# PHO-105: Introductory Quantum Information Theory

Mmukul Khedekar

January 14, 2026

Notes for the course *Introductory Quantum Information Theory* instructed by Prof. P.C. Srivastava at IIT Roorkee, Spring, 2026.

## Contents

# 1 Jan 12, 2026

The recommended book for this course is **Quantum Computation and Quantum Information**, *Neilsen and Chuang*. It is not meant to be followed linearly. Instead we will jump over topics and read only what is relevant.

## 1.1 The Six Postulates of Quantum Mechanics

1. The first postulate describes the state of a quantum mechanical system. It can be completely specified by a function $\psi$ known as the **wave function**. The wave function is a function of the coodinates in space and time, usually represented as
$$|\psi(\mathbf{r}, t)\rangle$$
This function has an important property that $(\psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)dV)$ is the probability that the particle lies in the volume element $dV$ located at $\mathbf{r}$ at time $t$. The wave function must also satisfy the following normalization condition due to the previous probabilistic interpretation,
$$\int_{-\infty}^{\infty} \psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t)dV = 1$$

2. The second postulate describes a correspondence between every observable quantity $A$ in classical mechanics to its quantum counter-part using the linear, Hermitian operator $\hat{A}$ in quantum mechanics.

3. The third postulate describes the measurements of an observable. The only values that will ever be observed for an observable associated with $\hat{A}$ are its eigenvalues
$$\hat{A}\psi = a\psi$$

4. The fourth postulate describes the expectation of an observable corresponding to $\hat{A}$, which is given by
$$\langle A \rangle = \int_{-\infty}^{\infty} \psi^* \hat{A}\psi dV$$
An immediate consequence of the fourth postulate is that, after the measurement of $\psi$, the wave functions immediately *collapses* into the corresponding eigenstate. In other words, the measurement affects the state of the system.

5. The fifth postulate describes the **time evolution** of a wave function. The time evolution of the wave function is governed by the **Schrödinger Equation**.
$$i\hbar\frac{\partial}{\partial t}|\psi(t)\rangle = \hat{H}|\psi(t)\rangle$$

6. The sixth, and the last postulate, states that the wave function is *symmetric* for particles with integer spin, called the **bosons**, and it is *antisymmetric* for particles with half-integer spin, called the **fermions**. The mathematical treatment of this postulate yields the **Pauli Exclusion Principle** which states that no two identical fermions can occupy the same quantum state.

# 2 Jan 14, 2026

## 2.1 Dirac Notation

For the simplicity of discussion, we assume that the dimension of the space is 2. We first define two linearly independent vectors in space.

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

We can now define the **ket notation** and the **bra notation**.

**Definition 2.1.** For a state $\psi$, the **ket notation** is defined as

$$|\psi\rangle = \psi_0 |0\rangle + \psi_1 |1\rangle = \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix}$$

**Definition 2.2.** For any given *ket notation*, there exists a corresponding *bra notation*. The **bra notation** is defined as the transpose of the complex conjugate of the ket notation.

$$\langle\psi| = \begin{pmatrix} \psi_0^* & \psi_1^* \end{pmatrix}$$

## 2.2 Inner product and Outer product

We introduce a few more operations with no real significance yet.

**Definition 2.3.** Suppose we have two states $\phi$ and $\psi$. The **inner product** of these two states is denoted and defined as

$$\begin{aligned} \langle \phi \,|\, \psi \rangle &= \begin{pmatrix} \phi_0^* & \phi_1^* \end{pmatrix} \begin{pmatrix} \psi_0 \\ \psi_1 \end{pmatrix} \\ &= \phi_0^* \psi_0 + \phi_1^* \psi_1 \end{aligned}$$

**Definition 2.4.** Suppose we have two states $\phi$ and $\psi$. The **outer product** of these two states is denoted and defined as

$$\begin{aligned} |\phi\rangle \langle\psi| &= \begin{pmatrix} \phi_0 \\ \phi_1 \end{pmatrix} \begin{pmatrix} \psi_0^* & \psi_1^* \end{pmatrix} \\ &= \begin{pmatrix} \phi_0\psi_0^* & \phi_0\psi_1^* \\ \phi_1\psi_0^* & \phi_1\psi_1^* \end{pmatrix} \end{aligned}$$

## 2.3 Unitary Operator

A unitary operator $\hat{U}$ is said to be linear if its inverse $\hat{U}^{-1}$ is equal to its **Hermitian Adjoint**.

$$\hat{U}\hat{U}^\dagger = \hat{U}^\dagger\hat{U} = \hat{I}$$