Monte Carlo Integration Techniques

Prepare a tutorial on Monte Carlo integration techniques and give several examples

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Abstract

The purpose of this project is to implement Monte Carlo Integration techniques to explore and simulate different hypothetical portfolios' returns over a specific length of time. Through different simulations, the project will showcase various techniques to implement Monte Carlo simulation based on their specific needs when it comes to investing. Due to the nature of Monte Carlo Simulation, this project only focuses on the performance of each stock, excluding dividends, reinvestment, and periodic contribution, while assuming that price reflects macroeconomic events. The findings showcase how to use Monte Carlo Simulation to model a few different hypothetical portfolio mixes to assess the risks and rewards in a medium timeframe horizon.

Background

I have always been fascinated with the stock markets. Since I started working a job after my undergraduate studies, I have started to pay more attention to it because now I can invest in 401K and IRA. There are many different theories about the market in general, but the one I like the most is the "random walk theory", which suggests that the changes in asset prices are truly random, and it's very challenging to time the market. **(Malkie, B., 2003)** Nevertheless, for an average investor(s) like myself with over 30 years time frame horizon, the question is if I participate in the market in some way, how can I assess the risks and the rewards of different portfolio mixes? Would a portfolio of the top 7 (MAG7) be better than the index (SPY)? Is there a balance between assuming a certain level of risk while still being able to capture exponential growth? In this sense, I'll run various Monte Carlo Simulation models on a different basket of stocks to estimate the growth, volatility, and risks.

Monte Carlo - Introduction

Monte Carlo Simulation is a mathematical technique used to estimate possible outcomes of uncertain events through repeated simulation. The methodology was invited by John von Neumann and Stanislaw Ulam during World War II to improve the decision-making process under uncertain conditions. **(IBM, 2024)** The core modeling approach is element of chance. Monte Carlo Simulation has been used to assess the impact of risks in many real-life scenarios, such as stock prices, sales forecasts, weather events, etc.

Flipping coin

One of the more common examples is flipping a coin; as we know, there are only two possible outcomes: head or tail. Each possible outcome has an even 50% chance. Nevertheless, as someone who previously tried my luck in the casino before, sometimes, it just never go my way. Here is a demo of flipping the coin 10 times versus 5000 times.

