Major Changes:

Given some developments in his research, my mentor had some new ideas of projects that might interest me, so we discussed some of them and decided to change project topic. At the moment, were running with a project on determining whether there exists a distribution over graphs on n vertices having a planted sqrt(n) coloring which are indistinguishable, from the point of view of low-degree polynomials, from random graphs (where each potential edge is taken with probability ½). Proving this claim false would get us very close to disproving a conjecture that Dr. Kothari suspects is indeed false. Proving the existence of such a distribution would be an interesting and new result, which, based on a fairly widely accepted conjecture, would imply the existence of an improvement to work Dr.Kothari recently did with one of his PhD students, and would likely make it easy to directly improve that work without assuming the conjecture. We will see how far we get with this idea and perhaps pivot to something else.

What You Have Accomplished Since Your Last Meeting:

Since the start of the semester, I've read materials relating to a handful of potential new project ideas, such as SDP applications to geometry (plane independence number, kissing number), maximum rectangle independent set, statistical methods for determining the number of clusters to be used in clustering algorithms, and made some progress on this planted coloring problem posed by Dr. Kothari. Specifically, I've considered a naïve class of distributions over sqrt(n) colorable graphs—those which randomly assign each vertex to a color in the range 1 through sqrt(n) and then, for every pair of edges across colors, takes this edge with probability p. I've concluded that on the one hand, it's possible to pass any one fixed test by adjusting p, however, for sufficiently large n, no distribution of this type can pass all of our tests. In particular, depending on the value of p, at least one of the triangle-counting polynomial and the 4-cycle-counting polynomial must be able to distinguish this graph from a fully random graph. So the distribution we seek has to be of some other form, and we also see that if there is some strong polynomial that's impossible to fool with any distribution, it can't be of the type "count k-cliques" for some k.

Meeting Your Milestone:

The milestone set in my original proposal was for a different project idea, however, I did accomplish the analogous goal of doing some preliminary research, but I made perhaps a bit less progress than I would have hoped due to having switched projects.

Surprises:

None

Looking Ahead:

Hopefully continue to think about this problem and see whether I can get some interesting results

Revisions to your future Milestones:

It's definitely somewhat difficult to come up concrete goals on a theory project, but here are some ambitious revised milestones:

February 15th: Consider other classes of distributions and come up with new partial results for these

March 1st: Come up with new partial results (depending on what is found in previous weeks)

March 15th: Come up with new partial results (depending on what is found in previous weeks)

March 29th: prove result for planted-n^k-coloring for some k < 0.5

April 12th: prove result for planted-n^0.5-coloring

April 26th: Investigate implications of positive or negative result on pseudo calibration conjecture or Dr.Kotharis work with Peter Manohar

Resources needed:

None