EGMO 2013 P4

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§1 Problem

Problem

Find all positive integers a and b for which there are three consecutive integers at which the polynomial

$$P(n) = \frac{n^5 + a}{b}$$

takes integer values.

§2 Solution 1 (Using Modular Arithmetics)

Proof. Suppose the three numbers at which $P(n)$ takes integer values are $x-1$, x and $x+1$. Then we must have

$$(x-1)^5 \equiv x^5 \equiv (x+1)^5 \equiv -a \pmod{b}$$

□