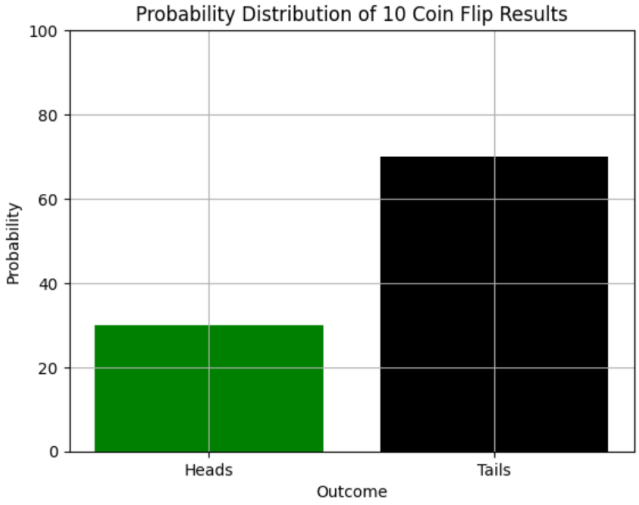

Figure 1: Flipping a coin 10 Times

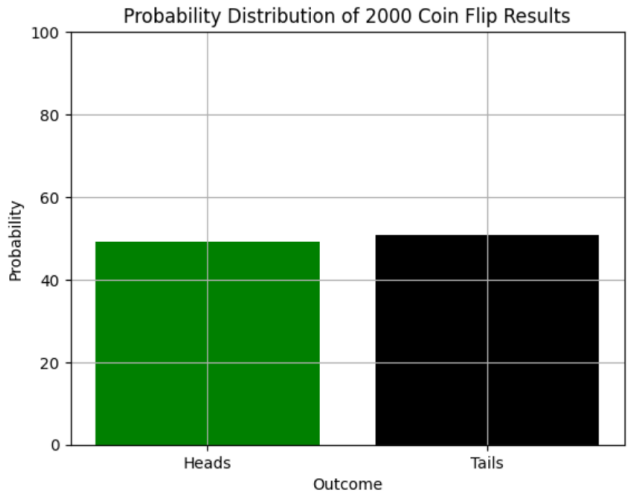

Figure 2: Flipping a coin 2000 Times

The simulation shows that because the true value (true probability) of either outcome is 50%, the more simulation we run, the closer it will get to the true values. Inherently, Monte Carlo Simulation employ a very powerful theory in statistic, the Law of Large Number.

Monte Carlo Simulation - Pulling the data

The code that are deployed as the backbone of this simulation has three portions. The first class is to collect all the necessary data of your portfolio basket. This will pull data directly from Yahoo Finance API and calculate the average daily change and average daily volatility of those individual stocks. Then the second part of that class will construct a portfolio based on equal distribution of the initial investment. Lastly, the output of the first two functions will feed into the simulation itself, producing a price matrix of the simulation based on the time frame horizon and number of runs. The second class will assist with printing out charts and statistics for analysis. In addition, I also include a helper function to easily re-run the simulation with new parameters.

Monte Carlo Simulation - SPY

SPY - Introduction

To begin, we will explore Monte Carlo simulation on one single stock: the S&P 500 Index, ticker symbol: **SPY**. The Standards and Poor's 500 Index is a market-capitalization-weighted index of the leading 500 publicly traded companies in the United States. Please note the index has 503 components due to three of them having two share classes listed. To qualify for the S&P 500, a company must be a U.S Companies that are publicly traded on the major exchange such as New York Stock Exchanges or Nasdaq. The company must also have a market capitalization of at least $8.2 billion, with at least 50 percent shares float, and have a daily volume of at least 250,000 in the six months prior to inclusion. **(Royal and Beers, 2024)**

I chose **SPY** for this part because the ETF aim to track a diversified, strong companies within the U.S Stock Market. Institutional index that aims to seek the return of S&P 500 have their own index, such as **VOO** (Vanguar S&P 500 ETF) or **IVV** (BlackRock iShare ETF).

The annualized return of the S&P 500 are as follows:

- **1993 - 2023 (30 years)**: 9.98% per year, adjusted for inflation is 7.33% per year.
- **2003 - 2023 (20 years)**: 10.20% per year, adjusted for inflation is 7.55% per year.
- **2013 - 2023 (10 years)**: 13.05% per year, adjusted for inflation is 10.24% per year.

(Lake, 2023)

Model with fixed parameters

First, I run a Monte Carlo Simulation based on fixed parameters.

- Initial price = $63.72
- Average annual return: 9.98%
- Average monthly volatility: 11.89%
- Number of simulations: 1000
- Time Frame: 1 year, 252 trading days per year

