Name:______________
School:______________
Compliments of my sister

Pre-Calculus Team 2019

Benjamin Franklin Tournament

1. Over what domain could the graph of $y = \csc x$ be restricted to guarantee that its inverse is a function?

2. Find the exact value of $\sin^{-1}(\cos(\frac{2\pi}{3}))$

3. Find the value of the x^8 coefficient of the expression $(2x+1)^9$.

4. For what values of x is it true that $\arcsin x = \arccos x$?

5. Larsen was making his famous strawberry-banana smoothie when Annie wondered, how does he get that perfect texture? Larsen said that there are $\sin(\tan^{-1}\frac{12}{5})$ parts strawberry to banana. Annie is dumb and doesn't know what this means. Help Annie figure out the ratio of strawberries to bananas.

6. Evaluate $(\sin^2(\frac{13\pi}{4}) + \cos^2(\frac{13\pi}{4}))(2\tan(\frac{5\pi}{4}))$

7. If $f(x) = \sin x$ and $f(a) = 1/3$, find the exact value of $f(a) + f(a+2\pi) + f(a+4\pi)$

8. Consider the function $f(x) = 3\cos(\frac{1}{2}x) + 4$, how many distinct points of intersection exist between the line $y = f(x)$ and $y = 5$ on $[-2\pi, 2\pi]$

9. How many ways can you get from the origin to the point (5,7) on a standard Cartesian plane if you can only go one unit up or right at a time and you must pass through the point (2,3)?

10. Annie, Anisha, and Grace are playing a game of Mafia with 12 other people. Each player in the game is assigned a unique role, and each role belongs to a specific team. There are 8 roles in the town team, 4 roles in the mafia team, and 3 roles in the neutral team. If all roles are randomly assigned with equal probability, what is the probability that Annie, Anisha, and Grace are all on the same team?

11. Convert 105° to radians.

12. Let $x = 2(\ln(6)) + 4(\ln(2)) + 4(\ln(3)) - 2(\ln(18))$. Compute e^x.

13. Solve the equation $\cos^2(x) + \sin(x) = 2$, $0 \leq x < 2\pi$ (write no solution if there is no real solution).

14. Solve the inequality $2x^2 < x + 10$

15. Find the sum of the squares of the roots of $f(x) = x^3 + x^2 - 26x + 24$

16. Compute: $3\cos(x) + 3 = 2\sin^2(x)$, $0 \leq x < 2\pi$.

17. Given: $y = \cos(x + \frac{\pi}{3}) + \sin(x + \frac{\pi}{6})$, what is the product of the amplitude and the period of y?

18. A class has four boys and three girls. If the students are called randomly to the office once at a time, what is the probability that they go in alternating boy/girl order?

19. Evaluate: $4\log_3 \frac{1}{3} + 2\log_9 27 + 6\log_{27} 3$

20. How many different integers satsify both $|3x - 4| \leq 10$ and $|3x + 2| > 4$?

Answers

1. $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2})]$ (or any interval of the form $[-\frac{\pi}{2} + \pi k, \pi k) \cup (\pi k, \frac{\pi}{2} + \pi k)]$, where k is an integer)
2. $-\frac{\pi}{6}$
3. 2304
4. $\frac{\sqrt{2}}{2}$
5. 12/13
6. 2
7. 1
8. 2
9. 350
10. 61/455
11. $\frac{7}{12}\pi$
12. 144
13. no solution
14. $(-2, \frac{5}{2})$
15. 53
16. $\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$
17. amplitude: 1
    period: 2π
18. 1/35
19. 1
20. 4 (The integers are 1, 2, 3, and 4)