Engr098 Lab4

Vincent Liu

Lab 4: Thévenin's Theorem

Objectives

1. Understand the experimental procedure to determine V_{Th} and R_{Th}.
2. Validate Thévenin's theorem through experimental measurements using two methods.
3. Validate the condition for maximum power transfer to a Thévenin circuit.

Equipment

Lab toolbox; Digital Multimeter (DMM); DC Power Supply.

Theory (Concise)

A linear two–terminal network can be replaced by an equivalent source in series with a single resistor:

$$\boxed{\text{Thévenin equivalent} \quad \longrightarrow \quad V_{\mathrm{Th}} \text{ in series with } R_{\mathrm{Th}}.}$$

- V_{Th}: the *open-circuit* voltage at the output terminals.
- R_{Th}: the equivalent resistance *seen into* the network with sources handled as below.

How to evaluate R_{Th} (two standard methods)

1. **Deactivated-sources method (Method-1).** Deactivate all *independent* sources:
   - Voltage sources $\rightarrow$ short circuits.
   - Current sources $\rightarrow$ open circuits.

   Then reduce the remaining resistor network as seen from the output terminals: $R_{\mathrm{Th}} = R_{\mathrm{eq}}$.

2. **Open-/short-circuit ratio (Method-2).**

$$\boxed{R_{\mathrm{Th}} = \frac{V_{\mathrm{oc}}}{I_{\mathrm{sc}}}}$$

   where V_{oc} is the open-circuit terminal voltage and I_{sc} is the short-circuit current.

Maximum power transfer For a load R_L connected to a Thévenin source,

$$R_L = R_{\text{Th}} \quad \Rightarrow \quad P_{L,\max} = \frac{V_{\text{Th}}^2}{4R_{\text{Th}}}.$$

Given Circuit

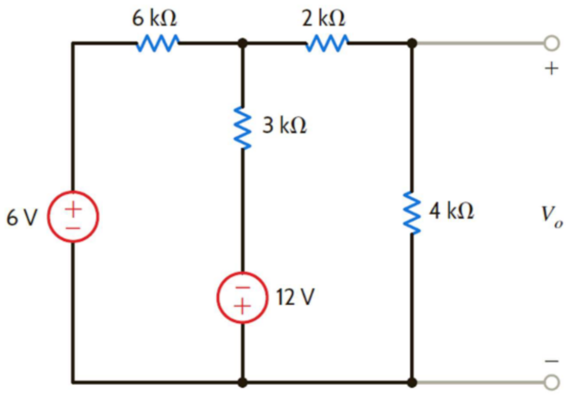

Figure 1: Two-source network with 6 kΩ, 3 kΩ, 2 kΩ, and 4 kΩ resistors. Output terminals at the right (+ at top).

Worked reduction for Method-1 on this circuit (preview of Part 1A)

Deactivate the independent voltage sources (6 V and 12 V $\Rightarrow$ shorts). The network seen from the terminals becomes:

- A direct branch 4 kΩ from the top terminal to ground.
- A second branch consisting of 2 kΩ in series with the parallel of 6 kΩ and 3 kΩ.

Compute step by step:

$$(6\ \text{k}\Omega \parallel 3\ \text{k}\Omega) = \frac{6k \cdot 3k}{6k + 3k} = 2\ \text{k}\Omega, \qquad R_{\text{path}} = 2\ \text{k}\Omega + 2\ \text{k}\Omega = 4\ \text{k}\Omega,$$

$$\boxed{R_{\text{Th}} = 4\ \text{k}\Omega \parallel 4\ \text{k}\Omega = 2\ \text{k}\Omega.}$$

Percent-difference metric (for later comparison)

Given a theoretical value $R_{\text{Th,thy}}$ and measured $R_{\text{Th,expt}}$,

$$\%\text{Difference} = \frac{|R_{\text{Th,thy}} - R_{\text{Th,expt}}|}{\frac{R_{\text{Th,thy}} + R_{\text{Th,expt}}}{2}} \times 100\%.$$

Part 1A: Method 1 — Thévenin Resistance with Deactivated Sources

Goal

Find the Thévenin resistance R_{Th} at the output terminals by *deactivating all independent sources* and reducing the resistor network. Then measure R_{Th} with a DMM and compare.