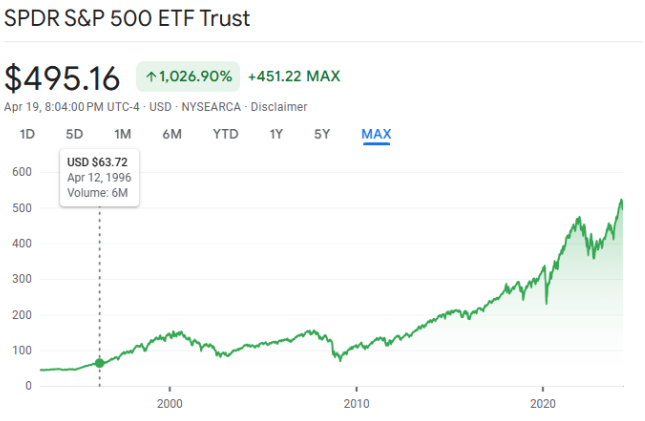

Figure 3: SPY

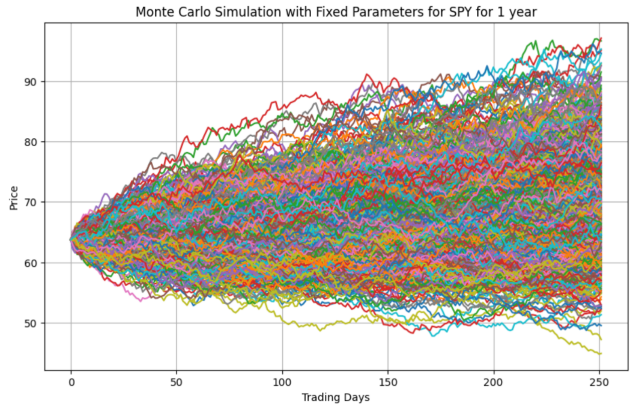

Figure 4: Simulation 1a

CAGR - Compound Annual Growth Rate

The annualized return shows that we are in line with the historical annualized return. Next, I'll run the model with a time frame of 30 years. The model and statistic are as followed:

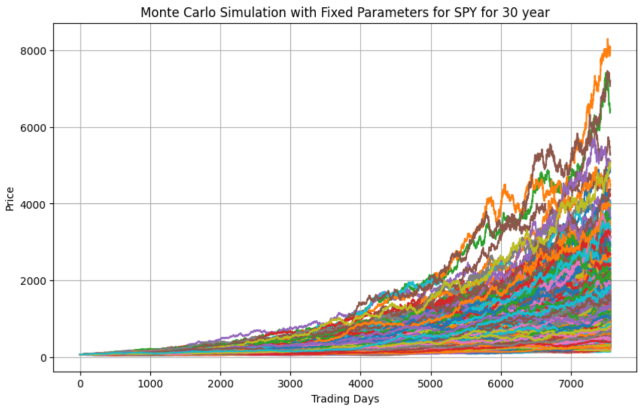

Figure 5: Simulation 1b

The average final price is significantly higher than the current price of SPY, which is $500 (7% CAGR) at the time of this writing. The simulation shows that even with a slight percentage difference, the compound return over a long period of time will be heavily affected exponentially.

Warren Buffet's portfolio

Unlike the previous example of running a simulation on one stock's price with fixed parameters, it would be more realistic to use the model to simulate a hypothetical portfolio. Since Warren Buffet is my idol, I'd like to "borrow" his ideas and top investments. *Note all of the hypothetical portfolio will be simulated with a 5 year timeframe due to limited computer resources*

Currently, the top 10 holdings of Berkshire Hathaway are as follows:

- Apple Inc. (AAPL) - 41.4%
- Bank of America Corp. (BAC) - 10.4%
- American Express Co. (AXP) - 9.3%
- Coca-Cola Co. (KO) - 9.3%
- Chevron Corp (CVX) - 6.5%
- Occidental Petroleum Corporation (OXY) - 4.5%
- Kraft Heinz Co. (KHC) - 3.3%
- Moody's Corporation (MCO) 2.6%
- Mitsubishi Corp (MTSUY) - 2.2%
- Mitsui & Co. (MITSY) - 1.6%

The following two simulations are run with equal allocation and custom allocation (model according to the actual distribution above). Note that custom allocation (CA) model adds up to 91.1% of the initial investment, leaving out a portion as cash.

