1st AGO Shortlist G1

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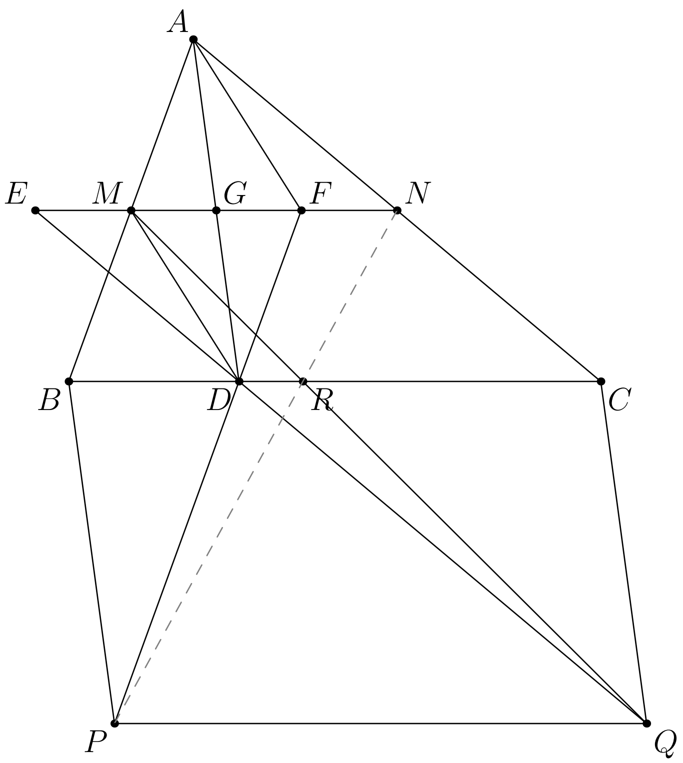
§1 Problem

Problem (1st AGO Shortlist G1)
Let ABC be a triangle and D a point on side BC. Points M and N are midpoints of sides AB and AC, respectively.P and Q are points such that $ABPD$ and $ACQD$ are parallelograms. Prove that lines BC, PN and QM concur.

§2 Solution



Proof. Suppose PD intersects MN at F. Then $\overline{DF} = \overline{BM} = \overline{AM}$. Thus $AMDF$ is a parallelogram. Let G be the midpoint of $\overline{MF}$ and E be the reflection of N over G. Since

$\overline{AG} = \overline{GD}$ and $\overline{GN} = \overline{GE} \implies ANDE$ is a parallelogram. But $\overline{DQ} \parallel \overline{AC}$, thus E lies on DQ. Let QM intersect BC at R, then

$$\frac{\overline{DR}}{\overline{FN}} = \frac{\overline{DR}}{\overline{EM}} = \frac{\overline{DQ}}{\overline{EQ}} = \frac{1}{\frac{\overline{EQ}}{\overline{DQ}}} = \frac{1}{1 + \frac{\overline{ED}}{\overline{DQ}}} = \frac{1}{1 + \frac{\overline{FD}}{\overline{DP}}} = \frac{1}{\frac{\overline{FP}}{\overline{DP}}} = \frac{\overline{DP}}{\overline{FP}}$$

which implies that $\triangle PDR \sim \triangle PFN \implies R$ lies on $\overline{PN}$. In other words, $\overline{PN}$, $\overline{QM}$ and $\overline{BC}$ are concurrent at R. $\square$