Procedure (Step-by-Step)

1. **Remove the load** at the output terminals.
2. **Deactivate independent sources:** voltage sources $\to$ short circuits; current sources $\to$ open circuits.
3. **Look into the terminals** and reduce the remaining resistors to get $R_{\mathrm{Th,\,thy}}$.
4. **Measure** $R_{\mathrm{Th,\,expt}}$ with the DMM (ohmmeter) across the open terminals (ensure the network is unpowered).
5. **Compare** using percent difference.

Deactivated Network (this circuit)

With the 6 V and 12 V sources shorted, the network seen from the terminals becomes two branches in parallel:

- Branch 1: a single 4 kΩ.
- Branch 2: a 2 kΩ in series with (6 kΩ ‖ 3 kΩ).

[european] (0,0) to[short,o-] (0,2); (5,0) node[ground] to[short] (5,0); (0,2) to[R=4 kΩ] (0,0); (0,2) to[short] (2,2) to[R=2 kΩ] (4,2) to[short] (4,2) to[short] (4,2); (4,2) to[R=6 kΩ] (4,0); (4,2) to[R=3 kΩ] (2,0); (2,0) to[short] (0,0);

Figure 2: Deactivated-source network seen from the output terminals.

Theoretical Calculation

Parallel and series reductions:

$$R_{63} = 6k\Omega \parallel 3k\Omega = \frac{6k \cdot 3k}{6k + 3k} = 2k\Omega,$$

$$R_{\mathrm{path}} = 2k\Omega + R_{63} = 2k\Omega + 2k\Omega = 4k\Omega,$$

$$\boxed{R_{\mathrm{Th,\,thy}} = R_{\mathrm{path}} \parallel 4k\Omega = \frac{4k \cdot 4k}{4k + 4k} = 2k\Omega.}$$

Check by test-source method (optional). Apply a 1 V test source at the terminals, compute input current:

$$I_{\text{in}} = \frac{1}{4k} + \frac{1}{2k + (6k \parallel 3k)} = \frac{1}{4k} + \frac{1}{4k} = \frac{1}{2k} \;\Rightarrow\; R_{\text{Th}} = \frac{V}{I} = \frac{1}{1/2k} = 2k\Omega.$$

Experimental Measurement

With the supplies off and sources disconnected (shorted), we measured resistance across the open terminals using the DMM (ohms mode).

