

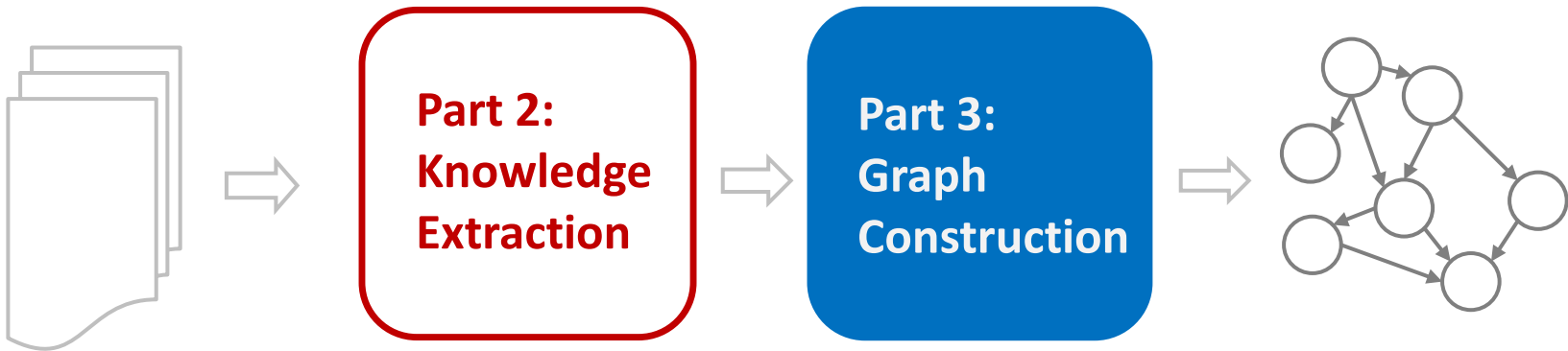
Knowledge Graph Construction from Text

AAAI 2017

JAY PUJARA, SAMEER SINGH, BHAVANA DALVI

Tutorial Overview

Part 1: Knowledge Graphs



Part 4: Critical Analysis

Tutorial Outline

1. Knowledge Graph Primer

[Jay]



2. Knowledge Extraction from Text

a. NLP Fundamentals

[Sameer]



b. Information Extraction

[Bhavana]



Coffee Break



3. Knowledge Graph Construction

a. Probabilistic Models

[Jay]



b. Embedding Techniques

[Sameer]



4. Critical Overview and Conclusion [Bhavana]



Knowledge Graph Construction

TOPICS:

PROBLEM SETTING

PROBABILISTIC MODELS

EMBEDDING TECHNIQUES

Knowledge Graph Construction

TOPICS:

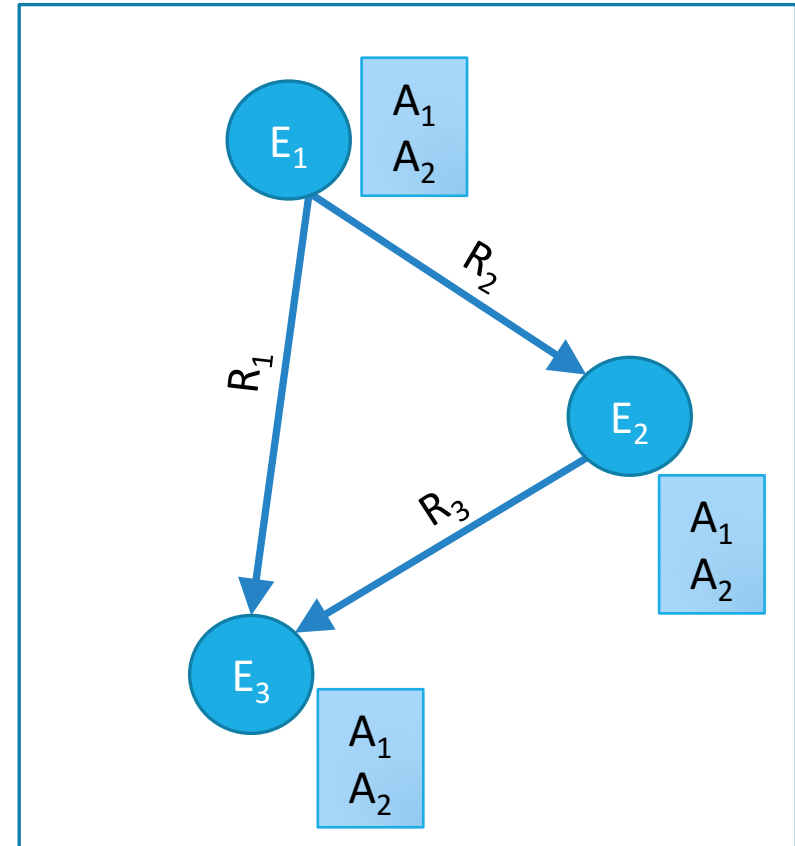
PROBLEM SETTING

PROBABILISTIC MODELS

EMBEDDING TECHNIQUES

Reminder: Basic problems

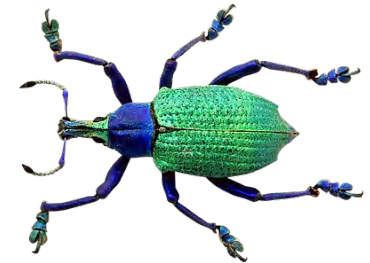
- **Who** are the entities (nodes) in the graph?
- **What** are their attributes and types (labels)?
- **How** are they related (edges)?



Graph Construction Issues

Extracted knowledge is:

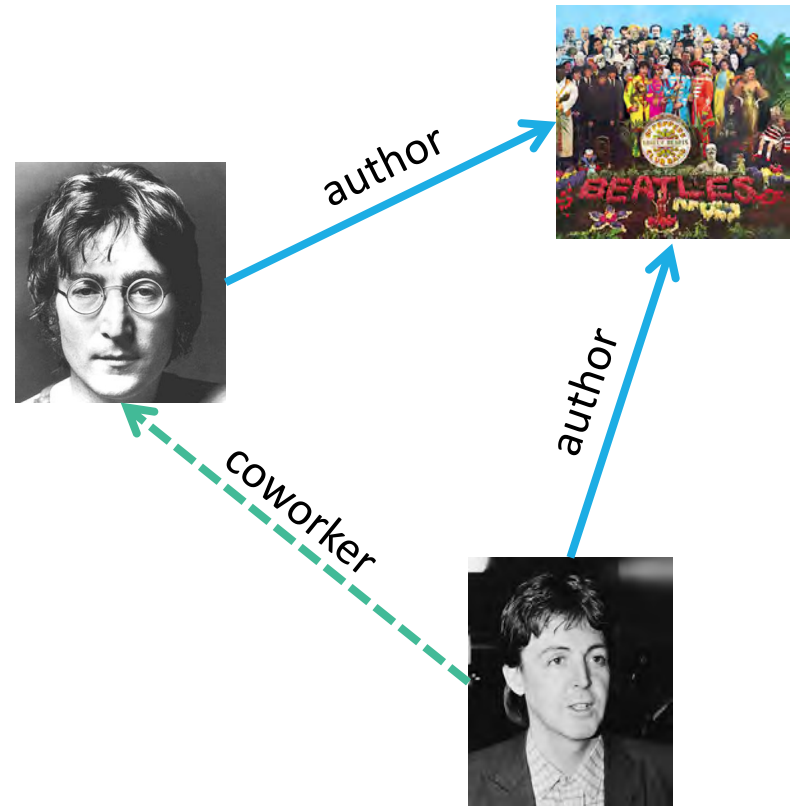
- ambiguous:
 - Ex: Beetles, beetles, Beatles
 - Ex: citizenOf, livedIn, bornIn



Graph Construction Issues

Extracted knowledge is:

- ambiguous
- incomplete
 - Ex: missing relationships
 - Ex: missing labels
 - Ex: missing entities



Graph Construction Issues

Extracted knowledge is:

- ambiguous
- incomplete
- inconsistent
 - Ex: Cynthia Lennon, Yoko Ono
 - Ex: exclusive labels (alive, dead)
 - Ex: domain-range constraints



spouse



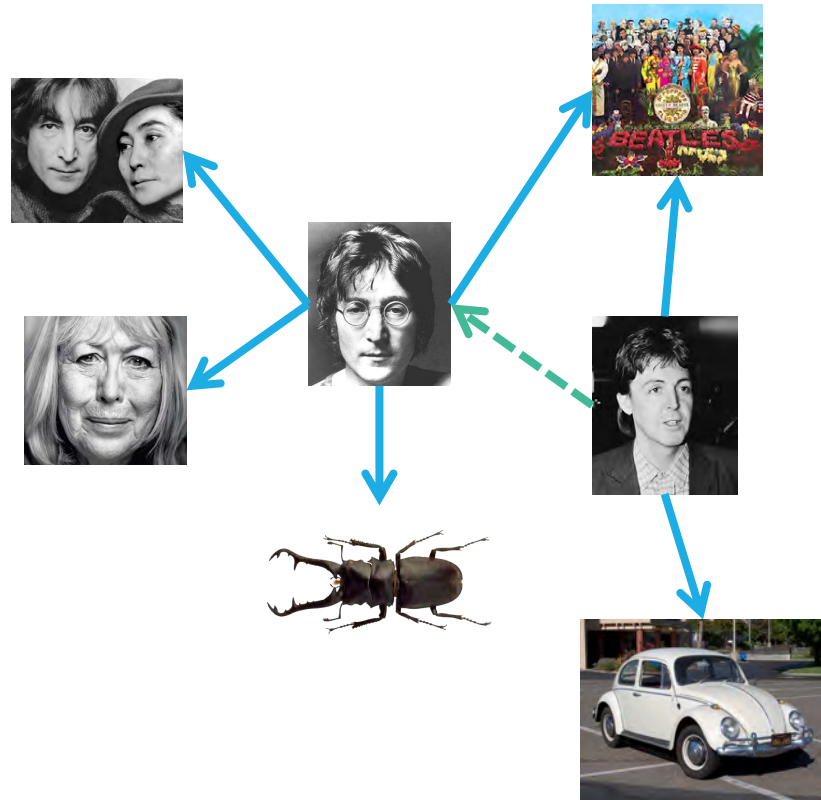
spouse



Graph Construction Issues

Extracted knowledge is:

- ambiguous
- incomplete
- inconsistent



Graph Construction approach

- Graph construction **cleans** and **completes** extraction graph
- Incorporate ontological constraints and relational patterns
- Discover statistical relationships within knowledge graph

Knowledge Graph Construction

TOPICS:

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EMBEDDING TECHNIQUES

Graph Construction Probabilistic Models

TOPICS:

OVERVIEW

GRAPHICAL MODELS

RANDOM WALK METHODS

Graph Construction

Probabilistic Models

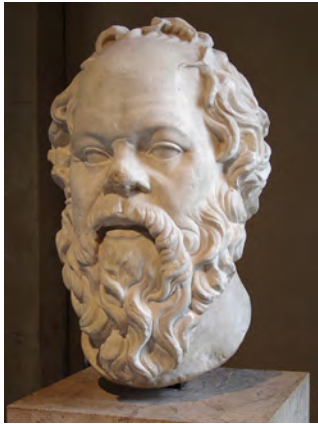
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Beyond Pure Reasoning



- Classical AI approach to knowledge: reasoning

$\text{Lbl}(\text{Socrates}, \text{Man}) \ \& \ \text{Sub}(\text{Man}, \text{Mortal}) \rightarrow \text{Lbl}(\text{Socrates}, \text{Mortal})$

Beyond Pure Reasoning



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- Reasoning difficult when extracted knowledge has errors

Beyond Pure Reasoning



- Classical AI approach to knowledge: reasoning

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- Reasoning difficult when extracted knowledge has errors
- Solution: probabilistic models

$P(\text{Lbl}(\text{Socrates}, \text{Mortal}) \mid \text{Lbl}(\text{Socrates}, \text{Man}) = 0.9)$

Graph Construction

Probabilistic Models

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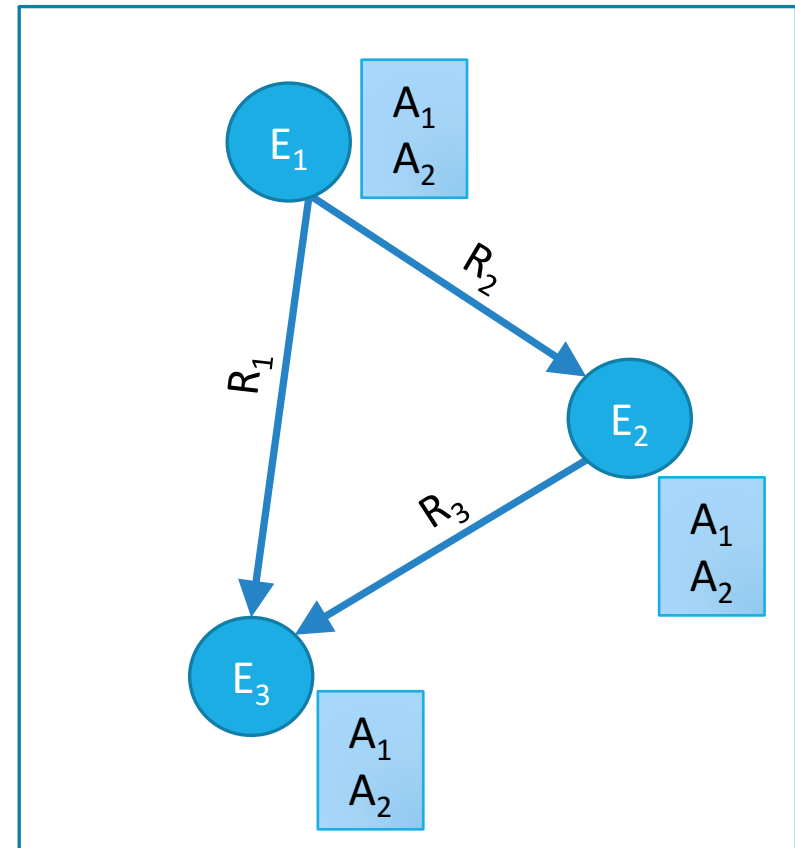
Graphical Models: Overview

