1st AGO Shortlist G5

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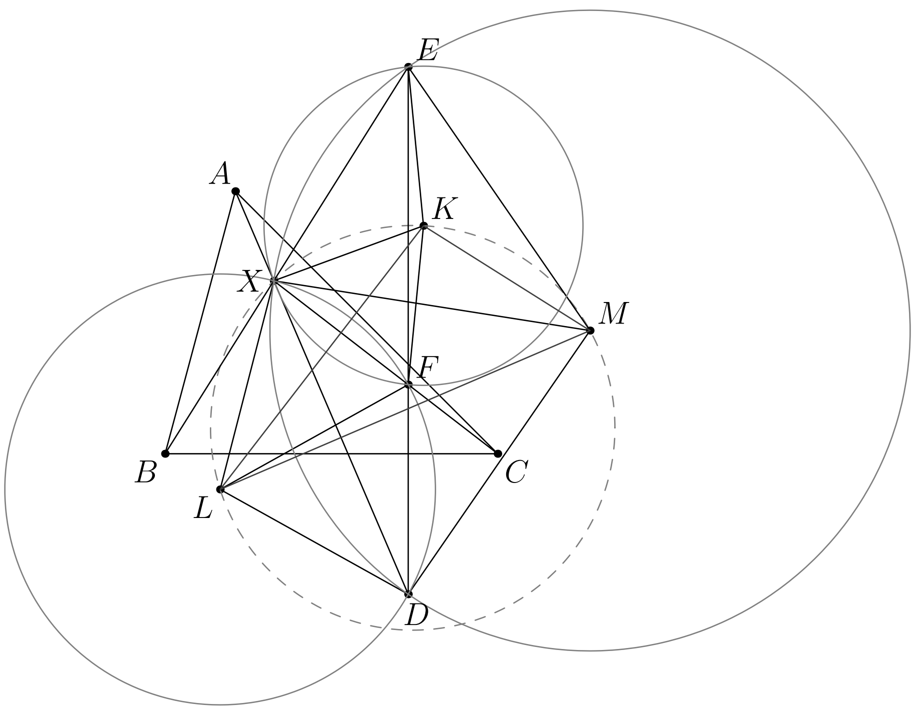
§1 Problem

Problem (1st AGO Shortlist G5)
Let ABC be a triangle and X be a point distinct from A, B, C. A line l intersects lines AX, BX, CX at D, E, F respectively. The perpendicular bisectors of segments DX, EX, FX define a triangle with circumcircle Θ. Prove that X lies on Θ.

§2 Solution



Proof. Let the center of the circles $\odot(XEF)$, $\odot(XDF)$ and $\odot(XFE)$ be K, L and M. So,

$$\angle XKL = \frac{1}{2}\angle XKF = \angle XEF = \angle XED = \frac{1}{2}\angle XMD = \angle XML$$

which implies that $XKML$ is a cyclic quadrilateral, as desired. □