IMO Shortlist 2020 G2

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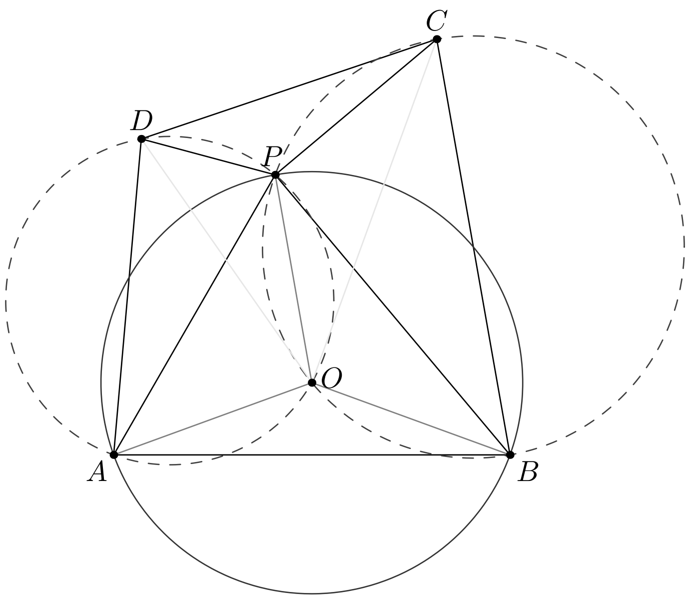
§1 Problem

Problem (IMO Shortlist 2020 G2)

Consider the convex quadrilateral $ABCD$. The point P is in the interior of $ABCD$. The following ratio equalities hold:

$$\angle PAD : \angle PBA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BAP : \angle BPC$$

Prove that the following three lines meet in a point: the internal bisectors of angles $\angle ADP$ and $\angle PCB$ and the perpendicular bisector of segment AB.



§2 Solution 1 (Construction of Circumcenter)

Proof. Let O be the circumcenter of $\odot(PAB)$. Due to the following chase

$$\begin{aligned}\angle AOP &= 2\angle PBA = 4\angle PAD \\ &= \angle PAD + \angle DPA \\ &= 180^\circ - \angle ADP\end{aligned}$$

we have that $\odot(APD)$ passes through O. Similarly $\odot(BPC)$ passes through point O. Since $\overline{OA} = \overline{OP} \implies \overline{OD}$ is the internal angle bisector of $\angle ADP$. Similarly $\overline{OC}$ is the internal angle bisector of $\angle BPC$. But O lies on the perpendicular bisector of segment $\overline{AB}$ which proves that the desired three lines are concurrent at point O. $\square$