- Define **joint probability distribution** on knowledge graphs
- Each candidate fact in the knowledge graph is a **variable**
- Statistical signals, ontological knowledge and rules parameterize the **dependencies** between variables
- Find most likely knowledge graph by **optimization/sampling**

Knowledge Graph Identification

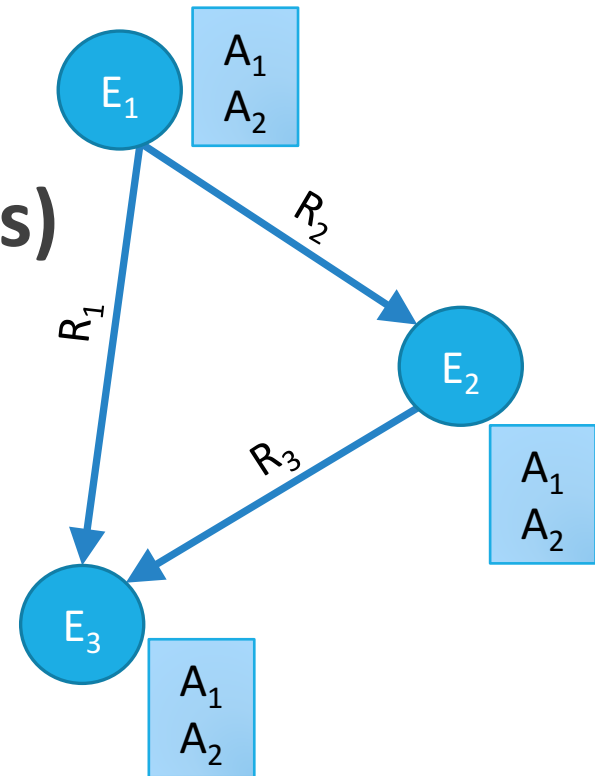
Define a graphical model to perform all three of these tasks simultaneously!

- **Who** are the entities (nodes) in the graph?
- **What** are their attributes and types (labels)?
- **How** are they related (edges)?



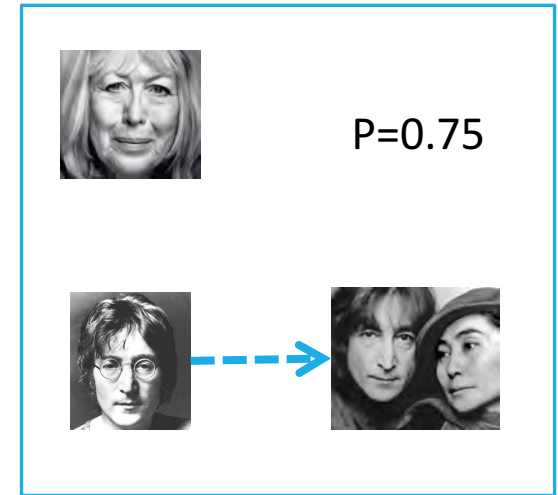
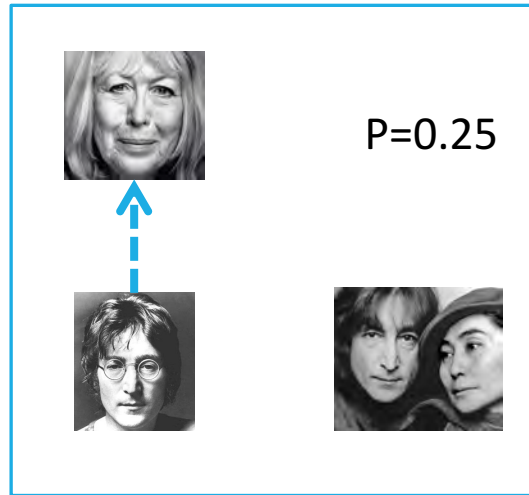
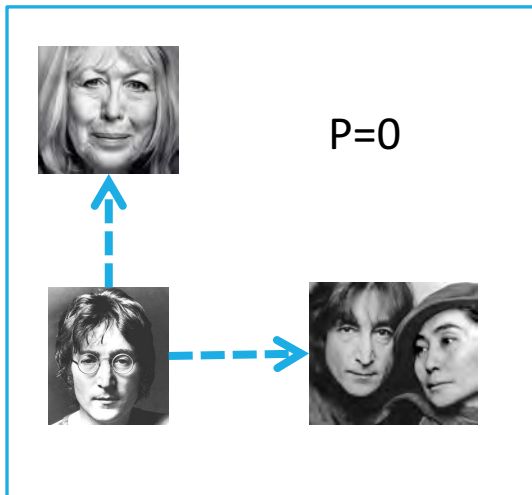
Knowledge Graph Identification

P(Who, What, How | Extractions)



Probabilistic Models

- Use dependencies between facts in KG
- Probability defined *jointly* over facts



What determines probability?

- **Statistical signals from text extractors and classifiers**

What determines probability?

- **Statistical signals from text extractors and classifiers**
 - $P(R(\text{John}, \text{Spouse}, \text{Yoko})) = 0.75$; $P(R(\text{John}, \text{Spouse}, \text{Cynthia})) = 0.25$
 - $\text{LevenshteinSimilarity}(\text{Beatles}, \text{Beetles}) = 0.9$

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- **Ontological knowledge about domain**

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- Statistical signals from text extractors and classifiers
- **Ontological knowledge about domain**
 - $\text{Functional}(\text{Spouse}) \ \& \ R(A, \text{Spouse}, B) \rightarrow !R(A, \text{Spouse}, C)$
 - $\text{Range}(\text{Spouse}, \text{Person}) \ \& \ R(A, \text{Spouse}, B) \rightarrow \text{Type}(B, \text{Person})$

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- Rules and patterns mined from data

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- Ontological knowledge about domain
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 - $R(A, \text{Spouse}, B) \ \& \ R(A, \text{Lives}, L) \rightarrow R(B, \text{Lives}, L)$
 - $R(A, \text{Spouse}, B) \ \& \ R(A, \text{Child}, C) \rightarrow R(B, \text{Child}, C)$

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Example: The Fab Four

THE
BEATLES



Illustration of KG Identification

Uncertain Extractions:

.5: Lbl(Fab Four, novel)

.7: Lbl(Fab Four, musician)

.9: Lbl(Beatles, musician)

.8: Rel(Beatles, AlbumArtist,
Abbey Road)

Illustration of KG Identification

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(Annotated) Extraction Graph

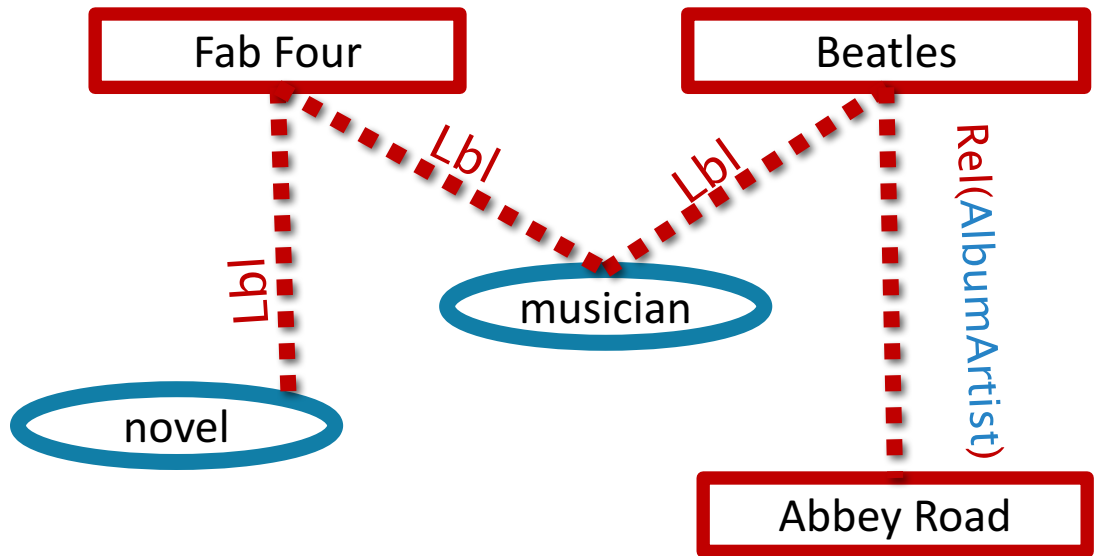


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Ontology:

Dom(albumArtist, musician)

Mut(novel, musician)

Extraction Graph

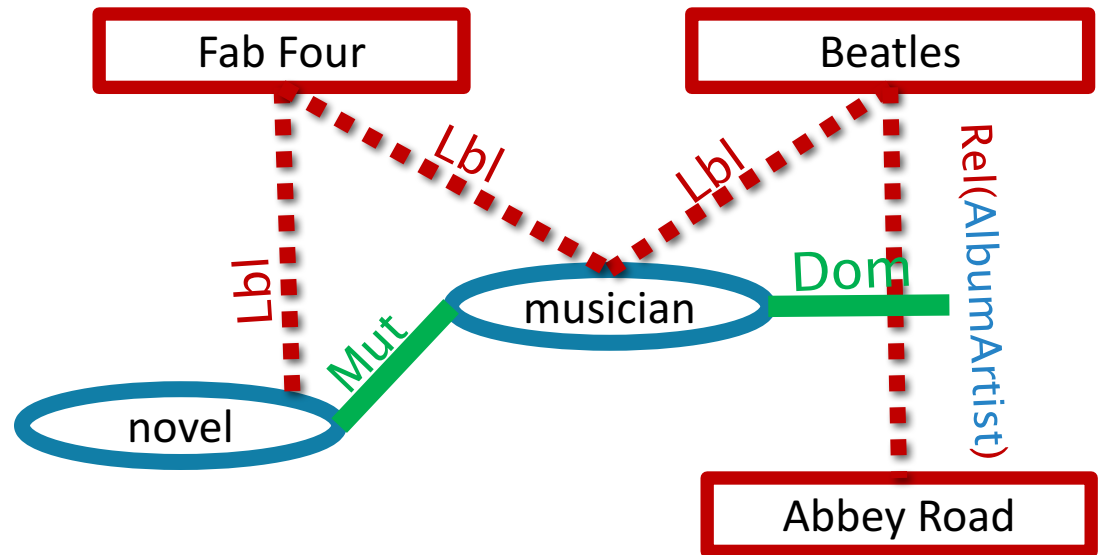


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Entity Resolution:

SameEnt(Fab Four, Beatles)

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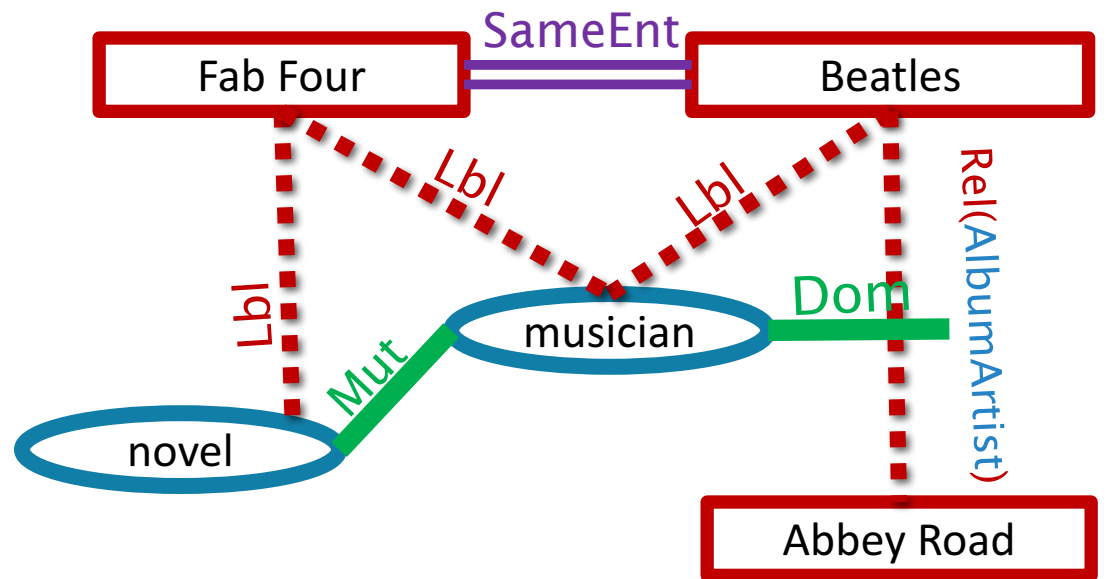


