Team notebook

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Contents

1 DP

1.1 LineContainer

```
bool Q;
struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return Q ? p < o.p
       : k < o.k; }
};
struct LineContainer : multiset<Line> {
  // ( for doubles , use inf = 1/.0 , div (a, b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a  b) < 0 && a % b);
  }
  bool isect(iterator x, iterator y) {
    if (y == end()) {
      x->p = inf;
      return false;
    }
    if (x->k == y->k)
      x->p = x->m > y->m ? inf : -inf;
    else
      x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y =
         erase(y));
    while ((y = x) != begin() && (--x)->p >= y->p) isect(x,
         erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    Q = 1;
    auto l = *lower_bound({0, 0, x});
    Q = 0;
    return l.k * x + l.m;
  }
};
```

2 DS

2.1 Fenwick2D

```
struct Fenwick2D {
#define gb(x) (x) & -(x)
  vector<vector<int>> nodes;
  vector<vector<int>> bit;
  int sx;
  void init(int _sx) {
    sx = _sx;
    nodes.resize(sx + 1);
    bit.resize(sx + 1);
  }
  void init_nodes() {
    for (int i = 1; i <= sx; ++i) {
      sort(all(nodes[i]));
      nodes[i].resize(unique(all(nodes[i])) -
           nodes[i].begin());
      bit[i].resize(sz(nodes[i]) + 1);
    }
  }
  void fake_update(int x, int y) {
    for (; x <= sx; x += gb(x)) nodes[x].push_back(y);
  }
  void fake_get(int x, int y) {
    for (; x > 0; x -= gb(x)) nodes[x].push_back(y);
  }
  void update(int x, int yy, int val) {
    for (; x <= sx; x += gb(x))
      for (int y = lower_bound(all(nodes[x]), yy) -
           nodes[x].begin() + 1;
          y <= sz(nodes[x]); y += gb(y))
        bit[x][y] = max(bit[x][y], val);
  }
  int get(int x, int yy) {
    int res = 0;
    for (; x > 0; x -= gb(x))
      for (int y = upper_bound(all(nodes[x]), yy) -
           nodes[x].begin(); y > 0;
          y -= gb(y))
        res = max(res, bit[x][y]);
    return res;
  }
};
```

2.2 ImplicitTreap

```
// Implicit Treap
// Tested: https://oj.vnoi.info/problem/sqrt_b
struct Treap {
  ll val;
  int prior, size;
  ll sum;
  Treap *left, *right;
  Treap(ll val)
      : val(val), prior(rng()), size(1), sum(val),
         left(NULL), right(NULL){};
};
int size(Treap *t) { return t == NULL ? 0 : t->size; }
void down(Treap *t) {
  // do lazy propagation here
}
void refine(Treap *t) {
  if (t == NULL) return;

  t->size = 1;
  t->sum = t->val;
  if (t->left != NULL) {
    t->size += t->left->size;
    t->sum += t->left->sum;
  }
  if (t->right != NULL) {
    t->size += t->right->size;
    t->sum += t->right->sum;
  }
}
void split(Treap *t, Treap *&left, Treap *&right, int val)
      {
  if (t == NULL) return void(left = right = NULL);
  down(t);
  if (size(t->left) < val) {
    split(t->right, t->right, right, val - size(t->left) -
         1);
    left = t;
  } else {
    split(t->left, left, t->left, val);
    right = t;
  }
  refine(t);
}
void merge(Treap *&t, Treap *left, Treap *right) {
  if (left == NULL) {
    t = right;
    return;
  }
  if (right == NULL) {
    t = left;
    return;
  }

  down(left);
  down(right);
  if (left->prior < right->prior) {
    merge(left->right, left->right, right);
    t = left;
  } else {
    merge(right->left, left, right->left);
    t = right;
  }
  refine(t);
}
array<Treap *, 2> split(Treap *root, int val) {
  array<Treap *, 2> t;
  split(root, t[0], t[1], val);
  return t;
}
array<Treap *, 3> split(Treap *root, int l, int r) {
  array<Treap *, 3> t;
  Treap *tmp;

  split(root, t[0], t[1], l - 1);
  tmp = t[1];
  split(tmp, t[1], t[2], r - l + 1);

  return t;
}
Treap *root;
```

2.3 MeldableHeap

```
mt19937 gen(0x94949);
template<typename T>
struct Node {
	Node *l, *r;
	T v;
	Node(T x): l(0), r(0), v(x){}
};
template<typename T>
Node<T>* Meld(Node<T>* A, Node<T>* B) {
	if(!A) return B; if(!B) return A;
	if(B->v < A->v) swap(A, B);
	if(gen()&1) A->l = Meld(A->l, B);
	else A->r = Meld(A->r, B);
	return A;
}
template<typename T>
struct Heap {
	Node<T> *r; int s;
	Heap(): r(0), s(0){}
	void push(T x) {
		r = Meld(new Node<T>(x), r);
		++s;
	}
	int size(){ return s; }
	bool empty(){ return s == 0;}
	T top(){ return r->v; }
	void pop() {
		Node<T>* p = r;
		r = Meld(r->l, r->r);
		delete p;
		--s;
	}
	void Meld(Heap x) {
		s += x->s;
		r = Meld(r, x->r);
	}
};
```

2.4 OrderStatisticTree

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
template <class T>
using Tree =
    tree<T, null_type, less<T>, rb_tree_tag,
         tree_order_statistics_node_update>;
void example() {
  Tree<int> t, t2;
  t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
}
```

2.5 PalindromeTree

```
template <int MAXC = 26>
struct PalindromicTree {
  PalindromicTree(const string& str) : _sz(str.size() +
      5), next(_sz, vector<int>(MAXC, 0)), link(_sz, 0),
      qlink(_sz, 0), cnt(_sz, 0), right_id(_sz, 0),
      len(_sz, 0), s(_sz, 0) {
    init();
    for (int i = 0; i < (int)str.size(); ++i) {
      add(str[i], i);
    }
    count();
  }
  int _sz;

  // returns vector of (left, right, frequency)
  vector<tuple<int, int, int>> get_palindromes() {
    vector<tuple<int, int, int>> res;
    dfs(0, res);
    dfs(1, res);
    return res;
  }

  void dfs(int u, vector<tuple<int, int, int>>& res) {
    if (u > 1) { // u = 0 and u = 1 are two empty nodes
      res.emplace_back(right_id[u] - len[u] + 1,
          right_id[u], cnt[u]);
    }
    for (int i = 0; i < MAXC; ++i) {
      if (next[u][i]) dfs(next[u][i], res);
    }
  }

  int last, n, p;
  vector<vector<int>> next, dlink;
  vector<int> link, qlink, cnt, right_id, len, s;

  int newnode(int l, int right) {
    len[p] = l;
    right_id[p] = right;
    return p++;
  }
  void init() {
    p = 0;
    newnode(0, -1), newnode(-1, -1);
    n = last = 0;
    s[n] = -1, link[0] = 1;
  }
  int getlink(int x) {
    while (s[n - len[x] - 1] != s[n]) {
      if (s[n - len[link[x]] - 1] == s[n])
        x = link[x];
      else
        x = qlink[x];
    }
    return x;
  }
  void add(char c, int right) {
    c -= 'a';
    s[++n] = c;
    int cur = getlink(last);
    if (!next[cur][(int)c]) {
      int now = newnode(len[cur] + 2, right);
      link[now] = next[getlink(link[cur])][(int)c];
      next[cur][(int)c] = now;
      if (s[n - len[link[now]]] == s[n -
          len[link[link[now]]]]) {
        qlink[now] = qlink[link[now]];
      } else {
        qlink[now] = link[link[now]];
      }
    }
    last = next[cur][(int)c];
    cnt[last]++;
  }
  void count() {
    for (int i = p - 1; i >= 0; i--) {
      cnt[link[i]] += cnt[i];
    }
  }
};
```

2.6 RMQ

```
ll a[N], st[LG + 1][N];

void pre() {
  for (int i = 1; i <= n; ++i) st[0][i] = a[i];
  for (int j = 1; j <= LG; ++j)
    for (int i = 1; i + (1 << j) - 1 <= n; ++i)
      st[j][i] = __gcd(st[j - 1][i], st[j - 1][i + (1 << (j
          - 1))]);
}

ll query(int l, int r) {
  int k = __lg(r - l + 1);
  return __gcd(st[k][l], st[k][r - (1 << k) + 1]);
}
```

2.7 RMQ

```
// RMQ O(1): 1-indexed
// remember to change the constants, types
using ll = long long;
#define For(i, j, k) for ( int i = (j) ; i <= (k) ; i++ )
#define Fol(i, j, k) for ( int i = (j) ; i >= (k) ; i-- )
namespace RMQ
{
	using T = int; constexpr int N = 2e6 + 6; // change
	    this
	inline bool cmp(T x, T y) { return x < y; } //
	    change to '>' to query max
	inline T calc(T x, T y) { return cmp(x, y) ? x : y;
	    }
	T val[N], pre[N], st[__lg(( N >> 5 ) + 9) + 1][( N
	    >> 5 ) + 9]; unsigned f[N];
```

```
        __attribute__((target("bmi" ))) inline int
              lb(unsigned x) { return __builtin_ctz(x) ; }
        __attribute__((target("lzcnt"))) inline int
              hb(unsigned x) { return __builtin_clz(x) ^ 31;
              }
        inline void build(int n, T *a)
        {
                int m = ( n - 1 ) >> 5, o = hb(m + 1),
                      stk[33]; copy(a + 1, a + n + 1, val);
                For(i, 0, n - 1) pre[i] = i & 31 ?
                      calc(pre[i - 1], val[i]) : val[i];
                For(i, 0, m) st[0][i] = pre[min(n - 1, i <<
                      5 | 31)];
                For(i, 1, o) For(j, 0, m + 1 - ( 1 << i ))
                        st[i][j] = calc(st[i - 1][j], st[i -
                              1][j + ( 1 << ( i - 1 ) )]);
                For(i, 0, n - 1)
                        if ( i & 31 )
                        {
                                f[i] = f[i - 1];
                                while ( o &&
                                      !cmp(val[stk[o]],
                                      val[i]) ) f[i] &= ~( 1u
                                      << ( stk[o--] & 31 ) );
                                f[i] |= 1u << ( ( stk[++o] =
                                      i ) & 31 );
                        }
                        else f[i] = 1u << ( ( stk[o = 1] = i
                              ) & 31 );
        }
        inline T qry(int l, int r)
        {
                if ( ( --l >> 5 ) == ( --r >> 5 ) ) return
                      val[l + lb(f[r] >> ( l & 31 ))];
                T z = calc(pre[r], val[l + lb(f[l | 31] >> (
                      l & 31 ))]);
                if ( ( l = ( l >> 5 ) + 1 ) == ( r >>= 5 ) )
                      return z;
                int t = hb(r - l); return calc(z,
                      calc(st[t][l], st[t][r - ( 1 << t )]));
        }
}
// build: RMQ::build(n, a), a is an array (not a vector!)
// query: RMQ::qry(l, r)
```

2.8 SegmentTree

```
struct Tree {
  typedef int T;
  static constexpr T unit = INT_MIN;
  T f(T a, T b) { return max(a, b); } // (any associative
      fn)
  vector<T> s;
  int n;
  Tree(int n = 0, T def = unit) : s(2 * n, def), n(n) {}
  void update(int pos, T val) {
    for (s[pos += n] = val; pos /= 2;) s[pos] = f(s[pos *
        2], s[pos * 2 + 1]);
  }
  T query(int b, int e) { // query [b , e)
    T ra = unit, rb = unit;
    for (b += n, e += n; b < e; b /= 2, e /= 2) {
      if (b % 2) ra = f(ra, s[b++]);
      if (e % 2) rb = f(s[--e], rb);
    }
    return f(ra, rb);
  }
};
```

2.9 WalkOnBIT

```
int bit[N]; // BIT array

int bit_search(int v) {
  int sum = 0;
  int pos = 0;

  for (int i = LOGN; i >= 0; i--) {
    if (pos + (1 << i) < N and sum + bit[pos + (1 << i)] <
        v) {
      sum += bit[pos + (1 << i)];
      pos += (1 << i);
    }
  }

  return pos + 1; // +1 because 'pos' will have position
      of largest value less than 'v'
}
```

3 Geometry

3.1 AngleBisector

```
// bisector vector of <abc
PT angle_bisector(PT &a, PT &b, PT &c){
  PT p = a - b, q = c - b;
  return p + q * sqrt(dot(p, p) / dot(q, q));
}
```

3.2 Centroid

```
// centroid of a (possibly non-convex) polygon,
// assuming that the coordinates are listed in a clockwise
    or
// counterclockwise fashion. Note that the centroid is
    often known as
// the "center of gravity" or "center of mass".
PT centroid(vector<PT> &p) {
    int n = p.size(); PT c(0, 0);
    double sum = 0;
    for (int i = 0; i < n; i++) sum += cross(p[i], p[(i +
        1) % n]);
    double scale = 3.0 * sum;
    for (int i = 0; i < n; i++) {
        int j = (i + 1) % n;
        c = c + (p[i] + p[j]) * cross(p[i], p[j]);
    }
    return c / scale;
}
```

3.3 Circle

```
struct circle {
  PT p;
  double r;
  circle() {}
  circle(PT _p, double _r) : p(_p), r(_r){};
  // center (x, y) and radius r
  circle(double x, double y, double _r) : p(PT(x, y)),
      r(_r){};
  // circumcircle of a triangle
  // the three points must be unique
  circle(PT a, PT b, PT c) {
    b = (a + b) * 0.5;
    c = (a + c) * 0.5;
    line_line_intersection(b, b + rotatecw90(a - b), c, c +
        rotatecw90(a - c), p);
    r = dist(a, p);
  }
  // inscribed circle of a triangle
  circle(PT a, PT b, PT c, bool t) {
    line u, v;
    double m = atan2(b.y - a.y, b.x - a.x), n = atan2(c.y -
        a.y, c.x - a.x);
    u.a = a;
    u.b = u.a + (PT(cos((n + m) / 2.0), sin((n + m) /
        2.0)));
    v.a = b;
    m = atan2(a.y - b.y, a.x - b.x), n = atan2(c.y - b.y,
        c.x - b.x);
    v.b = v.a + (PT(cos((n + m) / 2.0), sin((n + m) /
        2.0)));
    line_line_intersection(u.a, u.b, v.a, v.b, p);
    r = dist_from_point_to_seg(a, b, p);
  }
  bool operator==(circle v) { return p == v.p && sign(r -
      v.r) == 0; }
  double area() { return PI * r * r; }
  double circumference() { return 2.0 * PI * r; }
};
// 0 if outside, 1 if on circumference, 2 if inside circle
int circle_point_relation(PT p, double r, PT b) {
  double d = dist(p, b);
  if (sign(d - r) < 0) return 2;
  if (sign(d - r) == 0) return 1;
  return 0;
}
// 0 if outside, 1 if on circumference, 2 if inside circle
int circle_line_relation(PT p, double r, PT a, PT b) {
```

