Engr098 Lab3

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Part 1: DC Circuits Containing Independent Sources

Objective

Apply basic circuit analysis to the given DC circuit (sources: 24 V and 2 A; resistors: $3\,\Omega$ and $6\,\Omega$) to find v_6 (voltage across the $6\,\Omega$ resistor, positive at the top). Verify the result with an independent method and check power balance.

Given Circuit (text description)

Left branch: an ideal 24 V source (positive at top) in series with a $3\,\Omega$ resistor connected to the top node. Middle branch: a $6\,\Omega$ resistor from the top node to ground. Right branch: a 2 A current source from ground to the top node (arrow up). The bottom node is the reference (0 V). Let the top node voltage be V; then $v_6 = V$.

Key Formulas

- Ohm's law: $v = iR$.
- KCL at a node: $\sum i_{\text{leaving}} = \sum i_{\text{entering}}$.
- Voltage divider (for checking): $v_{\text{out}} = V_s \frac{R_{\text{load}}}{R_{\text{series}}+R_{\text{load}}}$.
- Parallel equivalent: $R_{\parallel} = \left(\sum \frac{1}{R_i}\right)^{-1}$.
- Power (passive sign convention): $p = vi$ is positive if current enters the element's + terminal.

Part 1A: Nodal Analysis (primary solution)

Define currents *leaving* the top node through each element. With V as the top node voltage:

$$i_{6\Omega} = \frac{V-0}{6},$$

$$i_{3\Omega} = \frac{V-24}{3},$$

Right branch current leaving top node $= -2\,\text{A}$ (because the source pushes 2 A *into* the node).

KCL at the top node:

$$\frac{V}{6} + \frac{V-24}{3} - 2 = 0 \implies 6\Big(\frac{V}{6} + \frac{V-24}{3} - 2\Big) = 0 \implies V + 2(V-24) - 12 = 0$$

$$3V - 60 = 0 \implies V = \boxed{20\,\text{V}}.$$

Thus,

$$v_6 = V = \boxed{20\,\text{V}}.$$

Branch currents (sign + = leaving top node)

$$i_{6\Omega} = \frac{20}{6} = \frac{10}{3} \approx 3.333\,\mathrm{A}\ (\downarrow), \qquad i_{3\Omega} = \frac{20-24}{3} = -\frac{4}{3} \approx -1.333\,\mathrm{A}.$$

The negative sign on $i_{3\Omega}$ means the actual current flows from the 24 V node *into* the top node with magnitude 1.333 A. The right current source injects 2 A upward into the top node, as given.