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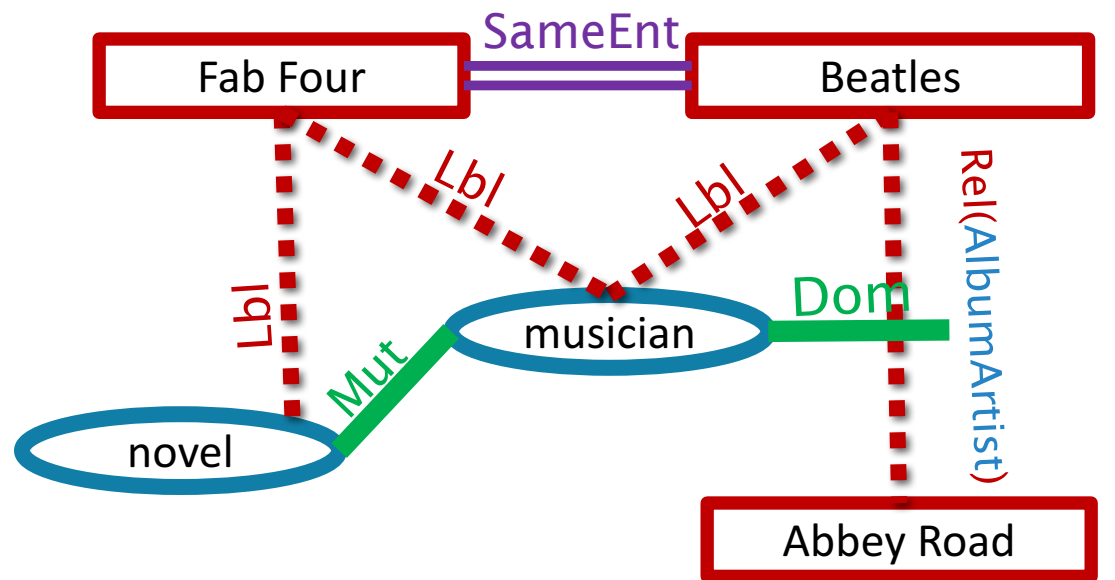
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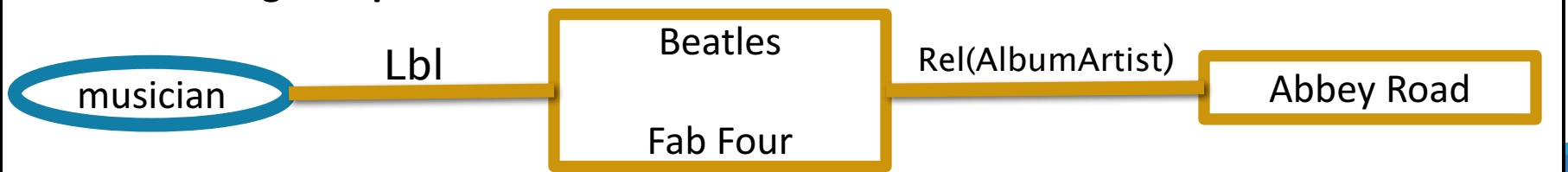
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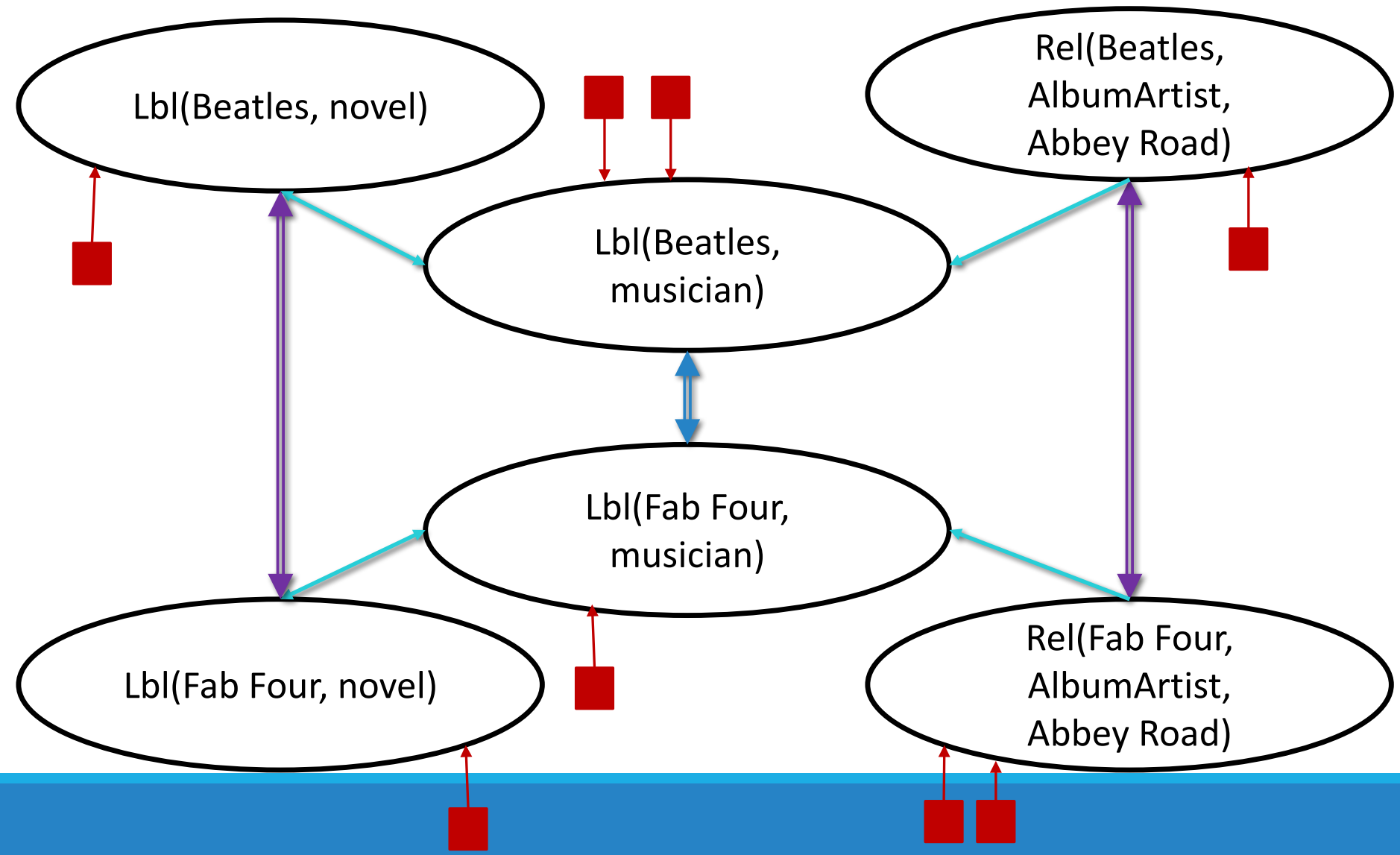
(Annotated) Extraction Graph



After Knowledge Graph Identification



Probabilistic graphical model for KG



Defining graphical models

- Many options for defining a graphical model
- We focus on two approaches, MLNs and PSL, that use **rules**
- **MLNs** treat facts as Boolean, use sampling for satisfaction
- **PSL** infers a “truth value” for each fact via optimization



Rules for KG Model

```
100: Subsumes(L1,L2) & Label(E,L1) -> Label(E,L2)
100: Exclusive(L1,L2) & Label(E,L1) -> !Label(E,L2)

100: Inverse(R1,R2) & Relation(R1,E,O) -> Relation(R2,O,E)
100: Subsumes(R1,R2) & Relation(R1,E,O) -> Relation(R2,E,O)
100: Exclusive(R1,R2) & Relation(R1,E,O) -> !Relation(R2,E,O)

100: Domain(R,L) & Relation(R,E,O) -> Label(E,L)
100: Range(R,L) & Relation(R,E,O) -> Label(O,L)

10: SameEntity(E1,E2) & Label(E1,L) -> Label(E2,L)
10: SameEntity(E1,E2) & Relation(R,E1,O) -> Relation(R,E2,O)

1: Label_NYT(E,L) -> Label(E,L)
1: Label_YouTube(E,L) -> Label(E,L)
1: Relation_LATimes(R,E,O) -> Relation(R,E,O)
1: -> !Relation(R,E,O)
1: -> !Label(E,L)
```

Rules to Distributions

- Rules are *grounded* by substituting literals into formulas

$\mathbf{w}_r : \text{SAMEENT}(\text{Fab Four}, \text{Beatles}) \wedge$

$\text{LBL}(\text{Beatles}, \text{musician}) \Rightarrow \text{LBL}(\text{Fab Four}, \text{musician})$

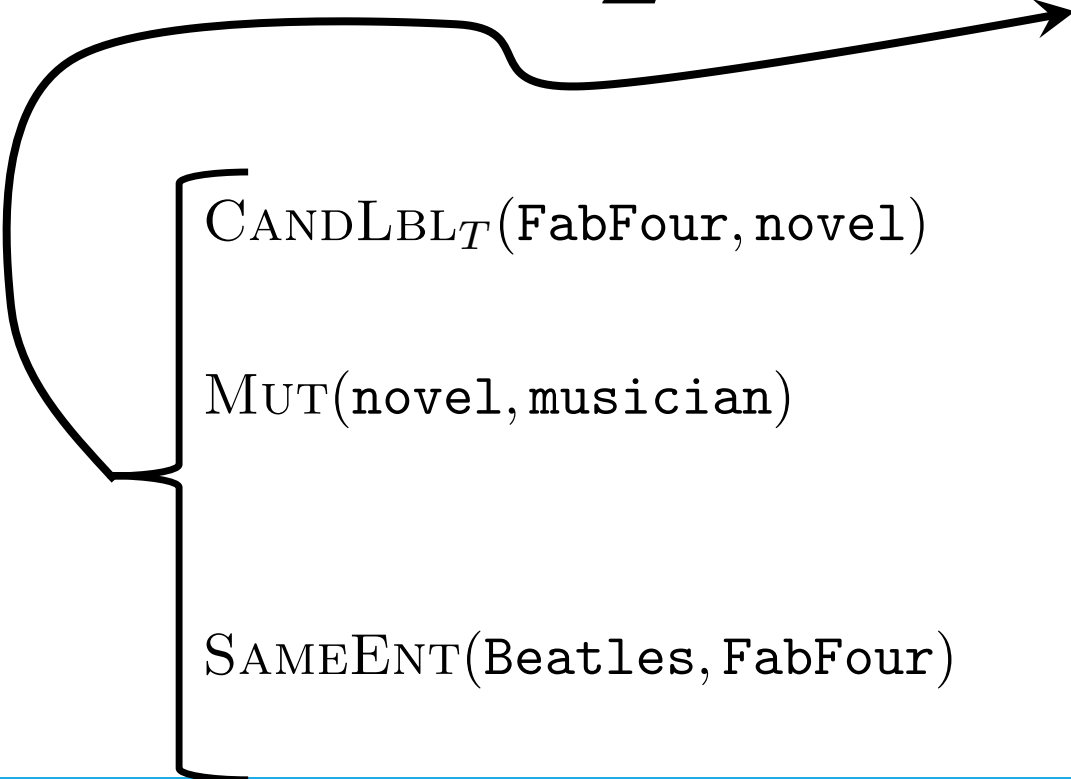
- Each ground **rule** has a **weighted satisfaction** derived from the formula's truth value

$$P(G|E) = \frac{1}{Z} \exp \left[\sum_{r \in R} w_r \phi_r(G, E) \right]$$

- Together, the ground rules provide a joint probability distribution over knowledge graph facts, conditioned on the extractions

Probability Distribution over KGs

$$P(G \mid E) = \frac{1}{Z} \exp \left[- \sum_{r \in R} w_r \varphi_r(G) \right]$$



CANDLBL_T(FabFour, novel)

\Rightarrow LBL(FabFour, novel)

MUT(novel, musician)

\wedge LBL(Beatles, novel)

$\Rightarrow \neg$ LBL(Beatles, musician)

SAMEENT(Beatles, FabFour)

\wedge LBL(Beatles, musician)

\Rightarrow LBL(FabFour, musician)

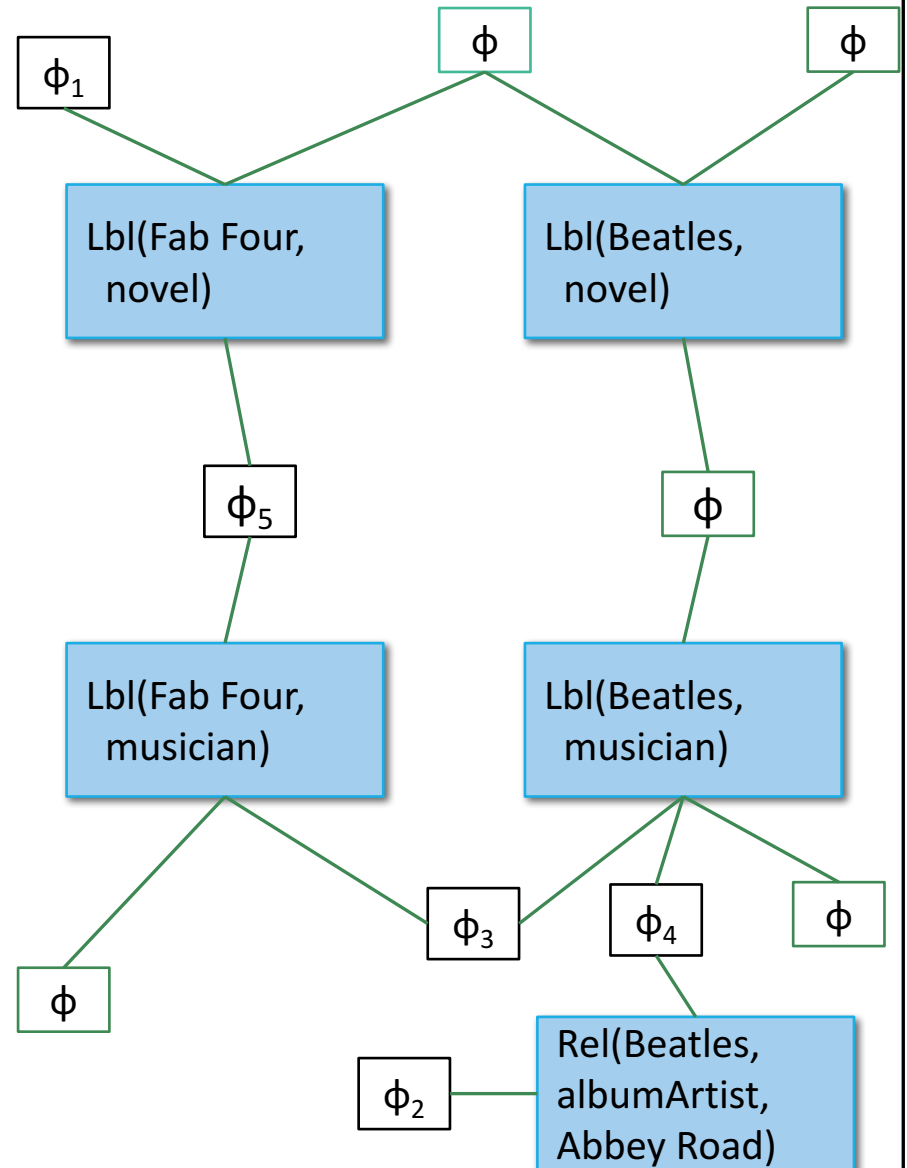
$[\phi_1]$ CANDLBL_{struct}(FabFour, novel)
 \Rightarrow LBL(FabFour, novel)