```
  double d = dist_from_point_to_line(a, b, p);
  if (sign(d - r) < 0) return 2;
  if (sign(d - r) == 0) return 1;
  return 0;
}
// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> circle_line_intersection(PT c, double r, PT a,
     PT b) {
  vector<PT> ret;
  b = b - a;
  a = a - c;
  double A = dot(b, b), B = dot(a, b);
  double C = dot(a, a) - r * r, D = B * B - A * C;
  if (D < -eps) return ret;
  ret.push_back(c + a + b * (-B + sqrt(D + eps)) / A);
  if (D > eps) ret.push_back(c + a + b * (-B - sqrt(D)) /
       A);
  return ret;
}
// 5 - outside and do not intersect
// 4 - intersect outside in one point
// 3 - intersect in 2 points
// 2 - intersect inside in one point
// 1 - inside and do not intersect
int circle_circle_relation(PT a, double r, PT b, double R)
     {
  double d = dist(a, b);
  if (sign(d - r - R) > 0) return 5;
  if (sign(d - r - R) == 0) return 4;
  double l = fabs(r - R);
  if (sign(d - r - R) < 0 && sign(d - l) > 0) return 3;
  if (sign(d - l) == 0) return 2;
  if (sign(d - l) < 0) return 1;
  assert(0);
  return -1;
}
vector<PT> circle_circle_intersection(PT a, double r, PT
     b, double R) {
  if (a == b && sign(r - R) == 0) return {PT(1e18, 1e18)};
  vector<PT> ret;
  double d = sqrt(dist2(a, b));
  if (d > r + R || d + min(r, R) < max(r, R)) return ret;
  double x = (d * d - R * R + r * r) / (2 * d);
  double y = sqrt(r * r - x * x);
  PT v = (b - a) / d;
  ret.push_back(a + v * x + rotateccw90(v) * y);
  if (y > 0) ret.push_back(a + v * x - rotateccw90(v) * y);
  return ret;
}
// returns two circle c1, c2 through points a, b and of
     radius r
// 0 if there is no such circle, 1 if one circle, 2 if two
     circle
int get_circle(PT a, PT b, double r, circle &c1, circle
     &c2) {
  vector<PT> v = circle_circle_intersection(a, r, b, r);
  int t = v.size();
  if (!t) return 0;
  c1.p = v[0], c1.r = r;
  if (t == 2) c2.p = v[1], c2.r = r;
  return t;
```

```
}
// returns two circle c1, c2 which is tangent to line u,
     goes through
// point q and has radius r1; 0 for no circle, 1 if c1 =
     c2 , 2 if c1 != c2
int get_circle(line u, PT q, double r1, circle &c1, circle
     &c2) {
  double d = dist_from_point_to_line(u.a, u.b, q);
  if (sign(d - r1 * 2.0) > 0) return 0;
  if (sign(d) == 0) {
    cout << u.v.x << ' ' << u.v.y << '\n';
    c1.p = q + rotateccw90(u.v).truncate(r1);
    c2.p = q + rotatecw90(u.v).truncate(r1);
    c1.r = c2.r = r1;
    return 2;
  }
  line u1 = line(u.a + rotateccw90(u.v).truncate(r1), u.b
       + rotateccw90(u.v).truncate(r1));
  line u2 = line(u.a + rotatecw90(u.v).truncate(r1), u.b +
       rotatecw90(u.v).truncate(r1));
  circle cc = circle(q, r1);
  PT p1, p2;
  vector<PT> v;
  v = circle_line_intersection(q, r1, u1.a, u1.b);
  if (!v.size()) v = circle_line_intersection(q, r1, u2.a,
       u2.b);
  v.push_back(v[0]);
  p1 = v[0], p2 = v[1];
  c1 = circle(p1, r1);
  if (p1 == p2) {
    c2 = c1;
    return 1;
  }
  c2 = circle(p2, r1);
  return 2;
}
// returns area of intersection between two circles
double circle_circle_area(PT a, double r1, PT b, double
     r2) {
  double d = (a - b).norm();
  if (r1 + r2 < d + eps) return 0;
  if (r1 + d < r2 + eps) return PI * r1 * r1;
  if (r2 + d < r1 + eps) return PI * r2 * r2;
  double theta_1 = acos((r1 * r1 + d * d - r2 * r2) / (2 *
       r1 * d)),
        theta_2 = acos((r2 * r2 + d * d - r1 * r1) / (2 *
             r2 * d));
  return r1 * r1 * (theta_1 - sin(2 * theta_1) / 2.) + r2
       * r2 * (theta_2 - sin(2 * theta_2) / 2.);
}
// tangent lines from point q to the circle
int tangent_lines_from_point(PT p, double r, PT q, line
     &u, line &v) {
  int x = sign(dist2(p, q) - r * r);
  if (x < 0) return 0; // point in cricle
  if (x == 0) {       // point on circle
    u = line(q, q + rotateccw90(q - p));
    v = u;
    return 1;
  }
  double d = dist(p, q);
  double l = r * r / d;
```

```
  double h = sqrt(r * r - l * l);
  u = line(q, p + ((q - p).truncate(l) + (rotateccw90(q -
       p).truncate(h))));
  v = line(q, p + ((q - p).truncate(l) + (rotatecw90(q -
       p).truncate(h))));
  return 2;
}
// returns outer tangents line of two circles
// if inner == 1 it returns inner tangent lines
int tangents_lines_from_circle(PT c1, double r1, PT c2,
     double r2, bool inner, line &u, line &v) {
  if (inner) r2 = -r2;
  PT d = c2 - c1;
  double dr = r1 - r2, d2 = d.norm2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0) {
    assert(h2 != 0);
    return 0;
  }
  vector<pair<PT, PT>> out;
  for (int tmp : {-1, 1}) {
    PT v = (d * dr + rotateccw90(d) * sqrt(h2) * tmp) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  u = line(out[0].first, out[0].second);
  if (out.size() == 2) v = line(out[1].first,
       out[1].second);
  return 1 + (h2 > 0);
}
// O(n^2 log n)
struct CircleUnion {
  int n;
  double x[2020], y[2020], r[2020];
  int covered[2020];
  vector<pair<double, double>> seg, cover;
  double arc, pol;
  inline int sign(double x) { return x < -eps ? -1 : x >
       eps; }
  inline int sign(double x, double y) { return sign(x -
       y); }
  inline double SQ(const double x) { return x * x; }
  inline double dist(double x1, double y1, double x2,
       double y2) {
    return sqrt(SQ(x1 - x2) + SQ(y1 - y2));
  }
  inline double angle(double A, double B, double C) {
    double val = (SQ(A) + SQ(B) - SQ(C)) / (2 * A * B);
    if (val < -1) val = -1;
    if (val > +1) val = +1;
    return acos(val);
  }
  CircleUnion() {
    n = 0;
    seg.clear(), cover.clear();
    arc = pol = 0;
  }
  void init() {
    n = 0;
    seg.clear(), cover.clear();
    arc = pol = 0;
  }
  void add(double xx, double yy, double rr) {
    x[n] = xx, y[n] = yy, r[n] = rr, covered[n] = 0, n++;
```

```
}
void getarea(int i, double lef, double rig) {
  arc += 0.5 * r[i] * r[i] * (rig - lef - sin(rig - lef));
  double x1 = x[i] + r[i] * cos(lef), y1 = y[i] + r[i] *
       sin(lef);
  double x2 = x[i] + r[i] * cos(rig), y2 = y[i] + r[i] *
       sin(rig);
  pol += x1 * y2 - x2 * y1;
}
double solve() {
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < i; j++) {
      if (!sign(x[i] - x[j]) && !sign(y[i] - y[j]) &&
           !sign(r[i] - r[j])) {
        r[i] = 0.0;
        break;
      }
    }
  }
  for (int i = 0; i < n; i++) {
    for (int j = 0; j < n; j++) {
      if (i != j && sign(r[j] - r[i]) >= 0 &&
          sign(dist(x[i], y[i], x[j], y[j]) - (r[j] -
               r[i])) <= 0) {
        covered[i] = 1;
        break;
      }
    }
  }
  for (int i = 0; i < n; i++) {
    if (sign(r[i]) && !covered[i]) {
      seg.clear();
      for (int j = 0; j < n; j++) {
        if (i != j) {
          double d = dist(x[i], y[i], x[j], y[j]);
          if (sign(d - (r[j] + r[i])) >= 0 || sign(d -
               abs(r[j] - r[i])) <= 0) {
            continue;
          }
          double alpha = atan2(y[j] - y[i], x[j] - x[i]);
          double beta = angle(r[i], d, r[j]);
          pair<double, double> tmp(alpha - beta, alpha +
               beta);
          if (sign(tmp.first) <= 0 && sign(tmp.second) <=
               0) {
            seg.push_back(pair<double, double>(2 * PI +
                 tmp.first, 2 * PI + tmp.second));
          } else if (sign(tmp.first) < 0) {
            seg.push_back(pair<double, double>(2 * PI +
                 tmp.first, 2 * PI));
            seg.push_back(pair<double, double>(0,
                 tmp.second));
          } else {
            seg.push_back(tmp);
          }
        }
      }
      sort(seg.begin(), seg.end());
      double rig = 0;
      for (vector<pair<double, double>>::iterator iter =
           seg.begin(); iter != seg.end(); iter++) {
        if (sign(rig - iter->first) >= 0) {
          rig = max(rig, iter->second);
        } else {
          getarea(i, rig, iter->first);
          rig = iter->second;
        }
      }
      if (!sign(rig)) {
        arc += r[i] * r[i] * PI;
      } else {
        getarea(i, rig, 2 * PI);
      }
    }
  }
  return pol / 2.0 + arc;
}
} CU;
```

3.4 ClosestPair

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  for (P p : v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
         d);
    for (; lo != hi; ++lo) ret = min(ret, {(*lo -
         p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
```

3.5 ConvexPolygon

```
vector<PT> convex_hull(vector<PT>& p) {
  if (p.size() <= 1) return p;
  vector<PT> v = p;
  sort(v.begin(), v.end());
  vector<PT> up, dn;
  for (auto& p : v) {
    while (up.size() > 1 && orientation(up[up.size() - 2],
         up.back(), p) >= 0)
      up.pop_back();
    while (dn.size() > 1 && orientation(dn[dn.size() - 2],
         dn.back(), p) <= 0)
      dn.pop_back();
    up.push_back(p);
    dn.push_back(p);
  }
  v = dn;
  if (v.size() > 1)
    v.pop_back();
  reverse(up.begin(), up.end());
  up.pop_back();
  for (auto& p : up)
    v.push_back(p);
  if (v.size() == 2 && v[0] == v[1])
    v.pop_back();
  return v;
}
// checks if convex or not
bool is_convex(vector<PT>& p) {
  bool s[3];
  s[0] = s[1] = s[2] = 0;
  int n = p.size();
  for (int i = 0; i < n; i++) {
    int j = (i + 1) % n;
    int k = (j + 1) % n;
    s[sign(cross(p[j] - p[i], p[k] - p[i])) + 1] = 1;
    if (s[0] && s[2])
      return 0;
  }
  return 1;
}
// -1 if strictly inside, 0 if on the polygon, 1 if
     strictly outside
// it must be strictly convex, otherwise make it strictly
     convex first
int is_point_in_convex(vector<PT>& p, const PT& x) { //
     O(log n)
  int n = p.size();
  assert(n >= 3);
  int a = orientation(p[0], p[1], x), b =
       orientation(p[0], p[n - 1], x);
  if (a < 0 || b > 0)
    return 1;
  int l = 1, r = n - 1;
  while (l + 1 < r) {
    int mid = l + r >> 1;
    if (orientation(p[0], p[mid], x) >= 0) l = mid;
    else r = mid;
  }
  int k = orientation(p[l], p[r], x);
  if (k <= 0)
    return -k;
  if (l == 1 && a == 0)
    return 0;
  if (r == n - 1 && b == 0)
    return 0;
  return -1;
}
```

3.6 ExtremeVertex

```
// id of the vertex having maximum dot product with z
// polygon must need to be convex
// top - upper right vertex
// for minimum dot prouct negate z and return -dot(z,
     p[id])
```

```
int extreme_vertex(vector<PT> &p, const PT &z, const int
     top) { // O(log n)
    int n = p.size();
    if (n == 1) return 0;
    double ans = dot(p[0], z); int id = 0;
    if (dot(p[top], z) > ans) ans = dot(p[top], z), id =
         top;
    int l = 1, r = top - 1;
    while (l < r) {
        int mid = l + r >> 1;
        if (dot(p[mid + 1], z) >= dot(p[mid], z)) l = mid +
             1;
        else r = mid;
    }
    if (dot(p[l], z) > ans) ans = dot(p[l], z), id = l;
    l = top + 1, r = n - 1;
    while (l < r) {
        int mid = l + r >> 1;
        if (dot(p[(mid + 1) % n], z) >= dot(p[mid], z)) l =
             mid + 1;
        else r = mid;
    }
    l %= n;
    if (dot(p[l], z) > ans) ans = dot(p[l], z), id = l;
    return id;
}
```

3.7 GeometricMedian

```
// it returns a point such that the sum of distances
// from that point to all points in p is minimum
// O(n log^2 MX)
PT geometric_median(vector<PT> p) {
    auto tot_dist = [&](PT z) {
        double res = 0;
        for (int i = 0; i < p.size(); i++) res +=
             dist(p[i], z);
        return res;
    };
    auto findY = [&](double x) {
        double yl = -1e5, yr = 1e5;
        for (int i = 0; i < 60; i++) {
            double ym1 = yl + (yr - yl) / 3;
            double ym2 = yr - (yr - yl) / 3;
            double d1 = tot_dist(PT(x, ym1));
            double d2 = tot_dist(PT(x, ym2));
            if (d1 < d2) yr = ym2;
            else yl = ym1;
        }
        return pair<double, double> (yl, tot_dist(PT(x,
             yl)));
    };
    double xl = -1e5, xr = 1e5;
    for (int i = 0; i < 60; i++) {
        double xm1 = xl + (xr - xl) / 3;
        double xm2 = xr - (xr - xl) / 3;
        double y1, d1, y2, d2;
        auto z = findY(xm1); y1 = z.first; d1 = z.second;
        z = findY(xm2); y2 = z.first; d2 = z.second;
        if (d1 < d2) xr = xm2;
        else xl = xm1;
    }
    return {xl, findY(xl).first };
}
```

3.8 GeometryTemplate

```
const long double PI = acos(-1);
struct Vector {
  using type = long long;
  type x, y;
  Vector operator-(const Vector &other) const {
    return {x - other.x, y - other.y};
  }
  type operator*(const Vector &other) const {
    return x * other.y - other.x * y;
  }
  type operator%(const Vector &other) const {
    return x * other.x + y * other.y;
  }
  bool operator==(const Vector &other) const {
    return x == other.x and y == other.y;
  }
  bool operator!=(const Vector &other) const { return
       !(*this == other); }
  friend type cross(const Vector &A, const Vector &B,
       const Vector &C) {
    return (B - A) * (C - A);
  }
  friend type dist(Vector A) { return A.x * A.x + A.y *
       A.y; }
  friend type dot(const Vector &A, const Vector &B, const
       Vector &C) {
    Vector u = (B - A), v = (C - A);
    return u % v;
  }
  friend istream &operator>>(istream &is, Vector &V) {
    is >> V.x >> V.y;
    return is;
  }
  friend ostream &operator<<(ostream &os, Vector &V) {
    os << V.x << ' ' << V.y;
    return os;
  }
  friend double angle(const Vector &A, const Vector &B,
       const Vector &C) {
    double x = dot(B, A, C) / sqrt(dist(A - B) * dist(C -
         B));
    return acos(min(1.0, max(-1.0, x))) * 180.0 / PI;
  }
};
using Point = Vector;
const Point origin = {0, 0};

long double area(Point A, Point B, Point C) {
  long double res =
      cross(origin, A, B) + cross(origin, B, C) +
            cross(origin, C, A);
  return abs(res) / 2.0;
}
```