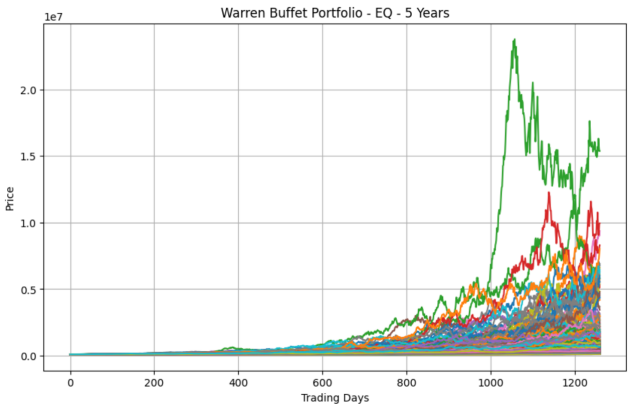

Figure 6: Warren Buffet's Portfolio - Equal allocation

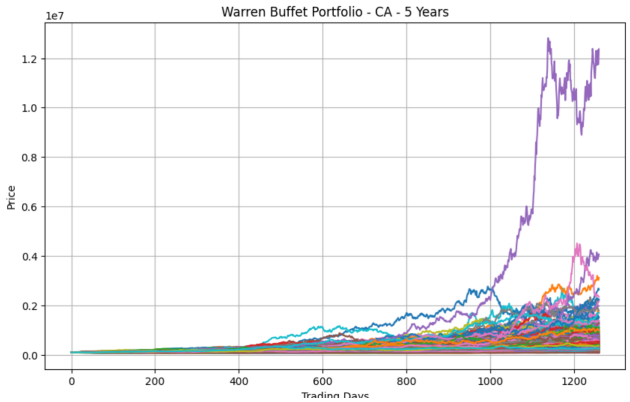

Figure 7: Warren Buffet's Portfolio: Custom Allocation

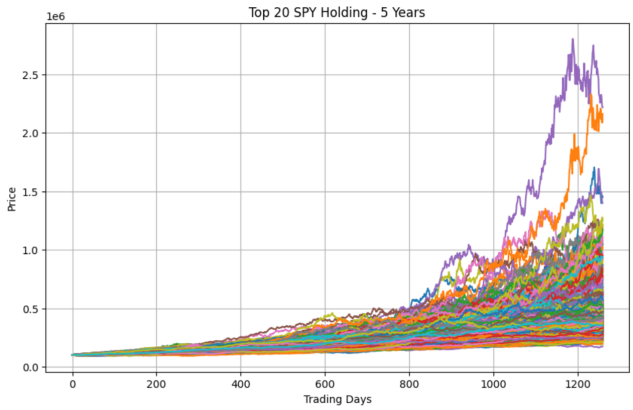

Figure 8: Top 20 SPY Holdings

Given the high annualized return and standard deviation (in the millions) of the first model, I'm genuinely surprised at this result. While it is not a definitive statement that one should invest all their funds, the model shows how powerful the difference between roughly $10,000 of initial investment could be in the long run. The differences in return (1̃7% CAGR) and volatility ($̃600,000 in standard deviation) is significant. Both portfolios have an average weighted beta** of roughly 1, closely tracking the market.

Beta *- a measure of volatility, or systematic risks, of a security or portfolio compared to the market as a whole. The S&P-500 has a beta of 1. Beta of 1 or higher can be interpreted as more volatile than the market.* (Kenton et al, 2024)

Strong companies versus Random companies

In the following few simulations, we will explore several ways to implement Monte Carlo Simulation to different baskets of stocks, and assess the risks versus rewards over a 5 years horizon.

Top 20 SPY Holding

I am picking the top 20 SPY holding by weighted for this basket. Over the years, these companies have proven to be the top performers of the S&P-500 and remain on top. The portfolio's beta is 0.9847

Including MSFT, AAPL, NVDA, AMZN, META, GOOGL, LLY, AVGO, JPM, XOM, UNH, TSLA, V, MA, PG, JNJ, HD, COST, MRK, ABBV

Random 20 SPY Holding

- Beta: 1.1014 From the list of 500 companies in the S&P-500, the script randomly picks 20 and runs the simulations. The first observation is the chart is very volatile as compared to the Top 20. Additionally, The annual growth rate is significantly less (34.73% versus 23.21%). From this analysis, one can base the case that by picking the top 20 SPY Holdings is a solid strategy.