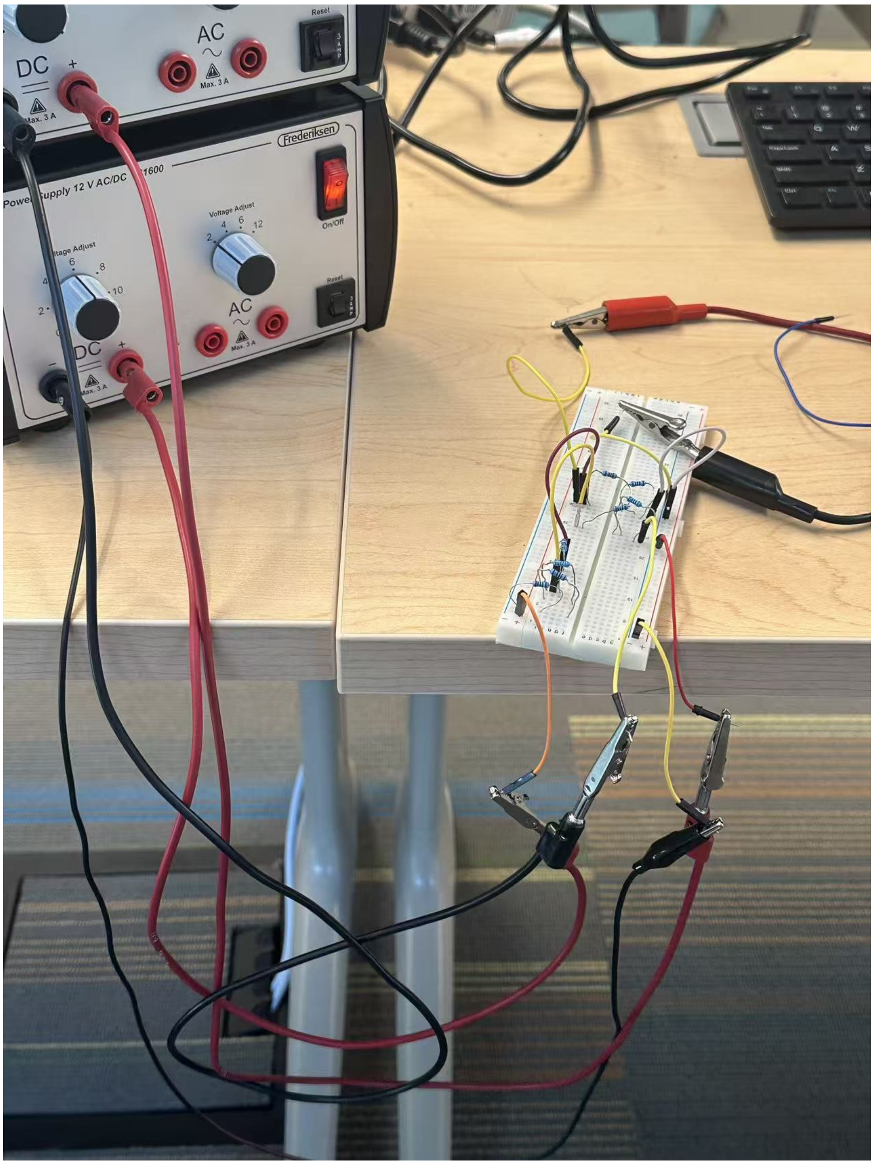

Figure 3: Bench setup: supply and breadboard during Thévenin testing.

Measured value:

$$\boxed{R_{\text{Th, expt}} \approx 2.00\,\text{k}\Omega} \quad \text{(reading from Fig. 4).}$$

