IMO Shortlist 2024 G2

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19 January 2026

https://artofproblemsolving.com/community/c6h3359767p31218657

Contents

§1 Problem

Problem (IMO Shortlist 2024 G2)
Let ABC be a triangle with $AB < AC < BC$. Let the incenter and incircle of triangle ABC be I and ω, respectively. Let X be the point on line BC different from C such that the line through X parallel to AC is tangent to ω. Similarly, let Y be the point on line BC different from B such that the line through Y parallel to AB is tangent to ω. Let AI intersect the circumcircle of triangle ABC at $P \neq A$. Let K and L be the midpoints of AC and AB, respectively. Prove that $\angle KIL + \angle YPX = 180°$.

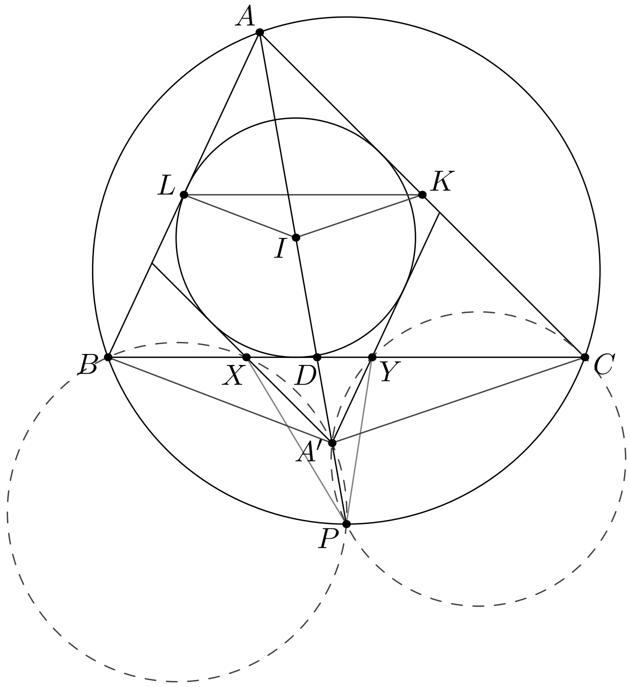
§2 Solution 1 (Using Homothety)

Proof. Let A' be the reflection of point A over point I.

Claim 2.1. $A'X \parallel AC$ and $A'Y \parallel AB$.

Proof. Construct point E on line segment AC such that ω touches AC at E. Reflect the point E over I to E'. By SAS congruency criterion $\triangle\ AIE \cong A'IE'$. Since, $\angle IE'A'$ $= 90° \implies A'E'$ is tangent to ω and $A'E' \parallel AC$. However, X lies on the line parallel to AC and tangent to $\omega \implies A'X \parallel AC$, and similarly $A'Y \parallel AB$ which proves the claim. $\square$

Claim 2.2. $BXA'P$ and $CYA'P$ are cyclic quadrilaterals.



Proof. Just angle chasing.

$$\angle CXA' = \angle BCA = \angle BPA = \angle BPA'$$

which proves that $BXA'P$ is cyclic. Similarly, we can show that $CYA'P$ is cyclic. □

> **Claim 2.3.** $\triangle\ KIL$ is homothetic to $\triangle\ CA'B$ from point A.

Proof. Since lines BL, $A'I$ and CK are concurrent at point A and $AB = 2AL$, $A'A = 2AI$ and $AC = 2AK \implies \triangle\ KIL \mapsto \triangle CA'B$ under a homothetic transformation with scaling factor $= 2$. □

Finally combining all the information from the proved claims,

$$\begin{aligned}\angle KIL + \angle YPX &= \angle BA'C + \angle XPY \\ &= \angle BA'C + \angle XPA' + \angle YPA' \\ &= \angle BA'C + \angle XBA' + \angle YCA' \\ &= \angle BA'C + \angle CBA' + \angle BCA' \\ &= 180^\circ\end{aligned}$$

as desired. □