3.9 HalfPlane

```
// contains all points p such that: cross(b - a, p - a) >=
     0
struct HP {
    PT a, b;
    HP() {}
    HP(PT a, PT b) : a(a), b(b) {}
    HP(const HP& rhs) : a(rhs.a), b(rhs.b) {}
    int operator < (const HP& rhs) const {
        PT p = b - a;
        PT q = rhs.b - rhs.a;
        int fp = (p.y < 0 || (p.y == 0 && p.x < 0));
        int fq = (q.y < 0 || (q.y == 0 && q.x < 0));
        if (fp != fq) return fp == 0;
        if (cross(p, q)) return cross(p, q) > 0;
        return cross(p, rhs.b - a) < 0;
    }
    PT line_line_intersection(PT a, PT b, PT c, PT d) {
        b = b - a; d = c - d; c = c - a;
        return a + b * cross(c, d) / cross(b, d);
    }
    PT intersection(const HP &v) {
        return line_line_intersection(a, b, v.a, v.b);
    }
};
int check(HP a, HP b, HP c) {
    return cross(a.b - a.a, b.intersection(c) - a.a) >
          -eps; //-eps to include polygons of zero area
          (straight lines, points)
}
// consider half-plane of counter-clockwise side of each
     line
// if lines are not bounded add infinity rectangle
// returns a convex polygon, a point can occur multiple
     times though
// complexity: O(n log(n))
vector<PT> half_plane_intersection(vector<HP> h) {
    sort(h.begin(), h.end());
    vector<HP> tmp;
    for (int i = 0; i < h.size(); i++) {
        if (!i || cross(h[i].b - h[i].a, h[i - 1].b - h[i -
             1].a)) {
            tmp.push_back(h[i]);
        }
    }
    h = tmp;
    vector<HP> q(h.size() + 10);
    int qh = 0, qe = 0;
    for (int i = 0; i < h.size(); i++) {
        while (qe - qh > 1 && !check(h[i], q[qe - 2], q[qe
             - 1])) qe--;
        while (qe - qh > 1 && !check(h[i], q[qh], q[qh +
             1])) qh++;
        q[qe++] = h[i];
    }
```

```
    while (qe - qh > 2 && !check(q[qh], q[qe - 2], q[qe -
        1])) qe--;
    while (qe - qh > 2 && !check(q[qe - 1], q[qh], q[qh +
        1])) qh++;
    vector<HP> res;
    for (int i = qh; i < qe; i++) res.push_back(q[i]);
    vector<PT> hull;
    if (res.size() > 2) {
        for (int i = 0; i < res.size(); i++) {
            hull.push_back(res[i].intersection(res[(i + 1)
                % ((int)res.size())]));
        }
    }
    return hull;
}
```

3.10 IsPoint

```
// -1 if strictly inside, 0 if on the polygon, 1 if
    strictly outside
int is_point_in_triangle(PT a, PT b, PT c, PT p) {
    if (sign(cross(b - a,c - a)) < 0) swap(b, c);
    int c1 = sign(cross(b - a,p - a));
    int c2 = sign(cross(c - b,p - b));
    int c3 = sign(cross(a - c,p - c));
    if (c1<0 || c2<0 || c3 < 0) return 1;
    if (c1 + c2 + c3 != 3) return 0;
    return -1;
}

bool is_point_on_polygon(vector<PT> &p, const PT& z) {
    int n = p.size();
    for (int i = 0; i < n; i++) {
        if (is_point_on_seg(p[i], p[(i + 1) % n], z))
            return 1;
    }
    return 0;
}

// returns 1e9 if the point is on the polygon
int winding_number(vector<PT> &p, const PT& z) { // O(n)
    if (is_point_on_polygon(p, z)) return 1e9;
    int n = p.size(), ans = 0;
    for (int i = 0; i < n; ++i) {
        int j = (i + 1) % n;
        bool below = p[i].y < z.y;
        if (below != (p[j].y < z.y)) {
            auto orient = orientation(z, p[j], p[i]);
            if (orient == 0) return 0;
            if (below == (orient > 0)) ans += below ? 1 :
                -1;
        }
    }
    return ans;
}
// -1 if strictly inside, 0 if on the polygon, 1 if
    strictly outside
int is_point_in_polygon(vector<PT> &p, const PT& z) { //
    O(n)
    int k = winding_number(p, z);
    return k == 1e9 ? 0 : k == 0 ? 1 : -1;
}
```

3.11 Line

```
struct line {
    PT a, b; // goes through points a and b
    PT v; double c; //line form: direction vec [cross] (x,
        y) = c
    line() {}
    //direction vector v and offset c
    line(PT v, double c) : v(v), c(c) {
        auto p = get_points();
        a = p.first; b = p.second;
    }
    // equation ax + by + c = 0
    line(double _a, double _b, double _c) : v({_b, -_a}),
        c(-_c) {
        auto p = get_points();
        a = p.first; b = p.second;
    }
    // goes through points p and q
    line(PT p, PT q) : v(q - p), c(cross(v, p)), a(p), b(q)
        {}
        pair<PT, PT> get_points() { //extract any two
            points from this line
        PT p, q; double a = -v.y, b = v.x; // ax + by = c
        if (sign(a) == 0) {
            p = PT(0, c / b);
            q = PT(1, c / b);
        }
        else if (sign(b) == 0) {
            p = PT(c / a, 0);
            q = PT(c / a, 1);
        }
        else {
            p = PT(0, c / b);
            q = PT(1, (c - a) / b);
        }
        return {p, q};
        }
    //ax + by + c = 0
    array<double, 3> get_abc() {
        double a = -v.y, b = v.x;
        return {a, b, c};
    }
    // 1 if on the left, -1 if on the right, 0 if on the
        line
    int side(PT p) { return sign(cross(v, p) - c); }
    // line that is perpendicular to this and goes through
        point p
    line perpendicular_through(PT p) { return {p, p +
        perp(v)}; }
    // translate the line by vector t i.e. shifting it by
        vector t
    line translate(PT t) { return {v, c + cross(v, t)}; }
    // compare two points by their orthogonal projection on
        this line
    // a projection point comes before another if it comes
        first according to vector v
    bool cmp_by_projection(PT p, PT q) { return dot(v, p) <
        dot(v, q); }
    line shift_left(double d) {
        PT z = v.perp().truncate(d);
        return line(a + z, b + z);
    }
};
```

3.12 LineLineIntersection

```
// intersection point between ab and cd assuming unique
    intersection exists
bool line_line_intersection(PT a, PT b, PT c, PT d, PT
    &ans) {
    double a1 = a.y - b.y, b1 = b.x - a.x, c1 = cross(a, b);
    double a2 = c.y - d.y, b2 = d.x - c.x, c2 = cross(c, d);
    double det = a1 * b2 - a2 * b1;
    if (det == 0) return 0;
    ans = PT((b1 * c2 - b2 * c1) / det, (c1 * a2 - a1 * c2)
        / det);
    return 1;
}
```

3.13 MaximumCircleCover

```
// find a circle of radius r that contains as many points
    as possible
// O(n^2 log n);
double maximum_circle_cover(vector<PT> p, double r, circle
    &c) {
    int n = p.size();
    int ans = 0;
    int id = 0; double th = 0;
    for (int i = 0; i < n; ++i) {
        // maximum circle cover when the circle goes
            through this point
        vector<pair<double, int>> events = {{-PI, +1}, {PI,
            -1}};
        for (int j = 0; j < n; ++j) {
            if (j == i) continue;
            double d = dist(p[i], p[j]);
            if (d > r * 2) continue;
            double dir = (p[j] - p[i]).arg();
            double ang = acos(d / 2 / r);
            double st = dir - ang, ed = dir + ang;
            if (st > PI) st -= PI * 2;
            if (st <= -PI) st += PI * 2;
            if (ed > PI) ed -= PI * 2;
            if (ed <= -PI) ed += PI * 2;
            events.push_back({st - eps, +1}); // take care
                of precisions!
            events.push_back({ed, -1});
            if (st > ed) {
                events.push_back({-PI, +1});
```

```
            events.push_back({+PI, -1});
        }
    }
    sort(events.begin(), events.end());
    int cnt = 0;
    for (auto &&e: events) {
        cnt += e.second;
        if (cnt > ans) {
            ans = cnt;
            id = i; th = e.first;
        }
    }
}
PT w = PT(p[id].x + r * cos(th), p[id].y + r * sin(th));
c = circle(w, r); //best_circle
return ans;
}
```

3.14 MaximumInscribedCircle

```
// radius of the maximum inscribed circle in a convex
    polygon
double maximum_inscribed_circle(vector<PT> p) {
    int n = p.size();
    if (n <= 2) return 0;
    double l = 0, r = 20000;
    while (r - l > eps) {
        double mid = (l + r) * 0.5;
        vector<HP> h;
        const int L = 1e9;
        h.push_back(HP(PT(-L, -L), PT(L, -L)));
        h.push_back(HP(PT(L, -L), PT(L, L)));
        h.push_back(HP(PT(L, L), PT(-L, L)));
        h.push_back(HP(PT(-L, L), PT(-L, -L)));
        for (int i = 0; i < n; i++) {
            PT z = (p[(i + 1) % n] - p[i]).perp();
            z = z.truncate(mid);
            PT y = p[i] + z, q = p[(i + 1) % n] + z;
            h.push_back(HP(p[i] + z, p[(i + 1) % n] + z));
        }
        vector<PT> nw = half_plane_intersection(h);
        if (!nw.empty()) l = mid;
        else r = mid;
    }
    return l;
}
```

3.15 MinimumEnclosingCircle

```
// given n points, find the minimum enclosing circle of
    the points
// call convex_hull() before this for faster solution
// expected O(n)
circle minimum_enclosing_circle(vector<PT> &p) {
    random_shuffle(p.begin(), p.end());
    int n = p.size();
    circle c(p[0], 0);
    for (int i = 1; i < n; i++) {
        if (sign(dist(c.p, p[i]) - c.r) > 0) {
            c = circle(p[i], 0);
            for (int j = 0; j < i; j++) {
                if (sign(dist(c.p, p[j]) - c.r) > 0) {
                    c = circle((p[i] + p[j]) / 2, dist(p[i],
                        p[j]) / 2);
                    for (int k = 0; k < j; k++) {
                        if (sign(dist(c.p, p[k]) - c.r) > 0) {
                            c = circle(p[i], p[j], p[k]);
                        }
                    }
                }
            }
        }
    }
    return c;
}
```

3.16 MinimumEnclosingRectangle

```
// minimum perimeter
double minimum_enclosing_rectangle(vector<PT> &p) {
    int n = p.size();
    if (n <= 2) return perimeter(p);
    int mndot = 0; double tmp = dot(p[1] - p[0], p[0]);
    for (int i = 1; i < n; i++) {
        if (dot(p[1] - p[0], p[i]) <= tmp) {
            tmp = dot(p[1] - p[0], p[i]);
            mndot = i;
        }
    }
    double ans = inf;
    int i = 0, j = 1, mxdot = 1;
    while (i < n) {
        PT cur = p[(i + 1) % n] - p[i];
        while (cross(cur, p[(j + 1) % n] - p[j]) >= 0) j =
            (j + 1) % n;
        while (dot(p[(mxdot + 1) % n], cur) >=
            dot(p[mxdot], cur)) mxdot = (mxdot + 1) % n;
        while (dot(p[(mndot + 1) % n], cur) <=
            dot(p[mndot], cur)) mndot = (mndot + 1) % n;
        ans = min(ans, 2.0 * ((dot(p[mxdot], cur) /
            cur.norm() - dot(p[mndot], cur) / cur.norm())
            + dist_from_point_to_line(p[i], p[(i + 1) %
            n], p[j])));
        i++;
    }
    return ans;
}
```

3.17 MinkowskiSum

```
// a and b are strictly convex polygons of DISTINCT points
// returns a convex hull of their minkowski sum with
    distinct points
vector<PT> minkowski_sum(vector<PT> &a, vector<PT> &b) {
    int n = (int)a.size(), m = (int)b.size();
    int i = 0, j = 0; //assuming a[i] and b[j] both are
        (left, bottom)-most points
    vector<PT> c;
    c.push_back(a[i] + b[j]);
    while (1) {
        PT p1 = a[i] + b[(j + 1) % m];
        PT p2 = a[(i + 1) % n] + b[j];
        int t = orientation(c.back(), p1, p2);
        if (t >= 0) j = (j + 1) % m;
        if (t <= 0) i = (i + 1) % n, p1 = p2;
        if (t == 0) p1 = a[i] + b[j];
        if (p1 == c[0]) break;
        c.push_back(p1);
    }
    return c;
}
```

3.18 MonotoneChain

```
// warning: different template
vector<Point> convex_hull(vector<Point> p, int n){
    sort(p.begin(), p.end(), [](const Point &A, const Point
        &B){
        return A.x != B.x ? A.x < B.x : A.y < B.y;
    });
    Point st = p[0], en = p[n - 1];
    vector<Point> up = {p[0]};
    vector<Point> down = {p[0]};
    for(int i = 1; i < n; ++i){
        // upper hull
        if(i == n - 1 or cross(st, p[i], en) < 0){
            while((int)up.size() >= 2 and
                cross(up[up.size() - 2], up.back(), p[i])
                >= 0)
                up.pop_back();
            up.push_back(p[i]);
        }
        // lower hull
        if(i == n - 1 or cross(st, p[i], en) > 0){
            while((int)down.size() >= 2 and
                cross(down[down.size() - 2], down.back(),
                p[i]) <= 0)
                down.pop_back();
            down.push_back(p[i]);
        }
    }
    p.clear();
    for(int i = 0; i < (int)up.size(); ++i)
        p.push_back(up[i]);
    for(int i = down.size() - 2; i >= 1; --i)
        p.push_back(down[i]);
    // return hull in clockwise order
    return p;
}
```

3.19 Point2D

```
const double inf = 1e100;
const double eps = 1e-9;
const double PI = acos((double)-1.0);
int sign(double x) { return (x > eps) - (x < -eps); }
struct PT {
    double x, y;
    PT() { x = 0, y = 0; }
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &a) const { return PT(x + a.x,
         y + a.y); }
    PT operator - (const PT &a) const { return PT(x - a.x,
         y - a.y); }
    PT operator * (const double a) const { return PT(x * a,
         y * a); }
    friend PT operator * (const double &a, const PT &b) {
         return PT(a * b.x, a * b.y); }
    PT operator / (const double a) const { return PT(x / a,
         y / a); }
    bool operator == (PT a) const { return sign(a.x - x) ==
         0 && sign(a.y - y) == 0; }
    bool operator != (PT a) const { return !(*this == a); }
    bool operator < (PT a) const { return sign(a.x - x) ==
         0 ? y < a.y : x < a.x; }
    bool operator > (PT a) const { return sign(a.x - x) ==
         0 ? y > a.y : x > a.x; }
    double norm() { return sqrt(x * x + y * y); }
    double norm2() { return x * x + y * y; }
    PT perp() { return PT(-y, x); }
    double arg() { return atan2(y, x); }
    PT truncate(double r) { // returns a vector with norm r
         and having same direction
        double k = norm();
        if (!sign(k)) return *this;
        r /= k;
        return PT(x * r, y * r);
    }
};
inline double dot(PT a, PT b) { return a.x * b.x + a.y *
     b.y; }
inline double dist2(PT a, PT b) { return dot(a - b, a -
     b); }
inline double dist(PT a, PT b) { return sqrt(dot(a - b, a
     - b)); }
inline double cross(PT a, PT b) { return a.x * b.y - a.y *
     b.x; }
inline double cross2(PT a, PT b, PT c) { return cross(b -
     a, c - a); }
inline int orientation(PT a, PT b, PT c) { return
     sign(cross(b - a, c - a)); }
PT perp(PT a) { return PT(-a.y, a.x); }
PT rotateccw90(PT a) { return PT(-a.y, a.x); }
PT rotatecw90(PT a) { return PT(a.y, -a.x); }
PT rotateccw(PT a, double t) { return PT(a.x * cos(t) -
     a.y * sin(t), a.x * sin(t) + a.y * cos(t)); }
PT rotatecw(PT a, double t) { return PT(a.x * cos(t) + a.y
     * sin(t), -a.x * sin(t) + a.y * cos(t)); }
double SQ(double x) { return x * x; }
double rad_to_deg(double r) { return (r * 180.0 / PI); }
double deg_to_rad(double d) { return (d * PI / 180.0); }
double get_angle(PT a, PT b) {
    double costheta = dot(a, b) / a.norm() / b.norm();
    return acos(max((double)-1.0, min((double)1.0,
         costheta)));
}
bool is_point_in_angle(PT b, PT a, PT c, PT p) { // does
     point p lie in angle <bac
    assert(orientation(a, b, c) != 0);
    if (orientation(a, c, b) < 0) swap(b, c);
    return orientation(a, c, p) >= 0 && orientation(a, b,
         p) <= 0;
}
```

