IMO Shortlist 2019 G1

Mmukul Khedekar

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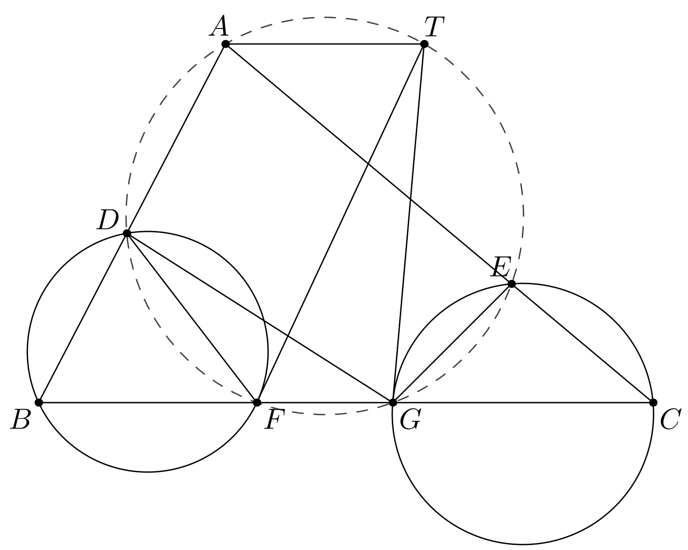
§1 Problem

Problem (IMO Shortlist 2019 G1)

Let ABC be a triangle. Circle Γ passes through A, meets segments AB and AC again at points D and E respectively, and intersects segment BC at F and G such that F lies between B and G. The tangent to circle BDF at F and the tangent to circle CEG at G meet at point T. Suppose that points A and T are distinct. Prove that line AT is parallel to BC.

§2 Solution 1 (Using Angle Chasing)



Claim 2.1. T lies on Γ.

Proof. Proof. We want to show that $ADFGET$ is cyclic. It suffices to show that $DFGT$ is cyclic. Notice that $\angle DFT = \angle ABC$ and $\angle TGE = \angle ACB$. Since,

$$\begin{aligned}\angle DGT &= \angle DGE - \angle TGE \\ &= 180 - \angle BAC - \angle ACB \\ &= \angle ABC = \angle DFT\end{aligned}$$

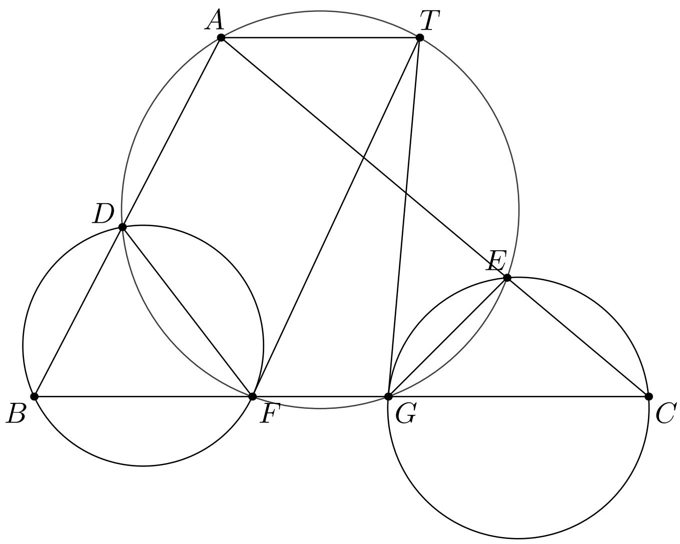
Hence $\angle DGT = \angle DFT \implies DFGT$ is cyclic. □

From here, it's easy to show that $\overline{AT} \parallel \overline{BC}$, which holds because

$$\angle TAC = \angle TAE = \angle TGE = \angle ACB$$

□

§3 Solution 2 (Using Reim's Theorem)



Proof. Suppose the tangents FT and GT meet Γ at T_1 and T_2. Applying reim's theorem on pairs of circles $\odot(BDF)$, Γ and $\odot(CGE)$, $\Gamma \implies \overline{AT_1} \parallel \overline{BF}$ and $\overline{AT_2} \parallel \overline{GC}$. Hence $T_1 = T_2$, which implies that $\overline{AT} \parallel \overline{BC}$. □