frac{1}{2\pi}\left.\frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{\cancel{2\pi}}\frac{2\pi^3}{3} = \frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \cos nx\,dx = \frac{1}{\pi}\left[\frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - 2\frac{\sin nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[\frac{2\cancel{\pi}\cos n\pi}{n^2} + \frac{2\cancel{\pi}\cos n\pi}{n^2}\right]$$

$$= \frac{4\cos n\pi}{n^2} = \frac{4(-1)^n}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2 \cdot \sin nx\,dx = 0$$

$$x^2 \sim \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n \cos nx}{n^2} = \frac{\pi^2}{3} + 4\left[-\frac{\cos x}{1} + \frac{\cos 2x}{2^2} - \frac{\cos 3x}{3^2} + \cdots\right]$$

a) $\cancel{\sum \frac{(-1)^n}{n^2}}$ $\sum_{n=1}^{\infty}\frac{1}{n^2}$ $\qquad x = \pi$

$$\pi^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{1}{n^2}$$

$$\frac{2\pi^2}{3\cdot 4} = \sum_{n=1}^{\infty}\frac{1}{n^2} \Rightarrow \frac{\pi^2}{6}$$

b) $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$ $\qquad x = 0 \qquad = \frac{\pi^2}{3} + 4\left[-1 + \frac{1}{2^2} - \frac{1}{3^2} \cdots\right]$

$$\frac{f(\pi) - f(0)}{2} = 4\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$$

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2} = \frac{\pi^2 - 0}{8}$$

c) $\sum_{n=1}^{\infty}\frac{1}{n^4}$ $\qquad$ Using Par

(+)

11) $f(x) = x - x^2 \qquad -\pi < x < \pi$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(x-x^2)\,dx = \frac{1}{2\pi}\left.\frac{x^2}{2}-\frac{x^3}{3}\right|_{-\pi}^{\pi} = \frac{1}{2\pi}\left[\frac{\pi^2}{2}-\frac{\pi^3}{3}-\left(\frac{\pi^2}{2}+\frac{\pi^3}{3}\right)\right]$$

$$= \frac{1}{2\pi}\left[\cancel{\frac{\pi^2}{2}}-\frac{\pi^3}{3}-\cancel{\frac{\pi^2}{2}}-\frac{\pi^3}{3}\right] = -\frac{\cancel{2}\pi^{\cancel{3}}}{3}\,\frac{1}{\cancel{2\pi}} = -\frac{\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\frac{\sin nx}{n} + (1-2x)\frac{\cos nx}{n^2} + \frac{2\sin nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{(1-2\pi)\cos n\pi}{n^2} - \frac{(1+2\pi)\cos n\pi}{n^2}\right]$$

$$= \frac{\cos n\pi}{\pi n^2}\;\cancel{1}-2\pi\,\cancel{-1}-2\pi = \frac{-4\cos n\pi}{n^2} = \frac{(-1)^n(-4)}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\cancel{x-x^2}\;\cancel{(1-x^2)}(x^2-x)\frac{\cos nx}{n} + (1-2x)\frac{\sin nx}{n^2} - \frac{2\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{x^2\cos nx}{n} - \frac{x\cos nx}{n} - \frac{2\cos nx}{n^3}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\cancel{\pi}}\left[\cancel{\frac{\pi^2\cos nx}{n}} - \cancel{\frac{\pi^2\cos n\pi}{n}} - \left[\frac{2\cancel{\pi}\cos n\pi}{n}\right] \cancel{\pi} - \left[\cancel{\frac{2\cos n\pi}{n^3}} - \cancel{\frac{2\cos n\pi}{n^3}}\right]\right]$$

$$= \frac{(-2)(-1)^n}{n}$$

$$x - x^2 \sim -\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{(-1)^n(-4)}{n^2}\cos nx + \frac{(-2)(-1)^n}{n}\sin nx$$

$x = 0$

$$0 = -\frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n(-1)}{n^2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n(-1)}{n^2} = \frac{\pi^2}{12}$$

12) $f(x) = e^{ax}$ $\quad f(x+2\pi) = f(x)$

$-\pi < x < \pi$

(7)