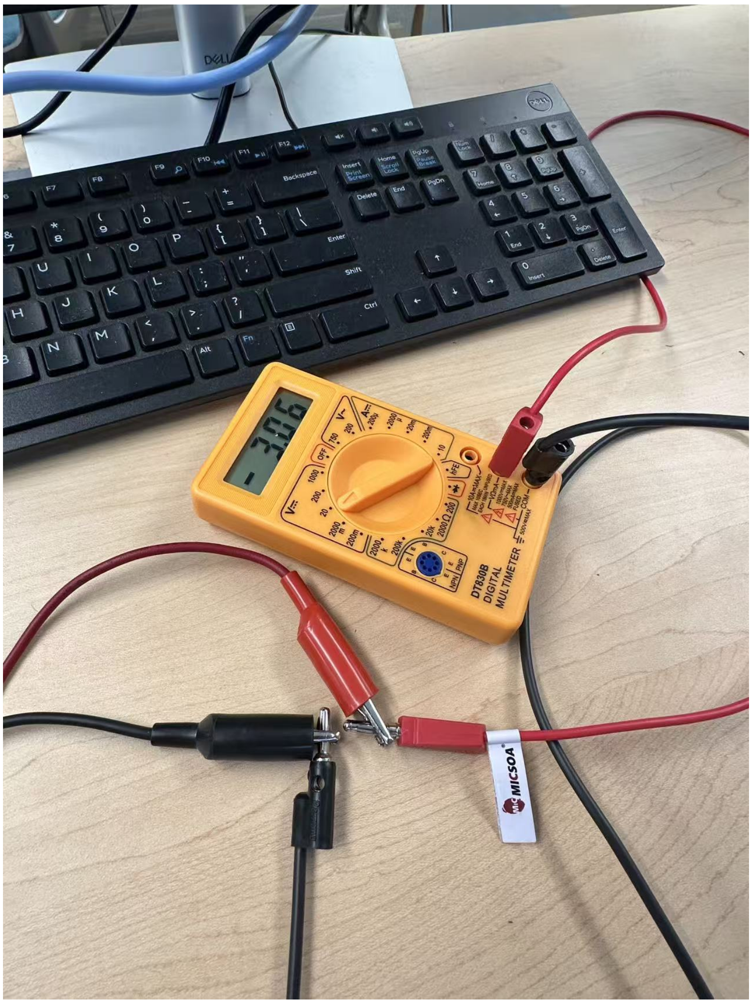

Figure 4: DMM measurement across the output terminals (sources deactivated).

Comparison

$$\%\text{Difference} = \frac{|R_{\text{Th, thy}} - R_{\text{Th, expt}}|}{\frac{R_{\text{Th, thy}} + R_{\text{Th, expt}}}{2}} \times 100\% = \frac{|2.00k - 2.00k|}{(2.00k + 2.00k)/2} \times 100\% \approx 0\%,$$

(If your meter reads 2.02 kΩ, the difference is about 0.99 %.)

Part 1B: Thévenin Resistance using v_{oc}/i_{sc}Voc/Isc

Short-Circuit Current i_{sc}

Theory (all sources *activated*, output shorted). Let the bottom rail be ground. With the output shorted, the output node voltage is $V_2 = 0$. Denote the center top node as V_1. Using KCL:

$$\underbrace{\frac{V_1 - 6}{6k}}_{\text{to 6V via 6k}} + \underbrace{\frac{V_1 - 12}{3k}}_{\text{to 12V via 3k}} + \underbrace{\frac{V_1 - V_2}{2k}}_{\text{to } V_2 \text{ via 2k}} = 0, \qquad V_2 = 0.$$

Solving gives

$$V_1 = 5\,\text{V}, \qquad i_{sc,\,\text{thy}} = \frac{V_1 - V_2}{2k\Omega} = \frac{5-0}{2000} = \boxed{2.50\,\text{mA}}.$$

Experiment. We inserted a short across the output terminals and measured the current using the DMM:

