Calculus Math Bowl

Haynes Mu Alpha Theta 2019

Calculus Math Bowl

SET 1

Toss-up 1: If there are any, determine the inflection points of the following polynomial:

$$-\sin x + \frac{x^2}{2} + 7x + 2$$

Bonus 1: Find $\frac{d^2}{dx^2}\left[\frac{10}{3x^3}\right]$

Toss-up 2: Evaluate the following expression: $\pi + \frac{d}{dk}[e^{3\pi x}]$, where x is a constant.

Bonus 2: If $\int_b^a f(x)dx = -7$, what is the average value of f(x) on $[a, b]$, where $b > a$ and the interval length is 5.

Toss-up 3: Find the average rate of change of f(x) on the interval [4,7], given that $f^{-1}(x) = g(x)$, $g(7) = 8,\ g(9) = 7,\ g(4) = 0,$ and $g(3) = 4$.

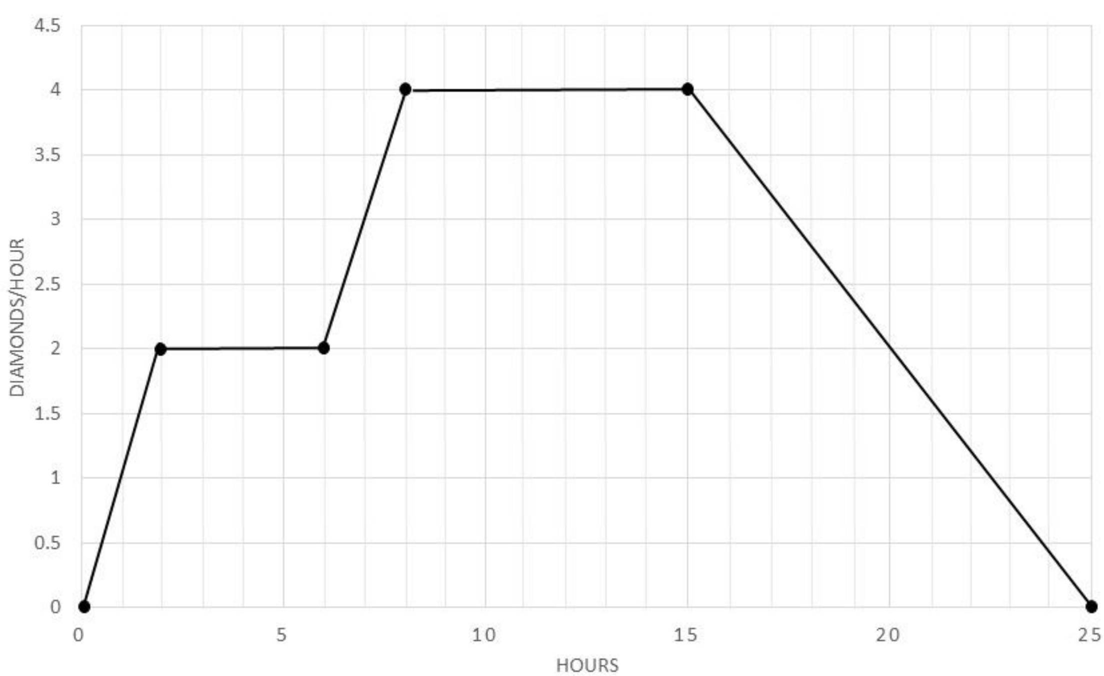
Bonus 3: Heath asks you to evaluate the following limit: $\lim_{x \to 5} \frac{x+5}{-x^2+3x+10}$

Toss-up 4: If f(x)= ln(x+4+e^{-3x}), find $f'(0)$.

Bonus 4: A bullet travels at an acceleration of 7 m/sec^2. The function $v(t)$ gives the bullet's velocity at any time $t \geq 0$. If $v(5) = 55$, what is the function $v(t)$?

Toss-up 5: Evaluate $\lim_{x \to 0} \frac{7e^{x+4} - 7e^4}{x}$

Bonus 5: The graph below shows the rate at which Andrew mines diamonds over a 25 hour day. If Andrew collected 64 diamonds by the end of the 25 hour period, how many diamonds did Andrew collect from 6 to 25 hours?



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Toss-up 6:

The following table lists the values of functions *g* and *h*, and of their derivatives, *g'* and *h'*, for x=-2. Let *H* be defined as *H(x)= g(x) • h(x)*. Find H'(2).

x	g(x)	g'(x)	h(x)	h'(x)
-2	2	-1	3	4

Bonus 6: The radius of our flat earth is decreasing at a rate of 1 meter per hour. At a certain instant, the radius is 40,000,000 meters. What is the rate of change of the area of our flat, circle Earth at that instant? Include units.

Toss-up 7: If $H(x) =- x - 5$ and $h(x) = H'(x)$, find $\frac{d}{dx}[\int_0^t h(x)dx]$, where t is a constant.

Bonus 7: Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

Toss-up 8: Let y= $(14 + 5x - 3x^2)^{\frac{3}{4}}$. Evaluate $\frac{dy}{dx}$ at x=1.

Bonus 8: If $x^2 + 2xy^2 = 420$, find $\frac{dy}{dx}$.

Toss-up 9: Let f(x)= $\frac{1}{2}x^4 - 4x^3$. For what x values does the graph of *f* have a point of inflection?

Bonus 9: Evaluate area under curve from 0 to 6π of $sinx$.

