

Name: _____
School: _____

Advanced Mathematics Individual 2018

St. Paul's Tournament

1. Find the inverse of the function $f(x) = \sqrt[3]{2x - 5}$
2. Factor $3x^3 + 8x^2 + 3x - 2$ given that $(x + 2)$ is a factor.
3. Write the quotient $\frac{2 + 3i}{4 - 2i}$ in standard form.
4. Simplify $-6i^3 + i^2$
5. Divide $x^4 - 3x^3 + 6x^2 + 2x - 60$ by $x^2 - 2x + 10$
6. Find the asymptotes of $f(x) = \frac{x^2 + 2x}{2x^2 - x}$
7. Solve: $\left(\frac{1}{2}\right)^x = 32$
8. Solve for x : $\ln x - \ln 2 = 0$
9. Simplify: $e^{\ln(5) \cdot 2}$
10. Solve: $e^{2x} - 4e^x - 5 = 0$
11. Solve $\log_4 x - \log_4(x - 1) = \frac{1}{2}$
12. Use sigma notation to write the sum: $\frac{1}{4} + \frac{3}{8} + \frac{7}{16} + \frac{15}{32} + \frac{31}{64}$

13. Find the sum: $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$

14. Find the 7th term of the geometric sequence 7, 21, 63, ...

15. Expand $(2x - 3)^4$

16. Find the standard form of the equation of the ellipse with vertices $(5, 0)$, $(5, 12)$ and endpoints of the minor axis $(0, 6)$, $(10, 6)$.

17. Evaluate AB if possible, given that $A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$

18. Evaluate: $\begin{vmatrix} 2 & -1 & 0 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{vmatrix}$

19. An auditorium has 20 rows of seats. There are 20 seats in the first row, 21 seats in the second row, 22 seats in the third row, and so on. How many seats are there in all 20 rows?

20. A truck traveled at an average speed of 50 miles an hour on a 200-mile trip. On the return trip, the average speed was 40 miles per hour. Find the average speed for the round trip. Round your answer to the nearest tenth.

21. Find the 52nd term: 19, 12, 5, -2, ...

22. Solve for x :

$$\frac{1}{r^3 - 2r^2} + \frac{x - 3}{r^2 - 2r} = \frac{1}{r}$$

23. Solve the system:

$$\begin{aligned}5x - 5y &= 0 \\ -3x + 3y + z &= -2 \\ 5x + 2y + 5z &= 11\end{aligned}$$

24. Solve and write answer in interval notation: $x^2 - x - 42 \leq 0$

25. Solve and write answer in interval notation: $\frac{2x - 24}{x - 2} \leq 4$

Answers

1. $\frac{x^3+5}{2}$
2. $(x+2)(3x-1)(x+1)$
3. $\frac{4}{5}i + \frac{1}{10}$
4. $6i - 1$
5. $x^2 - x - 6$
6. $x = \frac{1}{2}$ and $y = \frac{1}{2}$
7. $x = -5$
8. $x = 2$
9. 25
10. $x = \ln 5$
11. $x = 2$
12. $\sum_{k=2}^6 \frac{2^{k-1} - 1}{2^k}$
13. 1
14. 5103
15. $16x^4 - 96x^3 + 216x^2 - 216x + 81$
16. $\frac{(x-5)^2}{25} + \frac{(y-6)^2}{36} = 1$
17. 12
18. 0
19. 590
20. 44.4
21. -338
22. $x = r + 1 - \frac{1}{r}, \quad r \neq 0, 2$
23. $x = 3, y = 3, z = -2$
24. $[-6, 7]$
25. $(-\infty, -8] \cup (2, \infty)$