

Name: _____
School: _____

Mu B Individual 2012

Louisiana State Competition

1. Find the area of the region bounded by the graphs $y = xe^x$, $x = 0$, and $x = \ln 2$.
2. If $F(x) = \int_0^{x^2} \sqrt{t+3} \, dt$, what is $F'(x)$?
3. If $e^y \frac{dy}{dx} = 2x$ and $y(1) = 2$, then what is the particular solution $y(x)$?
4. A particle moves in the xy plane such that its position for time $t \geq 0$ is given by $x(t) = 3t^2 - 19t$ and $y(t) = e^{2t-7}$. What is the slope of the tangent to the path of the particle when $t = 4$?
5. If $f(x) = \sin(x^2)$, what are the first 3 terms of the Taylor series expansion about $x = 0$ for $f'(x)$.
6. If $\ln y = (\ln x)^2 + 2$, find $\frac{dy}{dx}$ in terms of y .
7. For time $0 \leq t \leq 10$, a particle moves along the x -axis with position given by $x(t) = t^3 - 7t^2 + 8t + 5$. During what time intervals is the speed of the particle increasing?
8. Selected values for the continuous function $f(x)$ are given in the table below.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$.48	1.25	1.07	0.53	0.27	1.04	3.56	2.18	2

Using 3 left-hand rectangles of equal width, find the approximation for $\int_{-3}^3 f(x) \, dx$

9. $\int_3^{12} (x-3)^{-\frac{1}{2}}$

10. Let $f(x) = x^2 e^x$ on the interval $-10 \leq x \leq 0$. Find the absolute maximum of $f(x)$.
11. $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{6} + h) - \frac{1}{2}}{h}$
12. Consider the curve defined by the parametric equations $x(t) = 3 + \sin t$ and $y(t) = 2t^2 + 5t + 1$. For time t , find $\frac{d^2 y}{dx^2}$.
13. Find the area enclosed by the polar curve $r \cos(\frac{1}{2}\theta) = 1$ in the interval $0 \leq \theta \leq \frac{\pi}{2}$.
14. $\lim_{x \rightarrow 0} \frac{\cos x - e^x}{\ln(1+x)}$
15. $\int_0^1 \frac{3}{x} dx$
16. Give the integral that expresses the volume of the solid generated by revolving the region enclosed between the graph of $y = 1 + x^2$ and the lines $y = 1$ and $x = 2$ about the x -axis.
17. A particle moves in the xy plane with position vector $\langle x(t), y(t) \rangle$ such that $x(t) = t^3 - 6t^2 + 9t + 1$ and $y(t) = -t^2 + 6t + 2$ in the time interval $0 \leq t \leq 5$. What is the average speed of the particle for the time interval?
18. $\int \frac{1}{16 + x^2} dx$
19. What is the slope of the curve $2xy^2 = 3x^2 - y^3$ at $(1, 1)$
20. What are all the values of a for which the series $\sum_{k=1}^{\infty} \frac{k^2}{k^{2a-3} + 4}$ converges?