IOQM Geometry Problems

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§1 2025

Problem 1.1. The area of an integer-sided rectangle is 20. What is the minimum possible value of its perimeter?

Problem 1.2. How many isoscleles integer-sided triangles are there with perimeter 23?

Problem 1.3. Four sides and a diagonal of a quadrilateral are of lengths $10, 20, 28, 50, 75$, not necessarily in that order. Which amongst them is the only possible length of the diagonal?

Problem 1.4. Three sides of a quadrilateral are $a = 4\sqrt{3}$, $b = 9$ and $c = \sqrt{3}$. The sides a and b enclose an angle of $30°$, and the sides b and c encose an angle of $90°$. If the acute angle between the diagonals is $x°$, what is the value of x?

Problem 1.5. $MTAI$ is a parallelogram of area $\frac{40}{41}$ square units such that $MI = 1/MT$. If d is the least possible length of the diagonal MA, and $d^2 = \frac{a}{b}$, where a, b are positive integers with $\gcd(a, b) = 1$, find $|a - b|$.

Problem 1.6. In triangle ABC, $\angle B = 90°, AB = 1$ and $BC = 2$. On the side BC there are two points D and E such that E lies between C and D and $DEFG$ is a square, where F lines of AC and G lies on the circle through B with centre A. If the area of $DEFG$ is $\frac{m}{n}$ where m and n are positive integers with $\gcd(m, n) = 1$, what is the value of $m + n$?

Problem 1.7. Let $ABCD$ be a rectangle and let M, N be points lying on the sides AB and BC respectively. Assume that $MC = CD$ and $MD = MN$, and the points C, D, M, N lie on a circle. if $(AB/BC)^2 = m/n$ where M and N are positive integers with $\gcd(m, n) = 1$, what is the value of $m + n$?

Problem 1.8. Let S be a circle of radius 10 with centre O. Suppose S_1 and S_2 are two circles which touch A internaly and intersect each other at two distinct points A and B. If $\angle OAB = 90°$ what is the sum of the radii of S_1 and S_2?

§2 2024

Problem 2.1. Let $ABCD$ be a quadrilateral with $\angle ADC = 70°$, $\angle ACD = 70°$, $\angle ACB = 10°$ and $\angle BAD = 110°$. The measure of $\angle CAB$ (in degrees) is:

Problem 2.2. Consider a square $ABCD$ of side length 16. Let E, F be points on CD such that $CE = EF = FD$. Let the line BF and AE meet in M. The area of $\triangle MAB$ is:

Problem 2.3. In a triangle ABC, $\angle BAC = 90^\circ$. Let D be the point on BC such that $AB + BD = AC + CD$. Suppose $BD : DC = 2 : 1$. if $\frac{AC}{AB} = \frac{m+\sqrt{p}}{n}$, Where m, n are relatively prime positive integers and p is a prime number, determine the value of $m + n + p$.

Problem 2.4. In a triangle ABC, a point P in the interior of ABC is such that

$$\angle BPC - \angle BAC = \angle CPA - \angle CBA = \angle APB - \angle ACB.$$

Suppose $\angle BAC = 30^\circ$ and $AP = 12$. Let D, E, F be the feet of perpendiculars from P on to BC, CA, AB respectively. If $m\sqrt{n}$ is the area of the triangle DEF where m, n are integers with n prime, then what is the value of the product mn?

Problem 2.5. Let ABC be a right-angled triangle with $\angle B = 90^\circ$. Let the length of the altitude BD be equal to 12. What is the minimum possible length of AC, given that AC and the perimeter of triangle ABC are integers?

§3 2023

Problem 3.1. In a triangle ABC, let E be the midpoint of AC and F be the midpoint of AB. The medians BE and CF intersect at G. Let Y and Z be the midpoints of BE and CF respectively. If the area of triangle ABC is 480 , find the area of triangle GYZ.

Problem 3.2. The ex-radii of a triangle are $10\frac{1}{2}, 12$ and 14. If the sides of the triangle are the roots of the cubic $x^3 - px^2 + qx - r = 0$, where p, q, r are integers , find the nearest integer to $\sqrt{p+q+r}$.

Problem 3.3. Let ABC be a triangle in the xy plane, where B is at the origin $(0,0)$. Let BC be produced to D such that $BC : CD = 1 : 1, CA$ be produced to E such that $CA : AE = 1 : 2$ and AB be produced to F such that $AB : BF = 1 : 3$. Let $G(32, 24)$ be the centroid of the triangle ABC and K be the centroid of the triangle DEF. Find the length GK.

Problem 3.4. Let $ABCD$ be a unit square. Suppose M and N are points on BC and CD respectively such that the perimeter of triangle MCN is 2 . Let O be the circumcentre of triangle MAN, and P be the circumcentre of triangle MON. If $\left(\frac{OP}{OA}\right)^2 = \frac{m}{n}$ for some relatively prime positive integers m and n, find the value of $m + n$

Problem 3.5. In the coordinate plane, a point is called a lattice point if both of its coordinates are integers. Let A be the point $(12, 84)$. Find the number of right angled triangles ABC in the coordinate plane B and C are lattice points, having a right angle at vertex A and whose incenter is at the origin $(0,0)$.

