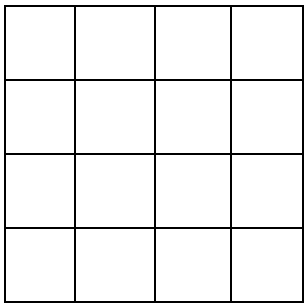
Interschool Test

Haynes Mu Alpha Theta 2019

Instructions

1. You have 50 minutes for this test.
2. No calculators allowed on this test.
3. Units are not required unless problem specifically says [units required]
4. Provide exact answers unless otherwise stated.
5. Put school name on answer sheet.
6. Good luck and have fun!

School_____

1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. 
8. _____
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18. _____
19. _____
20. _____

1) A number n whose square can be partitioned into two parts whose sum is n itself is called a *Kaprekar* number. For example:

$$9^2 = 81 \Rightarrow 8 + 1 = 9$$
$$999^2 = 998001 \Rightarrow 998 + 001 = 999$$
$$7777^2 = 60481729 \Rightarrow 6048 + 1729 = 7777$$

Find one of the two-digit *Kaprekar* numbers.

2) The base-30 system uses "0"-"9" to present values zero to nine, and "A"-"U" to represent values ten to thirty. Convert the base-10 number 603,887 to a base-30 number.

3) Each letter in the following sum stands for one of the digits 0 through 9 and no two letters stand for the same digit. Find the 6-digit numerical value of the sum's answer:

$$\begin{array}{r} \text{SINE} \\ +\text{COSINE} \\ \hline \text{HAYNES} \end{array}$$

4) At *Senyah Academy*, students are taught mathematics with different notations for some of their mathematical operations. Some sample equations from their school include:

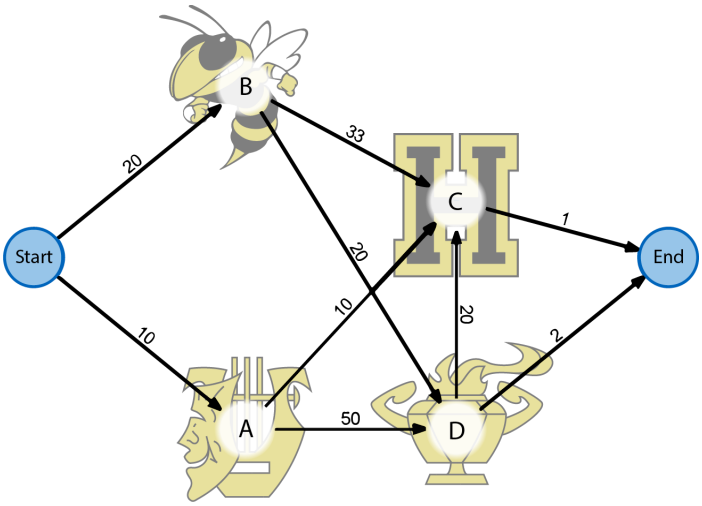
$80 \times 16 = 5$	$2 - 7 = 9$	$44 \$ 19 = 6$	$2 + 7 = 14$
$7 \div 4 = 3$	$49 \times 7 = 7$	$21 \div 16 = 5$	$97 \$ 3 = 1$
$3 + 5 = 15$	$8 - 19 = 27$	$2 + (14\$3) = 4$	$9 \div (32 \times 8) = 5$

Using this information, evaluate the following expression:

$$\{1\times[((5-14)\$(17\div 11))\times 3]\}\times 5\,.$$

5) A student is traversing *Haynesland*, Haynes's very own amusement park. She wants to get from *Start* to *End,* as shown in the graph below. The student can take on one route at a time, which requires her to use up a certain amount of energy as indicated. For example, the graph shows that traveling from *Start* to *Attraction B* takes up 20 units of energy. What is the least units of energy the student has to spend in order to get from *Start* to *End*?

Example Route: Student can take *Start* ⇒ *Attraction B* ⇒ *Attraction D* ⇒ *End,* which would take 20 + 20 + 2 = 42 units of energy. (This may not be the most energy-efficient path)



6) Knuth's up-arrow notation is a method of notation for very large integers. A single arrow means exponentiation (iterated multiplication); more than one arrow means iterating the operation associated with one few arrow. For example:

- single arrow is iterated multiplication (exponentiation)
  - $2 \uparrow 4 = 2 \times (2 \times (2 \times 2)) = 2^4 = 16$
- double arrow is iterated exponentiation (tetration)
  - $2 \uparrow\uparrow 4 = 2 \uparrow (2 \uparrow (2 \uparrow 2)) = 2^{2^{2^2}} = 65536$

Using this information, what is the value of $i^{93\uparrow\uparrow 93}$?

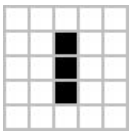
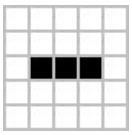
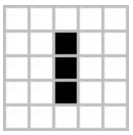
7) Max Pan the Math Magician is trying to perfect a new magic trick. In practicing the trick he stumbles across a magic square (shown below). All he knows is that the sum of the numbers in each row, column and diagonal must be 34. Can you help him solve the square?

			1
	11		14
3	10		
	5	9	4

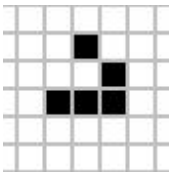
8) Conway's Game of Life is a *zero-player game*, meaning that the game progression is determined by its initial state, requiring no further input. Live and dead cells are represented by black and white squares, respectively. If a cell is a neighbor to another cell, it means that it is adjacent or diagonal to it. The rules are as follows:

1. Any live cell with <2 live neighbors dies.
2. Any live cell with 2-3 live neighbors lives on to the next generation.
3. Any live cell with >3 live neighbors dies.
4. Any dead cell with exactly 3 live neighbors becomes a live cell.

