# 1st AGO Shortlist G2

Mmukul Khedekar

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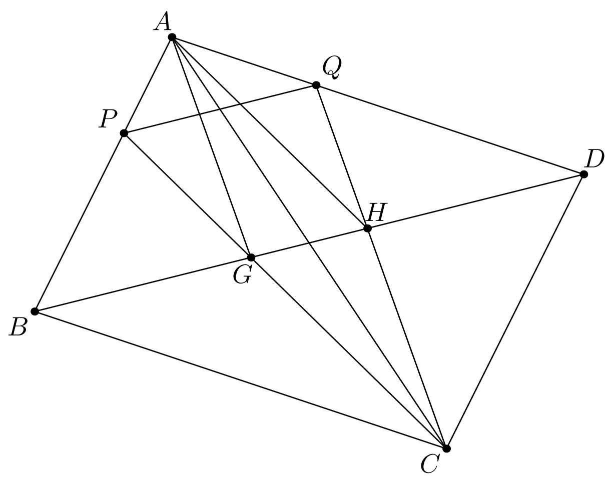
https://artofproblemsolving.com/community/c6h3336439p30900010

## §1 Problem

**Problem** (1st AGO Shortlist G2)
Let $P$ and $Q$ be points on the sides $AB$ and $AD$, respectively, of a convex quadrilateral $ABCD$ such that $PQ \parallel BD$. Let the segments $CP$ and $CQ$ intersect $BD$ at points $G$ and $H$, respectively. Prove that if the quadrilateral $AGCH$ is a parallelogram, then the quadrilateral $ABCD$ is a parallelogram.

## §2 Solution




*Proof.* If $AGCH$ is a parallelogram, then we can show that $\overline{BG} = \overline{HD}$ because

$$\frac{\overline{BG}}{\overline{GH}} = \frac{\overline{BP}}{\overline{PA}} = \frac{\overline{DQ}}{\overline{QA}} = \frac{\overline{HD}}{\overline{GH}}$$

Using this we can use SAS congruence criterion to show that $\triangle AGB \cong \triangle CHD$ and $\triangle ABH \cong \triangle CDG$. Therefore $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ which implies that $ABCD$ is a parallelogram. If $ABCD$ is a parallelogram, then $\overline{AC}$ bisects $\overline{BD} \implies \overline{AC}$ bisects $\overline{PQ} \implies \overline{AC}$ bisects $\overline{GH}$. Since $\overline{BD}$ bisects $\overline{AC} \implies \overline{GH}$ bisects $\overline{AC} \implies AGCH$ is a parallelogram. $\square$