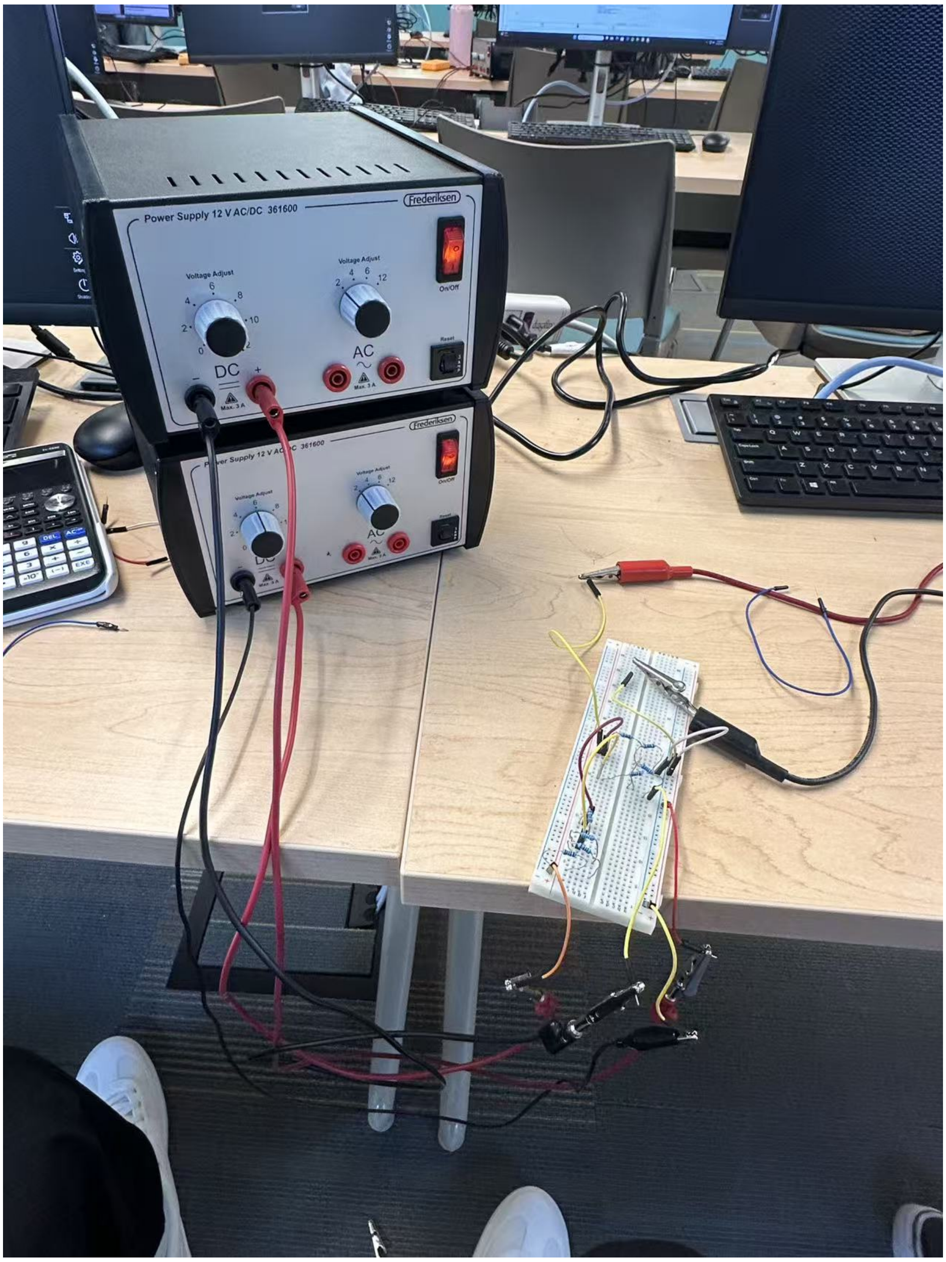

Figure 5: Bench setup with output short for i_{sc} measurement.

Thus,

$$\boxed{i_{sc,\,\text{expt}} \approx 1.49\,\text{mA}}.$$

Percent difference (current):

$$\%\text{Diff}_{i_{sc}} = \frac{|\,i_{sc,\text{thy}} - i_{sc,\text{expt}}\,|}{\frac{i_{sc,\text{thy}} + i_{sc,\text{expt}}}{2}} \times 100\% = \frac{|2.50 - 1.49|}{(2.50 + 1.49)/2} \times 100\% \approx \boxed{50.6\%}.$$

