

Subsets and binary strings

Theorem

A k -element set has 2^k subsets.

$k = 3$	\emptyset	$\{A\}$	$\{B\}$	$\{C\}$	$\{A,B\}$	$\{A,C\}$	$\{B,C\}$	$\{A,B,C\}$
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Proof

The subsets of a k -element set are encoded by binary strings of length k , i.e., by k -letter words in a 2-letter alphabet:

\emptyset	$\{A\}$	$\{B\}$	$\{C\}$	$\{A,B\}$	$\{A,C\}$	$\{B,C\}$	$\{A,B,C\}$
000	100	010	001	110	101	011	111

The number of such words is 2^k .