1st AGO Shortlist G3

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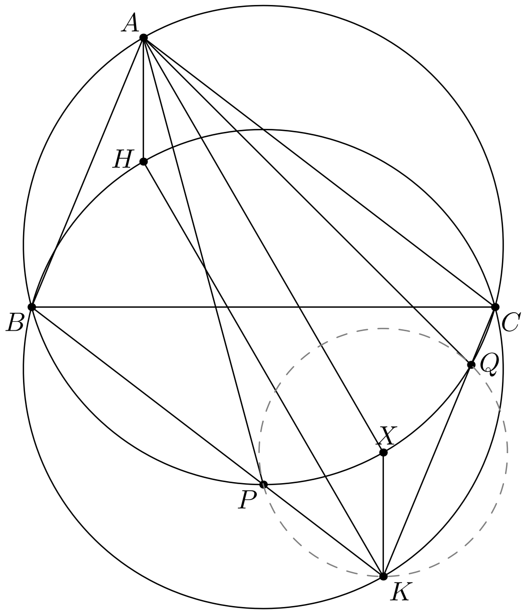
§1 Problem

Problem (1st AGO Shortlist G3)
Let ABC be an acute triangle with circumcircle Ω and K be a point such that $ABKC$ is a parallelogram. Lines BK and CK intersect Ω again at P and Q, respectively. The line that passes through K and perpendicular to BC intersects Ω at X such that X is closer to K. Prove that $XP = XQ$.

§2 Solution



Proof. Let H be the orthocenter of $\triangle ABC$. Since $\triangle ABC \cong \triangle BKC \implies \odot(ABC)$ is the reflection of $\odot(BKC)$ over $\overline{BC}$. Hence H lies on $\odot(BKC)$. Infact $\triangle ABC \cup H$ is mapped to $\triangle KCB \cup X$ under reflection over midpoint of $\overline{BC}$. Thus

$$\angle XPK = \angle BAX = \angle HAC = \angle BKX = \angle PKX$$

which implies that $\overline{XP} = \overline{XK}$. Similarly, we can show that $\overline{XK} = \overline{XQ} \implies \overline{XP} = \overline{XQ}$. $\square$