$[\phi_2]$ CANDREL_{pat}(Beatles, AlbumArtist, AbbeyRoad)
 \Rightarrow REL(Beatles, AlbumArtist, AbbeyRoad)

$[\phi_3]$ SAMEENT(Beatles, FabFour)
 \wedge LBL(Beatles, musician)
 \Rightarrow LBL(FabFour, musician)

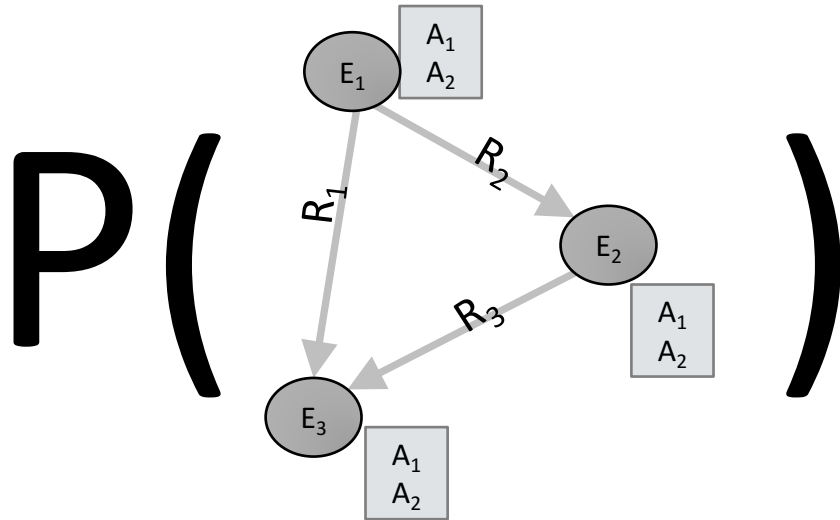
$[\phi_4]$ DOM(AlbumArtist, musician)
 \wedge REL(Beatles, AlbumArtist, AbbeyRoad)
 \Rightarrow LBL(Beatles, musician)

$[\phi_5]$ MUT(musician, novel)
 \wedge LBL(FabFour, musician)
 $\Rightarrow \neg$ LBL(FabFour, novel)

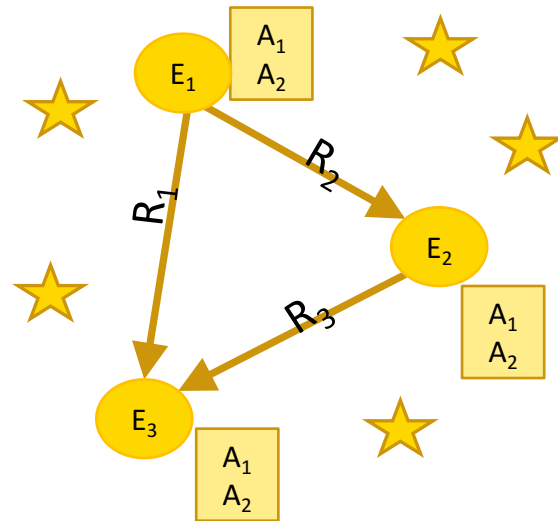


How do we get a knowledge graph?

Have: $P(\text{KG})$ for all KGs



Need: best KG



MAP inference: optimizing over distribution to find the best knowledge graph

Inference and KG optimization

- Finding the best KG satisfying weighed rules: NP Hard
- MLNs [discrete]: Monte Carlo sampling methods
 - Solution quality dependent on burn-in time, iterations, etc.
- PSL [continuous]: optimize convex linear surrogate
 - Fast optimization, $\frac{3}{4}$ -optimal MAX SAT lower bound

Graphical Models Experiments

Data: ~1.5M extractions, ~70K ontological relations, ~500 relation/label types

Task: Collectively construct a KG and evaluate on 25K target facts

Comparisons:

Extract	Average confidences of extractors for each fact in the NELL candidates
Rules	Default, rule-based heuristic strategy used by the NELL project
MLN	Jiang+, ICDM12 – estimates marginal probabilities with MC-SAT
PSL	Pujara+, ISWC13 – convex optimization of continuous truth values with ADMM

Running Time: Inference completes in 10 seconds, values for 25K facts

	AUC	F1
Extract	.873	.828
Rules	.765	.673
MLN (Jiang, 12)	.899	.836
PSL (Pujara, 13)	.904	.853

Graphical Models: Pros/Cons

BENEFITS

- Define probability distribution over KGs
- Easily specified via rules
- Fuse knowledge from many different sources

DRAWBACKS

- Requires optimization over all KG facts - overkill
- Dependent on rules from ontology/expert
- Require probabilistic semantics - unavailable

Graph Construction

Probabilistic Models

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RANDOM WALK METHODS

Random Walk Overview

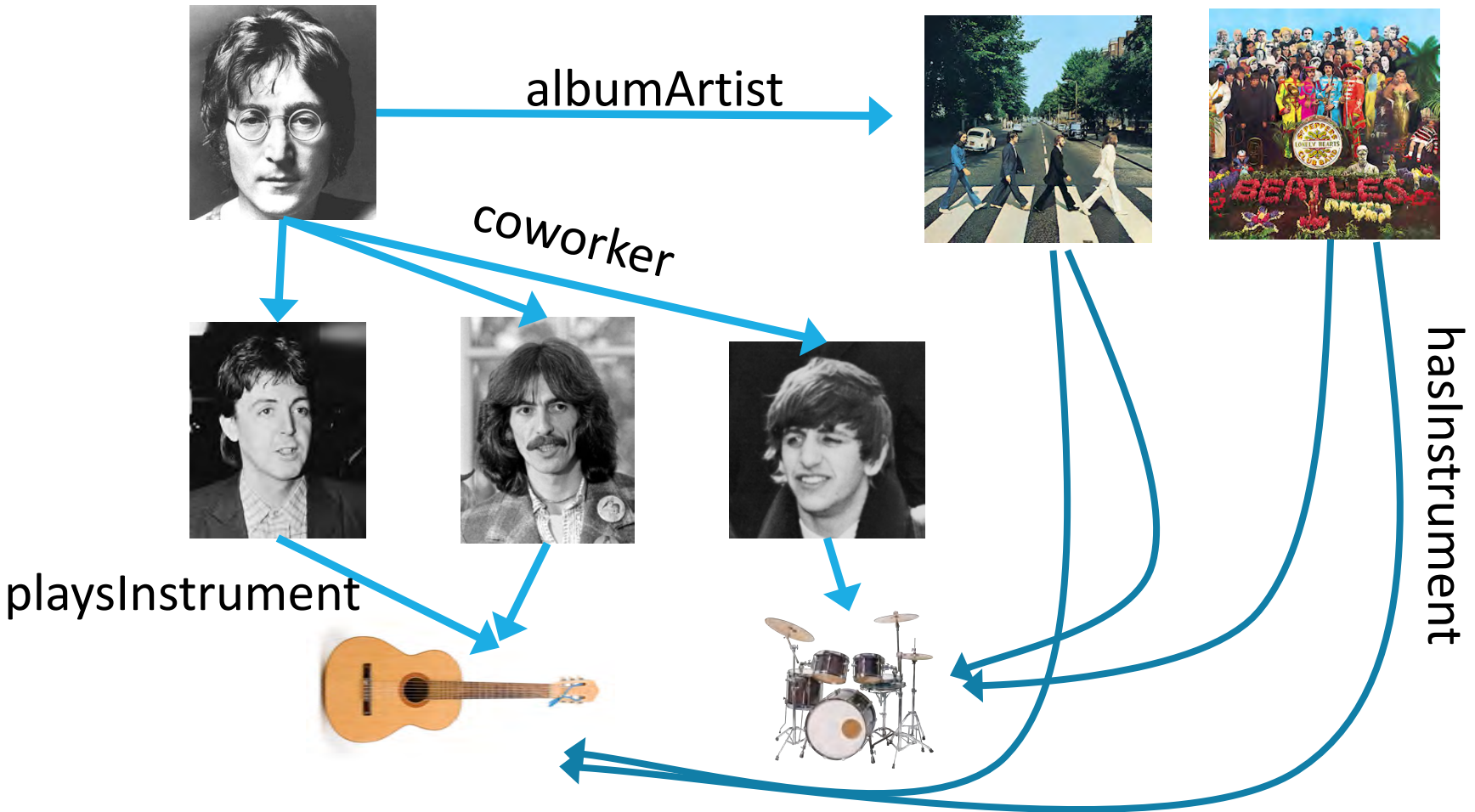
- Given: a query of an **entity** and **relation**
- Starting at the entity, **randomly walk** the KG
- Random walk ends when reaching an appropriate **goal**
- Learned **parameters** bias choices in the random walk
- Output **relative probabilities** of goal states

Random Walk Illustration

Query: R(Lennon, PlaysInstrument, ?)

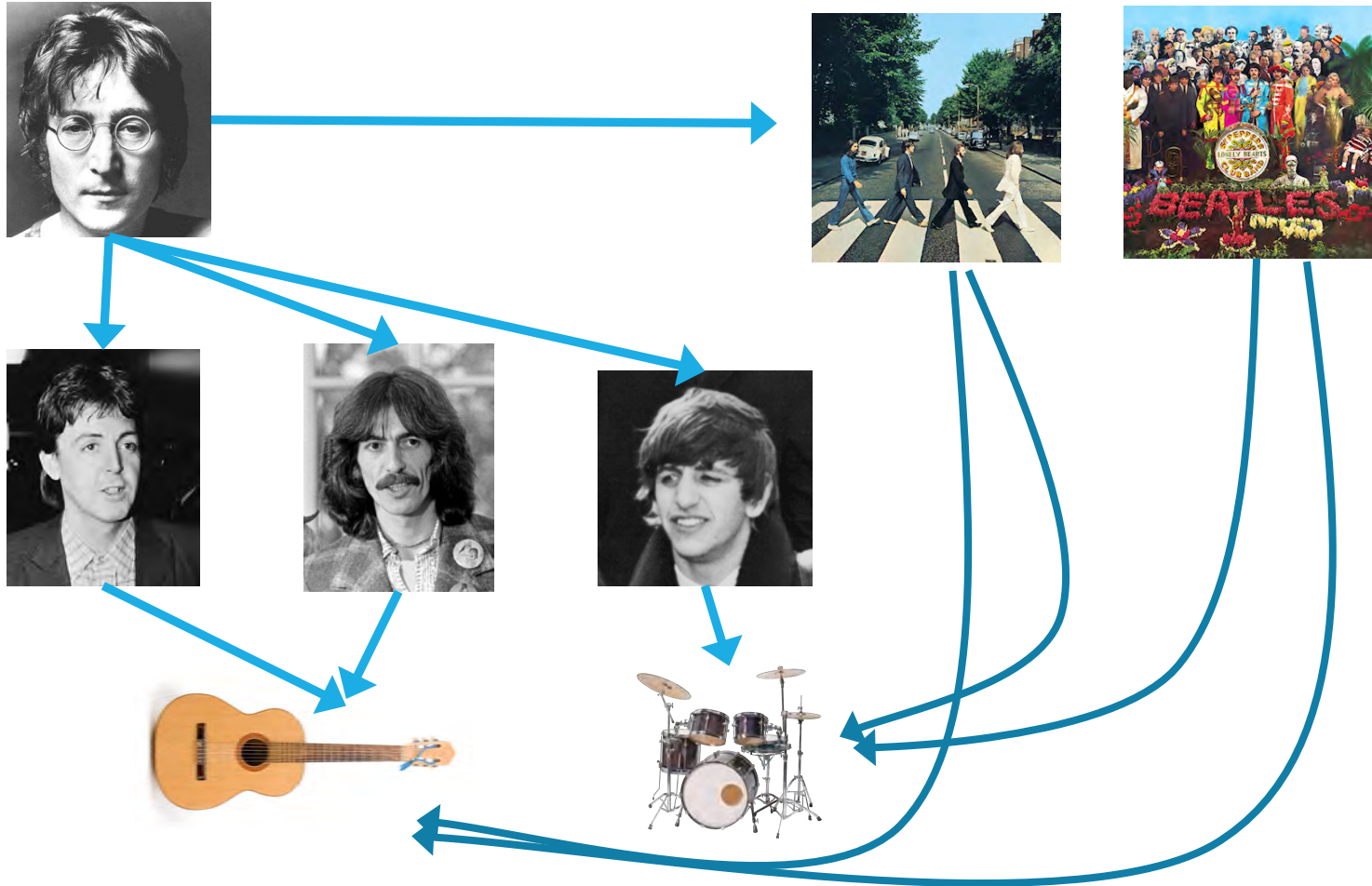
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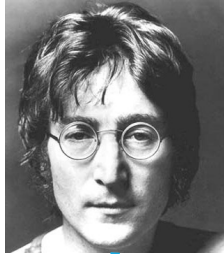
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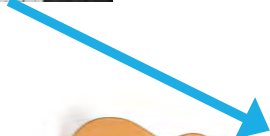
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Random Walk Illustration

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Path \rightarrow Weight of path

$P(Q | \pi = \langle \text{coworker}, \text{playsInstrument} \rangle) W_{\pi}$

Random Walk Illustration

Query Q: R(Lennon, PlaysInstrument, ?)