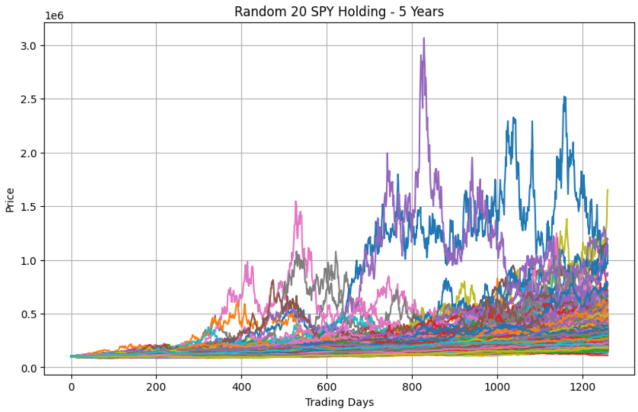

Figure 9: Random 20 SPY Holdings

Including BDX, EBAY, ALLE, TSN, BR, CDNS, TFX, REG, FFIV, HUBB, ZTS, BKR, LH, MRK, PXD, CZR, ABNB, BLK, HUM, NKE

Concentrated Portfolio

There are many ways to invest and build a portfolio. In recent years, there have been a few stocks that have

continued to grow at an exponential rate. As of 2024, the 7 stocks (dubbed Magnificent Seven) are GOOGL, AMZN, AAPL, META, MSFT, NVDA and TSLA. This simulation tries to see that if you invest in the top 7 companies in the world, would your reward be significantly higher, and would your portfolio's risks be also significantly higher due to the concentration in only seven stocks?

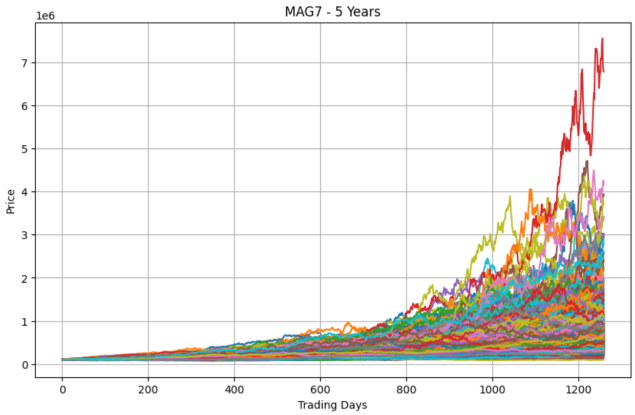

Figure 10: Magnificent Sevens

There are significantly volatility with the magnificent seven, when standard deviation is upward to the $500,000 range. However, the annualized return is also significantly higher at 47.08%

RESULT

The result is rather surprising for me as Warren Buffet's portfolio with equal allocation has a very high volatility with a standard deviation toward the millions. While the whole portfolio's average beta is roughly 1. The Nasdaq basket also has a very high standard deviation and growth rate, which a risk-averse investor, they should avoid. While investing means aiming for the best potential return, from the above simulations, we can also notice that the Top 20 S&P-500 has a very strong annual growth rate while maintaining medium volatility.

Model	Average final price	Median final price	Standard deviation	95% CI lower bound	95% CI upper bound	CI Range	Hypothetical Beta	
SPY	$173,437.00	$159,190.00	$76,141.00	$69,639.00	$358,141.00	$288,502.00	11.64%	1
Warren Buffet - Equal Allocation	$717,694.00	$383,569.00	$1,109,380.00	$148,965.00	$3,451,824.00	$3,302,859.00	48.32%	1.0659
Warren Buffet - Custom Allocation	$386,946.00	$278,454.00	$501,333.00	$111,793.00	$1,252,844.00	$1,141,051.00	31.08%	1.0659
Top 20 S&P 500	$443,868.00	$399,595.00	$183,871.00	$241,155.00	$241,155.00	$934,790.00	34.73%	0.9847
Random 20 S&P 500	$283,939.00	$257,949.00	$123,159.00	$159,862.00	$592,993.00	$433,131.00	23.21%	1.0753
Random 20 Nasdaq	$1,602,059.00	$507,454.00	$4,548,148.00	$216,204.00	$11,541,402.00	$11,325,198.00	74.15%	1.1456
MAG7	$688,247.00	$545,402.00	$524,014.00	$205,410.00	$2,068,730.00	$1,863,320.00	47.08%	1.4491