3.20 PointInsideHull

```
bool on_segment(const Point &A, const Point &B, const
     Point &C) { return cross(A, B, C) == 0 and dot(C, A,
     B) <= 0; }

bool check(vector<Point> &hull, Point &a) {
  int n = sz(hull);
  if (n == 1) return hull[0] == a;
  if (n == 2) return on_segment(hull[0], hull[1], a);
  if (cross(hull[0], hull[1], a) > 0) return 0;
  if (cross(hull[n - 1], hull[0], a) >= 0) return
       on_segment(hull[n - 1], hull[0], a);
  int l = 2, r = n - 1, ans = -1;
  while (l <= r) {
    int mid = (l + r) / 2;
    if (cross(hull[0], hull[mid], a) >= 0) {
      ans = mid;
      r = mid - 1;
    } else
      l = mid + 1;
  }
  debug(hull[0], hull[ans - 1], hull[ans], a, ans);
  return cross(hull[ans - 1], hull[ans], a) < 0 or
       on_segment(hull[ans - 1], hull[ans], a);
}
```

3.21 PointPolygonTangents

```
pair<PT, PT> convex_line_intersection(vector<PT> &p, PT a,
     PT b) {
    return {{0, 0}, {0, 0}};
}

pair<PT, int> point_poly_tangent(vector<PT> &p, PT Q, int
     dir, int l, int r) {
    while (r - l > 1) {
        int mid = (l + r) >> 1;
        bool pvs = orientation(Q, p[mid], p[mid - 1]) !=
             -dir;
        bool nxt = orientation(Q, p[mid], p[mid + 1]) !=
             -dir;
        if (pvs && nxt) return {p[mid], mid};
        if (!(pvs || nxt)) {
            auto p1 = point_poly_tangent(p, Q, dir, mid +
                 1, r);
            auto p2 = point_poly_tangent(p, Q, dir, l, mid
                 - 1);
            return orientation(Q, p1.first, p2.first) ==
                 dir ? p1 : p2;
        }
        if (!pvs) {
            if (orientation(Q, p[mid], p[l]) == dir) r =
                 mid - 1;
            else if (orientation(Q, p[l], p[r]) == dir) r =
                 mid - 1;
            else l = mid + 1;
        }
        if (!nxt) {
            if (orientation(Q, p[mid], p[l]) == dir) l =
                 mid + 1;
            else if (orientation(Q, p[l], p[r]) == dir) r =
                 mid - 1;
            else l = mid + 1;
        }
    }
    pair<PT, int> ret = {p[l], l};
    for (int i = l + 1; i <= r; i++) ret = orientation(Q,
         ret.first, p[i]) != dir ? make_pair(p[i], i) :
         ret;
    return ret;
}
// (cw, ccw) tangents from a point that is outside this
     convex polygon
// returns indexes of the points
pair<int, int> tangents_from_point_to_polygon(vector<PT>
     &p, PT Q){
    int cw = point_poly_tangent(p, Q, 1, 0, (int)p.size() -
         1).second;
    int ccw = point_poly_tangent(p, Q, -1, 0, (int)p.size()
         - 1).second;
    return make_pair(cw, ccw);
}
```

3.22 PolarSort

```
bool half(PT p) {
    return p.y > 0.0 || (p.y == 0.0 && p.x < 0.0);
}
void polar_sort(vector<PT> &v) { // sort points in
     counterclockwise
    sort(v.begin(), v.end(), [](PT a,PT b) {
        return make_tuple(half(a), 0.0, a.norm2()) <
             make_tuple(half(b), cross(a, b), b.norm2());
    });
}
void polar_sort(vector<PT> &v, PT o) { // sort points in
     counterclockwise with respect to point o
    sort(v.begin(), v.end(), [&](PT a,PT b) {
        return make_tuple(half(a - o), 0.0, (a -
             o).norm2()) < make_tuple(half(b - o), cross(a
```

```
            - o, b - o), (b - o).norm2());
    });
}
```

3.23 PolygonCircleIntersection

```
// intersection between a simple polygon and a circle
double polygon_circle_intersection(vector<PT> &v, PT p,
     double r) {
    int n = v.size();
    double ans = 0.00;
    PT org = {0, 0};
    for(int i = 0; i < n; i++) {
        int x = orientation(p, v[i], v[(i + 1) % n]);
        if(x == 0) continue;
        double area = triangle_circle_intersection(org, r,
             v[i] - p, v[(i + 1) % n] - p);
        if (x < 0) ans -= area;
        else ans += area;
    }
    return abs(ans);
}
```

3.24 PolygonCut

```
// returns a vector with the vertices of a polygon with
     everything
// to the left of the line going from a to b cut away.
vector<PT> cut(vector<PT> &p, PT a, PT b) {
    vector<PT> ans;
    int n = (int)p.size();
    for (int i = 0; i < n; i++) {
        double c1 = cross(b - a, p[i] - a);
        double c2 = cross(b - a, p[(i + 1) % n] - a);
        if (sign(c1) >= 0) ans.push_back(p[i]);
        if (sign(c1 * c2) < 0) {
            if (!is_parallel(p[i], p[(i + 1) % n], a, b)) {
                PT tmp; line_line_intersection(p[i], p[(i +
                     1) % n], a, b, tmp);
                ans.push_back(tmp);
            }
        }
    }
    return ans;
}
```

3.25 PolygonDiameter

```
// Maximum distance of 2 points
double diameter(vector<PT> &p) {
    int n = (int)p.size();
    if (n == 1) return 0;
    if (n == 2) return dist(p[0], p[1]);
    double ans = 0;
    int i = 0, j = 1;
    while (i < n) {
        while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n]
             - p[j]) >= 0) {
            ans = max(ans, dist2(p[i], p[j]));
            j = (j + 1) % n;
        }
        ans = max(ans, dist2(p[i], p[j]));
        i++;
    }
    return sqrt(ans);
}
```

3.26 PolygonDistances

```
// minimum distance from a point to a convex polygon
// it assumes point lie strictly outside the polygon
double dist_from_point_to_polygon(vector<PT> &p, PT z) {
    double ans = inf;
    int n = p.size();
    if (n <= 3) {
        for(int i = 0; i < n; i++) ans = min(ans,
             dist_from_point_to_seg(p[i], p[(i + 1) % n],
             z));
        return ans;
    }
    auto [r, l] = tangents_from_point_to_polygon(p, z);
    if(l > r) r += n;
    while (l < r) {
        int mid = (l + r) >> 1;
        double left = dist2(p[mid % n], z), right=
             dist2(p[(mid + 1) % n], z);
        ans = min({ans, left, right});
        if(left < right) r = mid;
        else l = mid + 1;
    }
    ans = sqrt(ans);
    ans = min(ans, dist_from_point_to_seg(p[l % n], p[(l +
         1) % n], z));
    ans = min(ans, dist_from_point_to_seg(p[l % n], p[(l -
         1 + n) % n], z));
    return ans;
}
// minimum distance from convex polygon p to line ab
// returns 0 is it intersects with the polygon
// top - upper right vertex
double dist_from_polygon_to_line(vector<PT> &p, PT a, PT
     b, int top) { //O(log n)
    PT orth = (b - a).perp();
    if (orientation(a, b, p[0]) > 0) orth = (a - b).perp();
    int id = extreme_vertex(p, orth, top);
    if (dot(p[id] - a, orth) > 0) return 0.0; //if orth and
         a are in the same half of the line, then poly and
         line intersects
    return dist_from_point_to_line(a, b, p[id]); //does not
         intersect
}
// minimum distance from a convex polygon to another
     convex polygon
// the polygon doesnot overlap or touch
// tested in https://toph.co/p/the-wall
double dist_from_polygon_to_polygon(vector<PT> &p1,
     vector<PT> &p2) { // O(n log n)
    double ans = inf;
    for (int i = 0; i < p1.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p2,
             p1[i]));
    }
    for (int i = 0; i < p2.size(); i++) {
        ans = min(ans, dist_from_point_to_polygon(p1,
             p2[i]));
    }
    return ans;
}
// maximum distance from a convex polygon to another
     convex polygon
double maximum_dist_from_polygon_to_polygon(vector<PT> &u,
     vector<PT> &v){ //O(n)
    int n = (int)u.size(), m = (int)v.size();
    double ans = 0;
    if (n < 3 || m < 3) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < m; j++) ans = max(ans,
                 dist2(u[i], v[j]));
        }
        return sqrt(ans);
    }
    if (u[0].x > v[0].x) swap(n, m), swap(u, v);
    int i = 0, j = 0, step = n + m + 10;
    while (j + 1 < m && v[j].x < v[j + 1].x) j++ ;
    while (step--) {
        if (cross(u[(i + 1)%n] - u[i], v[(j + 1)%m] - v[j])
             >= 0) j = (j + 1) % m;
        else i = (i + 1) % n;
        ans = max(ans, dist2(u[i], v[j]));
    }
    return sqrt(ans);
}
```

3.27 PolygonLineIntersection

```
// not necessarily convex, boundary is included in the
     intersection
// returns total intersected length
double polygon_line_intersection(vector<PT> p, PT a, PT b)
     {
    int n = p.size();
    p.push_back(p[0]);
    line l = line(a, b);
    double ans = 0.0;
    vector< pair<double, int> > vec;
    for (int i = 0; i < n; i++) {
        int s1 = sign(cross(b - a, p[i] - a));
        int s2 = sign(cross(b - a, p[i+1] - a));
        if (s1 == s2) continue;
        line t = line(p[i], p[i + 1]);
```

```
        PT inter = (t.v * l.c - l.v * t.c) / cross(l.v,
            t.v);
        double tmp = dot(inter, l.v);
        int f;
        if (s1 > s2) f = s1 && s2 ? 2 : 1;
        else f = s1 && s2 ? -2 : -1;
        vec.push_back(make_pair(tmp, f));
    }
    sort(vec.begin(), vec.end());
    for (int i = 0, j = 0; i + 1 < (int)vec.size(); i++){
        j += vec[i].second;
        if (j) ans += vec[i + 1].first - vec[i].first;
    }
    ans = ans / sqrt(dot(l.v, l.v));
    p.pop_back();
    return ans;
}
```

3.28 PolygonUnion

```
// calculates the area of the union of n polygons (not
    necessarily convex).
// the points within each polygon must be given in CCW
    order.
// complexity: O(N^2), where N is the total number of
    points
double rat(PT a, PT b, PT p) {
        return !sign(a.x - b.x) ? (p.y - a.y) / (b.y - a.y)
            : (p.x - a.x) / (b.x - a.x);
 };
double polygon_union(vector<vector<PT>> &p) {
    int n = p.size();
    double ans=0;
    for(int i = 0; i < n; ++i) {
        for (int v = 0; v < (int)p[i].size(); ++v) {
            PT a = p[i][v], b = p[i][(v + 1) % p[i].size()];
            vector<pair<double, int>> segs;
            segs.emplace_back(0, 0), segs.emplace_back(1,
                0);
            for(int j = 0; j < n; ++j) {
                if(i != j) {
                    for(size_t u = 0; u < p[j].size(); ++u) {
                        PT c = p[j][u], d = p[j][(u + 1) %
                            p[j].size()];
                        int sc = sign(cross(b - a, c - a)),
                            sd = sign(cross(b - a, d - a));
                        if(!sc && !sd) {
                            if(sign(dot(b - a, d - c)) > 0 &&
                                i > j) {
                                segs.emplace_back(rat(a, b,
                                    c), 1),
                                    segs.emplace_back(rat(a,
                                    b, d), -1);
                            }
                        }
                        else {
                            double sa = cross(d - c, a - c),
                                sb = cross(d - c, b - c);
                            if(sc >= 0 && sd < 0)
                                segs.emplace_back(sa / (sa -
                                sb), 1);
                            else if(sc < 0 && sd >= 0)
                                segs.emplace_back(sa / (sa -
                                sb), -1);
                        }
                    }
                }
            }
            sort(segs.begin(), segs.end());
            double pre = min(max(segs[0].first, 0.0), 1.0),
                now, sum = 0;
            int cnt = segs[0].second;
            for(int j = 1; j < segs.size(); ++j) {
                now = min(max(segs[j].first, 0.0), 1.0);
                if (!cnt) sum += now - pre;
                cnt += segs[j].second;
                pre = now;
            }
            ans += cross(a, b) * sum;
        }
    }
    return ans * 0.5;
}
```

3.29 PolygonWidth

```
// Maximum distance between 2 points IN the polygon
double width(vector<PT> &p) {
    int n = (int)p.size();
    if (n <= 2) return 0;
    double ans = inf;
    int i = 0, j = 1;
    while (i < n) {
        while (cross(p[(i + 1) % n] - p[i], p[(j + 1) % n]
            - p[j]) >= 0) j = (j + 1) % n;
        ans = min(ans, dist_from_point_to_line(p[i], p[(i +
            1) % n], p[j]));
        i++;
    }
    return ans;
}
```

3.30 Ray

```
// minimum distance from point c to ray (starting point a
    and direction vector b)
double dist_from_point_to_ray(PT a, PT b, PT c) {
    b = a + b;
    double r = dot(c - a, b - a);
    if (r < 0.0) return dist(c, a);
    return dist_from_point_to_line(a, b, c);
}
// starting point as and direction vector ad
bool ray_ray_intersection(PT as, PT ad, PT bs, PT bd) {
    double dx = bs.x - as.x, dy = bs.y - as.y;
    double det = bd.x * ad.y - bd.y * ad.x;
    if (fabs(det) < eps) return 0;
    double u = (dy * bd.x - dx * bd.y) / det;
    double v = (dy * ad.x - dx * ad.y) / det;
    if (sign(u) >= 0 && sign(v) >= 0) return 1;
    else return 0;
}
double ray_ray_distance(PT as, PT ad, PT bs, PT bd) {
    if (ray_ray_intersection(as, ad, bs, bd)) return 0.0;
    double ans = dist_from_point_to_ray(as, ad, bs);
    ans = min(ans, dist_from_point_to_ray(bs, bd, as));
    return ans;
}
```