Path

Weight of path

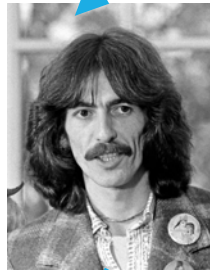
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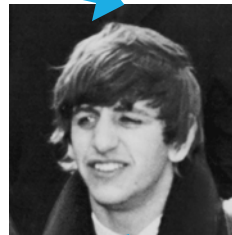


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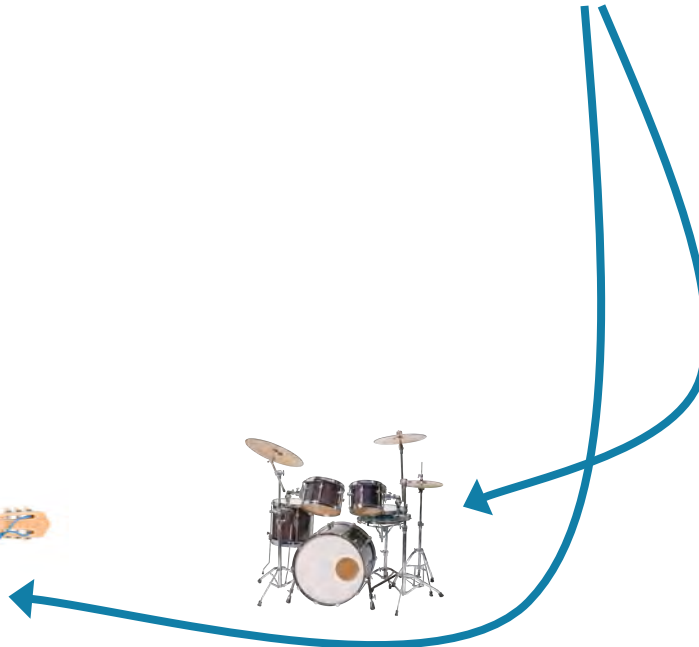


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Random Walk Illustration

Query Q: R(Lennon, PlaysInstrument, ?)



Random Walk Illustration

Query Q: R(Lennon, PlaysInstrument, ?)



$P(Q | \pi = \langle \text{albumArtist}, \text{hasInstrument} \rangle) W_{\pi}$

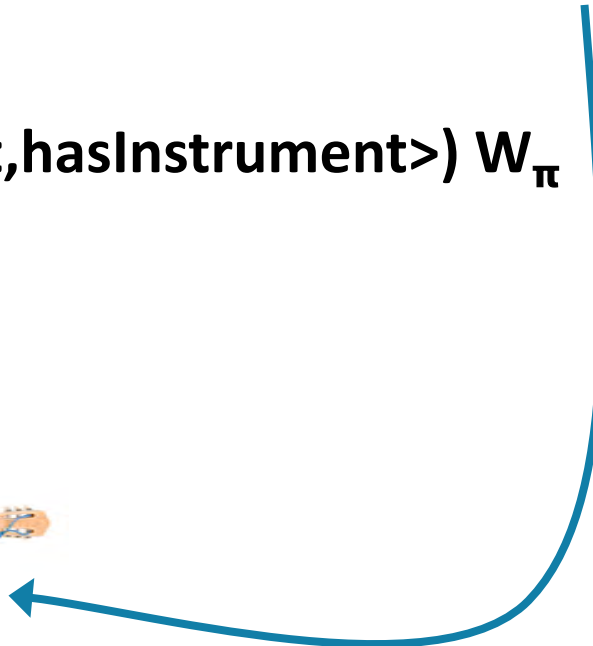


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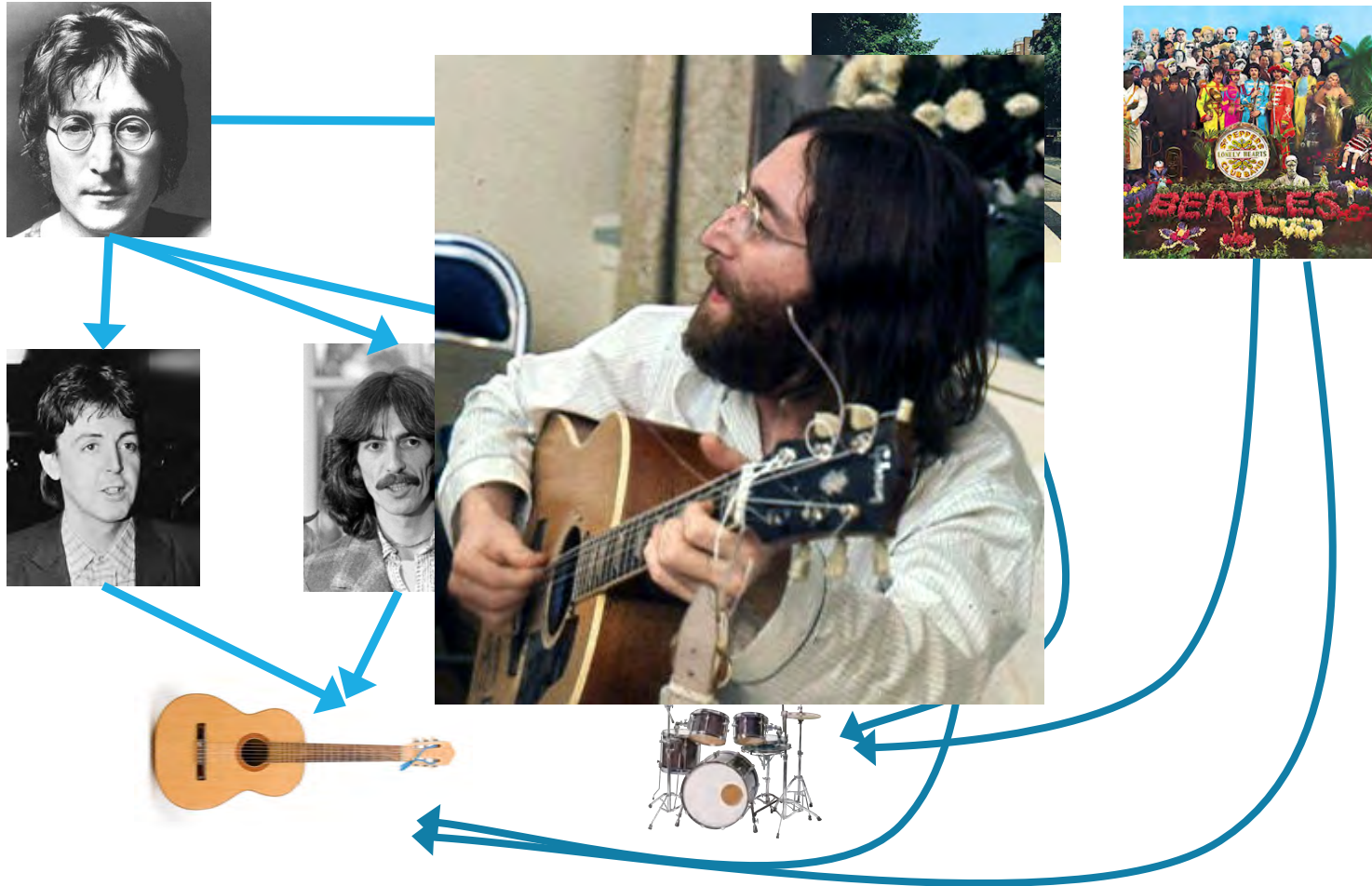


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Random Walk Illustration

Query: R(Lennon, PlaysInstrument, ?)



Recent Random Walk Methods

PRA: Path Ranking Algorithm

- Performs random walk of **imperfect knowledge graph**
- Estimates **transition probabilities** using KG
- For each relation, learns **parameters for paths** through the KG

ProPPR: Programming with Personalized PageRank

- Constructs **proof graph**
 - Nodes are partially-ground clauses with one or more facts
 - Edges are proof-transformations
- **Parameters** are learned for each **ground entity** and **rule**

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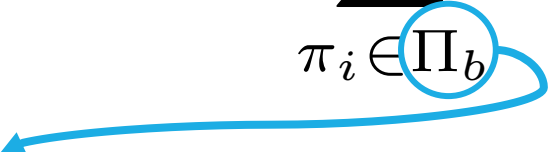
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PRA in a nutshell

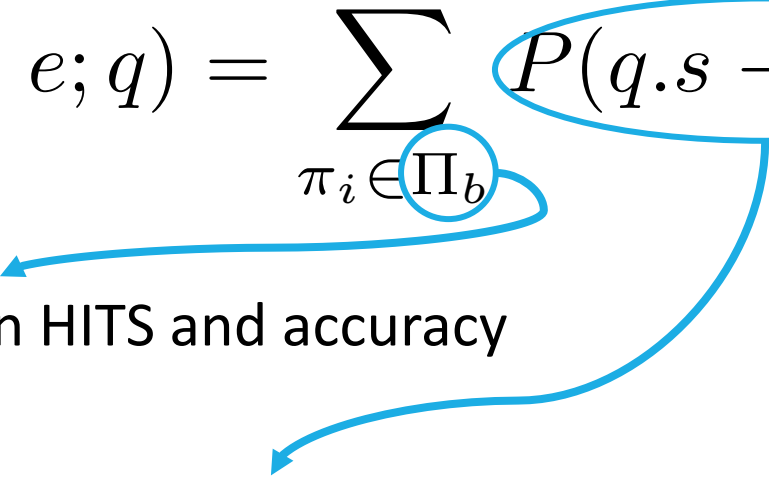
$$\text{score}(q.s \rightarrow e; q) = \sum_{\pi_i \in \Pi_b} P(q.s \rightarrow e; \pi_i) W_{\pi_i}$$

PRA in a nutshell

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Filter paths based on HITS and accuracy

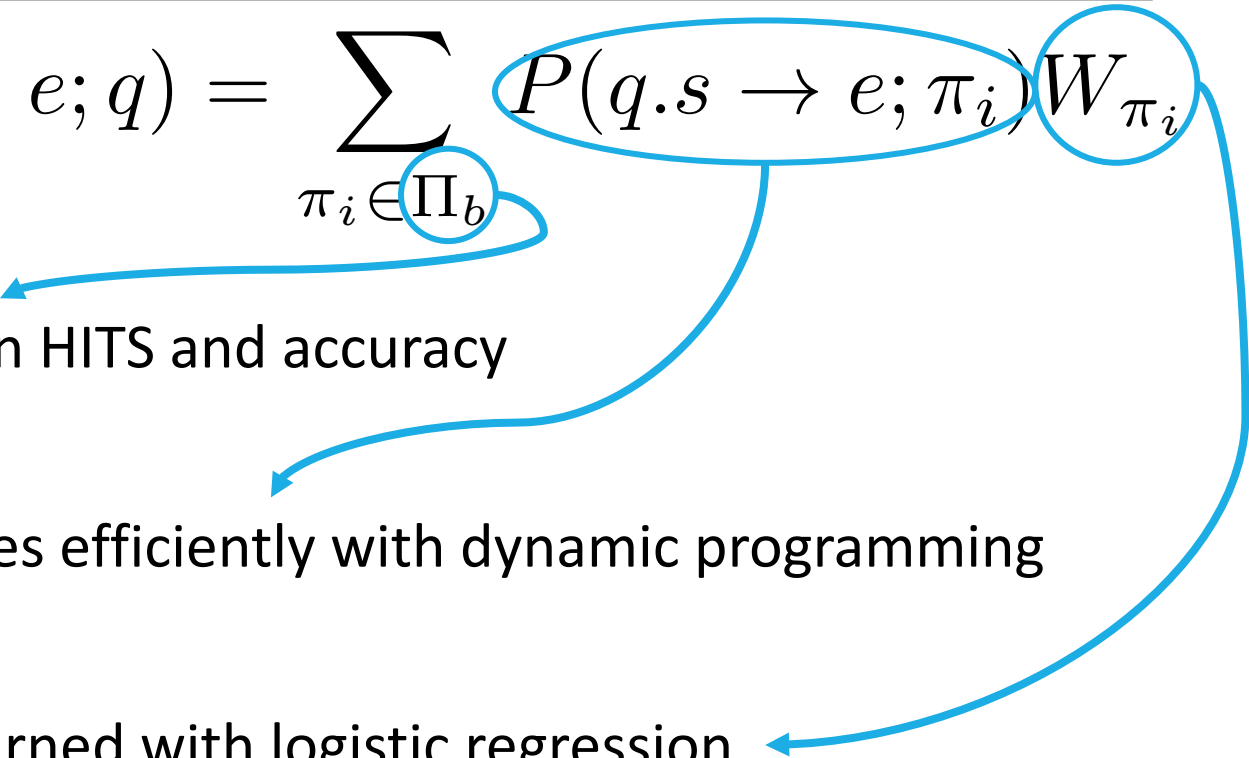
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Filter paths based on HITS and accuracy

Estimate probabilities efficiently with dynamic programming

PRA in a nutshell

$$\text{score}(q.s \rightarrow e; q) = \sum_{\pi_i \in \Pi_b} P(q.s \rightarrow e; \pi_i) W_{\pi_i}$$


