1st AGO Shortlist G4

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§1 Problem

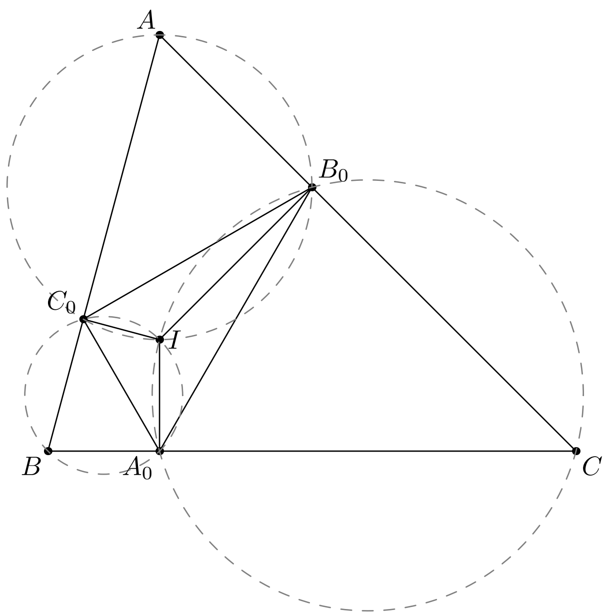
Problem (1st AGO Shortlist G4)

Given a triangle ABC, points A_0, B_0, C_0 lie on sides BC, CA, AB respectively. Triangle $A_0B_0C_0$ is called *reflective* with respect to triangle ABC if all of the following hold.

$$\angle BA_0C_0 = \angle CA_0B_0, \qquad \angle BC_0A_0 = \angle AC_0B_0, \qquad \angle AB_0C_0 = \angle CB_0A_0.$$

Prove that every acute triangle ABC has a unique reflective triangle with respect to triangle ABC.

§2 Solution



Proof. It's easy to verify that the orthic triangle satifies the condition. Now we will show that it is the only triangle that does. Suppose $\triangle A_0B_0C_0$ is not the orthic triangle of $\triangle ABC$. Suppose I is the incenter of $\triangle A_0B_0C_0$. Then $\overline{IA_0} \perp \overline{BC}$, $\overline{IB_0} \perp \overline{AC}$ and $\overline{IC_0} \perp \overline{AB} \implies AC_0IB_0$, BC_0IA_0 and CA_0IB_0 are cyclic. Moreover, A is the excenter opposite to vertex to A_0 which implies that A_0I passes through A. Similarly, B_0I passess through B and C_0I passes through C. Thus A_0, B_0 and C_0 are the feet of perpendiculars from A, B and $C \implies \triangle A_0B_0C_0$ is the orthic triangle, arriving at a contradiction to our initial assumption. $\square$