# Engr098 Lab7

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Circuit and convention. Current source  $I_s = 6$  A feeds a top node. A  $2\Omega$  resistor is from that node to ground (left branch). Through a switch, the node also connects to a  $4\Omega$  resistor in series with an inductor L = 3 H to ground (right branch). The inductor current i(t) is defined downward through L.

At t = 0 the switch *closes*. Thus:

$$i(t) = \begin{cases} i(0^{-}), & t < 0, \\ i(\infty) + [i(0^{+}) - i(\infty)]e^{-t/\tau}, & t \ge 0. \end{cases}$$

# 1. Initial current $i(0^-)$

For t < 0 the switch is *open*, so the RL branch is disconnected; no steady DC path exists through L. Therefore,

$$i(0^{-}) = 0 \text{ A}, \qquad \Rightarrow \qquad i(0^{+}) = i(0^{-}) = 0 \text{ A}.$$

(For reference, the left node voltage is  $V = I_s R = 6 \times 2 = 12$  V with only the  $2\Omega$  to ground.)

### 2. Final (steady) current $i(\infty)$

For  $t \to \infty$  with the switch closed, the inductor is a short. The right branch becomes a  $4\Omega$  to ground in parallel with the  $2\Omega$  to ground.

$$R_{\parallel} = 2 \parallel 4 = \frac{2 \cdot 4}{2 + 4} = \frac{4}{3} \Omega, \qquad V_{\text{node}} = I_s R_{\parallel} = 6 \cdot \frac{4}{3} = 8 \text{ V}.$$

Hence the DC current in the right branch (and through the inductor) is

$$i(\infty) = \frac{V_{\text{node}}}{4} = \frac{8}{4} = 2 \text{ A}.$$

#### 3. Time constant $\tau$

With the switch closed, find the Thevenin resistance seen by L with independent sources killed (current source opened). Looking into the inductor terminals, the path to ground is

$$R_{\rm th} = 4 \Omega + 2 \Omega = \boxed{6 \Omega}.$$

Thus

$$\tau = \frac{L}{R_{\rm th}} = \frac{3}{6} = 0.5 \text{ s}.$$

## 4. Complete response

$$i(t) = i(\infty) + [i(0) - i(\infty)]e^{-t/\tau} = 2 + (0 - 2)e^{-t/0.5} = 2(1 - e^{-2t}) A, \quad t \ge 0.$$

**Checks.**  $i(0^+) = 2(1-1) = 0$  A (continuous at t = 0).  $i(\infty) = 2(1-0) = 2$  A matches the DC analysis.

(Optional) Inductor voltage. With the passive sign convention (voltage drop top to bottom on L):

$$v_L(t) = L \frac{di}{dt} = 3 \cdot (4e^{-2t}) = \boxed{12e^{-2t} \text{ V}}, \quad t \ge 0.$$

**Plot tip.** In Desmos/PSPICE, use  $i(t) = 2(1 - e^{-2t})$  for  $t \ge 0$ ; a time window of  $0 \le t \le 2.5$  s  $(= 5\tau)$  shows the full rise.