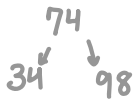
1. **AVL Trees (20 pts)**
   a. Insert the following values in the given order [34, 74, 98, 2, 64, 41, 86, 59, 40, 58, 50, 56, 45, 16, 44] into an empty AVL tree. Show the tree after all insertions.
   b. Add an element to the AVL tree such that it will cause a **right rotation** of the tree. State the element and show the tree after. Use the tree from part A.
   c. Add an element to the AVL tree such that it will cause a **left rotation** of the tree. State the element and show the tree after. Use the tree from part B.
   d. Delete an element from the AVL tree such that it will cause a **double rotation** to the tree. State the element and show the **final tree** after deletion. Use the tree from part C.

a) Empty AVL tree

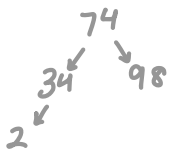
Insert 34

Insert 74

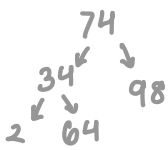
Insert 98



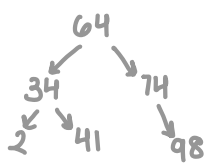
Insert 2



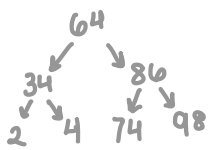
Insert 64



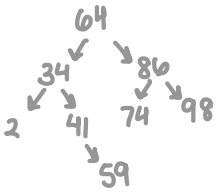
Insert 41



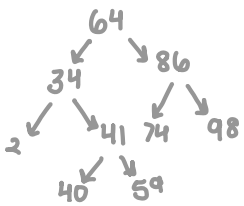
Insert 86



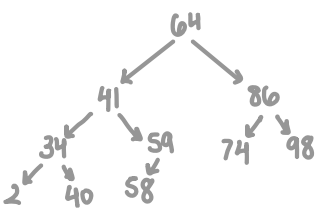
Insert 59



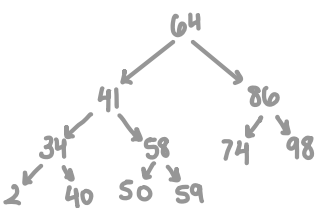
Insert 40



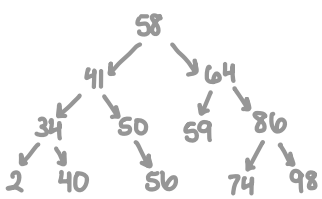
Insert 58



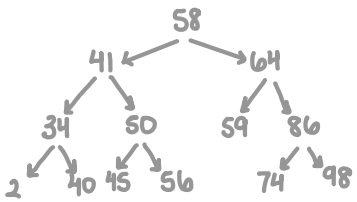
Insert 50



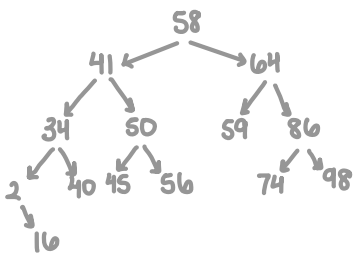
Insert 56



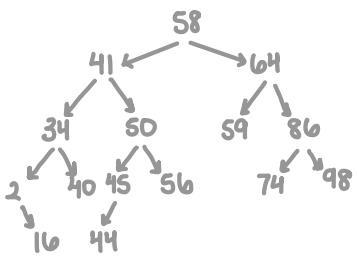
Insert 45



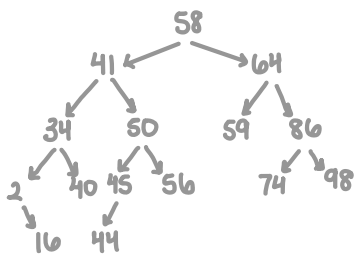
Insert 16



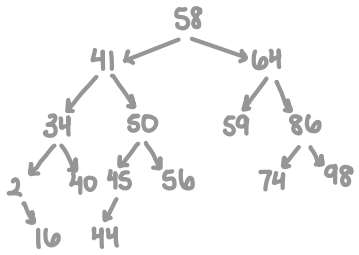
Insert 44



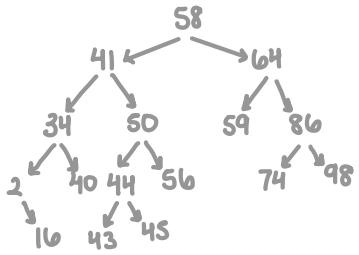
Final Tree



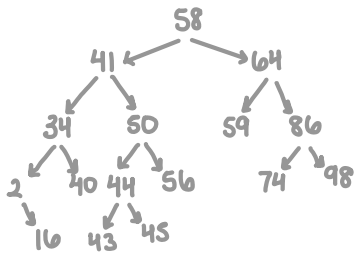
b) Right Rotation of tree



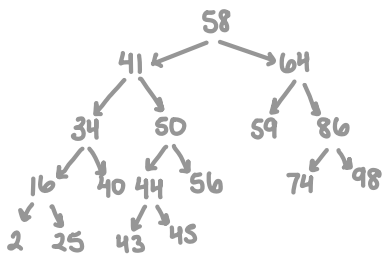
Insert 43



c) Left Rotation of tree



Insert 25



2. Heap (10 pts)

a. Create a binary max on top heap with the keys [11, 16, 33, 36, 26, 41, 17, 84, 8, 57, 30, 71, 53, 23, 87, 95, 62]. Show the heap after all insertions.

b. Show the heap when removing the max value 3 times. Show the steps.

