

**Major Changes:**

None

**What You Have Accomplished Since Your Last Meeting:**

There are two sides from which to make progress at this point: the original formulation, in which we find a distribution for which any (low-degree) polynomial achieves a low value, and the dual, in which we find a polynomial for which every ( $k$ -colorable) graph achieves a high value.

On the polynomial-seeking side I continued thinking about the projection of the best possible polynomial to see whether I can figure out whether it's "good" enough. I've also been thinking about whether the exact best polynomials (which I can compute for small values) might match a smoothed but analogous idea, since this best polynomial is not very "smooth", and intuitively, smoother polynomials have higher low-degree components. There doesn't seem to be a very natural best choice for what smoothed function to try first though.

On the distribution side, I'm thinking about whether we can construct some distributions that do well for small all tests of degree at most  $d$  for some small  $d$ , like 3 or 4. I already concluded a while ago that random color assignment and then picking all legal edges with the same fixed probability  $p$  will not work for  $d = 4$ , so a more general idea we're trying is to stick with these random groups (or even to force them to all be of the same size) but to allow the probability of an edge from group  $i$  to group  $j$  be different depending on  $i, j$ . This follows, more or less, the model from another paper which tackles a slightly different problem that deals only with regular graphs. I'm also trying to see whether we can take the distribution in that paper, convert it to a form which follows all of the parameters of the problem we're trying to solve, and then ask whether it's good at mimicking the uniform distribution over  $n/2$ -regular graphs (instead of the uniform distribution over all graphs, which is what we're concerned with). It seems like if the answer is "no", then it would seem like this worked in their paper only because of the other distinctions between their model and ours.

Also wrote the abstract for meeting of the minds.

**Meeting Your Milestone:**

Yes, worked on what I hoped to work on, making some distribution and polynomial progress.

**Surprises:**

None

**Looking Ahead:**

Hopefully will continue making progress on both directions, ideally resolving the "best projection polynomial question", at least for the empty graph entry.

**Revisions to your future Milestones:**

None

**Resources needed:**

None