Part 1B: Independent Verification

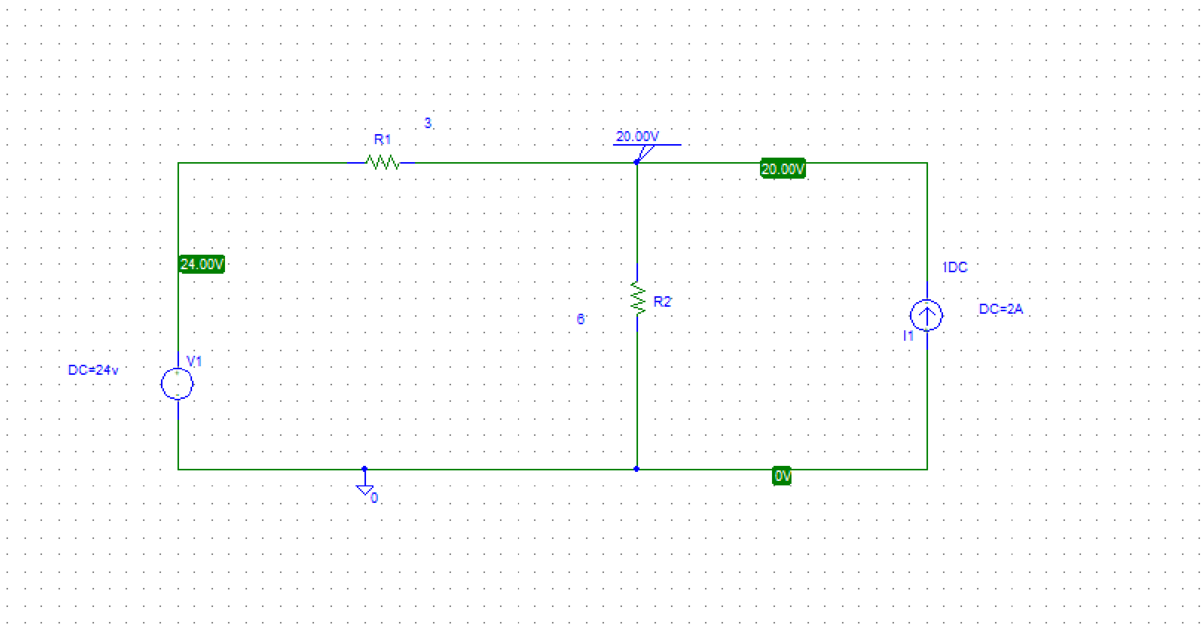

Figure 1: part 1 circuit

Method 1: Superposition

(a) Current source off (open circuit). The top node sees a divider of $3\,\Omega$ in series with $6\,\Omega$ driven by 24 V:

$$V_a = 24 \cdot \frac{6}{3+6} = 16\,\mathrm{V}.$$

(b) Voltage source off (short circuit). The top node sees the $3\,\Omega$ and $6\,\Omega$ in parallel, fed by a 2 A source:

$$R_{\parallel} = \frac{3 \cdot 6}{3+6} = 2\,\Omega, \qquad V_b = IR_{\parallel} = 2 \cdot 2 = 4\,\mathrm{V}.$$

Total: $V = V_a + V_b = 16 + 4 = \boxed{20\,\mathrm{V}}$ (matches nodal analysis and simulation).

Method 2: Power Check

Voltages (top minus bottom) and currents (positive entering + terminal) for each element:

$$v_{6\Omega} = 20\ \mathrm{V}, \quad i_{6\Omega} = +3.\overline{3}\ \mathrm{A}, \quad v_{3\Omega} = V - 24 = -4\ \mathrm{V}, \quad |v_{3\Omega}| = 4\ \mathrm{V}, \quad i_{3\Omega} = -1.\overline{3}\ \mathrm{A}.$$

Dissipations in resistors:

$$p_{6\Omega} = \frac{(20)^2}{6} = \frac{400}{6} \approx 66.667\,\mathrm{W}, \qquad p_{3\Omega} = \frac{(4)^2}{3} = \frac{16}{3} \approx 5.333\,\mathrm{W}.$$

Source powers (passive sign convention):

$$p_{V=24\,\mathrm{V}} = v\, i_{\mathrm{entering}\,+} = 24\,(-1.\overline{3}) \approx -32.0\,\mathrm{W},$$

$$p_{I=2\,\mathrm{A}} = v\, i_{\mathrm{entering}\,+} = 20\,(-2) = -40\,\mathrm{W}.$$

Negative values mean the sources *deliver* power. Totals:

$$p_{\mathrm{res}} = 66.667 + 5.333 = 72\,\mathrm{W}, \qquad p_{\mathrm{sources}} = -32 - 40 = -72\,\mathrm{W}.$$

Power balances (72 W absorbed = 72 W delivered).

Results Summary (tables)

Table 1: Node voltage and key currents

Quantity	Value	Note
v_6 (top node)	20 V	across 6 Ω (top positive)
$i_{6\Omega}$	3.333 A	downward (to ground)
$i_{3\Omega}$	−1.333 A	negative ⇒ 1.333 A flows from 24 V node **into** top node
Current source	2 A	injects into top node (arrow up)

Table 2: Element power (passive sign convention)

Element	Power (W)	Interpretation
6 Ω	+66.667	dissipated
3 Ω	+5.333	dissipated
24 V source	−32.0	delivered
2 A source	−40.0	delivered
Total	0	power conserved

Conclusion

Both nodal analysis and superposition give $v_6 = 20\,\mathrm{V}$. The branch currents are consistent with KCL, and the power check confirms conservation (sources deliver 72 W, resistors dissipate 72 W). These hand results agree with the PSpice measurement shown in the simulation (20 V at the top node).