3.31 Segment

```
// returns true if point p is on line segment ab
bool is_point_on_seg(PT a, PT b, PT p) {
    if (fabs(cross(p - b, a - b)) < eps) {
        if (p.x < min(a.x, b.x) || p.x > max(a.x, b.x))
            return false;
        if (p.y < min(a.y, b.y) || p.y > max(a.y, b.y))
            return false;
        return true;
    }
    return false;
}
// minimum distance point from point c to segment ab that
    lies on segment ab
PT project_from_point_to_seg(PT a, PT b, PT c) {
    double r = dist2(a, b);
    if (sign(r) == 0) return a;
    r = dot(c - a, b - a) / r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b - a) * r;
}
// minimum distance from point c to segment ab
double dist_from_point_to_seg(PT a, PT b, PT c) {
    return dist(c, project_from_point_to_seg(a, b, c));
}
// intersection point between segment ab and segment cd
    assuming unique intersection exists
bool seg_seg_intersection(PT a, PT b, PT c, PT d, PT &ans)
    {
    double oa = cross2(c, d, a), ob = cross2(c, d, b);
    double oc = cross2(a, b, c), od = cross2(a, b, d);
    if (oa * ob < 0 && oc * od < 0){
        ans = (a * ob - b * oa) / (ob - oa);
        return 1;
    }
    else return 0;
}
// intersection point between segment ab and segment cd
    assuming unique intersection may not exists
// se.size()==0 means no intersection
// se.size()==1 means one intersection
// se.size()==2 means range intersection
```

```
set<PT> seg_seg_intersection_inside(PT a, PT b, PT c, PT
     d) {
   PT ans;
   if (seg_seg_intersection(a, b, c, d, ans)) return {ans};
   set<PT> se;
   if (is_point_on_seg(c, d, a)) se.insert(a);
   if (is_point_on_seg(c, d, b)) se.insert(b);
   if (is_point_on_seg(a, b, c)) se.insert(c);
   if (is_point_on_seg(a, b, d)) se.insert(d);
   return se;
}
// intersection between segment ab and line cd
// 0 if do not intersect, 1 if proper intersect, 2 if
     segment intersect
int seg_line_relation(PT a, PT b, PT c, PT d) {
   double p = cross2(c, d, a);
   double q = cross2(c, d, b);
   if (sign(p) == 0 && sign(q) == 0) return 2;
   else if (p * q < 0) return 1;
   else return 0;
}
// intersection between segament ab and line cd assuming
     unique intersection exists
bool seg_line_intersection(PT a, PT b, PT c, PT d, PT
     &ans) {
   bool k = seg_line_relation(a, b, c, d);
   assert(k != 2);
   if (k) line_line_intersection(a, b, c, d, ans);
   return k;
}
// minimum distance from segment ab to segment cd
double dist_from_seg_to_seg(PT a, PT b, PT c, PT d) {
   PT dummy;
   if (seg_seg_intersection(a, b, c, d, dummy)) return 0.0;
   else return min({dist_from_point_to_seg(a, b, c),
        dist_from_point_to_seg(a, b, d),
      dist_from_point_to_seg(c, d, a),
           dist_from_point_to_seg(c, d, b)});
}
```

3.32 SmallestEnclosingCircle

```
double eps = 1e-9;
using Point = complex<double>;
struct Circle{ Point p; double r; };
double dist(Point p, Point q){ return abs(p-q); }
double area2(Point p, Point q){ return (conj(p)*q).imag();}
bool in(const Circle& c, Point p){ return dist(c.p, p) <
     c.r + eps; }
Circle INVAL = Circle{Point(0, 0), -1};
Circle mCC(Point a, Point b, Point c){
      b -= a; c -= a;
      double d = 2*(conj(b)*c).imag(); if(abs(d)<eps)
           return INVAL;
      Point ans = (c*norm(b) - b*norm(c)) * Point(0, -1)
           / d;
      return Circle{a + ans, abs(ans)};
}
Circle solve(vector<Point> p) {
      mt19937 gen(0x94949); shuffle(p.begin(), p.end(),
           gen);
      Circle c = INVAL;
      for(int i=0; i<p.size(); ++i) if(c.r<0 ||!in(c,
           p[i])){
            c = Circle{p[i], 0};
            for(int j=0; j<=i; ++j) if(!in(c, p[j])){
                  Circle ans{(p[i]+p[j])*0.5,
                        dist(p[i], p[j])*0.5};
                  if(c.r == 0) {c = ans; continue;}
                  Circle l, r; l = r = INVAL;
                  Point pq = p[j]-p[i];
                  for(int k=0; k<=j; ++k) if(!in(ans,
                        p[k])) {
                        double a2 = area2(pq,
                              p[k]-p[i]);
                        Circle c = mCC(p[i], p[j],
                              p[k]);
                        if(c.r<0) continue;
                        else if(a2 > 0 &&
                              (l.r<0||area2(pq,
                              c.p-p[i]) > area2(pq,
                              l.p-p[i]))) l = c;
                        else if(a2 < 0 &&
                              (r.r<0||area2(pq,
                              c.p-p[i]) < area2(pq,
                              r.p-p[i]))) r = c;
                  }
                  if(l.r<0&&r.r<0) c = ans;
                  else if(l.r<0) c = r;
                  else if(r.r<0) c = l;
                  else c = l.r<=r.r?l:r;
            }
      }
      return c;
}
```

3.33 TriangleCircleIntersection

```
// system should be translated from circle center
double triangle_circle_intersection(PT c, double r, PT a,
     PT b) {
   double sd1 = dist2(c, a), sd2 = dist2(c, b);
   if(sd1 > sd2) swap(a, b), swap(sd1, sd2);
   double sd = dist2(a, b);
   double d1 = sqrtl(sd1), d2 = sqrtl(sd2), d = sqrt(sd);
   double x = abs(sd2 - sd - sd1) / (2 * d);
   double h = sqrtl(sd1 - x * x);
   if(r >= d2) return h * d / 2;
   double area = 0;
   if(sd + sd1 < sd2) {
      if(r < d1) area = r * r * (acos(h / d2) - acos(h /
           d1)) / 2;
      else {
         area = r * r * ( acos(h / d2) - acos(h / r)) /
              2;
         double y = sqrtl(r * r - h * h);
         area += h * (y - x) / 2;
      }
   }
   else {
      if(r < h) area = r * r * (acos(h / d2) + acos(h /
           d1)) / 2;
      else {
         area += r * r * (acos(h / d2) - acos(h / r)) /
              2;
         double y = sqrtl(r * r - h * h);
         area += h * y / 2;
         if(r < d1) {
            area += r * r * (acos(h / d1) - acos(h / r))
                 / 2;
            area += h * y / 2;
         }
         else area += h * x / 2;
      }
   }
   return area;
}
```

3.34 Utilities

```
double perimeter(vector<PT> &p) {
   double ans=0; int n = p.size();
   for (int i = 0; i < n; i++) ans += dist(p[i], p[(i + 1)
        % n]);
   return ans;
}
double area(vector<PT> &p) {
   double ans = 0; int n = p.size();
   for (int i = 0; i < n; i++) ans += cross(p[i], p[(i +
        1) % n]);
   return fabs(ans) * 0.5;
}
double area_of_triangle(PT a, PT b, PT c) {
   return fabs(cross(b - a, c - a) * 0.5);
}
// 0 if cw, 1 if ccw
bool get_direction(vector<PT> &p) {
   double ans = 0; int n = p.size();
   for (int i = 0; i < n; i++) ans += cross(p[i], p[(i +
        1) % n]);
   if (sign(ans) > 0) return 1;
   return 0;
}
// find a point from a through b with distance d
PT point_along_line(PT a, PT b, double d) {
   assert(a != b);
   return a + (((b - a) / (b - a).norm()) * d);
}
// projection point c onto line through a and b assuming a
     != b
PT project_from_point_to_line(PT a, PT b, PT c) {
   return a + (b - a) * dot(c - a, b - a) / (b -
        a).norm2();
}
// reflection point c onto line through a and b assuming a
     != b
PT reflection_from_point_to_line(PT a, PT b, PT c) {
```

```
    PT p = project_from_point_to_line(a,b,c);
    return p + p - c;
}
// minimum distance from point c to line through a and b
double dist_from_point_to_line(PT a, PT b, PT c) {
    return fabs(cross(b - a, c - a) / (b - a).norm());
}
// 0 if not parallel, 1 if parallel, 2 if collinear
int is_parallel(PT a, PT b, PT c, PT d) {
    double k = fabs(cross(b - a, d - c));
    if (k < eps){
        if (fabs(cross(a - b, a - c)) < eps && fabs(cross(c
             - d, c - a)) < eps) return 2;
        else return 1;
    }
    else return 0;
}
// check if two lines are same
bool are_lines_same(PT a, PT b, PT c, PT d) {
    if (fabs(cross(a - c, c - d)) < eps && fabs(cross(b -
         c, c - d)) < eps) return true;
    return false;
}

// 1 if point is ccw to the line, 2 if point is cw to the
     line, 3 if point is on the line
int point_line_relation(PT a, PT b, PT p) {
    int c = sign(cross(p - a, b - a));
    if (c < 0) return 1;
    if (c > 0) return 2;
    return 3;
}
```

4 Graph

4.1 2pac

```
struct TwoSatSolver {
  int n_vars;
  int n_vertices;
  vector<vector<int>> adj, adj_t;
  vector<bool> used;
  vector<int> order, comp;
  vector<bool> assignment;
  TwoSatSolver(int _n_vars)
      : n_vars(_n_vars),
        n_vertices(2 * n_vars),
        adj(n_vertices),
        adj_t(n_vertices),
        used(n_vertices),
        order(),
        comp(n_vertices, -1),
        assignment(n_vars) {
    order.reserve(n_vertices);
  }
  void dfs1(int v) {
    used[v] = true;
    for (int u : adj[v]) {
      if (!used[u]) dfs1(u);
    }
    order.push_back(v);
  }
  void dfs2(int v, int cl) {
    comp[v] = cl;
    for (int u : adj_t[v]) {
      if (comp[u] == -1) dfs2(u, cl);
    }
  }
  bool solve_2SAT() {
    order.clear();
    used.assign(n_vertices, false);
    for (int i = 0; i < n_vertices; ++i) {
      if (!used[i]) dfs1(i);
    }

    comp.assign(n_vertices, -1);
    for (int i = 0, j = 0; i < n_vertices; ++i) {
      int v = order[n_vertices - i - 1];
      if (comp[v] == -1) dfs2(v, j++);
    }

    assignment.assign(n_vars, false);
    for (int i = 0; i < n_vertices; i += 2) {
      if (comp[i] == comp[i + 1]) return false;
      assignment[i / 2] = comp[i] > comp[i + 1];
    }
    return true;
  }
  void add_disjunction(int a, bool na, int b, bool nb) {
    // na and nb signify whether a and b are to be negated
    a = 2 * a ^ na;
    b = 2 * b ^ nb;
    int neg_a = a ^ 1;
    int neg_b = b ^ 1;
    adj[neg_a].push_back(b);
    adj[neg_b].push_back(a);
    adj_t[b].push_back(neg_a);
    adj_t[a].push_back(neg_b);
  }
  static void example_usage() {
    TwoSatSolver solver(3);                  // a, b, c
    solver.add_disjunction(0, false, 1, true); //  a  v
         not b
    solver.add_disjunction(0, true, 1, true); // not a v
         not b
    solver.add_disjunction(1, false, 2, false); // b  v
         c
    solver.add_disjunction(0, false, 0, false); // a  v
         a
    assert(solver.solve_2SAT() == true);
    auto expected = vector<bool>(True, False, True);
    assert(solver.assignment == expected);
  }
};
```

4.2 BiconnectedComponents

```
struct BiconnectedComponent {
  vector<int> low, num, s;
  vector<vector<int> > components;
  int counter;

  BiconnectedComponent() : low(n, -1), num(n, -1),
       counter(0) {
    for (int i = 0; i < n; i++)
      if (num[i] < 0) dfs(i, 1);
  }

  void dfs(int x, int isRoot) {
    low[x] = num[x] = ++counter;
    if (g[x].empty()) {
      components.push_back(vector<int>(1, x));
      return;
    }
    s.push_back(x);

    for (int i = 0; i < (int)g[x].size(); i++) {
      int y = g[x][i];
      if (num[y] > -1)
        low[x] = min(low[x], num[y]);
      else {
        dfs(y, 0);
        low[x] = min(low[x], low[y]);

        if (isRoot || low[y] >= num[x]) {
          components.push_back(vector<int>(1, x));
          while (1) {
            int u = s.back();
            s.pop_back();
            components.back().push_back(u);
            if (u == y) break;
          }
        }
      }
    }
  }
};
```

4.3 Dinic

```
const ll INF = 1e18;
struct Dinic {
  const static bool SCALING = false; // scaling = EV
       log(max C) with larger constant
  ll lim = 1;
  struct Edge {
    int u, v;
    ll cap, flow;
  };
  int n, s, t;
  vector<int> level, ptr;
  vector<Edge> e;
  vector<vector<int>> g;
  Dinic(int _n) : n(_n), level(_n), ptr(_n), g(_n) {
    e.clear();
    for (int i = 0; i < n; ++i) {
```

```
      ptr[i] = 0;
      g[i].clear();
    }
  }
  void add_edge(int u, int v, ll c) {
    debug(u, v, c);
    g[u].push_back(sz(e));
    e.push_back({u, v, c, 0});
    g[v].push_back(sz(e));
    e.push_back({v, u, 0, 0});
  }
  ll get_max_flow(int _s, int _t) {
    s = _s, t = _t;
    ll flow = 0;
    for (lim = SCALING ? (1 << 30) : 1; lim > 0; lim >>= 1)
        {
      while (1) {
        if (!bfs()) break;
        fill(all(ptr), 0);
        while (ll pushed = dfs(s, INF)) flow += pushed;
      }
    }
    return flow;
  }
 private:
  bool bfs() {
    queue<int> q;
    q.push(s);
    fill(all(level), -1);
    level[s] = 0;
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int id : g[u]) {
        if (e[id].cap - e[id].flow < 1) continue;
        if (level[e[id].v] != -1) continue;
        if (SCALING and e[id].cap - e[id].flow < lim)
             continue;
        level[e[id].v] = level[u] + 1;
        q.push(e[id].v);
      }
    }
    return level[t] != -1;
  }
  ll dfs(int u, ll flow) {
    if (!flow) return 0;
    if (u == t) return flow;
    for (; ptr[u] < sz(g[u]); ++ptr[u]) {
      int id = g[u][ptr[u]], to = e[id].v;
      if (level[to] != level[u] + 1) continue;
      ll pushed = dfs(to, min(flow, e[id].cap -
           e[id].flow));
      if (pushed) {
        e[id].flow += pushed;
        e[id ^ 1].flow -= pushed;
        return pushed;
      }
    }
    return 0;
  }
};
```

4.4 EulerPath

```
struct EulerUndirected {
  EulerUndirected(int _n) : n(_n), m(0), adj(_n), deg(_n,
       0) {}
  void add_edge(int u, int v) {
    adj[u].push_front(Edge(v));
    auto it1 = adj[u].begin();
    adj[v].push_front(Edge(u));
    auto it2 = adj[v].begin();

    it1->rev = it2;
    it2->rev = it1;

    ++deg[u];
    ++deg[v];
    ++m;
  }
  std::pair<bool, std::vector<int>> solve() {
    int cntOdd = 0;
    int start = -1;
    for (int i = 0; i < n; i++) {
      if (deg[i] % 2) {
        ++cntOdd;
        if (cntOdd > 2) return {false, {}};

        if (start < 0) start = i;
      }
    }
    // no odd vertex -> start from any vertex with positive
         degree
    if (start < 0) {
      for (int i = 0; i < n; i++) {
        if (deg[i]) {
          start = i;
          break;
        }
      }
      if (start < 0) {
        // no edge -> empty path
        return {true, {}};
      }
    }

    std::vector<int> path;
    find_path(start, path);

    if (m + 1 != static_cast<int>(path.size())) {
      return {false, {}};
    }

    return {true, path};
  }
  struct Edge {
    int to;
    std::list<Edge>::iterator rev;

    Edge(int _to) : to(_to) {}
  };
  // private:
  int n, m;
  std::vector<std::list<Edge>> adj;
  std::vector<int> deg;

  void find_path(int v, std::vector<int>& path) {
    while (adj[v].size() > 0) {
      int next = adj[v].front().to;
      adj[next].erase(adj[v].front().rev);
      adj[v].pop_front();
      find_path(next, path);
    }
    path.push_back(v);
  }
};
```