§4 2022

Problem 4.1. A triangle ABC with $AC = 20$ is inscribed in a circle ω. A tangent t to ω is drawn through B. The distance t from A is 25 and that from C is 16.If S denotes the area of the triangle ABC, find the largest integer not exceeding $\frac{S}{20}$

Problem 4.2. In a paralleogram $ABCD$, a point P on the segment AB is taken such that $\frac{AP}{AB} = \frac{61}{2022}$ and a point Q on the segment AD is taken such that $\frac{AQ}{AD} = \frac{61}{2065}$.If PQ intersects AC at T, find $\frac{AC}{AT}$ to the nearest integer

Problem 4.3. In a trapezoid $ABCD$, the internal bisector of angle A intersects the base BC(or its extension) at the point E. Inscribed in the triangle ABE is a circle touching the side AB at M and side BE at the point P. Find the angle DAE in degrees, if $AB : MP = 2$.

Problem 4.4. Two sides of an integer sided triangle have lengths 18 and x. If there are exactly 35 possible integer y such that $18, x, y$ are the sides of a non-degenerate triangle, find the number of possible integer values x can have.

Problem 4.5. Let AB be diameter of a circle ω and let C be a point on ω, different from A and B. The perpendicular from C intersects AB at D and ω at $E(\neq C)$. The circle with centre at C and radius CD intersects ω at P and Q. If the perimeter of the triangle PEQ is 24, find the length of the side PQ

Problem 4.6. Given $\triangle ABC$ with $\angle B = 60^\circ$ and $\angle C = 30^\circ$, let P, Q, R be points on the sides BA, AC, CB respectively such that $BPQR$ is an isosceles trapezium with $PQ \parallel BR$ and $BP = QR$. Find the maximum possible value of $\frac{2[ABC]}{[BPQR]}$ where $[S]$ denotes the area of any polygon S.

Problem 4.7. Let ABC be a triangle and let D be a point on the segment BC such that $AD = BC$. Suppose $\angle CAD = x^\circ, \angle ABC = y^\circ$ and $\angle ACB = z^\circ$ and x, y, z are in an arithmetic progression in that order where the first term and the common difference are positive integers. Find the largest possible value of $\angle ABC$ in degrees.

Problem 4.8. In a triangle ABC, the median AD divides $\angle BAC$ in the ratio $1 : 2$. Extend AD to E such that EB is perpendicular AB. Given that $BE = 3, BA = 4$, find the integer nearest to BC^2.

§5 2021

Problem 5.1. Three parallel lines L_1, L_2, L_2 are drawn in the plane such that the perpendicular distance between L_1 and L_2 is 3 and the perpendicular distance between lines L_2 and L_3 is also 3. A square $ABCD$ is constructed such that A lies on L_1, B lies on L_3 and C lies on L_2. Find the area of the square.

Problem 5.2. Consider the set $\mathcal{T}$ of all triangles whose sides are distinct prime numbers which are also in arithmetic progression. Let $\triangle \in \mathcal{T}$ be the triangle with least perimeter. If a° is the largest angle of $\triangle$ and L is its perimeter, determine the value of $\frac{a}{L}$.

Problem 5.3. In parallelogram $ABCD$, the longer side is twice the shorter side. Let $XYZW$ be the quadrilateral formed by the internal bisectors of the angles of $ABCD$. If the area of $XYZW$ is 10, find the area of $ABCD$

§6 2020

Problem 6.1. Let $ABCD$ be a trapezium in which $AB \parallel CD$ and $AB = 3CD$. Let E be then midpoint of the diagonal BD. If $[ABCD] = n \times [CDE]$, what is the value of n?

Problem 6.2. Let $ABCD$ be a rectangle in which $AB + BC + CD = 20$ and $AE = 9$ where E is the midpoint of the side BC. Find the area of the rectangle.

Problem 6.3. Let $\triangle ABC$ be a triangle with $AB = AC$. Let D be a point on the segment BC such that $BD = 48\frac{1}{61}$ and $DC = 61$. Let E be a point on AD such that

CE is perpendicular to AD and $DE = 11$. Find AE.

Problem 6.4. Let $\triangle ABC$ be a triangle with $AB = 5, AC = 4, BC = 6$. The internal angle bisector of C intersects the side AB at D. Points M and N are taken on sides BC and AC, respectively, such that $DM \parallel AC$ and $DN \parallel BC$. If $(MN)^2 = \frac{p}{q}$ where p and q are relatively prime positive integers then what is the sum of the digits of $|p - q|$?