Part 2: DC Circuit with a CCCS (Current-Controlled Current Source)

Goal

1) Find i, the current through the $6\,\Omega$ resistor in the left network. 2) A CCCS of value $3i$ drives the right network (a $5\,\Omega$ resistor). Find the voltage v across the $5\,\Omega$ resistor (top node +). 3) Verify by basic laws (KCL/KVL, power).

Given (text schematic)

- **Left network:** A 24 V source (top is +) feeds a $2\,\Omega$ series resistor into a top node. From that node to ground are two shunt resistors: $3\,\Omega$ and $6\,\Omega$. Define i as the current through the $6\,\Omega$ branch (downward to ground).
- **Right network:** A CCCS of value $3i$ is in parallel with a $5\,\Omega$ resistor; the voltage across the $5\,\Omega$ is labeled v (top node +).

Key formulas

Ohm's law $v = iR$; KCL $\sum i_{\text{leaving}} = 0$; Parallel: $R_{\parallel} = \left(\sum \frac{1}{R_k}\right)^{-1}$; Divider: $v_{\text{out}} = V_s \dfrac{R_{\text{load}}}{R_{\text{series}} + R_{\text{load}}}$; Power: $p = vi = i^2R = \dfrac{v^2}{R}$ (passive sign convention).

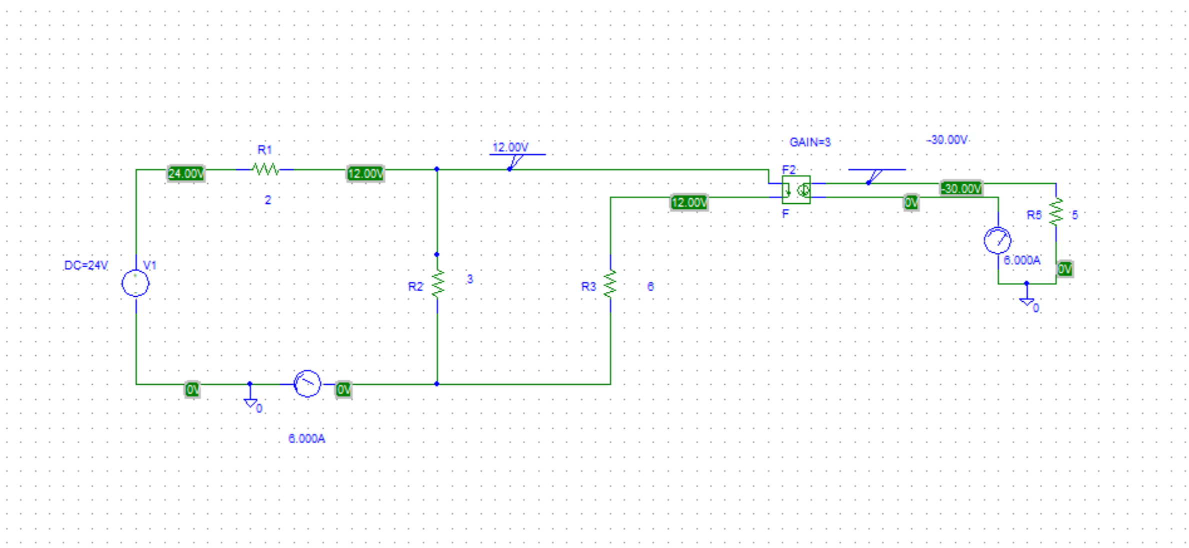

Figure 2: part 2 circuit

Step 1: Current i in the $6\,\Omega$ branch (left network)