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-ax}\,dx = \frac{1}{2\pi}\frac{e^{-ax}}{a}\Big]_{-\pi}^{\pi} = \frac{e^{\pi a} - e^{-\pi a}}{(\pi a)\,2} = \frac{\sinh \pi a}{\pi a}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\cos nx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}(-a\cos nx + n\sin nx)\right]_{-\pi}^{\pi}$$

$$= \frac{2}{\pi}\left[\frac{e^{\pi a} - e^{-\pi a}}{2}\right]\frac{\cos n\pi}{a^2+n^2} = \frac{2a(-1)^n \sinh \pi a}{\pi(a^2+n^2)}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-ax}\sin nx = \frac{1}{\pi}\left[\frac{e^{-ax}}{a^2+n^2}(-a\sin nx - b\cos nx)\right]_{-\pi}^{\pi}$$

$$= \frac{2b}{\pi(a^2+n^2)}\frac{(e^{\pi a} - e^{-\pi a})}{2}\cos n\pi = \frac{2b(-1)^n \sinh \pi a}{\pi(a^2+n^2)}$$

$$e^{ax} \approx \frac{\sinh \pi a}{\pi a} + \frac{2\sinh \pi a}{\pi}\sum_{n=1}^{\infty}\frac{a(-1)^n\cos nx + b(-1)^n \sin nx}{(a^2+n^2)}$$

$x = 0$

$$1 \approx \frac{\sinh \pi a}{\pi a} + \frac{2a\sinh \pi a}{\pi a}\sum_{n=1}^{\infty}\frac{(-1)^n}{a^2+n^2}$$

$$\frac{\pi a}{2a\sinh \pi a} - \frac{\sinh \pi a}{2a\sinh \pi a} = \sum_{n=1}^{\infty}\frac{a(-1)^n}{a^2+n^2}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n}{a^2+n^2} = \frac{\pi}{2\sinh \pi a} - \frac{1}{2a}$$

13) $f(t) = (1-t^2)$ $\quad -1 \le t \le 1$

$$a_0 = \frac{1}{2}\int_{-1}^{1}(1-t^2)\,dt = \int_0^1 (1-t^2)\,dt = t - \frac{t^3}{3} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$a_n = \int_{-1}^{1}(1-t^2)\cos n\pi t\,dt = (1-t^2)\left(\frac{\overset{0}{\sin n\pi t}}{n\pi}\right) - \frac{2t\cos n\pi t}{(n\pi)^2} + \frac{2\overset{0}{\sin n\pi t}}{(n\pi)^3}\Bigg|_{-1}^{1}$$

$$= \frac{2t\cos n\pi t}{(n\pi)^2}\Bigg|_{1}^{-1} = \frac{-4\cos n\pi}{(n\pi)^2}$$

$$= \frac{-4}{(n\pi)^2}(-1)^n$$

$$b_n = \int_{-1}^{1}(1-t^2)\sin n\pi t\,dt = 0$$

$$(1-t^2) \approx \sum_{n=1}^{\infty}\frac{-4(-1)^n}{(n\pi)^2}\cos nx$$

13. A)

$$f(x) = \begin{cases} 2 & 0 < x < 2\pi/3 \\ 1 & 2\pi/3 < x < 4\pi/3 \\ 0 & \frac{4\pi}{3} < x < 2\pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\left[2\left(\frac{2\pi}{3}\right) + 1\left(\frac{2\pi}{3}\right) + 0\right] = \frac{1}{\cancel{2\pi}} \frac{\cancel{6\pi}}{\cancel{3}} = 1$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi/3} 2\cos nx\,dx + \int_{2\pi/3}^{4\pi/3} \cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{2\sin nx}{n}\Big|_0^{2\pi/3} + \frac{\sin nx}{n}\Big|_{2\pi/3}^{4\pi/3}\right]$$

$$= \frac{1}{\pi}\ \frac{\sin\left(\frac{2n\pi}{3}\right)}{n} - 0 + \frac{\sin\left(\frac{4\pi n}{3}\right)}{n} - \frac{\sin\left(\frac{2n\pi}{3}\right)}{n}$$

$$= \frac{1}{\pi}\left[\frac{\sin}{n}\frac{2n\pi}{3} - \frac{\sin}{n}\frac{2n\pi}{3}\right] = 0$$