4.5 EulerPathDirected

```
struct EulerDirected {
  EulerDirected(int _n) : n(_n), adj(n), in_deg(n, 0),
       out_deg(n, 0) {}
  void add_edge(int u, int v) { // directed edge
    assert(0 <= u && u < n);
    assert(0 <= v && v < n);
    adj[u].push_front(v);
    in_deg[v]++;
    out_deg[u]++;
  }
  std::pair<bool, std::vector<int>> solve() {
    int start = -1, last = -1;
    for (int i = 0; i < n; i++) {
      // for all u, |in_deg(u) - out_deg(u)| <= 1
      if (std::abs(in_deg[i] - out_deg[i]) > 1) return
           {false, {}};

      if (out_deg[i] > in_deg[i]) {
        // At most 1 vertex with out_deg[u] - in_deg[u] = 1
             (start vertex)
        if (start >= 0) return {false, {}};
        start = i;
      }

      if (in_deg[i] > out_deg[i]) {
        // At most 1 vertex with in_deg[u] - out_deg[u] = 1
             (last vertex)
        if (last >= 0) return {false, {}};
        last = i;
      }
    }

    // can start at any vertex with degree > 0
    if (start < 0) {
      for (int i = 0; i < n; i++) {
        if (in_deg[i]) {
          start = i;
          break;
        }
      }
      // no start vertex --> all vertices have degree == 0
      if (start < 0) return {true, {}};
    }
```

```
    std::vector<int> path;
    find_path(start, path);
    std::reverse(path.begin(), path.end());

    // check that we visited all vertices with degree > 0
    std::vector<bool> visited(n, false);
    for (int u : path) visited[u] = true;

    for (int u = 0; u < n; u++) {
      if (in_deg[u] && !visited[u]) {
        return {false, {}};
      }
    }

    return {true, path};
  }
 private:
  int n;
  std::vector<std::list<int>> adj;
  std::vector<int> in_deg, out_deg;

  void find_path(int v, std::vector<int>& path) {
    while (adj[v].size() > 0) {
      int next = adj[v].front();
      adj[v].pop_front();
      find_path(next, path);
    }
    path.push_back(v);
  }
};
```

4.6 GeneralMatching

```
const int MAXN = 2020 + 1;
struct GM { // 1-based Vertex index
  int vis[MAXN], par[MAXN], orig[MAXN], match[MAXN],
      aux[MAXN], t, N;
  vector<int> conn[MAXN];
  queue<int> Q;
  void addEdge(int u, int v) {
    conn[u].push_back(v);
    conn[v].push_back(u);
  }
  void init(int n) {
    N = n;
    t = 0;
    for (int i = 0; i <= n; ++i) {
      conn[i].clear();
      match[i] = aux[i] = par[i] = 0;
    }
  }
  void augment(int u, int v) {
    int pv = v, nv;
    do {
      pv = par[v];
      nv = match[pv];
      match[v] = pv;
      match[pv] = v;
      v = nv;
    } while (u != pv);
  }
  int lca(int v, int w) {
    ++t;
    while (true) {
      if (v) {
        if (aux[v] == t) return v;
        aux[v] = t;
        v = orig[par[match[v]]];
      }
      swap(v, w);
    }
  }
  void blossom(int v, int w, int a) {
    while (orig[v] != a) {
      par[v] = w;
      w = match[v];
      if (vis[w] == 1) Q.push(w), vis[w] = 0;
      orig[v] = orig[w] = a;
      v = par[w];
    }
  }
  bool bfs(int u) {
    fill(vis + 1, vis + 1 + N, -1);
    iota(orig + 1, orig + N + 1, 1);
    Q = queue<int>();
    Q.push(u);
    vis[u] = 0;
    while (!Q.empty()) {
      int v = Q.front();
      Q.pop();
      for (int x : conn[v]) {
        if (vis[x] == -1) {
          par[x] = v;
          vis[x] = 1;
          if (!match[x]) return augment(u, x), true;
          Q.push(match[x]);
          vis[match[x]] = 0;
        } else if (vis[x] == 0 && orig[v] != orig[x]) {
          int a = lca(orig[v], orig[x]);
          blossom(x, v, a);
          blossom(v, x, a);
        }
      }
    }
    return false;
  }
  int Match() {
    int ans = 0;
    // find random matching (not necessary, constant
          improvement)
    vector<int> V(N - 1);
    iota(V.begin(), V.end(), 1);
    shuffle(V.begin(), V.end(), mt19937(0x94949));
    for (auto x : V)
      if (!match[x]) {
        for (auto y : conn[x])
          if (!match[y]) {
            match[x] = y, match[y] = x;
            ++ans;
            break;
          }
      }
    for (int i = 1; i <= N; ++i)
      if (!match[i] && bfs(i)) ++ans;
    return ans;
  }
};
```

4.7 GlobalMinCut

```
pair<int, vi> GetMinCut(vector<vi>& weights) {
  int N = sz(weights);
  vi used(N), cut, best_cut;
  int best_weight = -1;
  for (int phase = N - 1; phase >= 0; phase--) {
    vi w = weights[0], added = used;
    int prev, k = 0;
    rep(i, 0, phase) {
      prev = k;
      k = -1;
      rep(j, 1, N) if (!added[j] && (k == -1 || w[j] >
            w[k])) k = j;
      if (i == phase - 1) {
        rep(j, 0, N) weights[prev][j] += weights[k][j];
        rep(j, 0, N) weights[j][prev] = weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        rep(j, 0, N) w[j] += weights[k][j];
        added[k] = true;
      }
    }
  }
  return {best_weight, best_cut};
}
```

4.8 HopcroftKarp

```
struct maximum_bipartite_matching {
  int n, m;
  vector<int> matchX, matchY, dist;
  vector<vector<int>> g;
  int matched;
  maximum_bipartite_matching(int _n, int _m)
      : n(_n), m(_m), matchX(n + 1, -1), matchY(m + 1, -1),
          dist(n + 1, -1), g(n + 1), matched(0) {}

  void add_edge(int u, int v) { g[u].push_back(v); }
  void bfs() {
    queue<int> q;
    for (int i = 0; i < n; ++i) {
      if (matchX[i] == -1)
        q.push(i), dist[i] = 0;
```

```
      else
        dist[i] = -1;
    }
    while (!q.empty()) {
      int u = q.front();
      q.pop();
      for (int v : g[u]) {
        if (matchY[v] != -1 and dist[matchY[v]] == -1) {
          dist[matchY[v]] = dist[u] + 1;
          q.push(matchY[v]);
        }
      }
    }
  }
  bool dfs(int u) {
    for (int v : g[u]) {
      if (matchY[v] == -1) {
        matchX[u] = v, matchY[v] = u;
        return 1;
      }
    }
    for (int v : g[u]) {
      if (dist[matchY[v]] == dist[u] + 1 and
            dfs(matchY[v])) {
        matchX[u] = v, matchY[v] = u;
        return 1;
      }
    }
    return 0;
  }

  void match() {
    while (1) {
      bfs();
      int augment = 0;
      for (int i = 0; i < n; ++i)
        if (matchX[i] == -1) augment += dfs(i);
      if (!augment) break;
      matched += augment;
    }
  }
  vector<pii> get_edges() {
    vector<pii> res;
    for (int i = 0; i < n; ++i)
      if (matchX[i] != -1) res.push_back({i, matchX[i]});
    return res;
  }
};
```

4.9 KhopCau

```
#include <bits/stdc++.h>

using namespace std;

const int maxN = 10010;

int n, m;
bool joint[maxN];
int timeDfs = 0, bridge = 0;
int low[maxN], num[maxN];
vector <int> g[maxN];

void dfs(int u, int pre) {
  int child = 0; // So luong con truc tiep cua dinh u
      trong cy DFS
  num[u] = low[u] = ++timeDfs;
  for (int v : g[u]) {
    if (v == pre) continue;
    if (!num[v]) {
      dfs(v, u);
      low[u] = min(low[u], low[v]);
      if (low[v] == num[v]) bridge++;
      child++;
      if (u == pre) { // Neu u l dinh goc cua cy DFS
        if (child > 1) joint[u] = true;
      }
      else if (low[v] >= num[u]) joint[u] = true;
    }
    else low[u] = min(low[u], num[v]);
  }
}

int main() {
  cin >> n >> m;
  for (int i = 1; i <= m; i++) {
    int u, v;
    cin >> u >> v;
    g[u].push_back(v);
    g[v].push_back(u);
  }
  for (int i = 1; i <= n; i++)
    if (!num[i]) dfs(i, i);

  int cntJoint = 0;
  for (int i = 1; i <= n; i++) cntJoint += joint[i];

  cout << cntJoint << ' ' << bridge;
}
```

4.10 MCMF

```
#include <bits/extc++.h>
const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;
struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
  VL dist, pi;
  vector<pii> par;
  MCMF(int N) : N(N), ed(N), red(N), cap(N, VL(N)),
        flow(cap), cost(cap), seen(N), dist(N), pi(N),
        par(N) {}
  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }
  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0;
    ll di;
    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});
    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end())
          its[i] = q.push({-dist[i], i});
        else
          q.modify(its[i], {-dist[i], i});
      }
    };
    while (!q.empty()) {
      s = q.top().second;
      q.pop();
      seen[s] = 1;
      di = dist[s] + pi[s];
      trav(i, ed[s]) if (!seen[i]) relax(i, cap[s][i] -
            flow[s][i], cost[s][i], 1);
      trav(i, red[s]) if (!seen[i]) relax(i, flow[i][s],
            -cost[i][s], 0);
    }
    rep(i, 0, N) pi[i] = min(pi[i] + dist[i], INF);
  }
  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p, r, x = t; tie(p, r) = par[x], x != s; x =
            p) fl = min(fl, r ? cap[p][x] - flow[p][x] :
            flow[x][p]);
      totflow += fl;
      for (int p, r, x = t; tie(p, r) = par[x], x != s; x =
            p)
        if (r)
          flow[p][x] += fl;
        else
          flow[x][p] -= fl;
    }
    rep(i, 0, N) rep(j, 0, N) totcost += cost[i][j] *
          flow[i][j];
    return {totflow, totcost};
  }
  // I f some costs can be negative , call this before
        maxflow:
  void setpi(int s) { // (otherwise , leave this out)
    fill(all(pi), INF);
    pi[s] = 0;
    int it = N, ch = 1;
    ll v;
    while (ch-- && it--) rep(i, 0, N) if (pi[i] != INF)
          trav(to, ed[i]) if (cap[i][to]) if ((v = pi[i] +
```

```
        cost[i][to]) < pi[to]) pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
};
```

4.11 spfa

```
#include<bits/stdc++.h>
typedef pair<int, int> ii;
const int MaxN = 1e5 + 5;
const int Inf = 1e9;
vector<vector<ii>> AdjList;
int Dist[MaxN];
int Cnt[MaxN];
bool inqueue[MaxN];
int S;
int N;
queue<int> q;

bool spfa() {
    for(int i = 1 ; i <= N ; i++) {
        Dist[i] = Inf;
        Cnt[i] = 0;
        inqueue[i] = false;
    }
    Dist[S] = 0;
    q.push(S);
    inqueue[S] = true;
    while(!q.empty()) {
        int u = q.front();
        q.pop();
        inqueue[u] = false;

        for (ii tmp: AdjList[u]) {
            int v = tmp.first;
            int w = tmp.second;

            if (Dist[u] + w < Dist[v]) {
                Dist[v] = Dist[u] + w;
                if (!inqueue[v]) {
                    q.push(v);
                    inqueue[v] = true;
                    Cnt[v]++;
                    if (Cnt[v] > N)
                        return false;
                }
            }
        }
    }
    return true;
}
```

4.12 StronglyConnected

```
struct DirectedDfs {
  vector<vector<int>> g;
  int n;
  vector<int> num, low, current, S;
  int counter;
  vector<int> comp_ids;
  vector<vector<int>> scc;

  DirectedDfs(const vector<vector<int>>& _g)
      : g(_g),
        n(g.size()),
        num(n, -1),
        low(n, 0),
        current(n, 0),
        counter(0),
        comp_ids(n, -1) {
    for (int i = 0; i < n; i++) {
      if (num[i] == -1) dfs(i);
    }
  }

  void dfs(int u) {
    low[u] = num[u] = counter++;
    S.push_back(u);
    current[u] = 1;
    for (auto v : g[u]) {
      if (num[v] == -1) dfs(v);
      if (current[v]) low[u] = min(low[u], low[v]);
    }
    if (low[u] == num[u]) {
      scc.push_back(vector<int>());
      while (1) {
        int v = S.back();
        S.pop_back();
        current[v] = 0;
        scc.back().push_back(v);
        comp_ids[v] = ((int)scc.size()) - 1;
        if (u == v) break;
      }
    }
  }

  // build DAG of strongly connected components
  // Returns: adjacency list of DAG
  std::vector<std::vector<int>> build_scc_dag() {
    std::vector<std::vector<int>> dag(scc.size());
    for (int u = 0; u < n; u++) {
      int x = comp_ids[u];
      for (int v : g[u]) {
        int y = comp_ids[v];
        if (x != y) {
          dag[x].push_back(y);
        }
      }
    }
    return dag;
  }
};
```

4.13 TopoSort

```
std::pair<bool, std::vector<int>> topo_sort(const
    std::vector<std::vector<int>>& g) {
  int n = g.size();
  // init in_deg
  std::vector<int> in_deg(n, 0);
  for (int u = 0; u < n; u++) {
    for (int v : g[u]) {
      in_deg[v]++;
    }
  }

  // find topo order
  std::vector<int> res;
  std::queue<int> qu;
  for (int u = 0; u < n; u++) {
    if (in_deg[u] == 0) {
      qu.push(u);
    }
  }

  while (!qu.empty()) {
    int u = qu.front();
    qu.pop();
    res.push_back(u);
    for (int v : g[u]) {
      in_deg[v]--;
      if (in_deg[v] == 0) {
        qu.push(v);
      }
    }
  }

  if ((int)res.size() < n) {
    return {false, {}};
  }
  return {true, res};
}
```

5 Math

5.1 Euclid

```
// x, y such that ax + by = gcd(a, b)
ll gcd(ll a, ll b) { return __gcd(a, b); }
ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) {
    ll d = euclid(b, a % b, y, x);
    return y -= a / b * x, d;
  }
  return x = 1, y = 0, a;
}
```

5.2 Factorization

```
inline long long qpow(long long a, int b) {
```

```
  long long ans = 1;
  while (b) {
    if (b & 1) ans = ans * a % mod;
    a = a * a % mod;
    b >>= 1;
  }
  return ans;
}
inline long long rv(int x) { return qpow(x, mod - 2) %
     mod; }
bool is_prime(long long n) {
  if (n <= 1) return false;
  for (int a : {2, 3, 5, 13, 19, 73, 193, 407521,
       299210837}) {
    if (n == a) return true;
    if (n % a == 0) return false;
  }
  long long d = n - 1;
  while (!(d & 1)) d >>= 1;
  for (int a : {2, 325, 9375, 28178, 450775, 9780504,
       1795265022}) {
    long long t = d, y = ipow(a, t, n);
    while (t != n - 1 && y != 1 && y != n - 1) y = mul(y,
         y, n), t <<= 1;
    if (y != n - 1 && !(t & 1)) return false;
  }
  return true;
}

long long pollard(long n) {
  auto f = [n](long x) { return mul(x, x, n) + 1; };
  long long x = 0, y = 0, t = 0, prd = 2, i = 1, q;
  while (t++ % 40 || gcd(prd, n) == 1) {
    if (x == y) x = ++i, y = f(x);
    if ((q = mul(prd, max(x, y) - min(x, y), n))) prd = q;
    x = f(x), y = f(f(y));
  }
  return gcd(prd, n);
}
vector<long long> factor(long n) {
  if (n == 1) return {};
  if (is_prime(n)) return {n};
  long x = pollard(n);
  auto l = factor(x), r = factor(n / x);
  l.insert(l.end(), r.begin(), r.end());
  return l;
}
```

