

## Subsets and binary strings

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### Theorem

A  $k$ -element set has  $2^k$  subsets.

$k = 3$	$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{A,B\}$	$\{A,C\}$	$\{B,C\}$	$\{A,B,C\}$
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### Proof

The subsets of a  $k$ -element set are encoded by binary strings of length  $k$ , i.e., by  $k$ -letter words in a 2-letter alphabet:

$\emptyset$	$\{A\}$	$\{B\}$	$\{C\}$	$\{A,B\}$	$\{A,C\}$	$\{B,C\}$	$\{A,B,C\}$
000	100	010	001	110	101	011	111

The number of such words is  $2^k$ .