Haynes Mu Alpha Theta 2019

#### SET 1

**Toss-up 1:** What is the sum of the solutions to the equation  $3x^2 + 7x + 12 = 0$ ?

**Bonus 1:** Find the area of a square inscribed in a circle of radius 7.

**Toss-up 2:** A couple is going to have EXACTLY two children. If having a boy or a girl is equally likely, what is the probability of having one boy and one girl?

**Bonus 2:** Simplify  $\sqrt{x^2 - 10x + 25}$ .

**Toss-up 3:** What is the remainder when  $3x^4 - x^2 + 2x - 7$  is divided by x - 3?

**Bonus 3:** If f(x) = 4x + 7 and  $g(x) = x^2 - 8$ , compute f(g(x)).

**Toss-up 4:** Compute  $i^{2019} + i^{2018} + i^{2017} + i^{2016}$ .

**Bonus 4:** Solve for all real values of x in the equation  $x^3 + 3x^2 - 4x - 12 = 0$ .

**Toss-up 5:** Find *all* roots of the function:  $f(x) = x^4 - 16$ .

**Bonus 5:** Solve the equation:  $\frac{5}{r} - \frac{3}{r+4} = 2$ .

**Toss-up 6:** Solve for x:  $log_2 \frac{5x+7}{x} = 4$ .

**Bonus 6:** Consider z = 4.9 + 21i. Find Re(z) and Im(z).

**Toss-up 7:** Factor the following expression completely:  $k^2 + 7kp - 18p^2$ 

**Bonus 7:** For the function  $g(x) = \frac{x^2 + 5x - 14}{x^2 + 4x - 21}$  find the removable discontinuities, if any.

**Toss-up 8:** What is the domain of the function  $f(x) = \sqrt{3x+6}$ ?

**Bonus 8:** Solve the system of equations.

$$y = -3x + 4$$

$$x + 4y = -6$$

#### Haynes Academy MAO Tournament 2019

## Algebra II Math Bowl

**Toss-up 9:** Factor completely.  $16 - 49x^2$ 

**Bonus 9:** Simplify:  $\frac{\frac{3x}{y}}{\frac{2x}{7}}$ 

**Toss-up 10:** Solve  $x^2 + 12x + 32 = 0$ .

Bonus 10: Solve the given infinite geometric series.

$$1 + \frac{10}{11} + \frac{100}{121} \dots$$

SET 2

**Toss-up 1:** If  $(\sqrt[3]{7})^{5x} = 14$ , what is the value of  $(\sqrt[3]{7})^{10x-6}$ ?

**Bonus 1:** What is the center of a circle with the equation:  $(x - 17.34)^2 + (y + (-3.345))^2 = 100$ .

**Toss-up 2:** Write  $2 \log_3 x + \log_3 5$  as a single logarithmic expression.

**Bonus 2:** For what value of k does the equation  $-x^2 + 2kx - 4 = 0$  have *one real* solution?

**Toss-up 3:** Find the inverse of the function:  $f(x) = \log(-2x)$ .

**Bonus 3:** Evaluate the arithmetic series.  $\sum_{k=1}^{35} (5k - 2)$ .

**Toss-up 4:** Find the number of terms in the following arithmetic series with:  $a_1 = 16$ ,  $a_n = 163$ ,  $S_n = 4475$ . ( $S_n$  is the sum of the first n terms of the arithmetic sequence)

**Bonus 4:** John is drawing triangles for fun and he stumbles across a familiar triangle with side lengths 9, 40, and 41. Compute the area.

**Toss-up 5:** Simplify  $\frac{x^2+x-12}{x^2-6x+9}$ , if  $x \ne 3$ 

**Bonus 5:** The area of a square is  $2\sqrt{2} + 3$ . What is the side length of the square?

**Toss-up 6:** What is f(g(x)) if f(x) = 2x + 7 and  $g(x) = 3x^2 - 1$ ?

**Bonus 6:** A rectangular prism has a volume of  $8x^3 + 14x^2 + x - 2$  and a height of 2x+1. What is the expression for the area of the base of the prism?

**Toss-up 7:** Factor  $18x^2 - 39x + 20$ .

**Bonus 7:** If  $g(x) = \sqrt[3]{\frac{x+3}{4}}$ , for what value of x will g(2x) = 2(g(x))?

**Toss-up 8:** There are two classes of Physics at Senyah Academy. The normal class and the advanced class. Both classes are given an identical test. The advanced class makes an average score of 92 while the normal one makes an average score of 70. The combined average of both classes is 86. If the advanced class contains x students, and the normal class contains y students, what is the value of y/x.

**Bonus 8:** In interval notation, find all solutions x such that the inequality  $-2019|x^2+6x+8|>0$  is satisfied.

**Toss-up 9:**  $f(x) = \sqrt{x}$  and g(x) = 7x + b if f(g(4)) = 6 what is the value of b?

**Bonus 9:** Simplify  $\sqrt[3]{x}\sqrt[3]{x}\sqrt[3]{x}\sqrt[3]{x}$ . Express your answer in simplest radical form in terms of x.

**Toss-up 10:** Given that for the equation  $x^2 + mx + b = 0$  the only possible value of x is -3 and that m and b are integers what is the value of m?

**Bonus 10:** With no stops, it takes 3 hours for a motorist to make a trip driving at an average speed of 65 mph. When the motorist stops for gas and a sandwich on the same trip, he averages 60 mph. How many minutes long is the stop?

**Tiebreaker Tossup:** Billy can paint a room in 3 hours. John can paint the same room in 5 hours. Working together, how long will it take Billy and John to paint a room? Express your answer as a common fraction.

**Tiebreaker Bonus:** Find b if  $log_b 343 = -1.5$ .

## **Answer Key: Set 1**

**T1:** -7/3

**B1:** 98

**T2:** 1/2

**B2:** |x-5| (Must have absolute value)

**T3:** 233

**B3:**  $4x^2 - 25$ 

**T4:** 0

**B4:** x = 2, -2, -3

**T5:**  $x = \pm 2$ ,  $x = \pm 2i$ 

**B5:** x = -5, x = 2

**T6:**  $x = \frac{7}{11}$ 

**B6:** Re(z) = 4.9, Im(z) = 21 (not 21i)

**T7:** (k + 9p)(k - 2p)

**B7:** x = -7

**T8:**  $[-2, \infty)$ 

**B8:** (2, -2)

**T9:** (4 + 7x)(4 - 7x)

**B9:**  $\frac{21}{2y}$ 

**T10:** x = -4, x = -8

**B10:** 11

### **Answer Key: Set 2**

**T1:** 4

**B1:** (17.34, 3.345)

**T2:**  $log_3 5x^2$ 

**B2:** k = 2, k = -2

**T3:**  $y = -\frac{10^x}{2}$ 

**B3:** 3080

**T4:** 50 terms.

**B4:** 180

**T5:**  $\frac{x+4}{x-3}$ 

**B5:**  $\sqrt{2} + 1$ 

**T6:**  $6x^2 + 5$ 

**B6:**  $4x^2 + 5x - 2$ 

**T7:** (6x-5)(3x-4)

**B7:** x = -7/2

**T8:** 3/8

**B8:**  $(-\infty, -4)$  U  $(-2, \infty)$ 

**T9:** 8

**B9:**  $\sqrt{x}$ 

**T10:** 6

**B10:** 15

**TXX:** 15/8 hours

**BXX:** 1/49