Filter paths based on HITS and accuracy

Estimate probabilities efficiently with dynamic programming

Path weights are learned with logistic regression

Recent Random Walk Methods

PRA: Path Ranking Algorithm

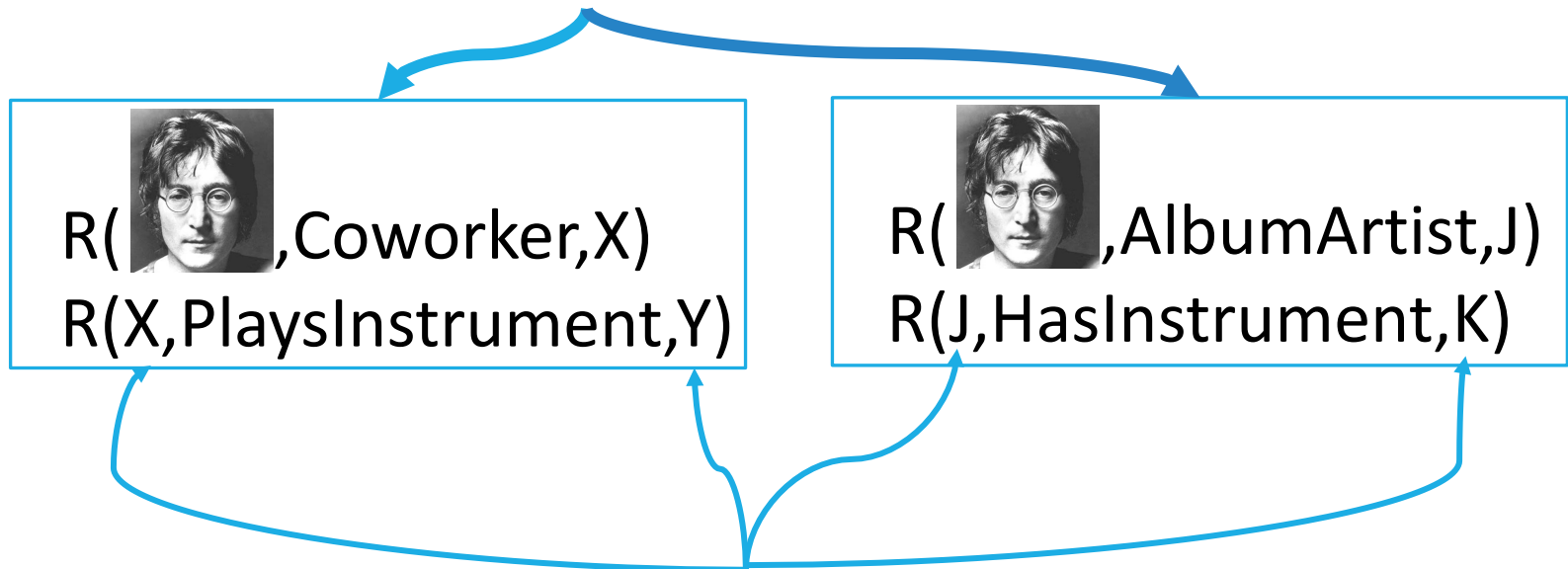
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ProPPR: ProbLog + Personalized PageRank

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- **Parameters** are learned for each **ground entity** and **rule**

ProPPR-ized PRA example

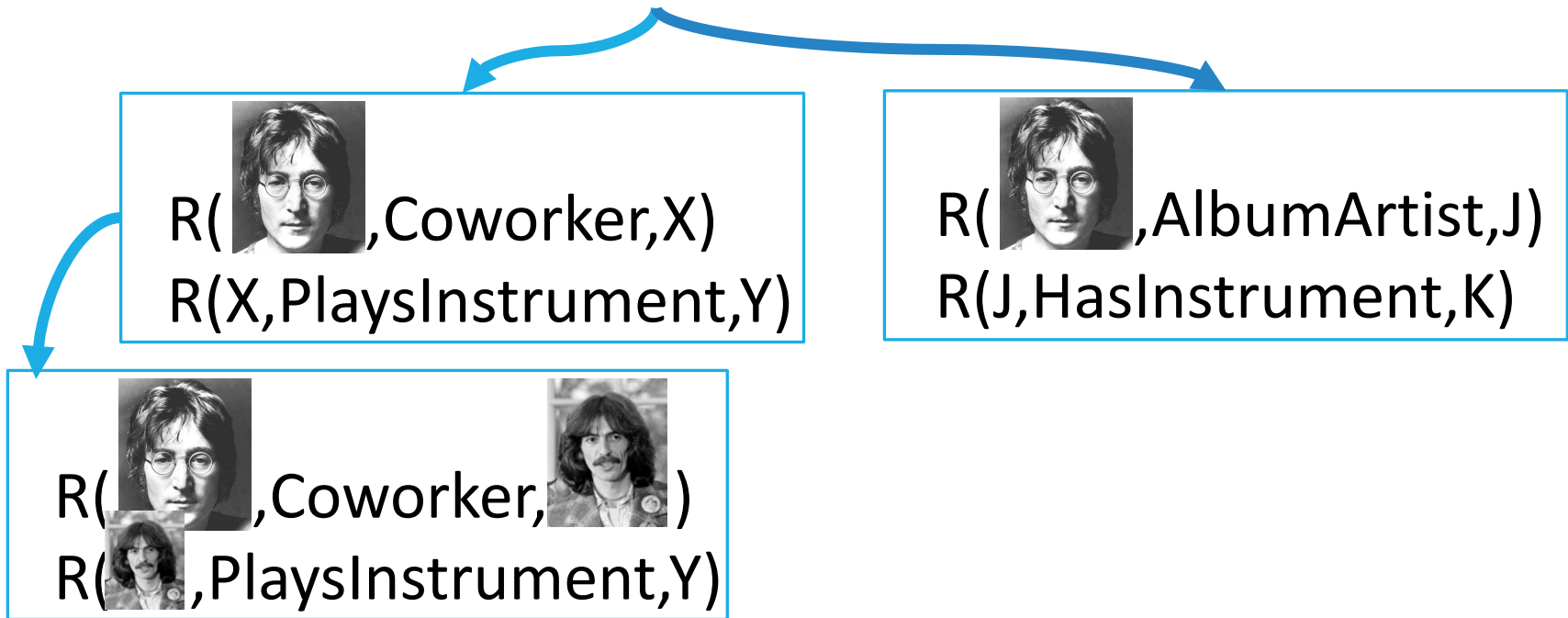
Query Q: R(Lennon, PlaysInstrument, ?)



Unbound variables in proof tree!

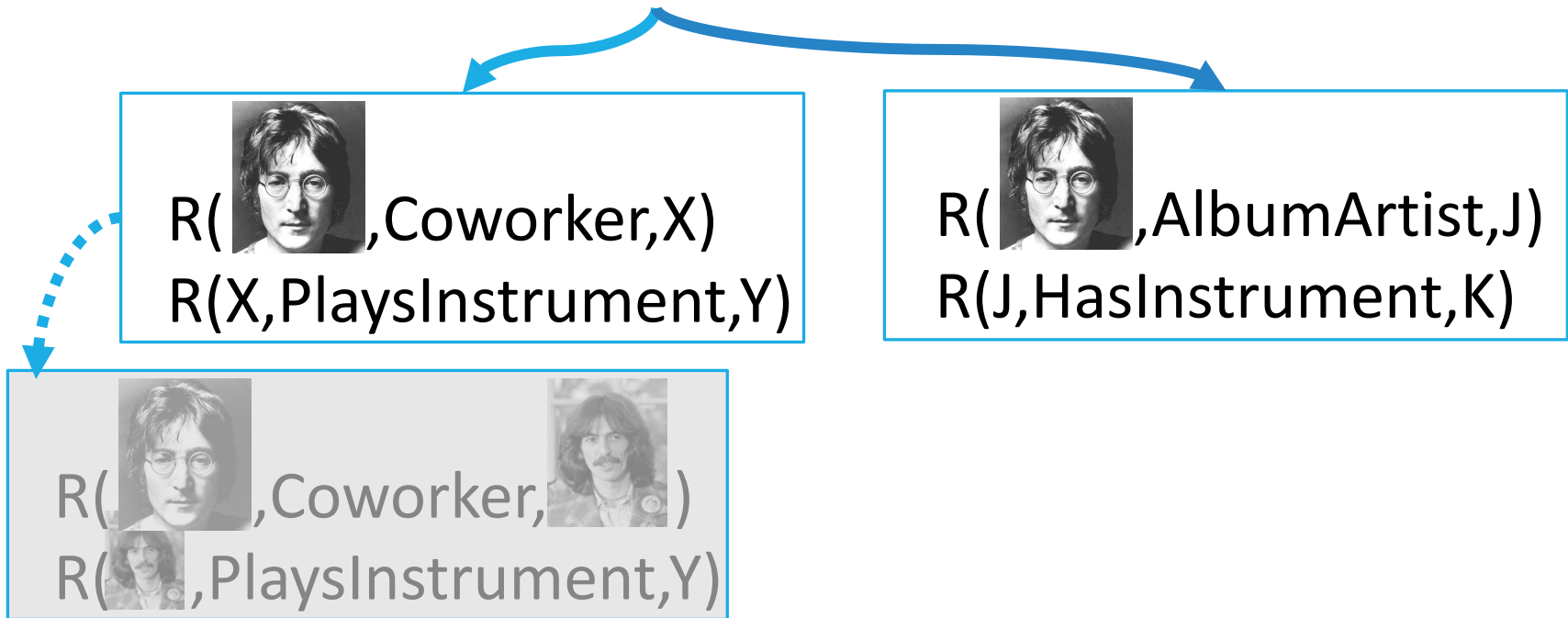
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



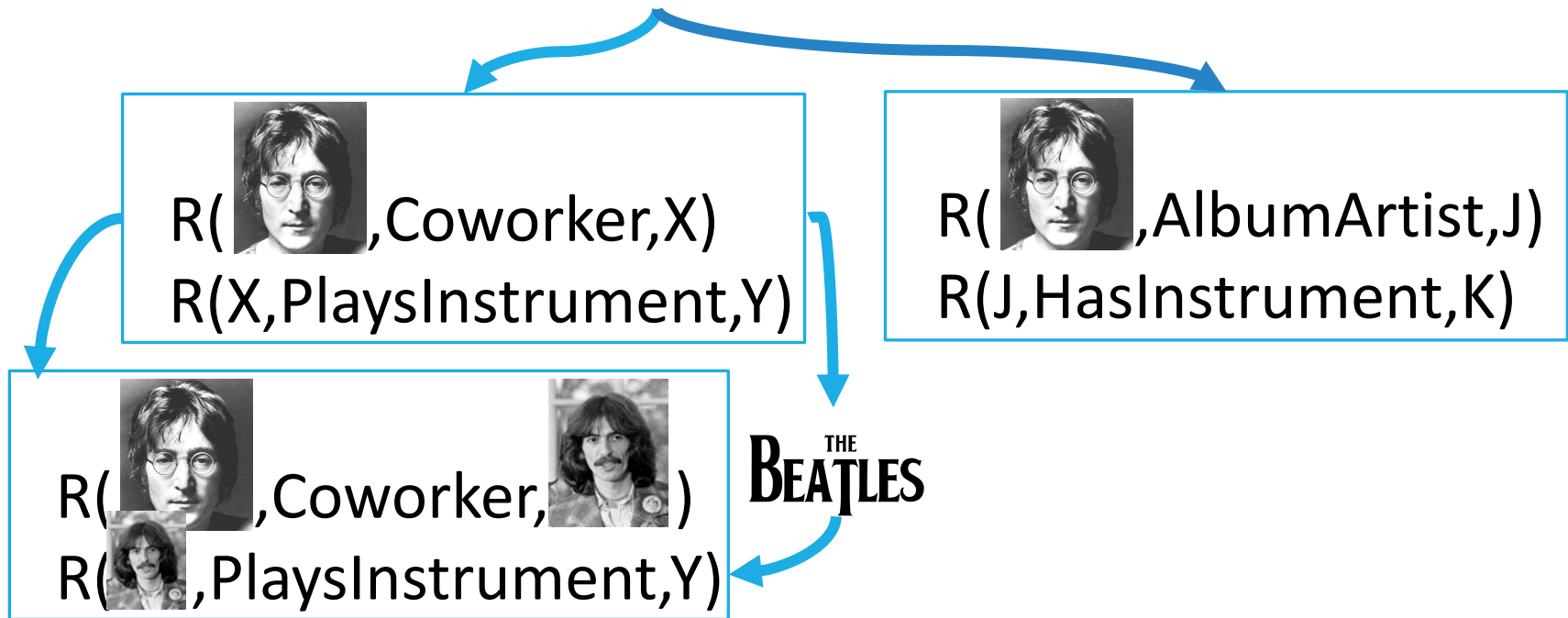
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