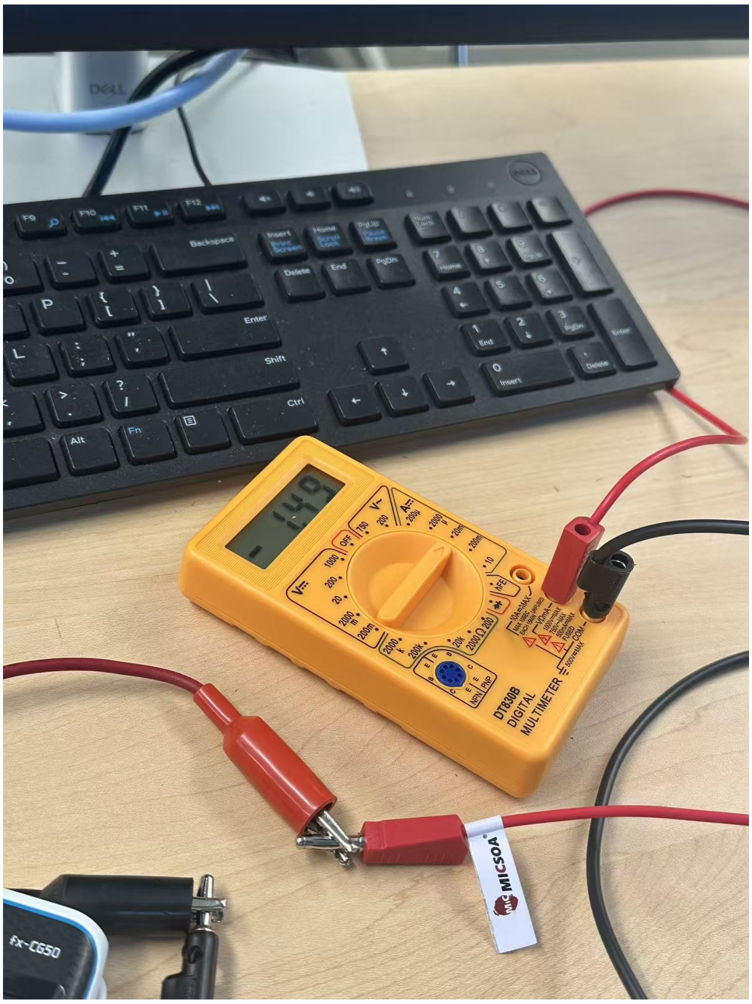

Figure 6: DMM reading for short-circuit current. Display shows `1.49` on the mA range.

Note. The large difference often indicates extra parallel paths (scope leads, supply not truly shorted/off, wiring across breadboard) or meter shunt effects. Ensure supplies are correctly configured for current measurement and that the short is low-resistance and local to the terminals.

Open-Circuit Voltage v_{oc} $(= V_{\mathrm{Th}})$

Theory (output open). With the output open, write KCL at V_1 and V_2:

$$\frac{V_1 - 6}{6k} + \frac{V_1 - 12}{3k} + \frac{V_1 - V_2}{2k} = 0,$$
$$\frac{V_2 - V_1}{2k} + \frac{V_2 - 0}{4k} = 0.$$

Solving,

$$V_1 = 7.5\,\mathrm{V}, \qquad \boxed{v_{oc} = V_2 = 5.00\,\mathrm{V}}.$$

Experiment. With the output open (no load), we measured the terminal voltage:

$$\boxed{v_{oc,\,\mathrm{expt}} \approx 5.0\,\mathrm{V}} \quad \text{(recorded on DMM)}.$$

Thévenin Resistance via Ratio

$$\boxed{R_{\text{Th, thy}-2} = \frac{v_{oc,\text{thy}}}{i_{sc,\text{thy}}} = \frac{5.00}{2.50\,\text{mA}} = 2.00\,\text{k}\Omega.}$$

Using the experimental numbers,

$$R_{\text{Th, expt}-2} = \frac{v_{oc,\text{expt}}}{i_{sc,\text{expt}}} = \frac{5.0}{1.49\,\text{mA}} \approx \boxed{3.36\,\text{k}\Omega}.$$

Comparison to Part 1A

From Part 1A:

$$R_{\text{Th, thy}-1} = 2.00\,\text{k}\Omega \quad \text{(deactivated-sources method)}.$$

Thus, theory agrees: $R_{\text{Th, thy}-1} = R_{\text{Th, thy}-2} = 2.00\,\text{k}\Omega$.

Using our experimental values here,

$$\%\text{Diff}_{R_{\text{Th}}} = \frac{\left|R_{\text{Th, thy}-1} - R_{\text{Th, expt}-2}\right|}{\frac{R_{\text{Th, thy}-1} + R_{\text{Th, expt}-2}}{2}} \times 100\% = \frac{|2.00 - 3.36|}{(2.00 + 3.36)/2} \times 100\% \approx \boxed{50.0\%}.$$