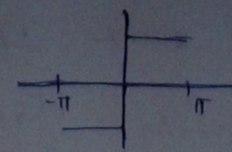
$$b_n = \frac{1}{\pi}\int_0^{2\pi/3} 2\sin nx\,dx + \int_{2\pi/3}^{4\pi/3} \sin nx\,dx$$

$$= \frac{1}{\pi}\ \frac{2\cos nx}{n}\Big|_{2\pi/3}^{0} + \frac{\cos nx}{n}\Big|_{4\pi/3}^{2\pi/3}$$

$$= \frac{1}{\pi}\left[\frac{2}{n} - \frac{2\cos}{n}\frac{2n\pi}{3} + \cancel{\frac{\cos}{n}}\frac{2n\pi}{3} - \frac{\cos\left(\frac{4n\pi}{3}\right)}{n}\right]$$

$$= \frac{1}{\pi}\left[\frac{2}{n} - \frac{2\cos}{n}\frac{2n\pi}{3}\right]$$

(14) $f(x) \begin{cases} -k & -\pi < x < 0 \\ k & 0 < x < \pi \end{cases}$ $\quad f(x) = f(x+2\pi)$



$f(x)$ is an odd function so a_n & $a_0 = 0$

(8)

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} k \sin nx\,dx + \int_{0}^{-\pi} k \sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{k\cos nx}{n}\Big|_{\pi}^{0} + \frac{k\cos nx}{n}\Big|_{-\pi}^{0}\right] = \frac{1}{\pi}\left[\frac{k}{n} - \frac{k\cos n\pi}{n} + \frac{k}{n} - \frac{k\cos n\pi}{n}\right]$$

$$= \frac{1}{\pi}\left[\frac{2k}{n} - \frac{2k\cos n\pi}{n}\right]$$

$$f(x) = \frac{2k}{\pi}\sum_{n=1}^{\infty}\left[\frac{1}{n} - \frac{(-1)^n}{n}\right]\sin nx$$

$$x = \pi/2$$

$$K = \frac{2K}{\pi}\sum_{n=1}^{\infty}\left[\frac{2}{(2n-1)}\sin(2n-1)\frac{\pi}{2}\right]$$

$$\frac{\pi}{4} = \sum_{n=1}^{\infty}\frac{\sin(2n-1)\pi/2}{(2n-1)} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \cdots$$

(15) $f(x) = x^2 \quad 0 < x < 2\pi \quad f(x) = f(x+2\pi)$

~~x^2 is a even function so $b_n = 0$~~

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x^2\,dx = \frac{1}{2\pi}\frac{8\pi^3}{3} = \frac{4\pi^2}{3}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \cos nx\,dx = \frac{1}{\pi}\left[\frac{x^2 \sin nx}{n} + \frac{2x\cos nx}{n^2} - \frac{2\sin nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left(\frac{4\pi}{n^2}\cos 2n\pi - \frac{4\pi}{n^2}\right) = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x^2 \sin nx\,dx = \frac{1}{\pi}\left[-\frac{x^2\cos nx}{n} + \frac{2x\sin nx}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{2\pi}$$

$$= \frac{1}{\pi} - \frac{4\pi^2\cos 2n\pi}{n} = -\frac{4\pi}{n}$$

$$f(x) \approx \frac{4\pi^2}{3} - 4\pi\sum_{n=1}^{\infty}\frac{\sin nx}{n}$$

(17) $f(t) = \{ \sin t \quad \text{for } 0 < t < \pi \qquad f(t+2\pi) = f(t)$

16) $f(x) = x \sin x \qquad \cancel{f(-x) = f(x)} \text{ so}$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x \sin x \, dx = \frac{1}{2\pi}\Big[-x\cos x + \sin x\Big]_0^{2\pi}$$

$$= \frac{1}{2\pi}\Big[-2\pi\Big] = -1$$

$$b_n = \frac{1}{2\pi}\int_0^{2\pi} x \sin x \sin nx \, dx = \frac{1}{2\pi}\int_0^{2\pi}\Big[x\cos(n-1)x - x\cos(n+1)x\Big]dx$$