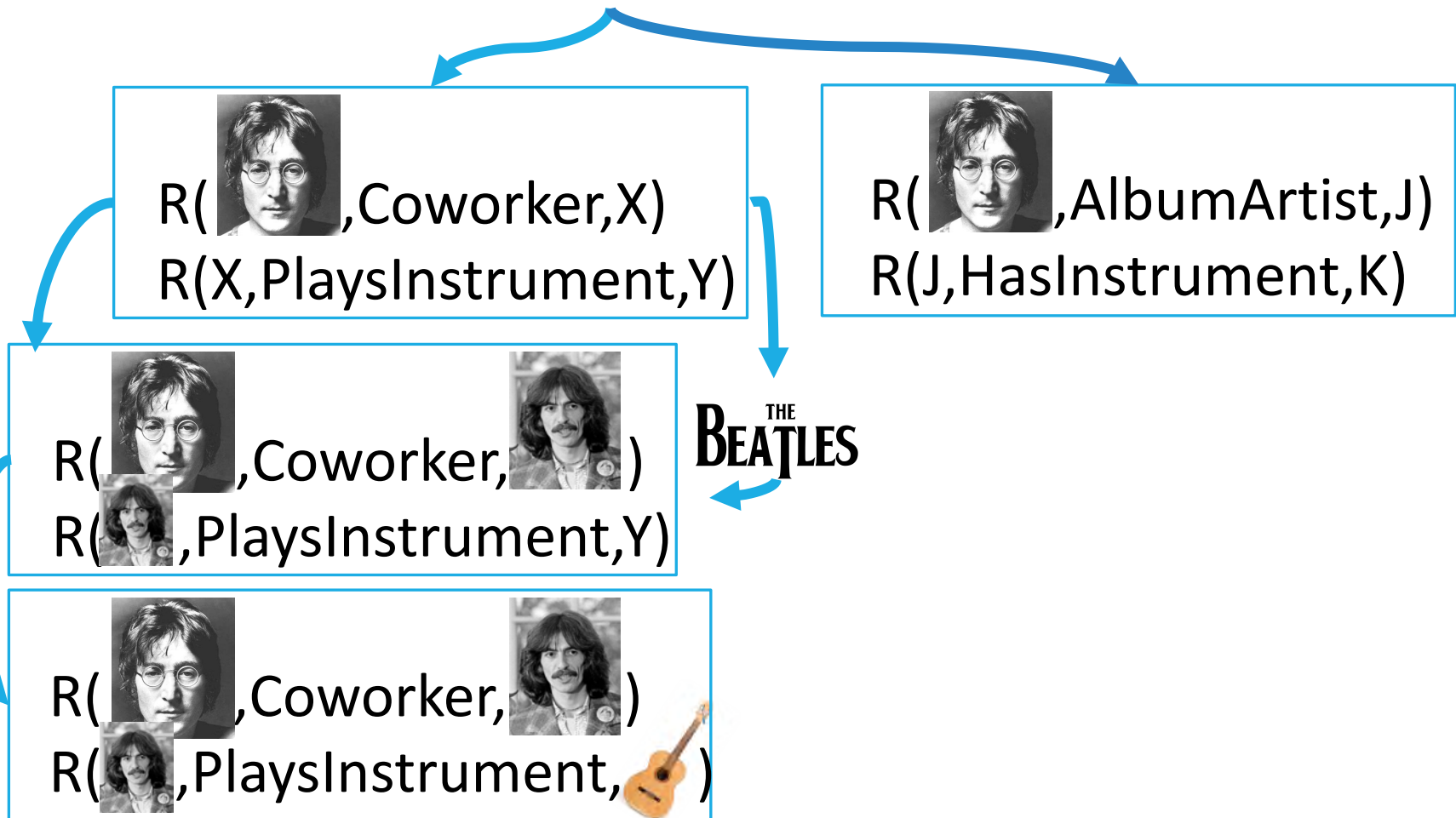
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



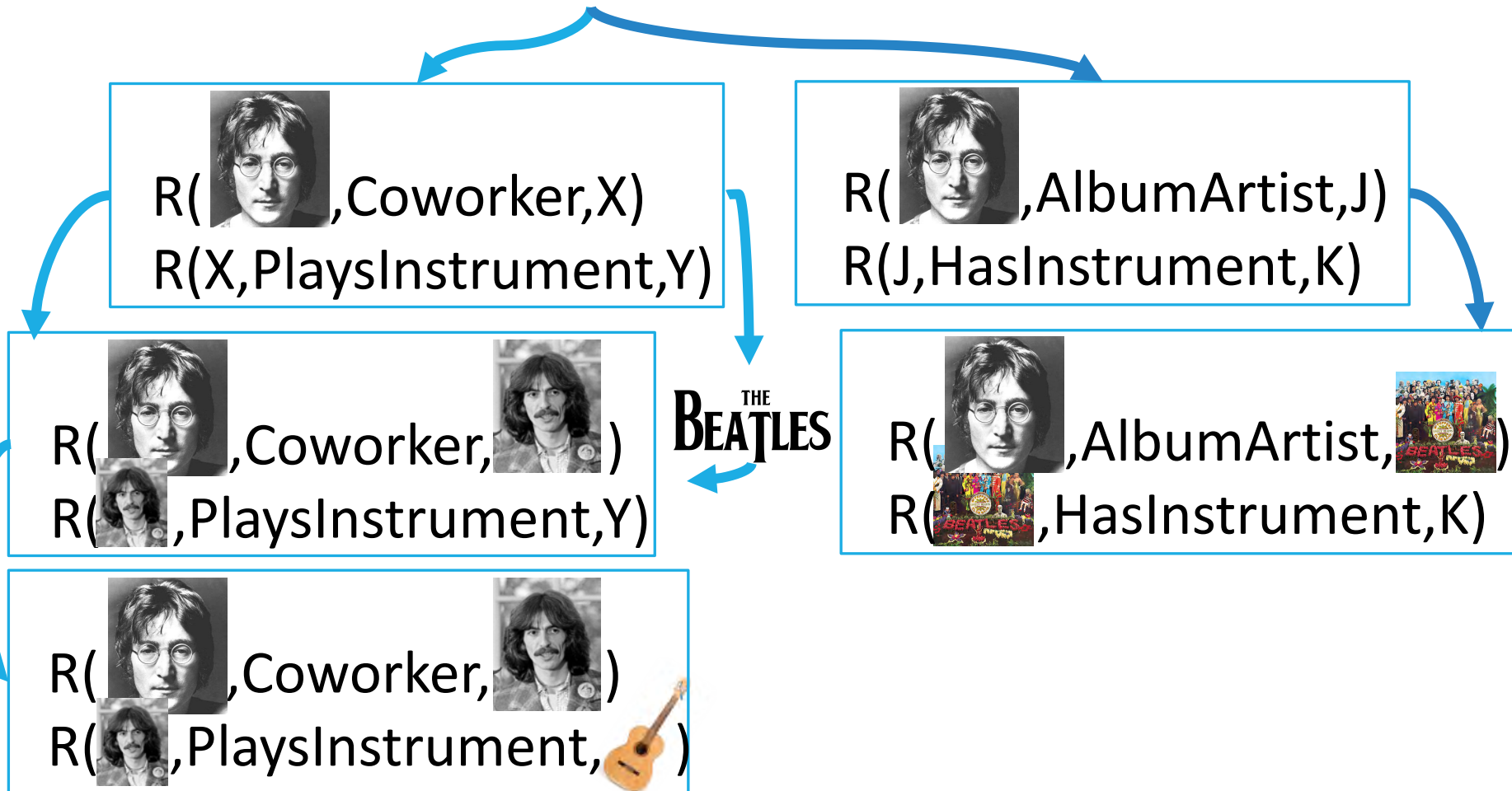
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



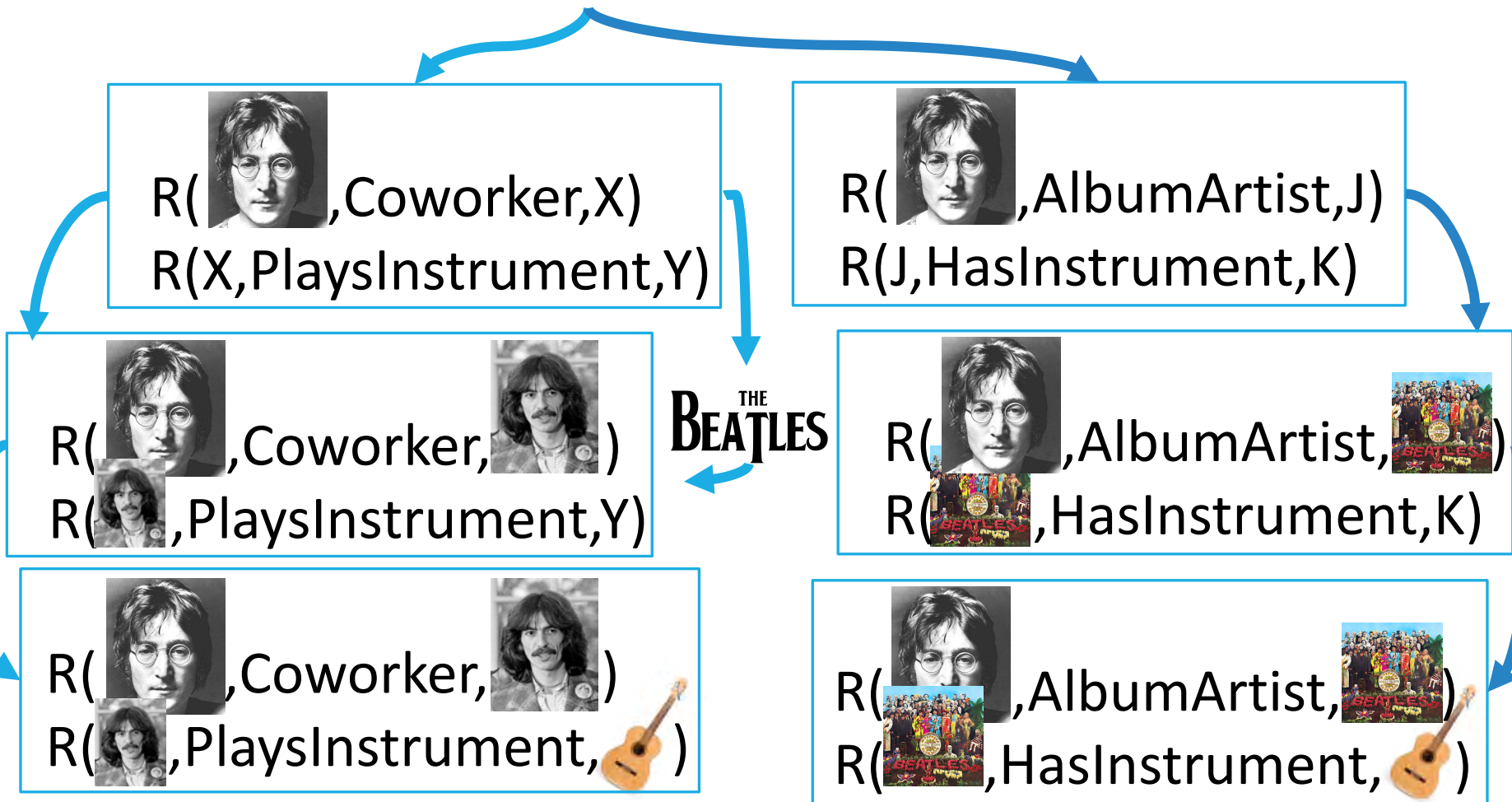
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



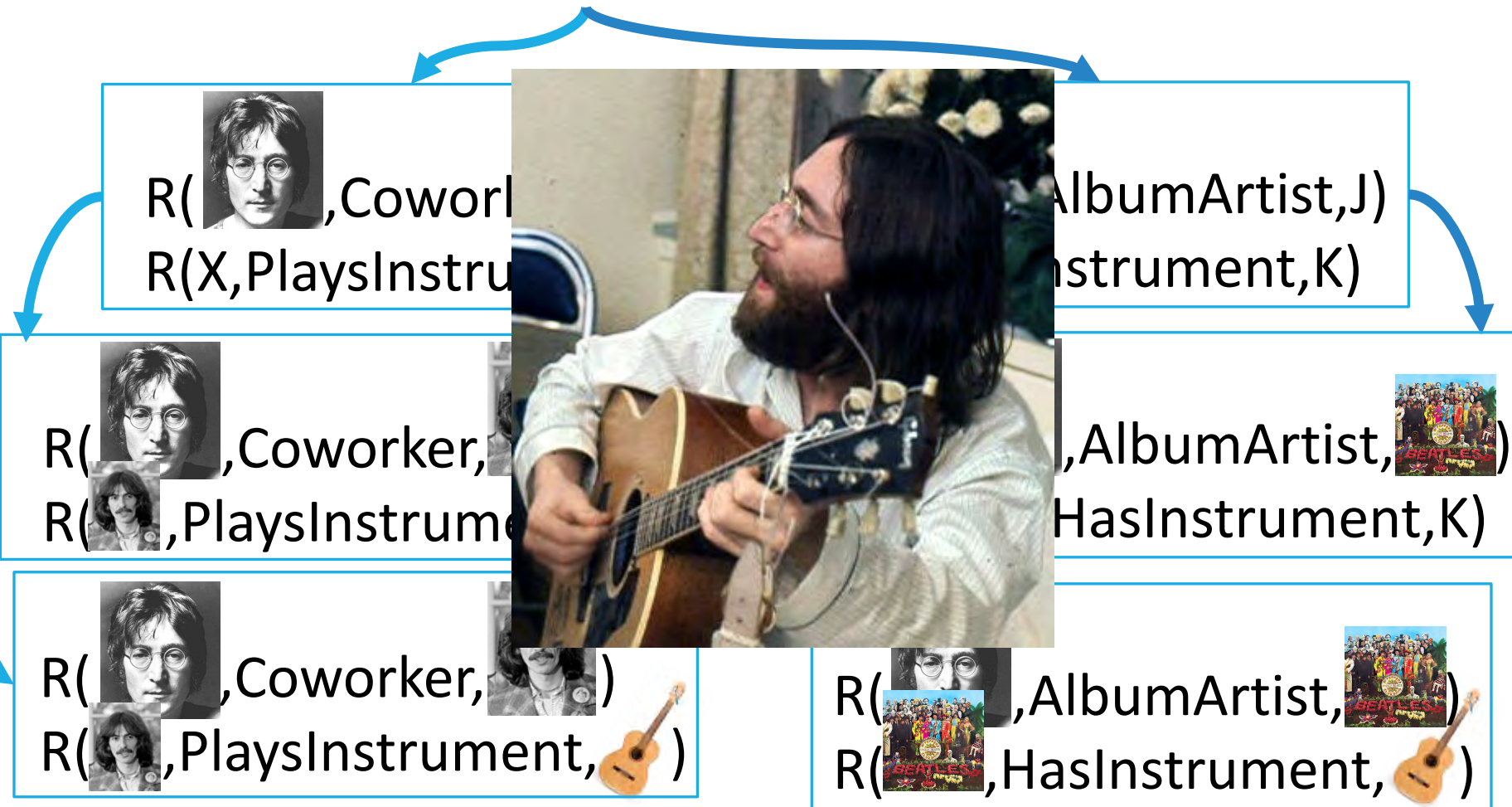
ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)

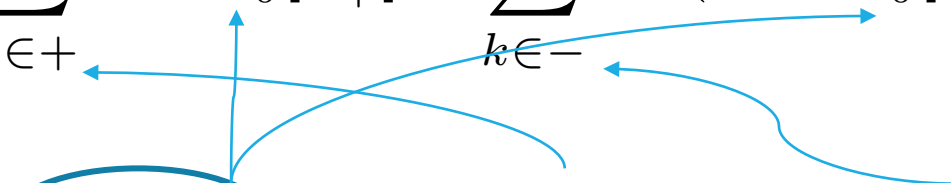


ProPPR-ized PRA example

Query Q: R(Lennon, PlaysInstrument, ?)



