The Isogonal Conjugate of Feuerbach Point

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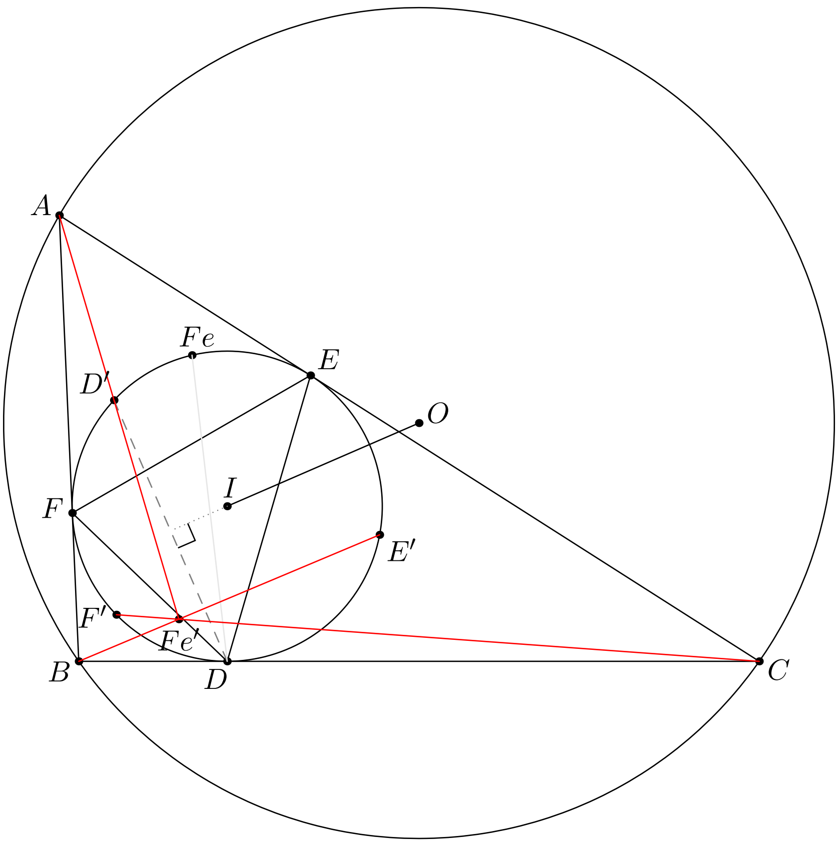
In this article, we shall explore some properties of the **Isogonal Conjugate** of the **Feuerbach Point**. Check out this article for some pre-requisites.

§1 Characterization 1

Problem 1.1 (Well Known)

Given a $\triangle ABC$, its incenter I and circumcenter O. Suppose $\triangle DEF$ is the contact triangle of $\triangle ABC$. Reflect D over $\overline{OI}$ to D', and similarly define E' and F'. Then $\overline{AD'}$, $\overline{BE'}$ and $\overline{CF'}$ concur at the isogonal conjugate of the feuerbach point.



Proof. It's well known that the isogonal line of $\overline{DFe}$ with respect to $\angle FDE$ is perpendicular to $\overline{OI}$. Therefore, $D'FEFe$ is an isosceles trapezium $\implies$ $\overline{AD'}$ and $\overline{AFe}$ are symmetric about $\overline{AI}$. Hence, $\overline{AD'}$, $\overline{BE'}$ and $\overline{CF'}$ concur at the isogonal conjugate of the feuerbach point. □

§2 Characterization 2

Problem 2.1 (INMO 2019 Mock P5 by Shantanu Nene)

Given $\triangle ABC$ with circumcenter O. Suppose $\triangle M_AM_BM_C$ is the medial triangle of $\triangle ABC$. Let ω_A, ω_B and ω_C be the A-, B- and C-mixtilinear excircles of $\triangle AM_BM_C$, $\triangle BM_CM_A$ and $\triangle CM_AM_B$. Show that

1. ω_A, ω_B and ω_C are tangent to $\odot(BOC)$, $\odot(COA)$ and $\odot(AOB)$.
2. Suppose X_A, X_B and X_C are the tangency points, then $\overline{AX_A}$, $\overline{BX_B}$ and $\overline{CX_C}$ are concurrent at the isogonal conjugate of the feuerbach point of $\triangle ABC$.

Proof. Performing a $\sqrt{\frac{bc}{2}}$ inversion around point A followed by a reflection along the angle bisector of $\angle BAC$, maps ω_A to the incircle and $\odot(BOC)$ to the nine-point circle. Therefore, the point of tangency X_A is the image of the feuerbach point under inversion $\implies$ $\overline{AX_A}$ is isogonal to $\overline{AFe}$ in $\angle BAC$. Therefore, $\overline{AX_A}$, $\overline{BX_B}$ and $\overline{CX_C}$ concur at the isogonal conjugate of the feuerbach point. □