$$= \frac{1}{2\pi}\Big[\frac{x\sin(n-1)x}{(n-1)} + \frac{\cos(n-1)x}{(n-1)^2} - \frac{x\sin(n+1)x}{(n+1)} - \frac{\cos(n+1)x}{(n+1)^2}\Big]_0^{2\pi}$$

$$\cancel{= \frac{1}{2\pi}\Big[\frac{\cos(n-1)2\pi}{(n-1)} - \frac{\cos(n+1)2\pi}{(n+1)} - \frac{1}{n-1} + \frac{1}{n+1}\Big]}$$

$$= \frac{1}{2\pi}\Big[\frac{1}{(n-1)^2} - \frac{1}{(n+1)^2} - \frac{1}{(n-1)^2} + \frac{1}{(n+1)^2}\Big] = 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x \sin x \cos nx \, dx = \frac{1}{2\pi}\int_0^{2\pi}\Big[x\sin(n+1)x - x\sin(n-1)x\Big]dx$$

$$= \frac{1}{2\pi}\Big[-\frac{x\cos(n+1)x}{(n+1)} + \frac{\sin(n+1)x}{(n+1)^2} + \frac{x\cos(n-1)x}{(n-1)} - \frac{\sin(n-1)x}{(n-1)^2}\Big]_0^{2\pi}$$

$$= \frac{1}{2\pi}\Big[-\frac{2\pi\cos(n+1)2\pi}{(n+1)} + \frac{2\pi\cos(n-1)2\pi}{(n-1)}\Big]$$

$$= \Big(\frac{1}{n-1}\Big) - \Big(\frac{1}{n+1}\Big) = \frac{n+1-n+1}{n^2-1} = \frac{2}{n^2-1}$$

$$f(x) \approx -1 + \sum_{n=1}^{\infty}\frac{2}{n^2-1}\cos nx$$

1) $f(t) = \begin{cases} \sin t & \text{for } 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases}$ $\quad f(t+2\pi) = f(t)$

$$a_0 = \frac{1}{2\pi}\int_0^{\pi} \sin t \, dt = \frac{1}{2\pi}\int_{\pi}^{0} \cos t \, dt = \frac{2}{2\pi} = \frac{1}{\pi}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \cos nt \sin t \, dt = \frac{1}{2\pi}\int_0^{\pi}\left[\sin(n+1)t - \sin(n-1)t\right] dt$$

$$= \frac{1}{2\pi}\left[-\frac{\cos(n+1)t}{(n+1)} + \frac{\cos(n-1)t}{(n-1)}\right]_0^{\pi}$$

(a)

$$= \frac{1}{2\pi}\left[\frac{\cos(n-1)\pi}{(n-1)} - \frac{\cos(n+1)\pi}{(n+1)} + \frac{1}{n+1} - \frac{1}{n-1}\right]$$

$$= \frac{1}{2\pi}\left[\begin{array}{ll} n \text{ is odd} & 0 \\ n \text{ is even} & \frac{-2}{n-1} + \frac{2}{n+1} \end{array}\right. = \frac{1}{2\pi}\left[\frac{2}{n+1} - \frac{2}{n-1}\right] = \frac{n-1-n-1}{(n+1)(n-1)}$$

$$= \frac{-2}{\pi(n+1)(n-1)} \quad n \text{ is even}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} \sin nt \cdot \sin t \, dt = \frac{1}{2\pi}\int_0^{\pi} \cos(n-1)t - \cos(n+1)t \, dt$$

$$= \frac{1}{2\pi}\left[\frac{\sin(n-1)t}{(n-1)} - \frac{\sin(n+1)t}{(n+1)}\right]_0^{\pi}$$

$$= \underline{0}$$

~~sint~~ $f(t) = \frac{1}{\pi} + \sum_{n=1}^{\infty} \frac{-2 \cos(2n-1)t}{\pi(2n+1)(2n-1)}$

d) $t = 0$

$$0 = \frac{1}{\pi} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n-1)}$$

$$\frac{1}{2} = \sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n-1)}$$

b)

