Problem Set 1

Mmukul Khedekar

30 December 2025

Some practice problems based on everything I've taught so far.

§1 Instructions

1. Try out each problem on paper for a sufficient amount of time before using GeoGebra.
2. Ideally you should continue working on the problem until you have exhausted all ideas and are unsure how to proceed further.
3. You may then use GeoGebra to identify key observations that may help you in solving the problem.
4. Feel free to ask queries or hints if you have made substantial progress and feel stuck. I will release the hints for each problem soon.

§2 Practice Problems

Exercise 2.1. Suppose H and O are the orthocenter and circumcenter of $\triangle ABC$. Let O_A, O_B and O_C be the reflections of O over the sides $\overline{BC}$, $\overline{CA}$ and $\overline{AB}$. Show that

1. O is the orthocenter and H is the circumcenter of $\triangle O_AO_BO_C$.
2. O_A, O_B and O_C are the circumcenters of $\triangle BHC$, $\triangle CHA$ and $\triangle AHB$.

Exercise 2.2 (IMO Shortlist 1989)**.** The vertex A of the acute triangle ABC is equidistant from the circumcenter O and the orthocenter H. Determine all possible values for the measure of angle A.

Exercise 2.3 (RMO 2025)**.** Let ABC be an acute-angled triangle with $AB < AC$, orthocenter H and circumcircle Ω . Let M be the midpoint of the minor arc BC of Ω . Suppose the MH is equal to the radius of Ω. Prove that $\angle BAC = 60^\circ$

Exercise 2.4. Let $\triangle ABC$ be an acute triangle such that $\angle A = 60^\circ$. Prove that $IH = IO$, where I, H and O are the incenter, orthocenter and circumcenter.

Exercise 2.5 (APMO 2007)**.** Let ABC be an acute angled triangle with $\angle BAC = 60^\circ$ and $AB > AC$. Let I be the incenter, and H the orthocenter of the triangle ABC . Prove that $2\angle AHI = 3\angle ABC$.

Exercise 2.6. Let triangle ABC satisfy $2BC = AB + AC$ and have incenter I and circumcircle ω. Let D be the intersection of AI and ω (with A, D distinct). Prove that I is the midpoint of AD.

Exercise 2.7 (IMO 2020)**.** Consider the convex quadrilateral $ABCD$. The point P is in the interior of $ABCD$. The following ratio equalities hold:

$$\angle PAD : \angle PBA : \angle DPA = 1 : 2 : 3 = \angle CBP : \angle BAP : \angle BPC$$

Prove that the following three lines meet in a point: the internal bisectors of angles $\angle ADP$ and $\angle PCB$ and the perpendicular bisector of segment AB.

Exercise 2.8. $\triangle ABC$ is inscribed in circle ω. A circle with chord BC intersects segments AB and AC again at S and R, respectively. Segments BR and CS meet at L, and rays LR and LS intersect ω at D and E, respectively. The internal angle bisector of $\angle BDE$ meets line ER at K. Prove that if $BE = BR$, then

1. R is the incenter of $\triangle CDE$.
2. $\angle ELK = \frac{1}{2}\angle BCD$.

Exercise 2.9 (Japan 2017)**.** Let ABC be an acute-angled triangle with the circumcenter O. Let D, E and F be the feet of the altitudes from A, B and C, respectively, and let M be the midpoint of BC. AD and EF meet at X, AO and BC meet at Y, and let Z be the midpoint of XY. Prove that A, Z, M are collinear.

Exercise 2.10 (IGO 2021 Advanced)**.** H is the orthocenter of the acute-angled triangle ABC. ($AB < AC$). Call the point where the perpendicular bisector of BC meets the sides AB and AC as P and Q, respectively. Consider M and N as the midpoints of segments BC and PQ. Prove that lines HM and AN intersect on the circumcircle of ABC.

Exercise 2.11 (IMO Shortlist 2024)**.** Let ABC be a triangle with $AB < AC < BC$. Let the incenter and incircle of triangle ABC be I and ω, respectively. Let X be the point on line BC different from C such that the line through X parallel to AC is tangent to ω. Similarly, let Y be the point on line BC different from B such that the line through Y parallel to AB is tangent to ω. Let AI intersect the circumcircle of triangle ABC at $P \neq A$. Let K and L be the midpoints of AC and AB, respectively. Prove that $\angle KIL + \angle YPX = 180°$.