LOUISIANA MU ALPHA THETA – SCHOLARSHIP TEST 2017

1. Find $g^{-1}(3)$ given that $g(x)=\dfrac{3x+1}{2x+g(x)}$. (answer exactly)

2. What is $\left(i-i^{-1}\right)^{-1}$?

3. Let $g(t)=2t^2-8\sqrt{5}t+25$. Find all values of t such that $g(t)=-13$.

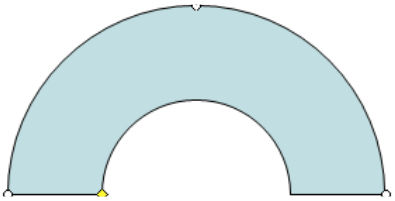
4. Find the area enclosed in the graph of $x^2+y^2=16x+32y$. (answer exactly)

5. Find a polynomial with integral coefficients that has $\sqrt{3+\sqrt{7}}$ as a root.

6. Find the sum of the series $1+\dfrac{1}{2}+\dfrac{1}{10}+\dfrac{1}{20}+\dfrac{1}{100}+\ldots$, where we alternately multiply by $\dfrac{1}{2}$ and $\dfrac{1}{5}$ to get successive terms. (answer exactly)

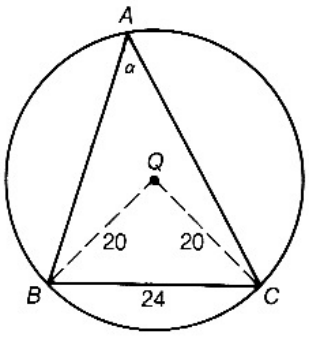
7. A certain radioactive isotope has a half-life of 4.3 seconds. If 4.2 grams of the isotope remain 12 seconds after an experiment starts, how many grams were present when the experiment began?

8. A room is built in the shape of the region between two semicircles with the same center and parallel diameters such that the smaller semicircle is contained within the larger. The room is shown below. The farthest distance between two points with a clear line of sight is 12 m. What is the area of the room? (answer exactly)



9. Find the exact value of $\log_2(2)\cdot\log_2(4)\cdot\log_2(8)\cdot\ldots\cdot\log_2\left(2^n\right)$

10. Triangle ABC is inscribed in a circle of radius 20 having center Q as shown. Find the measure of $\angle BAC$ to the nearest tenth of a degree



11. How long is the edge of a cube if after a slice 1 inch thick is cut from one side the volume of the remaining figure is 100 cubic inches?

12. Find the maximum perimeter of a rectangle whose top vertices are on the curve $y = 12 - 0.2x^2$ and whose bottom vertices are on the x axis.

13. If one semicircular arc of the unit circle extends from π to 2π, find the coordinates of a Quadrant III point that separates that arc into two sections, the ratio of whose lengths is 1:5.

14. A box contains chips, each of which is red, white, or blue. The number of blue chips is at least half the number of white chips, and at most one third the number of red chips. The number of chips that are white or blue is at least 55. What is the minimum possible number of red chips?

15. Kai has programmed his calculator to accept as inputs two positive integers, a and b, that have no common divisors. The calculator then compares 30 times the cube of $\frac{a}{b}$ plus 11 times the square of $\frac{a}{b}$ to the number that is 12 more than $\frac{59a}{b}$. When these two quantities are equal, the calculator lets him play a game. Unfortunately, Kai has forgotten what two integers to input to be able to play his game. What integers should Kai input?

SCHOLARSHIP TEST 2017 SOLUTIONS

1. $-\frac{8}{3}$

2. $-\frac{1}{2}i$

3. $2\sqrt{5} \pm 1$

4. 320π

5. $x^4 - 6x^2 + 2$

6. $\frac{5}{3}$

7. 29.1

8. 18π

9. $n!$

10. 36.9°

11. 5 inches

12. 34

13. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

14. 57

15. 3 and 4