1st AGO Shortlist G0

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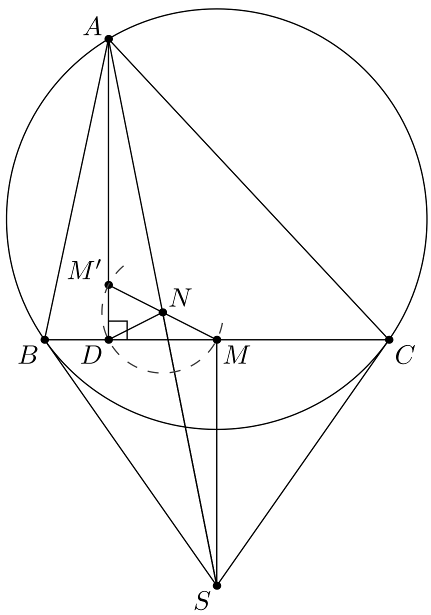
§1 Problem

Problem (1st AGO Shortlist G0)

Let ABC be an acute triangle, and let S be the intersection of the tangents from B and C to circumcircle of ABC. Let D be the projection of A onto line BC and M be the midpoint of BC, N be the midpoint of AS. Prove that $ND = NM$.

§2 Solution



Proof. Let M' be the reflection of M over N. Since $\overline{M'N} = \overline{NM}$ and $\overline{AN} = \overline{NS} \implies \triangle ANM' \cong \triangle SNM$ by SAS congruence criterion. This implies that $\overline{AM'} \parallel \overline{SM}$, but $\overline{SM} \perp \overline{BC} \implies \overline{AM'} \perp \overline{BC}$. From here we conclude that M' lies on $\overline{AD}$, but N is the midpoint of the hypotenuse of right-angled triangle $\triangle M'DM \implies N$ is the center of $\odot(M'DM)$. Thus, $\overline{ND} = \overline{DM}$. $\square$