Major Changes: None

What You Have Accomplished Since Your Last Meeting:

Since our last meeting, I've transitioned from a combinatorial distribution hunt to a higher-level attempt to simplify the problem so as to be able to perform computations on small instances in hopes of discovering some sort of pattern. This transition started from the realization that if p is a polynomial test and d is our probability distribution, we can express the value of the polynomial under our distribution as $\mathbf{E}_{g\sim\mathbb{F}_2^n}[d(x)p(x)]$. From here we realize that for a particular distribution, we can easily compute the degree-d polynomial test on which the distribution does worst by projecting its fourier coefficients onto the set of sets of size at most D, in which case above expression evaluates to the norm of the fourier coefficients of d corresponding to sets of size at most D. Using this idea we're no longer solving a mini-max problem, but rather just a convex minimzation one. In particular, using a submatrix of a Hadammard matrix, we can express this as just minimizing the norm of a particular matrix multiplied by a vector with its $L1$ norm constrained to be 1. If we can get this norm to be smaller than 1, this a "good" distribution in that it passes all degree-d tests. This makes it much easier to solve instances of this problem, and I was able to do this with Mathematica (i.e., compute for tuples (n, k, d) what the best distribution on k-colorable n-vertex graphs against tests of degree d is and how many standard deviations away from the best polynomial test it is). Unfortunately the size of the matrix involved in this computation is linearin the product of the number of k-colorable graphs and size-d subgraphs of K_n, which grows rather quickly, so I was only able to compute this for n, k, d up to around 4. Using this new view, though, I was able to prove that the best distribution must assign the same probability to isomorphic graphs and thus the best polynomial has the same coefficient for isomorphic graphs. This shrinks our matrix quite a bit and gives us more room to solve larger problems in a reasonable amount of time, so I incorporated this idea into my Mathemtica program and was able to get more data for n, k, d up to around 6.

I also recently realized that if we flip our problem around, we get a dual problem in which we're attempting to maximize a parameter α such that there's a distribution over degree-d polynomials (which is luckily just the same as finding a single degree-d polynomial, p) such that all k-colorable graphs, when plugged into p is at least α away from the expected value of p over random graphs. If we can achieve $\alpha \geq 1$, then we've found a degree-d polynomial for which any distribution over k-colorable graphs must have expectation at least 1 standard deviation away from that of a truly random graph, so there is no "good" distribution. Using the same idea as above we get a new concrete and numerically solvable optimization problem but now we're minimizing a linear function and have $L2$ constraints, which could possibly lead to more insights or be faster to solve numerically than the original formulation, but haven't coded this up yet.

Meeting Your Milestone: Yes, I definitely was able to conclude that basic fourier analysis does help with out problem and came up with some computational "best distribution results" as well as theoretical results (e.g. best distribution is the same over isomorphic graphs).

Surprises:

None

Looking Ahead:

Hopefully continue to think about some of the ideas described above, such as analyzing and looking for patterns in the optimal distributions for particular values of n, k, and d and seeing whether our new dual view inspires any new ideas. Also should look into a paper my mentor suggested looking at.

Revisions to your future Milestones:

None

Resources needed:

None