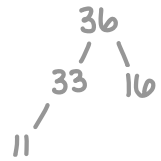
a) Empty Heap

Insert 11

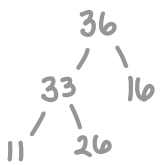
Insert 16

Insert 33

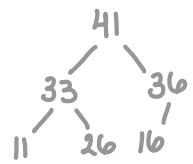
Insert 36



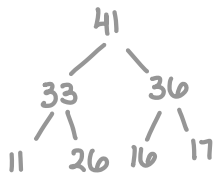
Insert 26



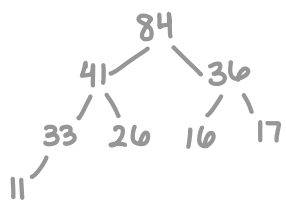
Insert 41



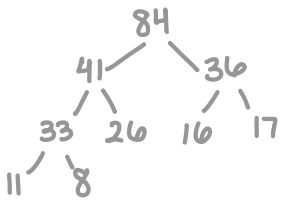
Insert 17



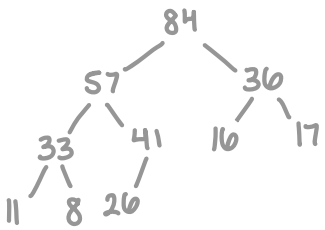
Insert 84



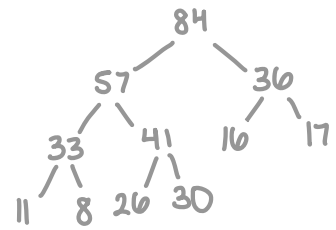
Insert 8



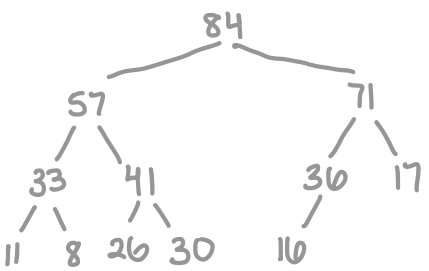
Insert 57



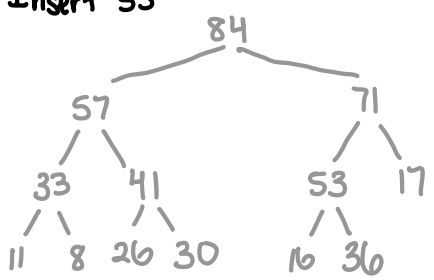
Insert 30



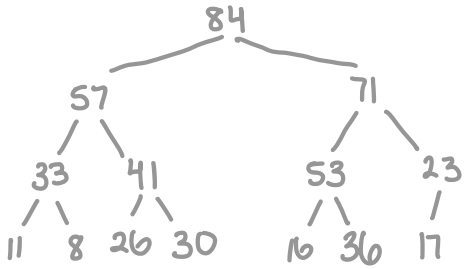
Insert 71



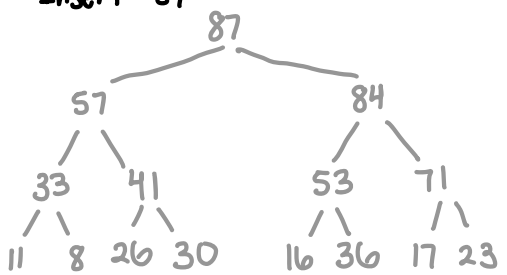
Insert 53



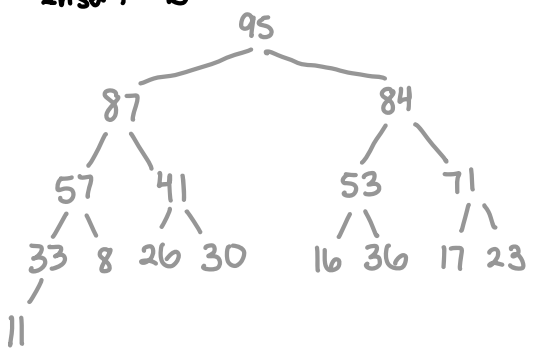
