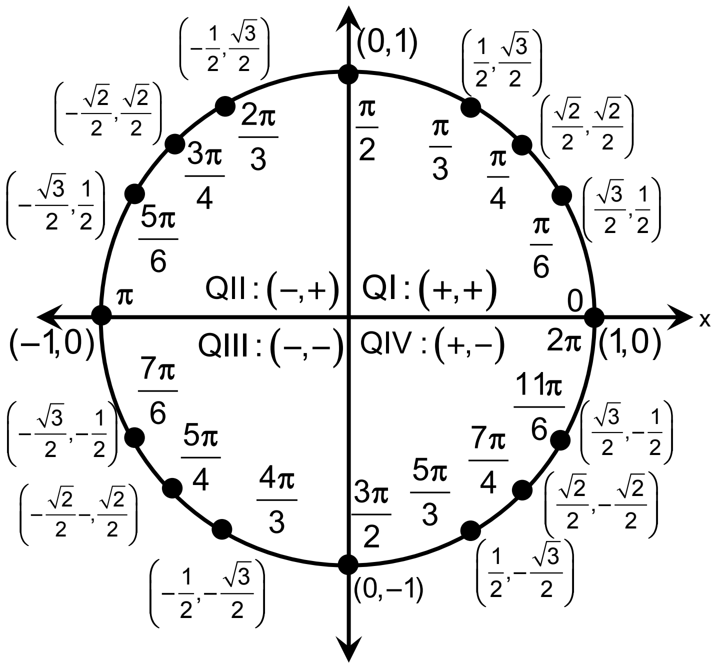
Trigonometric Identities:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	$\sin^2\theta + \cos^2\theta = 1$	$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$
$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$	$\tan^2\theta + 1 = \sec^2\theta$	$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$
$\csc\theta = \dfrac{1}{\sin\theta}$	$\cot^2\theta + 1 = \csc^2\theta$	$\sin(-\theta) = -\sin\theta$
$\sec\theta = \dfrac{1}{\cos\theta}$	$\sin(2\theta) = 2\sin\theta\cos\theta$	$\cos(-\theta) = \cos\theta$
$\cot\theta = \dfrac{1}{\tan\theta}$	$\cos(2\theta) = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= 1 - 2\sin^2\theta$	$\tan(-\theta) = -\tan\theta$

The Unit Circle:



Each point is (cos(θ),sin(θ))

Limit Definition of a Derivative

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Alternate Definition (at a Point)

$$f'(a) = \lim_{b\to a}\frac{f(b)-f(a)}{b-a}$$

Product Rule:

$$\frac{d}{dx}(fg) = f\frac{dg}{dx} + g\frac{df}{dx}$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{Hi}{Lo}\right) = \frac{Lo\ dHi - Hi\ dLo}{Lo\ Lo}$$

Chain Rule:

$$(f\circ g)'(x) = f'(g(x))\bullet g'(x) \quad \text{OR} \quad \frac{dy}{dx} = \frac{dy}{du}\bullet\frac{du}{dx}$$

Power Rule:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Derivative of a Constant:

$$\frac{d}{dx}(c) = 0$$

Exponential/Logarithmic Derivatives:

$$\frac{d}{dx}(a^u) = \ln(a)a^u u' \qquad \frac{d}{dx}(e^u) = \frac{du}{dx}e^u$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{\ln(a)}\frac{u'}{u} \qquad \frac{d}{dx}(\ln u) = \frac{u'}{u}$$

Inverse Function:

$$\frac{d}{dx}\left(f^{-1}(x)\right) = \frac{1}{f'(f^{-1}(x))}$$

Intermediate Value Theorem: If a and b are any two points in an interval on which f is **continuous**, then **f** takes on every value between f(a) and f(b).

Intermediate Value Theorem for Derivatives: If a and b are any two points in an interval on which f is **differentiable**, then the derivative **f '** takes on every value between f '(a) and f '(b).

