Engr098 Lab8

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Capacitor $C = 0.1$ F is in series between two nodes. The measured voltage is $v(t) = V_{\text{left}}(t) - V_{\text{right}}(t)$ ("+" on the left plate). Two switches change at $t = 0$:

- For $t < 0$: left switch *closed* (the $6\,\Omega$ branch present), right switch *open* (the $3\,\Omega$ branch absent).
- For $t > 0$: left switch *open*, right switch *closed*.

Sources: a $30\,\text{V}$ source through $12\,\Omega$ on the left; a $4\,\text{A}$ current source to the right node (upwards).

1. Initial voltage $v(0^-) = V_0$ (steady DC for $t < 0$)

At steady state, the capacitor is an open circuit, so the left and right sides are independent.

Left node voltage V_{L^-}

Voltage divider: $12\,\Omega$ in series feeding a $6\,\Omega$ load to ground.

$$V_{L^-} = 30\,\frac{6}{12+6} = \boxed{10\text{ V}}.$$

Right node voltage V_{R^-}

Only a $6\,\Omega$ to ground and a $4\,\text{A}$ current source into the node. KCL at steady DC:

$$V_{R^-} = I\,R = 4 \times 6 = \boxed{24\text{ V}}.$$

Initial capacitor voltage

$$V_0 = v(0^-) = V_{L^-} - V_{R^-} = 10 - 24 = \boxed{-14\text{ V}}.$$

2. Final voltage $v(\infty) = V_s$ (steady DC for $t > 0$)

Again the capacitor is an open circuit at steady DC.

Left node $V_{L\infty}$

Left switch is open $\Rightarrow$ the only path is a $12\,\Omega$ from the $30\,\text{V}$ source to an open node; current is 0, so no drop:

$$V_{L\infty} = \boxed{30\text{ V}}.$$

Right node $V_{R\infty}$

Right switch is closed, so $6\,\Omega \parallel 3\,\Omega$ to ground:

$$R_{\text{right,eq}} = 6 \parallel 3 = \frac{6 \cdot 3}{6+3} = \boxed{2\ \Omega}, \qquad V_{R\infty} = I\,R = 4 \times 2 = \boxed{8\ \text{V}}.$$

Thus

$$V_s = v(\infty) = V_{L\infty} - V_{R\infty} = 30 - 8 = \boxed{22\ \text{V}}.$$

3. Time constant τ for $t > 0$

Find the Thevenin resistance seen by the capacitor *with $t > 0$ topology and sources killed* (short the 30 V source, open the 4 A source).

$$R_{\text{left}} = 12\ \Omega, \qquad R_{\text{right}} = 6 \parallel 3 = 2\ \Omega, \qquad R_{\text{th}} = R_{\text{left}} + R_{\text{right}} = 12 + 2 = \boxed{14\ \Omega}.$$

$$\tau = R_{\text{th}}\,C = 14 \times 0.1 = \boxed{1.4\ \text{s}}.$$

4. Complete response

Using the standard first-order form

$$v(t) = \begin{cases} V_0, & t < 0, \\ V_s + \big(V_0 - V_s\big)e^{-t/\tau}, & t \geq 0, \end{cases}$$

and substituting $V_0 = -14$ V, $V_s = 22$ V, $\tau = 1.4$ s:

$$\boxed{v(t) = \begin{cases} -14\ \text{V}, & t < 0, \\ 22 - 36\,e^{-t/1.4}\ \text{V}, & t \geq 0. \end{cases}}$$

(Optional) Capacitor current. With current defined from left plate to right plate,

$$i_C(t) = C\,\frac{dv}{dt} = C\,\frac{36}{1.4}\,e^{-t/1.4} = \boxed{2.5714\,e^{-t/1.4}\ \text{A}}, \quad t \geq 0.$$

Quick checks. $v(0^+) = 22 - 36 = -14$ V (continuous across C). $v(\infty) = 22$ V (matches DC final values).