Major Changes:

None

What You Have Accomplished Since Your Last Meeting:

Since the last milestone meeting, I've mostly been thinking about the dual formulation of the problem I came up with before Spring break. I first coded this dual solver into Mathemtica and for any (n, k, d) triple was able to compute the "advantage" of the best degree-d polynomial against the best distribution over k-colorable graphs on n vertices. This gave me a way of computing the best test polynomials for small input instances. Since these polynomials didn't immediately reveal themselves as following a basic pattern, I then began thinking about what would make a good polynomial (where, per the formulation of the dual, a "good" polynomial maximizes the minimum value it assigns to a k-colorable graph). Naturally, the best polynomial of any degree assigns the same large value to k-colorable graphs and the same small value to all other graphs. The low-degree projection of this polynomial doesn't seem to agree with the computed best polynomials, but has a simple formulation that makes it an enticing one to analyze to see whether it suffices. I spent some time trying to figure out a whether we can come up with various bounds on the entries of the relevant sub-hadammard matrix to see whether this polynomial is good enough.

I also spent some time reading parts of a couple related papers and thinking about whether we can come up with projections of other polynomials representing basic criteria for non-k-colorability or whether we can use some spectral colorability bounds relating to smallest/largest eigenvectors (which we could approximate using log n rounds power iteration, yielding a degree d = log n polynomial). I also considered whether a different class of polynomials that would solve our problem (those which assign -1 to half of graphs and 1 to the other half, where all k-colorable ones are in the second half).

Meeting Your Milestone:

I ended up going in a different direction to what I had predicted I would last week due to some conversations with my research advisor, but definitely read some interesting related literature and came up with some interesting computational results as well as theoretical ideas this week.

Surprises:

None

Looking Ahead:

Hopefully will continue analyzing projection polynomial to see whether it yields anything useful and think about the question of constructing distributions beating some low-degree tests (edge count and triangle parity), as suggested by my advisor.

Revisions to your future Milestones:

None

Resources needed:

None