Average Rate of Change of a function f on [a,b]

$$\frac{f(b)-f(a)}{b-a}$$

Instantaneous Rate of Change of a function of f at x = a:

$$\lim_{b\to a}\frac{f(b)-f(a)}{b-a} = f'(a)$$

Mean Value Theorem (for Derivatives): If y = f(x) is continuous at every point of the closed interval [a,b] and differentiable at every point of its interior (a,b), then there is at least one point c in (a,b) such that

$$f'(c) = \lim_{b\to a}\frac{f(b)-f(a)}{b-a} \quad \text{(at some point "AROC" = "IROC")}$$

Top 20 Integrals (1-10):

1) $\int a\,dx = ax + c$
2) $\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + c \quad (n \neq -1)$
3) $\int \dfrac{1}{x}\,dx = \ln|x| + c$
4) $\int e^x\,dx = e^x + c$
5) $\int a^x\,dx = \dfrac{a^x}{\ln a} + c$
6) $\int \ln x\,dx = x\ln x - x + c$
7) $\int \sin x\,dx = -\cos x + c$
8) $\int \cos x\,dx = \sin x + c$
9) $\int \tan x\,dx = \ln|\sec x| + c$
10) $\int \cot x\,dx = \ln|\sin x| + c$

Top 20 Integrals (11-20):

11) $\int \sec x\,dx = \ln|\sec x + \tan x| + c$
12) $\int \csc x\,dx = -\ln|\csc x - \cot x| + c$
13) $\int \sec^2 x\,dx = \tan x + c$
14) $\int \sec x\tan x\,dx = \sec x + c$
15) $\int \csc^2 x\,dx = -\cot x + c$
16) $\int \csc x\cot x\,dx = -\csc x + c$
17) $\int \tan^2 x\,dx = \tan x - x + c$
18) $\int \dfrac{1}{a^2+x^2}\,dx = \dfrac{1}{a}\tan^{-1}\dfrac{x}{a} + c$
19) $\int \dfrac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\dfrac{x}{a} + c$
20) $\int \dfrac{1}{x\sqrt{x^2-a^2}}\,dx = \dfrac{1}{a}\sec^{-1}\left|\dfrac{x}{a}\right| + c$

Average Value of a Function on an Interval [a,b]:

$$\text{Average Value of } f(x) \text{ on } [a,b] = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Mean Value Theorem (for Definite Integrals):

If f is continuous on [a,b], then at some point c in [a,b],

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx$$

(At some point, the function equals its average value on the interval.)

Fundamental Theorem of Calculus (Part 1):

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x) \quad \text{OR} \quad \frac{d}{dx}\int_a^u f(t)\,dt = u'f(u)$$

Fundamental Theorem of Calculus (Part 2):

$$\int_a^b f(x)\,dx = F(b) - F(a) \qquad [F(x) \text{ is an antiderivative of } f(x)]$$

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

Properties of Definite Integrals:

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

$$\int_a^a f(x)\,dx = 0$$

$$\int_a^b k\cdot f(x)\,dx = k\int_a^b f(x)\,dx$$

$$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b \left(f(x) \pm g(x)\right)dx = \int_b^a f(x)\,dx \ \pm \ \int_b^a g(x)\,dx$$

$$\int_a^c f(x)\,dx = \int_a^b f(x)\,dx + \int_b^c f(x)\,dx$$

Improper Integrals, f(x) Continuous on [a,∞):

$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$$

L'Hopital's Rule for Indeterminate Limits:

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Exponential Growth/Decay:

Differential Equation: $\dfrac{dy}{dt} = ky$

Exponential Growth Equation: $y = y_0 e^{kt}$

Logistic Growth/Decay:

Differential Equation: $\dfrac{dP}{dt} = kP(M-P)$

Logistic Equation: $P(t) = \dfrac{M}{1+Ae^{-(Mk)t}}$

(k is the growth/decay constant)

(M is the carrying capacity)

(A is a constant you must solve for)

