IMO Shortlist 2021 G1

Mmukul Khedekar

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§1 Problem

Problem (IMO Shortlist 2021 G1)
Let $ABCD$ be a parallelogram with $AC = BC$. A point P is chosen on the extension of ray AB past B. The circumcircle of ACD meets the segment PD again at Q. The circumcircle of triangle APQ meets the segment PC at R. Prove that lines CD, AQ, BR are concurrent.

§2 Solution 1 (Using Angle Chasing)

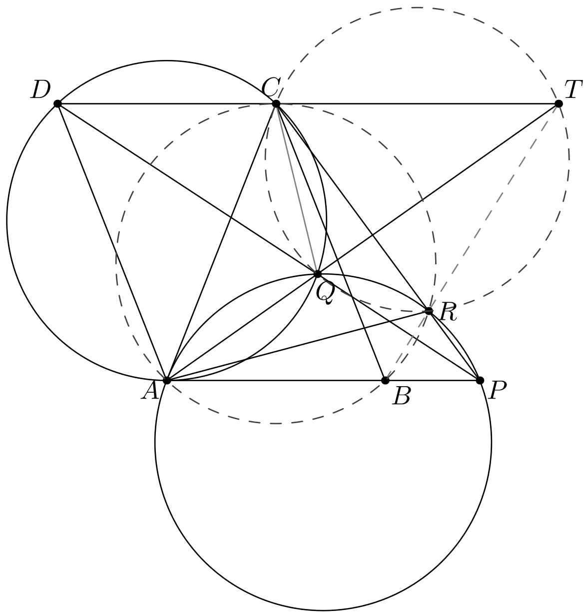
Proof. Define $CD \cap AQ$ as T. We want to show that BR passes through T too.

Claim 2.1. Quadrilateral $CABR$ is cyclic.

Proof. This is just angle chasing.

$$\begin{aligned}\angle CRA &= \angle CRQ + \angle QRA \\ &= \angle QAP + \angle QPA \\ &= \angle QAP + \angle QDC \\ &= \angle QAP + \angle QAC \\ &= \angle CAB = \angle ABC\end{aligned}$$

This implies that $CABR$ is a cyclic quadrilateral. □



> **Claim 2.2.** Quadrilateral $CQRT$ is cyclic.

Proof. Again we use angle chasing to prove this.

$$\begin{aligned}\angle CRQ &= \angle QAP \\ &= \angle QTC\end{aligned}$$

which implies that $CQRT$ is a cyclic quadrilateral. □

As a result, we have

$$\begin{aligned}\angle CRB + \angle CRT &= 180^\circ - \angle CAB + \angle CQT \\ &= 180^\circ - \angle CBA + \angle CDA \\ &= 180^\circ\end{aligned}$$

Therefore, points B, R and T are collinear. □

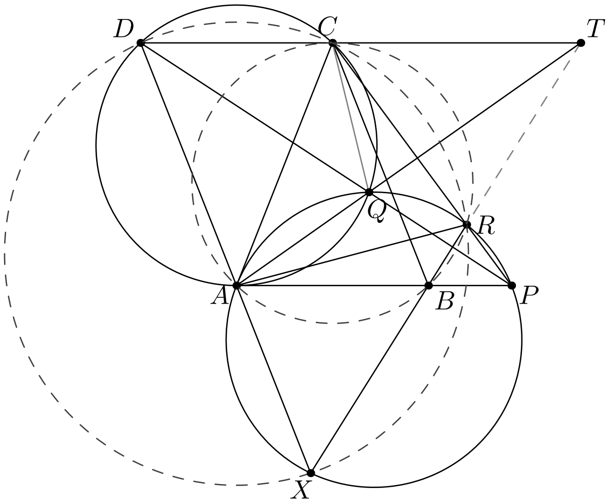
§3 Solution 2 (Using Radical Axis Theorem)

Proof. Define $BR \cap \odot(AQP) = X$.

> **Claim 3.1.** Quadrilateral $CABR$ is cyclic.

Proof. Same as Claim 1.1. □

> **Claim 3.2.** X lies on the line AD.



Proof. To prove the collinearity, we simply angle chase.

$$\begin{aligned}\angle DAB + \angle XAB &= 180^\circ - \angle ABC + \angle XRP \\ &= 180^\circ - \angle CAB + \angle CAB \\ &= 180^\circ\end{aligned}$$

This implies that X lies on AD. $\square$

> **Claim 3.3.** Quadrilateral $CDXR$ is cyclic.

Proof. This is again angle chasing.

$$\angle CDX = \angle CBA = \angle CAB = 180^\circ - \angle CRB$$

which implies that $CDXR$ is a cyclic quadrilateral. $\square$

Applying the radical axis theorem on $\odot(AQCD)$, $\odot(AQRP)$ and $\odot(CDXR)$, we get that CD, AQ and BR are concurrent. $\square$