5.3 FastSubsetTransform

```
// fast and/or/xor convolution
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j, i, i +
         step) {
        int &u = a[j], &v = a[j + step];
        tie(u, v) = inv ? pii(v - u, u) : pii(v, u + v); //
             AND
        inv ? pii(v, u - v) : pii(u + v, u);          // OR
        pii(u + v, u - v);                            // XOR
      }
  }
  if (inv)
    for (int& x : a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0);
  FST(b, 0);
  rep(i, 0, sz(a)) a[i] *= b[i];
  FST(a, 1);
  return a;
}
```

5.4 FFT

```
using ld = double;
// Can use std::complex<ld> instead to make code shorter
     (but it will be slightly slower)
struct Complex {
  ld x[2];

  Complex() { x[0] = x[1] = 0.0; }
  Complex(ld a) { x[0] = a; }
  Complex(ld a, ld b) {
    x[0] = a;
    x[1] = b;
  }
  Complex(const std::complex<ld>& c) {
    x[0] = c.real();
    x[1] = c.imag();
  }

  Complex conj() const { return Complex(x[0], -x[1]); }

  Complex operator+(const Complex& c) const {
    return Complex{
        x[0] + c.x[0],
        x[1] + c.x[1],
    };
  }
  Complex operator-(const Complex& c) const {
    return Complex{
        x[0] - c.x[0],
        x[1] - c.x[1],
    };
  }
  Complex operator*(const Complex& c) const { return
       Complex(x[0] * c.x[0] - x[1] * c.x[1], x[0] *
       c.x[1] + x[1] * c.x[0]); }

  Complex& operator+=(const Complex& c) { return *this =
       *this + c; }
  Complex& operator-=(const Complex& c) { return *this =
       *this - c; }
  Complex& operator*=(const Complex& c) { return *this =
       *this * c; }
};
void fft(vector<Complex>& a) {
  int n = a.size();
  int L = 31 - __builtin_clz(n);
  static vector<Complex> R(2, 1);
  static vector<Complex> rt(2, 1);
  for (static int k = 2; k < n; k *= 2) {
    R.resize(n);
    rt.resize(n);
    auto x = Complex(polar(ld(1.0), acos(ld(-1.0)) / k));
    for (int i = k; i < 2 * k; ++i) {
      rt[i] = R[i] = i & 1 ? R[i / 2] * x : R[i / 2];
    }
  }
  vector<int> rev(n);
  for (int i = 0; i < n; ++i) rev[i] = (rev[i / 2] | (i &
       1) << L) / 2;
  for (int i = 0; i < n; ++i)
    if (i < rev[i]) swap(a[i], a[rev[i]]);

  for (int k = 1; k < n; k *= 2) {
    for (int i = 0; i < n; i += 2 * k) {
      for (int j = 0; j < k; ++j) {
        auto x = (ld*)&rt[j + k].x, y = (ld*)&a[i + j +
             k].x;
        Complex z(x[0] * y[0] - x[1] * y[1], x[0] * y[1] +
             x[1] * y[0]);
        a[i + j + k] = a[i + j] - z;
        a[i + j] += z;
      }
    }
  }
}
vector<ld> multiply(const vector<ld>& a, const vector<ld>&
     b) {
  if (a.empty() || b.empty()) return {};
  vector<ld> res(a.size() + b.size() - 1);
  int L = 32 - __builtin_clz(res.size()), n = 1 << L;
  vector<Complex> in(n), out(n);

  for (size_t i = 0; i < a.size(); ++i) in[i].x[0] = a[i];
  for (size_t i = 0; i < b.size(); ++i) in[i].x[1] = b[i];

  fft(in);
  for (Complex& x : in) x *= x;

  for (int i = 0; i < n; ++i) out[i] = in[-i & (n - 1)] -
       in[i].conj();
  fft(out);

  for (size_t i = 0; i < res.size(); ++i) res[i] =
       out[i].x[1] / (4 * n);
  return res;
}
long long my_round(ld x) {
  if (x < 0) return -my_round(-x);
  return (long long)(x + 1e-2);
}
vector<long long> multiply(const vector<int>& a, const
     vector<int>& b) {
  vector<ld> ad(a.begin(), a.end());
  vector<ld> bd(b.begin(), b.end());
  auto rd = multiply(ad, bd);
  vector<long long> res(rd.size());
```

```
  for (int i = 0; i < (int)res.size(); ++i) {
    res[i] = my_round(rd[i]);
  }
  return res;
}
```

5.5 Interpolate

```
const int mod = 1e9 + 7;
const int N = 1e6 + 6;

long long inv[N], po[N], pre[N], suf[N], dakdak[N];
long long ans, num;

inline long long qpow(long long a, int b) {
  long long ans = 1;
  while (b) {
    if (b & 1) ans = ans * a % mod;
    a = a * a % mod;
    b >>= 1;
  }
  return ans;
}
inline long long rv(int x) { return qpow(x, mod - 2) %
     mod; }
void prec() {
  inv[0] = 1;
  for (int i = 1; i <= k + 1; ++i) {
    inv[i] = (1LL * inv[i - 1] * rv(i)) % mod;
    po[i] = (po[i - 1] + qpow(i, k)) % mod;
  }
  for (int i = 1; i <= k + 1; ++i) {
    dakdak[i] = (inv[i] * inv[k + 1 - i]) % mod;
  }
}
inline long long interpolate(int x, int k, bool bf =
     false) {
  if (k == 0) return x;
  if (x <= k + 1 || bf) {
    return po[x];
  }
  pre[0] = x;
  suf[k + 1] = x - (k + 1);
  for (int i = 1; i <= k; i++) pre[i] = (pre[i - 1] * (x -
       i)) % mod;
  for (int i = k; i >= 1; i--) suf[i] = (suf[i + 1] * (x -
       i)) % mod;
  ans = 0;
  for (int i = 0; i <= k + 1; i++) {
    if (i == 0)
      num = suf[1];
    else if (i == k + 1)
      num = pre[k];
    else
      num = (pre[i - 1] * suf[i + 1]) % mod; // numerator

    if ((i + k) & 1)
      ans = (ans + ((po[i] * num % mod) * dakdak[i])) % mod;
    else
      ans = (ans - ((po[i] * num % mod) * dakdak[i])) % mod;

    ans = (ans + mod) % mod;
  }
  return ans;
}
```

5.6 Lucas

```
ll lucas(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] %
         p;
    n /= p;
    m /= p;
  }
  return c;
}
```

5.7 Matrix

```
template <class T, int N>
struct Matrix {
  typedef Matrix M;
  array<array<T, N>, N> d{};
  M operator*(const M& m) const {
    M a;
    rep(i, 0, N) rep(j, 0, N) rep(k, 0, N) a.d[i][j] +=
         d[i][k] * m.d[k][j];
    return a;
  }
  vector<T> operator*(const vector<T>& vec) const {
    vector<T> ret(N);
    rep(i, 0, N) rep(j, 0, N) ret[i] += d[i][j] * vec[j];
    return ret;
  }
  M operator (ll p) const {
    assert(p >= 0);
    M a, b(*this);
    rep(i, 0, N) a.d[i][i] = 1;
    while (p) {
      if (p & 1) a = a * b;
      b = b * b;
      p >>= 1;
    }
    return a;
  }
};
```

5.8 MillerRabin

```
inline uint64_t mod_mult64(uint64_t a, uint64_t b,
     uint64_t m) { return __int128_t(a) * b % m; }
uint64_t mod_pow64(uint64_t a, uint64_t b, uint64_t m) {
  uint64_t ret = (m > 1);
  for (;;) {
    if (b & 1) ret = mod_mult64(ret, a, m);
    if (!(b >>= 1)) return ret;
    a = mod_mult64(a, a, m);
  }
}

// Works for all primes p < 2^64
bool is_prime(uint64_t n) {
  if (n <= 3) return (n >= 2);
  static const uint64_t small[] = {
    2,  3,  5,   7,   11,  13,  17,  19,  23,  29,  31,  37,
         41,  43,  47,  53,  59,  61,  67,  71,  73,  79,  83,
    89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139,
         149, 151, 157, 163, 167, 173, 179, 181, 191,
         193, 197, 199,
  };
  for (size_t i = 0; i < sizeof(small) / sizeof(uint64_t);
       ++i) {
    if (n % small[i] == 0) return n == small[i];
  }

  // Makes use of the known bounds for Miller-Rabin
       pseudoprimes.
  static const uint64_t millerrabin[] = {
    2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37,
  };
  static const uint64_t A014233[] = {
    // From OEIS.
    2047LL, 1373653LL, 25326001LL, 3215031751LL,
         2152302898747LL, 3474749660383LL,
         341550071728321LL, 341550071728321LL,
         3825123056546413051LL, 3825123056546413051LL,
         3825123056546413051LL, 0,
  };
  uint64_t s = n - 1, r = 0;
  while (s % 2 == 0) {
    s /= 2;
    r++;
  }
  for (size_t i = 0, j; i < sizeof(millerrabin) /
       sizeof(uint64_t); i++) {
    uint64_t md = mod_pow64(millerrabin[i], s, n);
    if (md != 1) {
      for (j = 1; j < r; j++) {
        if (md == n - 1) break;
        md = mod_mult64(md, md, n);
      }
      if (md != n - 1) return false;
    }
    if (n < A014233[i]) return true;
  }
  return true;
}
```

5.9 Mobius

```
mobius[1] = 1;
for (int i = 2; i < N; ++i) {
  --mobius[i];
  for (int j = i + i; j < N; j += i) mobius[j] -=
       mobius[i];
}
```

5.10 ModInverse

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
for(ll i = 2; i < LIM; ++i) inv[i] = mod - (mod / i) *
     inv[mod % i] % mod;
```

5.11 ModMulLL

```
typedef unsigned long long ull;
const int bits = 10; // i f a l l numbers are less than
     2^k , set bits = 64k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
  ull x = a * (b & (po - 1)) % c;
  while ((b >>= bits) > 0) {
    a = (a << bits) % c;
    x += (a * (b & (po - 1))) % c;
  }
  return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
  if (b == 0) return 1;
  ull res = mod_pow(a, b / 2, mod);
  res = mod_mul(res, res, mod);
  if (b & 1) return mod_mul(res, a, mod);
  return res;
}
```

5.12 ModularArithmetic

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) %
       mod); }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1);
    return Mod((x + mod) % mod);
  }
  Mod operator(ll e) {
    if (!e) return Mod(1);
    Mod r = *this (e / 2);
    r = r * r;
    return e & 1 ? *this * r : r;
  }
};
```

5.13 Notes

5.13.1 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n)\frac{x^n}{n!} = \exp\left(\sum_{n\in S}\frac{x^n}{n}\right)$$

5.13.2 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor\frac{n!}{e}\right\rceil$$

5.13.3 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = Z_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

5.13.4 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in Z\setminus\{0\}} (-1)^{k+1}p(n-k(3k-1)/2)$$

$$p(n) \sim 0.145/n\cdot\exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	~2e5	~2e8

5.13.5 Lucas' Theorem

Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k}\binom{n_i}{m_i} \pmod{p}$.

5.13.6 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able). $B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k}B_k\cdot(n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!}f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.13.7 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\textstyle\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.13.8 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

5.13.9 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

5.13.10 Bell numbers

Total number of partitions of n distinct elements. $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots$. For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

5.13.11 Labeled unrooted trees

\# on n vertices: n^{n-2}
\# on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

5.13.12 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.13.13 Hockey Stick Identity

$$\sum_{k=r}^{n}\binom{k}{r} = \binom{n+1}{r+1}$$

5.14 NTT

```
/* NTT with modulo 998244353
notes:
NTT with mod m
g is any primitive root modulo m (g = 3 works well for
    998244353)
n divides m - 1 evenly
wn = g^((m - 1) / n)
https://codeforces.com/blog/entry/75326
*/

const int N = 1 << 21;
const ll mod = 998244353;
const ll g = 3;
int rev[N];
ll w[N], iw[N], wt[N], inv_n;

ll binpow(ll a, ll b) {
  ll res = 1;
  for (; b; b >>= 1, a = (1ll * a * a) % mod)
    if (b & 1) res = (1ll * res * a) % mod;
  return res;
}

void precalc(int lg) {
  int n = 1 << lg;
  inv_n = binpow(n, mod - 2);
  for (int i = 0; i < n; ++i) {
    rev[i] = 0;
    for (int j = 0; j < lg; ++j)
      if (i & (1 << j)) rev[i] |= (1 << (lg - j - 1));
  }
  ll wn = binpow(g, (mod - 1) / n);
  w[0] = 1;
  for (int i = 1; i < n; ++i) w[i] = (1ll * w[i - 1] * wn)
      % mod;
  ll iwn = binpow(wn, mod - 2);
  iw[0] = 1;
  for (int i = 1; i < n; ++i) iw[i] = (1ll * iw[i - 1] *
      iwn) % mod;
}

void ntt(vector<ll> &a, int lg, bool inv = 0) {
  int n = (1 << lg);
  for (int i = 0; i < n; ++i)
    if (i < rev[i]) swap(a[i], a[rev[i]]);
  for (int len = 2; len <= n; len <<= 1) {
    int d = n / len;
    for (int j = 0; j < (len >> 1); ++j) wt[j] = (inv ?
        iw[d * j] : w[d * j]);
    for (int i = 0; i < n; i += len) {
      for (int j = 0; j < (len >> 1); ++j) {
        ll x = a[i + j], y = (1ll * a[i + j + (len >> 1)] *
            wt[j]) % mod;
        a[i + j] = (x + y) % mod;
        a[i + j + (len >> 1)] = (x - y + mod) % mod;
      }
    }
  }

  if (inv)
    for (int i = 0; i < n; ++i) a[i] = (1ll * a[i] * inv_n)
        % mod;
}

vector<ll> multiply(vector<ll> a, vector<ll> b) {
  int n = 1, lg = 0;
  int na = sz(a), nb = sz(b);
  while (n < na + nb) n <<= 1, ++lg;
  precalc(lg);
  a.resize(n);
  b.resize(n);
  ntt(a, lg);
  ntt(b, lg);
  for (int i = 0; i < n; ++i) a[i] = (1ll * a[i] * b[i]) %
      mod;
  ntt(a, lg, 1);
  vector<ll> c;
  for (int i = 0; i < na + nb - 1; ++i) c.push_back(a[i]);

  // while(!c.empty() and c.back() == 0)
  //    c.pop_back();

  return c;
}
```

5.15 PhiFunction

```
const int LIM = 5000000;
int phi[LIM];
void calculatePhi() {
  rep(i, 0, LIM) phi[i] = i & 1 ? i : i / 2;
  for (int i = 3; i < LIM; i += 2)
    if (phi[i] == i)
      for (int j = i; j < LIM; j += i) (phi[j] /= i) *= i -
          1;
```