Figure 11: Result Table

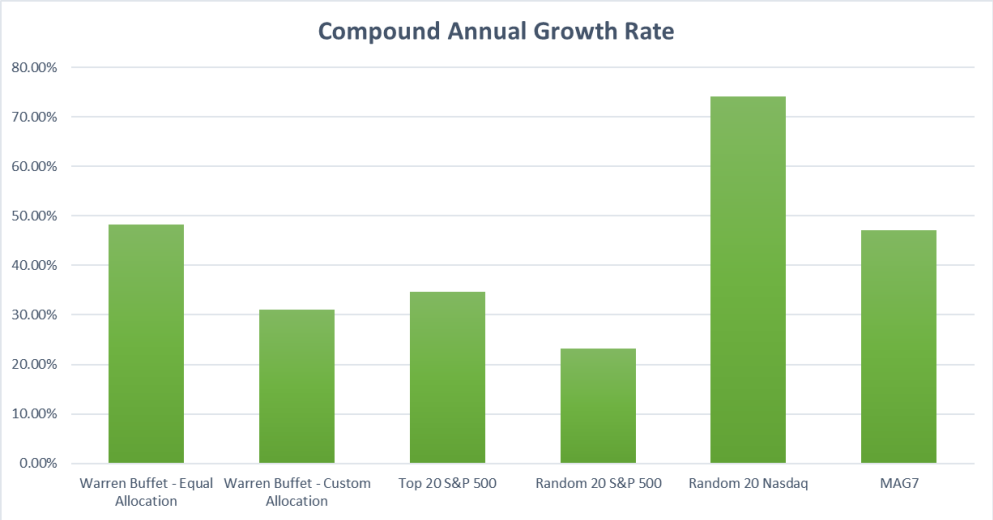

Figure 12: CAGR Chart

To summarize, the purpose of the project is to employ various Monte Carlo Integration Techniques and run a few different simulations across different sets of baskets in the financial market to determine the characteristics of each basket. While it is arguable that no one can accurately predict the market based on random-walk theory, Monte Carlo Simulation offers a different perspective to look at risks versus rewards in investing. As such, a young investor may look toward high returns with high risks, can invest in Technology sectors, or Magnificent Seven, or even modeling Warren Buffet's portfolio, whereas a more seasoned investor who looks for modest rewards with limited downside risks could opt for just investing in the index.

Future Consideration

There are a couple of areas that I'd like to explore more given a chance, such as significant sampling, where the emphasis of the model is based upon certain macroeconomic events, such as a period of low interest rate or a period of high interest rate, to accurately reflect the risks for each portfolio.

REFERENCES

[1] C. Davis. “About this investment calculator”. In: (2023). URL: https://www.nerdwallet.com/calculator/investment-calculator.

[2] IBM. “What is Monte Carlo Simulation?” In: (2024). URL: https://www.ibm.com/topics/monte-carlo-simulation.

[3] W Kenton, M James, and S. Kvilhaug. “Monte Carlo Simulation: What It Is, History, How It Works, and 4 Key Steps”. In: (2023). URL: https://www.investopedia.com/terms/m/montecarlosimulation.asp.

[4] W. Kenton, R. Kelly, and A. Bellucco-Chatham. “Beta: Definition, Calculation, and Explanation for Investors”. In: (2024). URL: https://www.investopedia.com/terms/b/beta.asp.

[5] R. Lake. “What Is the Average Stock Market Return?” In: (2023). URL: https://www.sofi.com/learn/content/average-stock-market-return/#:~:text=Looking%20at%20the%20S%26P%20500%20for%20the%20years%201993%20to,rates%20for%20five%20consecutive%20years..

[6] B. Malkie. *A random walk down Wall Street : the time-tested strategy for successful investing.* New York :W.W. Norton.

[7] J. Royal and B. Beers. “What is the SP 500?” In: (2024). URL: https://www.bankrate.com/investing/what-is-sp-500/#:~:text=Must%20be%20a%20U.S.%20company,York%20Stock%20Exchange%20or%20Nasdaq..