Problem 6.5. Given a pair of concentric circles, chords $AB, BC, CD, \ldots$ of the outer circle are drawn such that they all touch the inner circle. If $\angle ABC = 75°$, how many chords can be drawn before returning to the starting point ?

Problem 6.6. The sides x and y of a scalene triangle satisfy $x + \frac{2\Delta}{x} = y + \frac{2\Delta}{y}$, where Δ is the area of the triangle. If $x = 60, y = 63$, what is the length of the largest side of the triangle?

Problem 6.7. Let $ABCD$ be a parallelogram. Let E and F be the midpoints of sides AB and BC respectively. The lines EC and FD intersect at P and form four triangles APB, BPC, CPD, DPA. If the area of the parallelogram is 100, what is the maximum area of a triangles among these four triangles?

Problem 6.8. In triangle ABC, let P and R be the feet of the perpendiculars from A onto the external and internal bisectors of $\angle ABC$, respectively; and let Q and S be the feet of the perpendiculars from A onto the internal and external bisectors of $\angle ACB$, respectively. If $PQ = 7, QR = 6$ and $RS = 8$, what is the area of triangle ABC?

Problem 6.9. The incircle Γ of a scalene triangle ABC touches BC at D, CA at E and AB at F. Let r_A be the radius of the circle inside ABC which is tangent to Γ and the sides AB and AC. Define r_B and r_C similarly. If $r_A = 16, r_B = 25$ and $r_C = 36$, determine the radius of Γ.

Problem 6.10. A light source at the point $(0, 16)$ in the co-ordinate plane casts light in all directions. A disc(circle along ith it's interior) of radius 2 with center at $(6, 10)$ casts a shadow on the X-axis. The length of the shadow can be written in the form $m\sqrt{n}$ where m, n are positive integers and n is squarefree. Find $m + n$.

Problem 6.11. A bug travels in the co-ordinate plane moving along only the lines that are parallel to the X and Y axes.Let $A = (-3, 2)$ and $B = (3, -2)$. Consider all possible paths of the bug from A to B.How many lattice points lie on at least one of these paths.

Problem 6.12. If $ABCD$ is a rectangle and P is a point inside it such that $AP = 33$, $BP = 16$, $DP = 63$. Find CP.

Problem 6.13. Let ABC be an isosceles triangle with $AB = AC$ and incentre I. If $AI = 3$ and the distance from I to BC is 2, what is the square of length on BC?

Problem 6.14. Let $ABCD$ be a square with side length 100. A circle with centre C and radius CD is drawn. Another circle of radius r, lying inside $ABCD$, is drawn to touch this circle externally and such that the circle also touches AB and AD. If $r = m + n\sqrt{k}$, where m, n are integers and k is a prime number, find the value of $\frac{m+n}{k}$.

Problem 6.15. The sides of triangle are x, $2x + 1$ and $x + 2$ for some positive rational x. Angle of triangle is 60 degree. Find perimeter.

Problem 6.16. Let ABC be an equilateral triangle with side length 10. A square $PQRS$ is inscribed in it, with P on AB, Q, R on BC and S on AC. If the area of the square

$PQRS$ is $m + n\sqrt{k}$ where m, n are integers and k is a prime number then determine the value of $\sqrt{\frac{m+n}{k^2}}$.

Problem 6.17. Two sides of a regular polygon of n sides when extended meet at 28 degrees. What is smallest possible value of n.

Problem 6.18. Let D, E, F be points on the sides BC, CA, AB of a triangle ABC, respectively. Suppose AD, BE, CF are concurrent at P. If $PF/PC = 2/3, PE/PB = 2/7$ and $PD/PA = m/n$, where m, n are positive integers with $gcd(m, n) = 1$, find $m + n$.

Problem 6.19. A semicircular paper is folded along a chord such that the folded circular arc is tangent to the diameter of the semicircle. The radius of the semicircle is 4 units and the point of tangency divides the diameter in the ratio $7:1$. If the length of the crease (the dotted line segment in the figure) is ℓ then determine ℓ^2.

Problem 6.20. Let ABC be a triangle with $\angle BAC = 90^o$ and D be the point on the side BC such that $AD \perp BC$. Let r, r_1, and r_2 be the inradii of triangles ABC, ABD, and ACD, respectively. If r, r_1, and r_2 are positive integers and one of them is 5, find the largest possible value of $r + r_1 + r_2$.

Problem 6.21. Two circles S_1 and S_2, of radii 6 units and 3 units respectively, are tangent to each other, externally. Let AC and BD be their direct common tangents with A and B on S_1, and C and D on S_2. Find the area of quadrilateral $ABDC$ to the nearest Integer.

Problem 6.22. Let ABC be an acute-angled triangle and P be a point in its interior. Let P_A, P_B and P_c be the images of P under reflection in the sides BC, CA, and AB, respectively. If P is the orthocentre of the triangle $P_AP_BP_C$ and if the largest angle of the triangle that can be formed by the line segments$PA, PB.$ and PC is x^o, determine the value of x.