5.16 PollardFactorize

```
using ll = long long;
using ull = unsigned long long;
using ld = long double;
ll mult(ll x, ll y, ll md) {
  ull q = (ld)x * y / md;
  ll res = ((ull)x * y - q * md);
  if (res >= md) res -= md;
  if (res < 0) res += md;
  return res;
}

ll powMod(ll x, ll p, ll md) {
  if (p == 0) return 1;
  if (p & 1) return mult(x, powMod(x, p - 1, md), md);
  return powMod(mult(x, x, md), p / 2, md);
}

bool checkMillerRabin(ll x, ll md, ll s, int k) {
  x = powMod(x, s, md);
  if (x == 1) return true;
  while (k--) {
    if (x == md - 1) return true;
    x = mult(x, x, md);
    if (x == 1) return false;
  }
  return false;
}
bool isPrime(ll x) {
  if (x == 2 || x == 3 || x == 5 || x == 7) return true;
  if (x % 2 == 0 || x % 3 == 0 || x % 5 == 0 || x % 7 ==
      0) return false;
  if (x < 121) return x > 1;
  ll s = x - 1;
```

```
  int k = 0;
  while (s % 2 == 0) {
    s >>= 1;
    k++;
  }
  if (x < 1LL << 32) {
    for (ll z : {2, 7, 61}) {
      if (!checkMillerRabin(z, x, s, k)) return false;
    }
  } else {
    for (ll z : {2, 325, 9375, 28178, 450775, 9780504,
         1795265022}) {
      if (!checkMillerRabin(z, x, s, k)) return false;
    }
  }
  return true;
}

ll gcd(ll x, ll y) { return y == 0 ? x : gcd(y, x % y); }

void pollard(ll x, vector<ll> &ans) {
  if (isPrime(x)) {
    ans.push_back(x);
    return;
  }
  ll c = 1;
  while (true) {
    c = 1 + get_rand(x - 1);
    auto f = [&](ll y) {
      ll res = mult(y, y, x) + c;
      if (res >= x) res -= x;
      return res;
    };
    ll y = 2;
    int B = 100;
    int len = 1;
    ll g = 1;
    while (g == 1) {
      ll z = y;
      for (int i = 0; i < len; i++) {
        z = f(z);
      }
      ll zs = -1;
      int lft = len;
      while (g == 1 && lft > 0) {
        zs = z;
        ll p = 1;
        for (int i = 0; i < B && i < lft; i++) {
          p = mult(p, abs(z - y), x);
          z = f(z);
        }
        g = gcd(p, x);
        lft -= B;
      }
      if (g == 1) {
        y = z;
        len <<= 1;
        continue;
      }
      if (g == x) {
        g = 1;
        z = zs;
        while (g == 1) {
          g = gcd(abs(z - y), x);
          z = f(z);
        }
      }
      if (g == x) break;
      assert(g != 1);
      pollard(g, ans);
      pollard(x / g, ans);
      return;
    }
  }
}
// return list of all prime factors of x (can have
     duplicates)
vector<ll> factorize(ll x) {
  vector<ll> ans;
  for (ll p : {2, 3, 5, 7, 11, 13, 17, 19}) {
    while (x % p == 0) {
      x /= p;
      ans.push_back(p);
    }
  }
  if (x != 1) {
    pollard(x, ans);
  }
  sort(ans.begin(), ans.end());
  return ans;
}
// return pairs of (p, k) where x = product(p^k)
vector<pair<ll, int>> factorize_pk(ll x) {
  auto ps = factorize(x);
  ll last = -1, cnt = 0;
  vector<pair<ll, int>> res;
  for (auto p : ps) {
    if (p == last)
      ++cnt;
    else {
      if (last > 0) res.emplace_back(last, cnt);
      last = p;
      cnt = 1;
    }
  }
  if (cnt > 0) {
    res.emplace_back(last, cnt);
  }
  return res;
}
vector<ll> get_all_divisors(ll n) {
  auto pks = factorize_pk(n);

  vector<ll> res;
  function<void(int, ll)> gen = [&](int i, ll prod) {
    if (i == static_cast<int>(pks.size())) {
      res.push_back(prod);
      return;
    }

    ll cur_power = 1;
    for (int cur = 0; cur <= pks[i].second; ++cur) {
      gen(i + 1, prod * cur_power);
      cur_power *= pks[i].first;
    }
  };

  gen(0, 1LL);
  sort(res.begin(), res.end());
  return res;
}
```

5.17 PrimitiveRoot

```
// Primitive root of modulo n is integer g iff for all a <
     n & gcd(a, n) == 1, there exist k: g^k = a mod n
// k is called discrete log of a (in case P is prime, can
     find in O(sqrt(P)) by noting that (P-1) is divisible
     by k)
//
// Exist if:
// - n is 1, 2, 4
// - n = p^k for odd prime p
// - n = 2*p^k for odd prime p
int powmod (int a, int b, int p) {
    int res = 1;
    while (b)
        if (b & 1)
            res = int (res * 1ll * a % p), --b;
        else
            a = int (a * 1ll * a % p), b >>= 1;
    return res;
}

int generator (int p) {
    vector<int> fact;
    int phi = p-1, n = phi;
    for (int i=2; i*i<=n; ++i)
        if (n % i == 0) {
            fact.push_back (i);
            while (n % i == 0)
                n /= i;
        }
    if (n > 1)
        fact.push_back (n);

    for (int res=2; res<=p; ++res) {
        bool ok = true;
        for (size_t i=0; i<fact.size() && ok; ++i)
            ok &= powmod (res, phi / fact[i], p) != 1;
        if (ok) return res;
    }
    return -1;
}
```

5.18 TernarySearch

```
// Find the smallest i in [a; b] that maximizes f(i),
     assuming
// that f(a) < .. < f(i) >= ... >= f(b)
```

```
// Usage: int ind = ternSearch(0,n-1,[&](int i){return
     a[i];});
template <class F>
int ternSearch(int a, int b, F f) {
  assert(a <= b);
  while (b - a >= 5) {
    int mid = (a + b) / 2;
    if (f(mid) < f(mid + 1))
      a = mid; // (A)
    else
      b = mid + 1;
  }
  rep(i, a + 1, b + 1) if (f(a) < f(i)) a = i; // (B)
  return a;
}
```

5.19 XorBasis

```
struct Basis {
  const int LGX = 19;
  vector<int> a;
  Basis() : a(LGX + 1, 0) {}
  void add(int x) {
    for (int i = LGX; i >= 0; --i) {
      if (x & (1 << i)) {
        if (a[i])
          x ^= a[i];
        else {
          a[i] = x;
          break;
        }
      }
    }
  }
  void add(Basis o) {
    for (int i = LGX; i >= 0; --i) add(o.a[i]);
  }
  bool is_spannable(int x) {
    for (int i = LGX; i >= 0; --i)
      if (x & (1 << i)) x ^= a[i];
    return (x == 0);
  }
};
```

6 String

6.1 AhoCorasick

```
template <int MAXC = 26>
struct AhoCorasick {
  vector<array<int, MAXC>> C;
  vector<int> F;
  vector<vector<int>> FG;
  vector<bool> E;
  int node() {
    int r = C.size();
    E.push_back(0);
    F.push_back(-1);
    C.emplace_back();
    fill(C.back().begin(), C.back().end(), -1);
    return r;
  }
  int ctrans(int n, int c) {
    if (C[n][c] == -1) C[n][c] = node();
    return C[n][c];
  }
  int ftrans(int n, int c) const {
    while (n && C[n][c] == -1) n = F[n];
    return C[n][c] != -1 ? C[n][c] : 0;
  }
  AhoCorasick(vector<vector<int>> P) {
    node();
    for (int i = 0; i < (int)P.size(); i++) {
      int n = 0;
      for (int c : P[i]) n = ctrans(n, c);
      E[n] = 1;
    }
    queue<int> Q;
    F[0] = 0;
    for (int c : C[0])
      if (c != -1) Q.push(c), F[c] = 0;
    while (!Q.empty()) {
      int n = Q.front();
      Q.pop();
      for (int c = 0; c < MAXC; ++c)
        if (C[n][c] != -1) {
          int f = F[n];
          while (f && C[f][c] == -1) f = F[f];
          F[C[n][c]] = C[f][c] != -1 ? C[f][c] : 0;
          Q.emplace(C[n][c]);
        }
    }
    FG.resize(F.size());
    for (int i = 1; i < (int)F.size(); i++) {
      FG[F[i]].push_back(i);
      if (E[i]) Q.push(i);
    }
    while (!Q.empty()) {
      int n = Q.front();
      Q.pop();
      for (int f : FG[n]) E[f] = 1, Q.push(f);
    }
  }
  bool check(vector<int> V) {
    if (E[0]) return 1;
    int n = 0;
    for (int c : V) {
      n = ftrans(n, c);
      if (E[n]) return 1;
    }
    return 0;
  }
};
```

6.2 KMP

```
// prefix function: *length* of longest prefix which is
     also suffix:
//   pi[i] = max(k: s[0..k-1] == s[i-(k-1)..i]
//
// KMP {{{
template <typename Container>
std::vector<int> prefix_function(const Container& s) {
  int n = s.size();
  std::vector<int> pi(n);
  for (int i = 1; i < n; ++i) {
    int j = pi[i - 1];
    while (j > 0 && s[i] != s[j]) j = pi[j - 1];
    if (s[i] == s[j]) ++j;
    pi[i] = j;
  }
  return pi;
}

// Tested: https://oj.vnoi.info/problem/substr
// Return all positions (0-based) that pattern ‘pat‘
     appears in ‘text‘
std::vector<int> kmp(const std::string& pat, const
     std::string& text) {
  auto pi = prefix_function(pat + '\0' + text);
  std::vector<int> res;
  for (size_t i = pi.size() - text.size(); i < pi.size();
       ++i) {
    if (pi[i] == (int)pat.size()) {
      res.push_back(i - 2 * pat.size());
    }
  }
  return res;
}

// Tested: https://oj.vnoi.info/problem/icpc22_mt_b
// Returns cnt[i] = # occurrences of prefix of length-i
// NOTE: cnt[0] = n+1 (0-length prefix appears n+1 times)
std::vector<int> prefix_occurrences(const string& s) {
  int n = s.size();
  auto pi = prefix_function(s);
  std::vector<int> res(n + 1);
  for (int i = 0; i < n; ++i) res[pi[i]]++;
  for (int i = n - 1; i > 0; --i) res[pi[i - 1]] += res[i];
  for (int i = 0; i <= n; ++i) res[i]++;
  return res;
}
```

6.3 Manacher

```
vector<int> manacher_odd(string s) {
  int n = s.size();
  s = "$" + s + "^";
  vector<int> p(n + 2);
  int l = 1, r = 1;
  for (int i = 1; i <= n; i++) {
    p[i] = max(0, min(r - i, p[l + (r - i)]));
```

```
    while (s[i - p[i]] == s[i + p[i]]) {
      p[i]++;
    }
    if (i + p[i] > r) {
      l = i - p[i], r = i + p[i];
    }
  }
  return vector<int>(begin(p) + 1, end(p) - 1);
}
vector<int> manacher(string s) {
  string t;
  for (auto c : s) {
    t += string("#") + c;
  }
  auto res = manacher_odd(t + "#");
  return vector<int>(begin(res) + 1, end(res) - 1);
}
```

6.4 StringHashing

```
const int MOD1 = 127657753, MOD2 = 987654319;
const int p1 = 137, p2 = 277;
```

6.5 SuffixArray

```
/**
 * Author:         , chilli
 * Date: 2019-04-11
 * License: Unknown
 * Source: Suffix array - a powerful tool for dealing with
      strings
 * (Chinese IOI National team training paper, 2009)
 * Description: Builds suffix array for a string.
 * \texttt{sa[i]} is the starting index of the suffix which
 * is $i$'th in the sorted suffix array.
 * The returned vector is of size $n+1$, and \texttt{sa[0]
      = n}.
 * The \texttt{lcp} array contains longest common prefixes
      for
 * neighbouring strings in the suffix array:
 * \texttt{lcp[i] = lcp(sa[i], sa[i-1])}, \texttt{lcp[0] =
      0}.
 * The input string must not contain any zero bytes.
 * Time: O(n \log n)
 * Status: stress-tested
 */
struct SuffixArray {
        vi sa, lcp;
        SuffixArray(string& s, int lim=256) { // or
              basic_string<int>
                int n = sz(s) + 1, k = 0, a, b;
                vi x(all(s)), y(n), ws(max(n, lim));
                x.push_back(0), sa = lcp = y, iota(all(sa),
                      0);
                for (int j = 0, p = 0; p < n; j = max(1, j *
                      2), lim = p) {
                        p = j, iota(all(y), n - j);
                        rep(i,0,n) if (sa[i] >= j) y[p++] =
                              sa[i] - j;
                        fill(all(ws), 0);
                        rep(i,0,n) ws[x[i]]++;
                        rep(i,1,lim) ws[i] += ws[i - 1];
                        for (int i = n; i--;)
                              sa[--ws[x[y[i]]]] = y[i];
                        swap(x, y), p = 1, x[sa[0]] = 0;
                        rep(i,1,n) a = sa[i - 1], b = sa[i],
                              x[b] =
                                (y[a] == y[b] && y[a + j] ==
                                      y[b + j]) ? p - 1 : p++;
                }
                for (int i = 0, j; i < n - 1; lcp[x[i++]] =
                      k)
                        for (k && k--, j = sa[x[i] - 1];
                                      s[i + k] == s[j + k];
                                            k++);
        }
};
int64_t cnt_distinct_substrings(const std::string& s) {
    auto lcp = LCP(s, suffix_array(s, 0, 255));
    return s.size() * (int64_t) (s.size() + 1) / 2
        - std::accumulate(lcp.begin(), lcp.end(), 0LL);
}
```

6.6 Z

```
vector<int> zfunc(const string& s) {
  int n = (int)s.length();
  vector<int> z(n);
  z[0] = n;
  for (int i = 1, l = 0, r = 0; i < n; ++i) {
    if (i <= r) z[i] = min(r - i + 1, z[i - l]);
    while (i + z[i] < n && s[z[i]] == s[i + z[i]]) ++z[i];
    if (i + z[i] - 1 > r) l = i, r = i + z[i] - 1;
  }
  return z;
}
```

7 Utilities

7.1 FastInput

```
inline char gc() { // l ike getchar ()
  static char buf[1 << 16];
  static size_t bc, be;
  if (bc >= be) {
    buf[0] = 0, bc = 0;
    be = fread(buf, 1, sizeof(buf), stdin);
  }
  return buf[bc++]; // returns 0 on EOF
}
int readInt() {
  int a, c;
  while ((a = gc()) < 40)
    ;
  if (a == '-') return -readInt();
  while ((c = gc()) >= 48) a = a * 10 + c - 480;
  return a - 48;
}
```

7.2 multivec

```
template<int D, typename T> struct Vec : public
      vector<Vec<D - 1, T>> { template<typename... Args>
      Vec(int n = 0, Args... args) : vector < Vec < D - 1,
      T >> (n, Vec < D - 1, T > (args...)) {} };
template<typename T> struct Vec<1, T> : public vector<T> {
      Vec(int n = 0, const T &val = T()) : vector<T>(n,
      val) {} };
```

7.3 template

```
#include "bits/stdc++.h"
using namespace std;

using ll = long long;
using pii = pair<int, int>;

#define F first
#define S second
#define sz(x) (int)((x).size())
#define all(x) (x).begin(), (x).end()

mt19937_64
      rng(chrono::steady_clock::now().time_since_epoch().count());
ll get_rand(ll l, ll r) {
    assert(l <= r);
    return uniform_int_distribution<ll> (l, r)(rng);
}

void solve(){

}

int32_t main() {
    cin.tie(nullptr)->sync_with_stdio(0);
    int test = 1;
    while(test--) solve();
    #ifdef LOCAL
        cerr << "\n[Time]: " << 1000.0 * clock() /
              CLOCKS_PER_SEC << " ms.\n";

    return 0;
}
```