15) $f(t) = t^2 + t \quad 0 \le t \le \pi$ half range sine series.

$$f(t) = \begin{cases} t^2 + t & 0 < t < \pi \\ -t^2 + t & -\pi < t < 0 \end{cases}$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} (-t^2+t)\,dt + \int_0^{\pi} (t^2+t)\,dt\right]$$

$$= \frac{1}{2\pi}\left[-\frac{t^3}{3} + \frac{t^2}{2}\Big|_{-\pi}^{0} + \frac{t^3}{3} + \frac{t^2}{2}\Big|_0^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\cancel{\frac{-\pi^3}{3}} - \cancel{\frac{\pi^2}{2}} + \cancel{\frac{\pi^3}{3}} + \cancel{\frac{\pi^2}{2}}\right] = 0$$

$a_n = 0$

$$b_n = \frac{1}{\pi} \int_0^{\pi} (t^2 \sin nt + t \sin nt)\,dt + \int_{-\pi}^{0} (-t^2+t) \sin nt\,dt$$

$$= \frac{1}{\pi}\left[-\frac{(t^2+t)\cos nt}{n} + \frac{(2t+1)\sin nt}{n^2} + \frac{2\cos nt}{n^3}\right]_0^{\pi}$$

$$+ \frac{1}{\pi}\left[-\frac{(t-t^2)\cos nt}{n} + \frac{(1-2t)\sin nt}{n^2} - \frac{2\cos nt}{n^3}\right]_{-\pi}^{0}$$

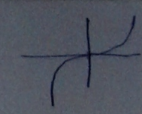
$$= \frac{1}{\pi}\left[-\frac{(t^2+t)\cos n\pi}{n} + \frac{2\cos n\pi}{n^3} - \frac{2}{n^3} - \frac{2}{n^3} - \frac{(t-t^2)\cos n\pi}{n} + \frac{2\cos n\pi}{n^3}\right]$$

$$= \frac{1}{\pi}\left[-\cancel{\frac{t^2\cos n\pi}{n}} - \cancel{\frac{\pi\cos n\pi}{n}} + \frac{4\cos n\pi}{n^3} - \frac{4}{n^3} + \cancel{\frac{\pi\cos n\pi}{n}} + \cancel{\frac{t^2\cos n\pi}{\pi}}\right]$$

$$= \frac{1}{\pi}\left[\frac{4(-1)^n}{n^3} - \frac{4}{n^3}\right]$$

$$f(t) = \frac{-8}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3}$$

19) $f(x) \begin{cases} x^2 & 0 \le x \le \pi \\ -x^2 & -\pi \le x \le 0 \end{cases}$



$a_n = 0$

(10)

$a_0 = 0$

$$b_n = \frac{1}{\pi}\left[\int_0^{\pi} x^2 \sin nx\, dx + \int_0^{-\pi} x^2 \sin x\, dx\right] = \frac{1}{\pi}\left[-\frac{x^2\cos nx}{n} + \frac{2x \overset{0}{\sin nx}}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{\pi}$$

$$+ \frac{1}{\pi}\left[-\frac{x^2\cos nx}{n} + \frac{2x\overset{0}{\sin nx}}{n^2} + \frac{2\cos nx}{n^3}\right]_0^{-\pi}$$

$$= \frac{1}{\pi}\left[\cancel{\frac{-\pi^2\cos n\pi}{n}} + \frac{2\cos n\pi}{n^3} - \frac{2}{n^3} - \cancel{\frac{\pi^2\cos n\pi}{n}} + \frac{2\cos n\pi}{n^3} - \frac{2}{n^3}\right]$$

$$= \frac{1}{\pi}\left[\frac{-2\pi^2}{n}\cos n\pi + \frac{4\cos n\pi}{n^3} - \frac{4}{n^3}\right] \ldots$$

n even $\rightarrow$ $-\frac{2\pi}{n}\cancel{\cos n\pi}(-1)^n$

odd $\rightarrow$ $-\frac{2\pi}{n}(-1)^n - \frac{8}{\pi n^3}$

$$f(x) \sim \sum_{n=1}^{\infty} -\frac{2\pi(-1)^n}{n} - \frac{8}{\pi(2n-1)^3}$$