Here is an example game:

			This pattern repeats every two generations, so it has a period of 2.
Initial (Generation 0)	Generation 1	Generation 2	

Using this information, what is the period of the following configuration?



9) There is a sphere of radius n. Inside of the sphere there is an inscribed cube. And inside of that cube is an inscribed sphere. And inside of that inscribed sphere is another inscribed cube! This pattern continues until there are a total of 4 spheres and 3 cubes (3 inscribed cubes and 3 inscribed spheres). What is the ratio of the volume of the outermost sphere to the volume of the innermost sphere?

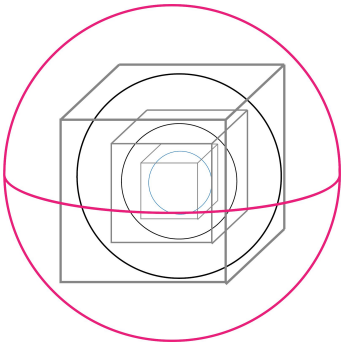


Figure not drawn to scale

10) Jonathan Poss and Adam Guillory are playing Pit-sniping on *Super Smash Bros. Ultimate*! They both choose Pit, set their damage to 300%, their stock to 1, and take turns shooting arrows at each other. Jonathan has a 40% chance of a successful snipe; Adam, being a better *Smash* player, has a 60% chance of a successful snipe. Jonathan and Adam alternate sniping; the first one to get a successful snipe wins. All snipes are independent. Jonathan goes first. What is the probability that Jonathan wins? (give your answer as a fraction in simplified form)

11) The planet *Heinz* does not follow the typical Gregorian calendar. The differences in their calendar are as follows:

- each year consists of 314 days
- each week consists of 6 days
- the days, in order, are H-day, A-day, Y-day, N-day, E-day, and S-day
- a leap year occurs every 6 years, where the year will have an extra day (315 days)

If year n starts on H-day and ends on Y-day, what is the 271th day of year $n+13$?

12) You have been given some ciphertext:

"Dolylcly aolyl pz ubtily, aolyl pz ilhbaf"

Convert it back into plaintext.

13) Suppose there is a function $f(H,\ A,\ Y,\ N,\ E,\ S) = A^{N/S} \times H^{Y/E}$. Evaluate $f(81,\ 8,\ -1,\ 1,\ 4,\ 3)$.

14) Patrick Taylor Mathletes Christian Otero and Alex Gelpi are single-handedly attempting to carry their school to victory on the Haynes Interschool Test. However, they decide to "risk it for the biscuit" by competing in Calculus Math Bowl with a two-man team. If Christian solves problems at a rate of 30 problems per hour, and Alex solves problems at a rate of 40 problem sets per day (A problem set consists of 12 problems). If they must solve each of the 40 Math Bowl questions (Toss-ups and bonuses) working together, how long (in minutes) will it be until they can leave Math Bowl to carry Interschool?

15) Each of the 100 students in Haynes Mu Alpha Theta know either algebra, geometry, or number theory. Some of them know more than one subject, but no one knows all three. There are 38 students who don't know algebra, 54 students who don't know geometry, and 45 who don't know number theory. How many students know two subjects?

16) Geoffrey is walking on the line $y = 3x - 2$. Starting from (-5, -17), he walks in the positive x-direction. After Geoffrey passes a certain point on the line, his distance from the origin starts increasing. What are the coordinates of that point?

17) For how many real values of x is $\sqrt{2019 - \sqrt{x}}$ an integer?

18) Jessica makes N sandwiches for a fundraiser. For each sandwich she uses B globs of peanut butter at 4 cents per glob and J blobs of jam at 5 cents per blob. The cost of the peanut butter and jam to make all the sandwiches is $2.53. Assume that B, J, and N are positive integers with N > 1. What is the cost of the jam Jessica uses to make the sandwiches?

19) A school bus travels from Baton Rouge to New Orleans. There are four children in the bus. Each child has four backpacks with him. There are four dogs sitting in each backpack. Every dog has four puppies with her. All these dogs have four legs, with four toes at each leg. Given that none of the students can drive, compute the number of toes in the bus.

20) Suraj, Neel, Kha, Kevin, and Jeff are creating an extraordinary Interschool Test for the first Haynes MAO Tournament. After question 19, they realize that they have trolled enough and wish to conclude the test with a simple, straightforward problem. Following countless seconds of thinking and planning, they devise the following problem:

Calculate the integer value of the following expression: $\frac{1}{n} \sin x$.

Answer Key

1. 45, 55, 99
2. MATH
3. 206,748 (A=0, C=1, E=4, H=2, I= 3, N=7, O=9, S=8, Y=6)
4. 3/5
5. 21 units of energy (*Start* $\Rightarrow A \Rightarrow C \Rightarrow$ *End)*
6. i
7.

13	8	12	1
2	11	7	14
3	10	6	15
16	5	9	4

8. 4
9. $81\sqrt{3}:1$
10. 10/19
11. S-day
12. “Wherever there is number, there is beauty”
13. 2/3
14. 48 minutes
15. 63
16. $(3/5, -1/5)$
17. 45
18. $1. 65
19. 5170 /Solution: 4096+1024+40+10
20. Six = 6