- The population is growing the fastest when P is half of the carrying capacity!
- As t tends to infinity, the population tends to the carrying capacity:

$$\lim_{t\to\infty} P(t) = M$$

Volume (Discs):

$$V_{\text{discs about x-axis}} = \pi\int_a^b f(x)^2\,dx$$

$$V_{\text{discs about y-axis}} = \pi\int_c^d f(y)^2\,dy$$

Volume (Shells):

$$V_{\text{shells about x-axis}} = 2\pi\int_c^d y\,f(y)\,dy$$

$$V_{\text{shells about y-axis}} = 2\pi\int_a^b x\,f(x)\,dx$$

Volume (Cross Sections):

$$V_{\text{cross sections}\perp x\text{-axis}} = \int_a^b A(x)\,dx$$

$$V_{\text{cross sections}\perp y\text{-axis}} = \int_c^d A(y)\,dy$$

Arc Length (y a function of x):

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Arc Length (x a function of y):

$$L = \int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Arc Length (Parameterized Curve):

$$L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

Finding dy/dx (Slope) for a Parametric Curve:

For $\frac{dx}{dt} \neq 0$, we have:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad \left(\frac{dx}{dt}\neq 0\right)$$

Surface Area (Rotate About x-axis):

$$SA = 2\pi\int_a^b f(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

Surface Area (Rotate About y-axis):

$$SA = 2\pi\int_c^d f(y)\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Surface Area (Rotate Parameterized Curve About The x-axis):

$$SA = 2\pi\int_{t_1}^{t_2} y\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

Finding the Second Derivative (Parametrics):

Let $y' = \frac{dy}{dx}$, then for $\frac{dx}{dt} \neq 0$:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(y') = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

Position, Velocity, and Acceleration Vectors:

$$r(t) = \langle x(t), y(t)\rangle$$

$$v(t) = \langle x'(t), y'(t)\rangle$$

$$a(t) = \langle x''(t), y''(t)\rangle$$

Speed at time t_0:

$$|v(t_0)| = \sqrt{x'(t_0)^2 + y'(t_0)^2}$$

Total Distance, $t \in [a,b]$:

$$\text{Total Distance} = \int_a^b |v(t)|\,dt = \int_a^b \sqrt{(v_1(t))^2+(v_2(t))^2}\,dt$$

Displacement, $t \in [a,b]$:

$$\left\langle \int_a^b v_1(t)\,dt, \int_a^b v_2(t)\,dt \right\rangle$$

Final Position, $t \in [a,b]$:

$$\langle x(a), y(a)\rangle + \left\langle \int_a^b v_1(t)\,dt, \int_a^b v_2(t)\,dt \right\rangle$$

Linearization:

If f is differentiable at x = a, then the equation of the tangent line,

$$L(x) = f(a) + f'(a)(x-a),$$

defines the linearization of f at a. The standard linear approximation of f at a is $f(x) \approx L(x)$. The point x = a is the center of the approximation. This is just a Degree 1 Taylor Series approximation of f at a!!

Newton's Method for Approximating a Solution to f(x) = 0:

1. Guess a first approximation to a solution of the equation f(x) = 0.
2. Use the first approximation to get a second, the second to get a third, and so on, using the formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Euler's Method for Approximating f(a):

1. Start at a point (x, y) specified by an initial condition.
2. Use the differential equation to find the slope dy/dx at (x, y)
3. Move by a small increment, Δx, and use this to determine Δy using Δy = (dy/dx)Δx.
4. Use the new point, (x + Δx, y + Δy), then repeat from Step 2.
5. Continue until you have your approximation.