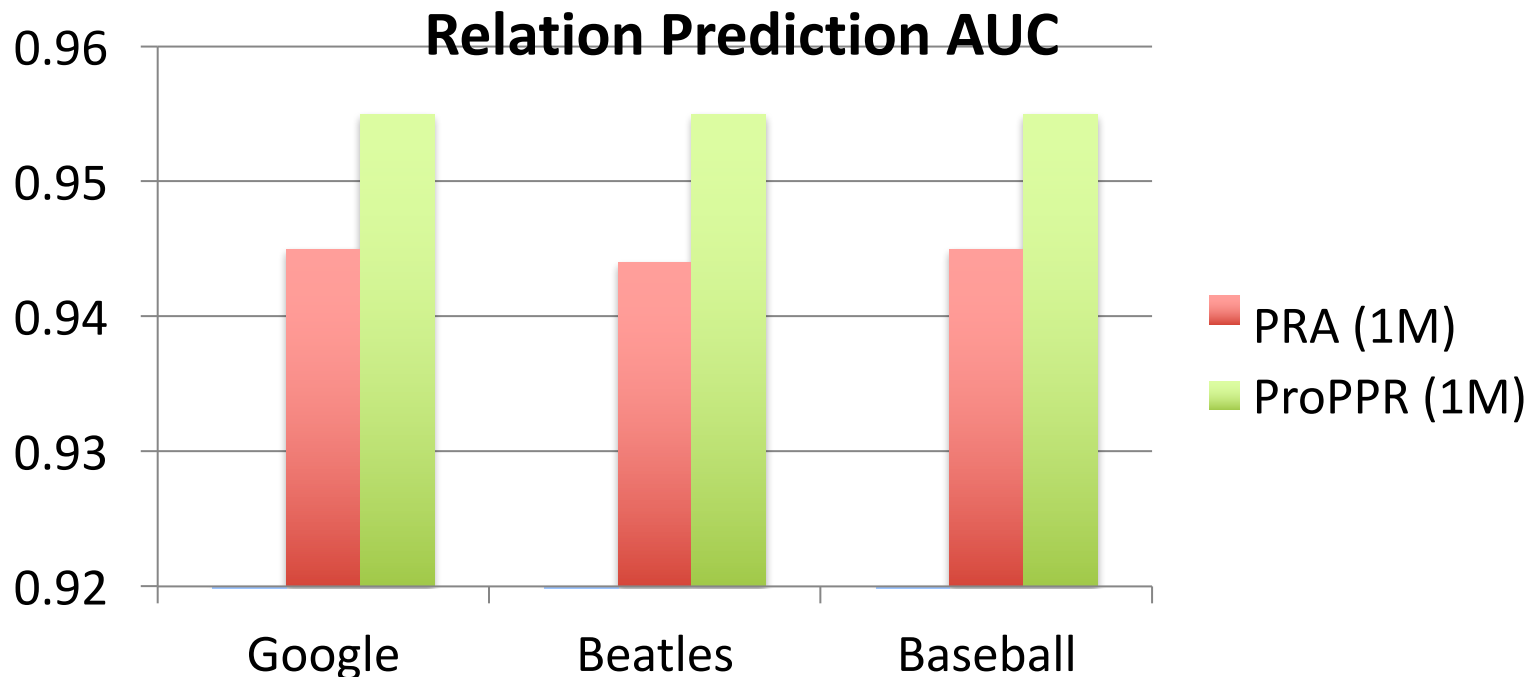
ProPPR in a nutshell

$$\min_{\mathbf{w}} - \left(\sum_{k \in +} \log \mathbf{p}_{\nu_0}[u_+^k] + \sum_{k \in -} \log(1 - \mathbf{p}_{\nu_0}[u_-^k]) \right) + \mu \|\mathbf{w}\|_2^2$$


- Input: queries, positive answers, negative answers
- Goal: $\mathbf{p}_{\nu_0}[u_+^k] \geq \mathbf{p}_{\nu_0}[u_-^k]$ (page rank from RW)
- Learn: random walk weights
- Train via stochastic gradient descent

Results from PRA and ProPPR

- Task:
 - 1M extractions for 3 domains;
 - ~100s of training queries
 - ~1000s of test queries
 - AUC of extractions alone is 0.7



Random Walks: Pros/Cons

BENEFITS

- KG query estimation independent of KG size
- Model training produces interpretable, logical rules
- Robust to noisy extractions through probabilistic form

DRAWBACKS

- Full KG completion task inefficient
- Training data difficult to obtain at scale
- Input must follow probabilistic semantics

Two classes of Probabilistic Models

GRAPHICAL MODELS

- Possible facts in KG are variables
- Logical rules relate facts
- Probability \propto satisfied rules
- Universally-quantified

RANDOM WALK METHODS

- Possible facts posed as queries
- Random walks of the KG constitute “proofs”
- Probability \propto path lengths/transitions
- Locally grounded

Path Ranking Algorithm (PRA)

(Lao & Cohen, MLJ 2010)

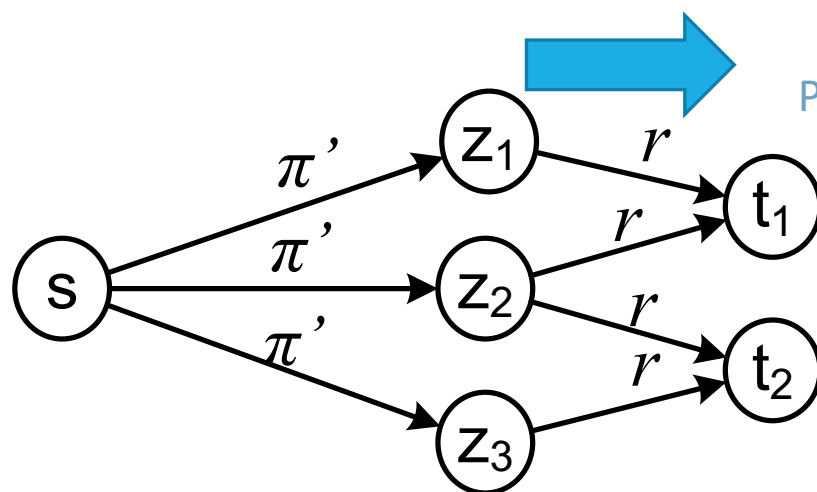
$$\textit{score}(s, t) = \sum_{\pi \in B} P(s \rightarrow t; \pi) \theta_{\pi}$$

a weight

e.g. $\pi = \langle \textit{coworker}, \textit{playsInstrument} \rangle$

Random Walk Calculation

$$score(s, t) = \sum_{\pi \in B} P(s \rightarrow t; \pi) \theta_{\pi}$$



*e.g. $\pi' = \langle \text{coworker} \rangle$
 $r = \text{playsInstrument}$*

$$P(s \rightarrow t; \pi) = \sum_z P(s \rightarrow z; \pi') P(z \rightarrow t; r)$$

Pruning paths using labeled data

$$score(s, t) = \sum_{\pi \in B} P(s \rightarrow t; \pi) \theta_{\pi}$$

given training query set $\{(s_i, G_i)\}$

$$hits(f) = \sum_i I \left[\sum_{j \in G_i} f(s_i, t_j) \right] \geq h$$

$$accuracy(f) = \frac{1}{N} \sum_i \left[\frac{\sum_{j \in G_i} f(s_i, t_j)}{\sum_j f(s_i, t_j)} \right] \geq a$$

$I()$: the indicator function

N : total number of queries

Estimating θ via Logistic Regression

$$score(s, t) = \sum_{\pi \in B} P(s \rightarrow t; \pi) \theta_{\pi}$$

for a relation r

generate positive and negative node pairs $\{(s_i, t_i)\}$

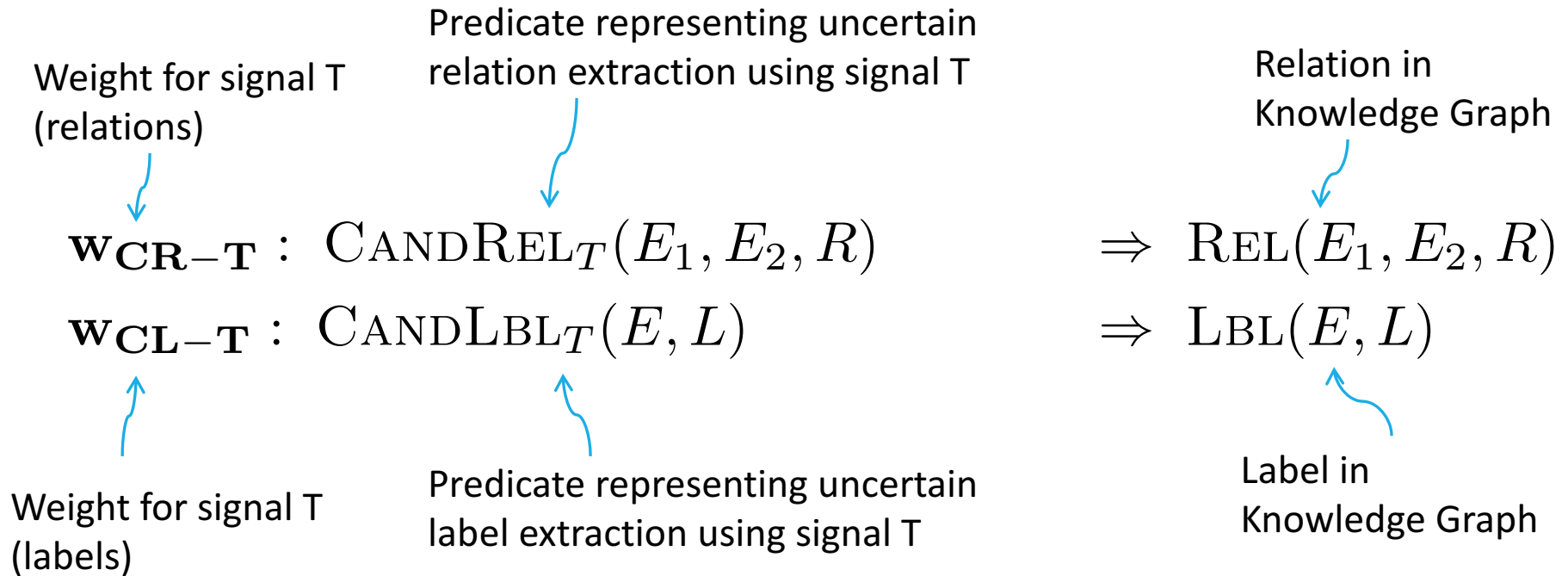
for each (s_i, t_i) generate (x_i, y_i)

x_i is a vector of RW features of different paths π

y_i is a binary label $r(s_i, t_i)$

estimate θ by L1/L2 regularized (elastic-net) **logistic regression**

Rules: Uncertain Extractions




Rules: Entity Resolution


$\mathbf{w_{EL}} : \text{SAMEENT}(E_1, E_2) \wedge \text{LBL}(E_1, L) \Rightarrow \text{LBL}(E_2, L)$

$\mathbf{w_{ER}} : \text{SAMEENT}(E_1, E_2) \wedge \text{REL}(E_1, E, R) \Rightarrow \text{REL}(E_2, E, R)$

$\mathbf{w_{ER}} : \text{SAMEENT}(E_1, E_2) \wedge \text{REL}(E, E_1, R) \Rightarrow \text{REL}(E, E_2, R)$



SameEnt predicate captures confidence that entities are co-referent

- 
- Rules require co-referent entities to have the same labels and relations
 - Creates an *equivalence class* of co-referent entities

Rules: Ontology

Inverse:

$$w_O : \text{INV}(R, S) \quad \wedge \quad \text{REL}(E_1, E_2, R) \quad \Rightarrow \quad \text{REL}(E_2, E_1, S)$$

Selectional Preference:

$$w_O : \text{DOM}(R, L) \quad \wedge \quad \text{REL}(E_1, E_2, R) \quad \Rightarrow \quad \text{LBL}(E_1, L)$$

$$w_O : \text{RNG}(R, L) \quad \wedge \quad \text{REL}(E_1, E_2, R) \quad \Rightarrow \quad \text{LBL}(E_2, L)$$

Subsumption:

$$w_O : \text{SUB}(L, P) \quad \wedge \quad \text{LBL}(E, L) \quad \Rightarrow \quad \text{LBL}(E, P)$$

$$w_O : \text{RSUB}(R, S) \quad \wedge \quad \text{REL}(E_1, E_2, R) \quad \Rightarrow \quad \text{REL}(E_1, E_2, S)$$

Mutual Exclusion:

$$w_O : \text{MUT}(L_1, L_2) \quad \wedge \quad \text{LBL}(E, L_1) \quad \Rightarrow \quad \neg \text{LBL}(E, L_2)$$

$$w_O : \text{RMUT}(R, S) \quad \wedge \quad \text{REL}(E_1, E_2, R) \quad \Rightarrow \quad \neg \text{REL}(E_1, E_2, S)$$