Combine the shunt resistors:

$$R_p = 3 \parallel 6 = \frac{3 \cdot 6}{3+6} = 2\,\Omega.$$

Seen from the source, this is a divider of 2 Ω (series) and $R_p = 2$ Ω (to ground). Node voltage:

$$V_{\text{node}} = 24 \cdot \frac{R_p}{2 + R_p} = 24 \cdot \frac{2}{2+2} = \boxed{12\,\text{V}}.$$

Hence the $6\,\Omega$-branch current

$$i = \frac{V_{\text{node}}}{6} = \frac{12}{6} = \boxed{2\,\text{A}} \text{ (downward).}$$

(For reference: the $3\,\Omega$ branch carries $12/3 = 4$ A and the series $2\,\Omega$ carries 6 A; KCL checks: $2 + 4 = 6$.)

Step 2: Voltage v across the $5\,\Omega$ (right network)

The CCCS delivers a current $3i = 3 \times 2 = 6$ A into the two-node network. With only a $5\,\Omega$ in parallel, that entire current flows through the resistor (KCL).

$$v = (3i) \cdot 5 = 6 \times 5 = \boxed{30\,\text{V}} \quad \text{(top node positive).}$$

Verification

KCL/KVL and splits (left). Total source current $I_s = 24/(2+2) = 6\,\text{A}$; drop on series $2\,\Omega$: $6 \times 2 = 12\,\text{V}$; node is at $12\,\text{V}$. Branch currents $i_{3\Omega} = 4\,\text{A}$ and $i_{6\Omega} = 2\,\text{A}$ sum to $6\,\text{A}$.

Power check. Left network: resistor powers

$$p_{2\Omega} = I_s^2 \cdot 2 = 6^2 \cdot 2 = 72\,\text{W}, \quad p_{3\Omega} = 4^2 \cdot 3 = 48\,\text{W}, \quad p_{6\Omega} = 2^2 \cdot 6 = 24\,\text{W}.$$

Total absorbed by resistors $= 144\,\text{W}$; source delivers $24 \times 6 = 144\,\text{W}$ (balances).

Right network: resistor absorbs

$$p_{5\Omega} = (3i)^2 \cdot 5 = 6^2 \cdot 5 = \boxed{180\,\text{W}},$$

so the CCCS delivers $180\,\text{W}$ (equal magnitude, opposite sign), which is consistent with $v = 30\,\text{V}$ and current 6 A through the resistor.

PSpice capture notes (CCCS)

To realize a CCCS controlled by branch current i:

1. Place a *zero-volt* voltage source (`VSENSE`) *in series* with the $6\,\Omega$ branch to sense i (its current becomes the control variable).
2. Place an `F` element (CCCS) with gain 3 and reference it to `VSENSE`. Connect it in parallel with the $5\,\Omega$ resistor.
3. Probe the node voltage across the $5\,\Omega$ (v) and the current through the $6\,\Omega$ (the current through `VSENSE`). You should read $i = 2\,\text{A}$ and $v = 30\,\text{V}$.

Results (tables)

Table 3: Left network ($24\,\text{V}$ source, $2\,\Omega$ series, shunt $3\,\Omega$ and $6\,\Omega$)

Quantity	Value	Note
Node voltage	$12\,\text{V}$	across each shunt resistor
i through $6\,\Omega$	$2\,\text{A}$	downward
Current through $3\,\Omega$	$4\,\text{A}$	downward
Series current ($2\,\Omega$)	$6\,\text{A}$	left to right

Table 4: Right network (CCCS in parallel with $5\,\Omega$)

Quantity	Value	Note
Control current i	$2\,\text{A}$	from left network
CCCS value $3i$	$6\,\text{A}$	upward arrow
v across $5\,\Omega$	$30\,\text{V}$	top node positive
Power in $5\,\Omega$	$180\,\text{W}$	equals CCCS delivery

Conclusion

The simple calculations give

$$\boxed{i = 2\,\text{A}}, \qquad \boxed{v = 30\,\text{V}}.$$

These values should match PSpice when the CCCS is referenced to the sensed current through the $6\,\Omega$ branch. Power and KCL/KVL checks confirm correctness.