(x,y)	dy/dx	Δx	Δy = (dy/dx)Δx	(x + Δx, y + Δy)

Geometric Series:

$$\sum_{n=1}^{\infty} a_n r^{n-1} = a_1 + a_2 r + a_3 r^2 + \ldots + a_n r^{n-1} + \ldots = \frac{a_1}{1-r} \textbf{ for } |r|<1$$

Taylor Series for $f(x)$ **centered at** $x = a$**:**

$$f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2+\ldots+\frac{f^{(n)}(a)}{n!}(x-a)^n+\ldots=\sum_{n=0}^{\infty}\frac{f^{(n)}(a)}{n!}(x-a)^n$$

MacLaurin for $f(x)$ **(always centered at** $x = 0$**):**

$$f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots+\frac{f^{(n)}(0)}{n!}x^n+\ldots=\sum_{n=0}^{\infty}\frac{f^{(n)}(0)}{n!}x^n$$

Actual Error (Actual Minus Estimate):

$$|R_n(x)| = |f(x) - P_n(x)|$$

LaGrange Error Bound on $[a,b]$**:**

$$|R_n(x)| < \left|\frac{f^{(n+1)}(c)x^{n+1}}{(n+1)!}\right|$$

(Pick c to **maximize** $f^{(n+1)}(c)$)

Alternating Series:

The error is no more than the next term!

MacLaurin Series To Memorize (Part 1):

$$\frac{1}{1-x} = 1 + x + x^2 + \ldots + x^n + \ldots \qquad (|x|<1)$$

$$\frac{1}{1+x} = 1 - x + x^2 - \ldots + (-x)^n + \ldots \qquad (|x|<1)$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots = \sum_{n=0}^{\infty}\frac{x^n}{n!} \qquad (\textbf{for all real } x)$$

MacLaurin Series To Memorize (Part 2):

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \qquad (\textbf{for all real } x)$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots + (-1)^n\frac{x^{2n}}{(2n)!} + \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} \qquad (\textbf{for all real } x)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n-1}\frac{x^n}{n} + \ldots = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^n}{n} \qquad (-1 < x \le 1)$$

$$\tan^{-1}(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots + (-1)^n\frac{x^{2n+1}}{(2n+1)} + \ldots = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)} \qquad (|x| \le 1)$$

Convergence at Endpoints: When you have an infinite series involving *x*, use Ratio Test to find an open interval of convergence. Then use other tests at endpoints!

Ratio Test:

For $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = L$:

$L<1$: the series converges.

$L>1$: the series diverges.

$L=1$: test is inconclusive.

Limit Comparison:

If $\lim_{n\to\infty}\frac{a_n}{b_n} = 0$ and $\sum b_n$ converges, then so does $\sum a_n$.

If $\lim_{n\to\infty}\frac{a_n}{b_n} = c$, then both converge or diverge.

Alternating Series

1) Show terms alternate
2) Show $a_n \ge a_{n+1}$
3) Show $\lim_{n\to\infty}|a_n| = 0$

If so, then the series converges.

p-series Test:

$\sum \frac{1}{n^p}$ converges if $p>1$

Geometric Series:

$\sum_{n=1}^{\infty} a_n r^{n-1}$ converges if $|r|<1$

n^{th} Term Test:

If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges.

Direct Comparison:

If $\sum c_n$ converges and $a_n \le c_n$, then so does $\sum a_n$.

If $\sum d_n$ diverges and $a_n \ge d_n$, then so does $\sum a_n$.

Integral Test:

If $f(n) = a_n$ is a decreasing sequence, then $\sum a_n$ and $\int f(x)\,dx$ both converge or diverge.

Telescoping Series:

Use partial fraction decomposition to separate into two sequences, then group terms and cancel!

Polar Coordinates:

$x = r\cos\theta$	$y = r\sin\theta$	$r^2 = x^2 + y^2$	$\tan\theta = \frac{y}{x}$

Parameterize the Polar Equation r = f(Θ):

$$x(\theta) = f(\theta)\cos(\theta)$$

$$y(\theta) = f(\theta)\sin(\theta)$$

Finding dy/dx (Slope) for a Polar Curve:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} \quad \left(\frac{dx}{d\theta}\neq 0\right)$$

Finding Area for a Polar Curve:

$$A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta = \frac{1}{2}\int_\alpha^\beta f(\theta)^2\,d\theta$$