Toss-up 10: Evaluate $\int_{-2}^{8} \pi x dx$

Bonus 10: Einstein pours water into a broken tank that initially had 2300 gallons. Water enters the tank at $f(t) = 300t^2$ gallons per hour. Water leaves the tank at 250 gallons per hour. How much water is there at $t = 3$ hours? Include units.

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SET 2

Toss-up 1: Evaluate $\lim\limits_{x\to\infty} \frac{x^3-2x^2+3x-4}{4x^3-3x^2+2x-1}$

Bonus 1: Find $\int_{\pi/4}^{7\pi/6} (sin^2 x + cos^2 x)\, dx$

Toss-up 2: Find two positive numbers whose sum is 138 and whose product is a maximum.
Bonus 2: What is the equation (in slope intercept form) of the tangent line to the function $w(x) = e^x + 2$ at $x = ln3$.

Toss-up 3: On a roof, Hassan kicks a ball upward. The ball's height as a function of time is $h(t) = -t^2 + 2t + 8$ meters after t seconds. What is the ball's velocity at the moment of impact?
Bonus 3: While Hassan climbs a 10 ft ladder, it starts to fall down the side of a house at a rate of 3 ft/s. The base of the ladder is 8 feet from the house. At what rate is the base moving away from the house?

Toss-up 4: Find the values of x that satisfy the Mean Value Theorem for derivatives for the function $f(x) = 4x^3 + 2$ on the interval $[1, 4]$.
Bonus 4: $\lim\limits_{x\to 0} \frac{sin3x}{4x}$

Toss-up 5: Evaluate $\lim\limits_{x\to -7} \frac{x^2-49}{x+7}$

Bonus 5: Find $\frac{d^2y}{dx^2}$ of the function y=3 $cos^2(2x)$.

Toss-up 6: A cone has a height that is 3 times its radius and a radius that is increasing at a rate of 3 ft per minute. Find the rate of change of volume when the radius is 9 ft, given that the formula for the volume of a cone is V=$\frac{1}{3}\pi r^2 h$.
Bonus 6: A sphere was measured and the radius was found to be 15 inches with a possible error of no more than 0.02 inches. What is the maximum possible error in the volume? (Volume of a sphere is V=$\frac{4}{3}\pi r^3$)

Toss-up 7: Amie wants to enclose a rectangular field with fence to keep the normies inside. She only has 600 meters of fencing, and she also creates a fort on one side of the field so that side would not need any fencing. Determine the dimensions of the field that will enclose the largest area.

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Bonus 7: The expression $\frac{1}{20}(\sqrt{\frac{1}{20}} + \sqrt{\frac{2}{20}} + \sqrt{\frac{3}{20}} ... + \sqrt{\frac{20}{20}})$ is a Riemann Sum approximation for what integral?

Toss-up 8: How many values of k are there for which $\int_2^k x^2 dx = 0$?

Bonus 8: If $\int_a^b f(x)dx = a + 2b$, then what does $\int_a^b (f(x) + 3)dx$ equal? Answer in terms of a and b.

Toss-up 9: The perimeter of a square is increasing at a constant rate of 12 meters/minute. What is the rate of change in the area of the square when the side length is 10 meters?

Bonus 9: If f is a linear function and $0 < a < b$, then what is $\int_a^b f''(x)dx$?

Toss-up 10: $\int(e^{-x}tan(e^{-x}))dx$

Bonus 10: If you rotate a semi-circle with a diameter of 6 around the x-axis, what is the volume of the shape created?

Tiebreaker Toss-up: $\int(t(\sqrt[3]{t}))dt$

Tiebreaker Bonus: If Suraj Zaveri makes Taylor, Amie, Andrew, and Kha make all of the Calc AB Math Bowl questions and he admits to running an oligarchy of a club, what is his ACT score and where is planning on going to college?

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Answer Key

SET 1

T1: No inflection points.

B1: $\frac{40}{x^5}$ ($40x^{-5}$ also acceptable)

T2: π

B2: 7/5

T3: 2

B3: DNE ($\lim_{x\to 5^+} f(x) \neq \lim_{x\to 5^-} f(x)$, therefore limit does not exist. ∞ *or* $-\infty$ are **not** acceptable answers.)

T4: $\frac{-2}{5}$

B4: $v(t) = 7t + 20$

T5: $7e^4$

B5: 54 diamonds

T6: 5

B6: -80,000,000 π meters^2/hour

T7: 0

B7: $\frac{1}{4}$

T8: $\frac{-3}{8}$

B8: $\frac{-x-y^2}{2xy}$,also acceptable: $\frac{-2x-2y^2}{4xy}$

T9: x= 0,4

B9: 12

T10: 30π

B10: 4250 gallons

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Answer Key

SET 2

T1: $\frac{1}{4}$

B1: $11\pi/12$

T2: 69 and 69

B2: y=3x-3ln3+5

T3: -6 meters per second

B3: 9/4 ft/s

T4: $\sqrt{7}$

B4: 3/4

T5: -14

B5: $-24cos(4x)$ or $24-48cos^2(2x)$ or $48sin^2(2x)-24$ or $24sin^2(2x)-24cos^2(2x)$

T6: $729\pi \quad ft^3/min$

B6: $\pm 18\pi \quad in^3$ check dis

T7: 150 by 300

B7: $\int_0^1 \sqrt{x}dx$

T8: 1

B8: 5b-2a

T9: $60\ m^2/min$

B9: 0

T10: $\ln|cos(e^{-x})|$+C

B10: 36π

TXX: $\frac{3}{7}t^{\frac{7}{3}}+C$

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BXX: 36; LSU