Insert 23



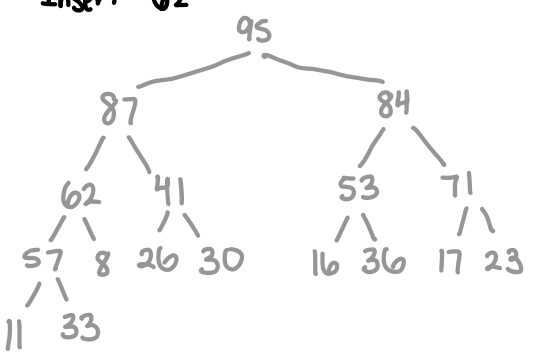
Insert 87



Insert 95

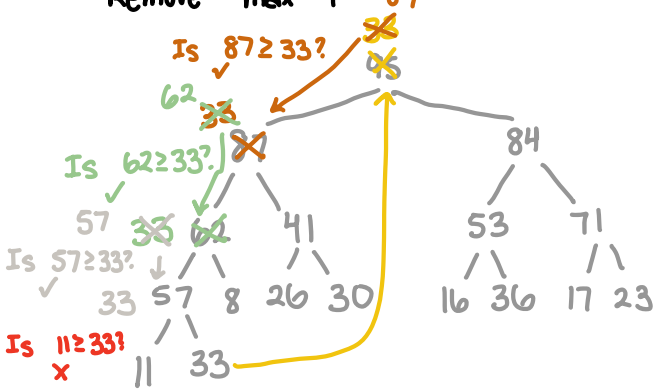


Insert 62

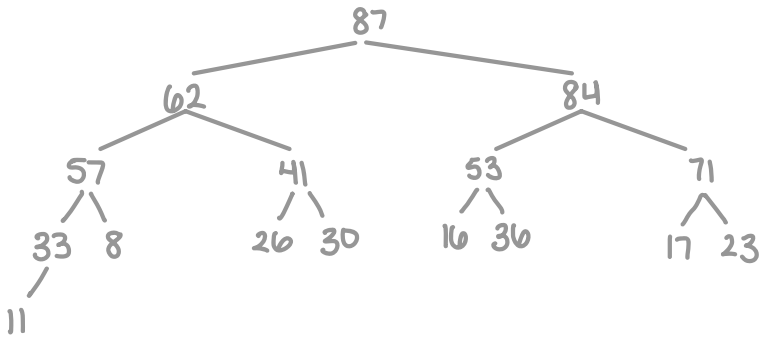


b) Remove max 3 times

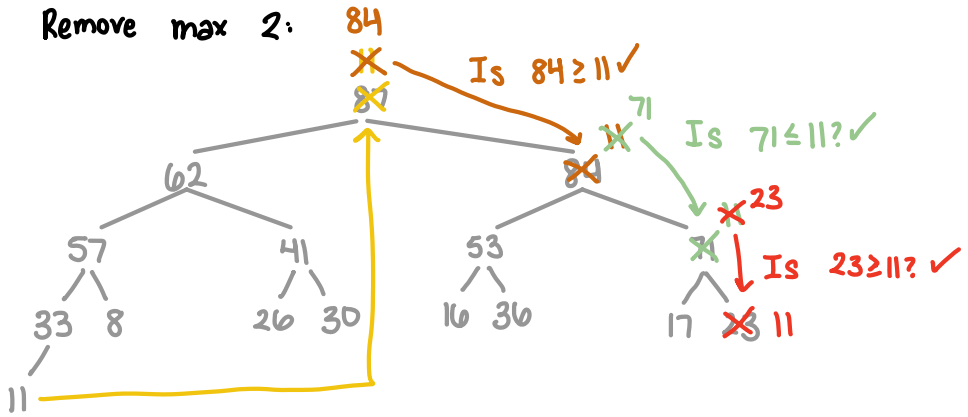
Remove max 1: 87



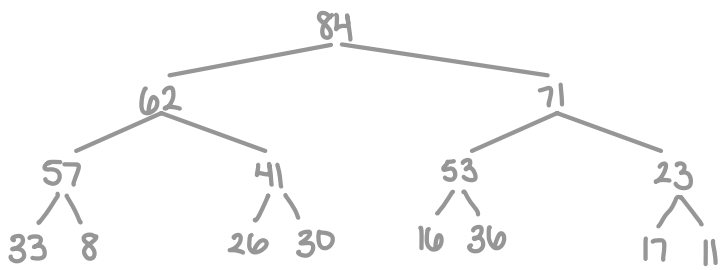
Tree after extracting:



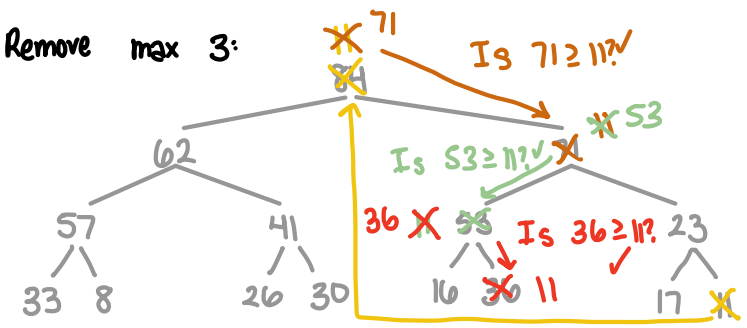
Remove max 2: 84



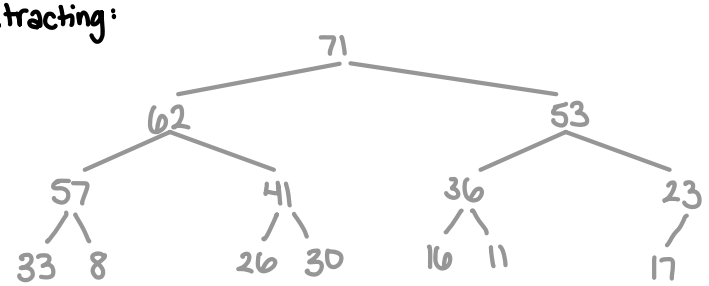
Tree after extracting:



Remove max 3: 71

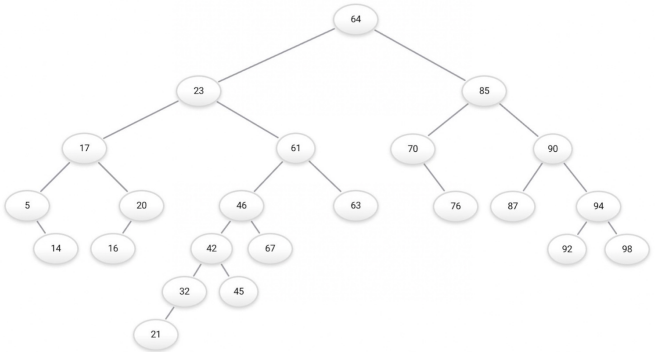


Tree after extracting:



3. Binary Trees (20 pts)

Given the following tree:



a. Give post-order, pre-order of the tree.
b. Give level-order and in-order of the tree
c. When looking for the value 67, give the path traced by breadth first search and depth first search algorithm
d. Given a tree with the preorder: [E, A, B, F, H, I, D, J, C, G, K] and the inorder: [B, A, F, H, I, E, D, J, C, G, K], draw the binary tree.

a) Post order:

14,5,16,20,17,21,32,45,42,67,46,63,61,23,76,70,87,92,98,94,90,85,64

Preorder:

64,23,17,5,14,20,16,61,46,42,32,21,45,67,63,85,70,76,90,87,94,92,98

b) Level-order:

64,23,85,17,61,70,90,5,20,46,63,76,87,94,14,16,42,67,92,98,32,45,21

In-order:

14,5,17,16,20,23,21,32,42,45,46,67,61,63,64,76,70,85,87,90,92,94,98

c) BFS:

64→23→85→17→61→70→90→5→20→46→63→76→87→94→14→16→42→67

DFS:

64→23→17→5→14→20→16→61→46→42→32→21→45→67

d)