IMO Shortlist 2020 G3

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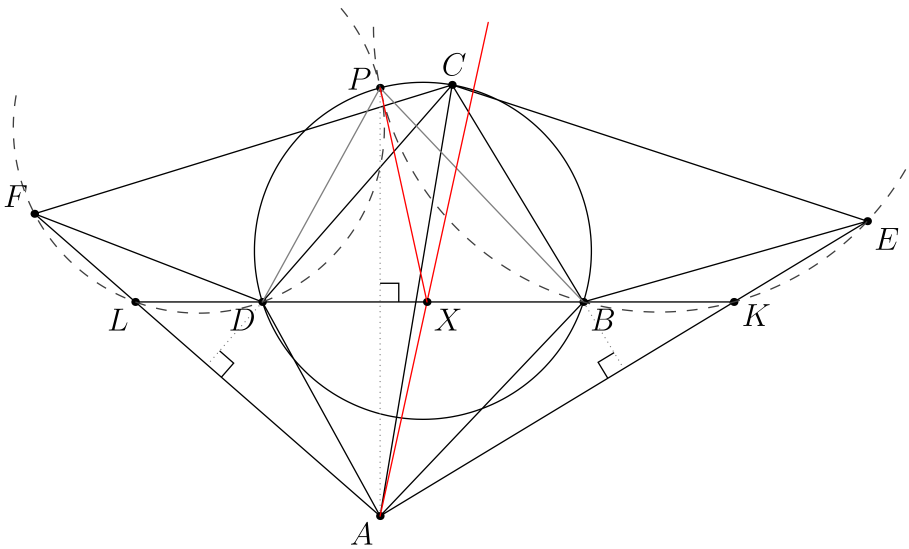
Contents

§1 Problem

Problem (IMO Shortlist 2020 G3)

Let $ABCD$ be a convex quadrilateral with $\angle ABC > 90$, $CDA > 90$ and $\angle DAB = \angle BCD$. Denote by E and F the reflections of A in lines BC and CD, respectively. Suppose that the segments AE and AF meet the line BD at K and L, respectively. Prove that the circumcircles of triangles BEK and DFL are tangent to each other.

§2 Solution 1 (Using Steiner Line)



Proof. Define P as the reflection of A over $\overline{BD}$. We will show that P is the point of tangency of the circumcircles $\odot(BEK)$ and $\odot(DFL)$.

Claim 2.1. P lies on the circle $\odot(BCD)$.

Proof. Since $\angle BPD = \angle BAD = \angle BCD \implies P$ lies on $\odot(BCD)$. □

Claim 2.2. Quadrilaterals $DLFP$ and $BKEP$ are cyclic.

Proof. Since $\angle DFL = \angle DAL = \angle DPL \implies DLFP$ is a cyclic quadrilateral. Similarly, we can show that $BKEP$ is a cyclic quadrilateral too. □

Draw the steiner line of P wrt $\triangle BCD$ and let it intersect BD at X. Then,

$$\angle XPD = \angle XAD = \angle DFP$$

which follows due to XA being the reflection of FP over $\overline{CD}$. Hence, $\overline{XP}$ is tangent to $\odot(DLFP)$ at P. Similarly, we can show that $\overline{XP}$ is tangent to $\odot(BKEP)$ at P. Thus, $\odot(BEK)$ and $\odot(DFL)$ are tangent to each other at P. □