

Calculus BC Individual

Haynes Mu Alpha Theta 2019

Instructions

1. You have 50 minutes for this test.
2. No calculators allowed on this test.
3. Do all scratch work on your test.
4. Units are not required unless problem specifically says [units required]
5. Provide exact answers unless otherwise stated.
6. Put name and school code on answer sheet.
7. Good luck and have fun!

Name_____

School_____

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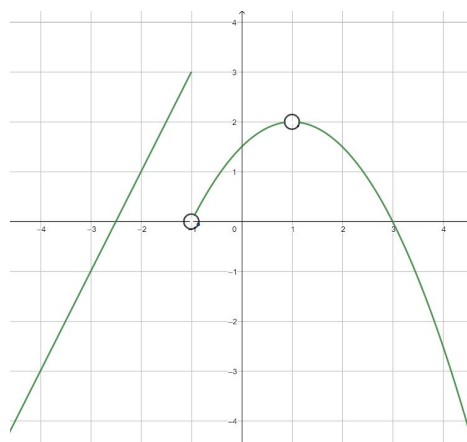
18. _____

19. _____

20. _____

1) Find $\lim_{x \rightarrow 1} \frac{x^3 + \ln(x) - 1}{4x^2 - x - 3}$

- 2) Using the graph of the function f on the right, find $\lim_{x \rightarrow -1^-} f(f(x))$



- 3) Find the 74th derivative of $-74\cos(74x)$ (Note: do NOT simplify exponents).

4) Evaluate $\int_{-\pi/2}^{\pi/2} \sin^6(x)\cos(x) dx$.

- 5) Find the equation of the line tangent to the graph of $x^2 + (y - x)^3 = 9$ at $x = 1$ in slope-intercept form.

- 6) On what interval(s) is the function $f(x) = 2x^3 + 4x^2 - 1$ decreasing and concave down?

- 7) The circumference of a circle is increasing at the rate of 5 meters/min. What is the rate of change of the area of the circle when the radius is 4 meters? [units required]

- 8) Jeffery may have had one too many liters of “punch” on Halloween. Because of his intoxication, he began to hand out money from under his mattress rather than candy to trick-or-treaters. After $\ln 5$ minutes, Jeffery’s life savings dropped from \$200,000 to \$8,000, at an exponential decline. If the exponential decline continues, how much moolah will Jeffery have left after $\ln 50$ minutes?

- 9) Many people say that Amie Sigur is a complete square. However, she can evaluate

$$\int \frac{dx}{x^2+2x+2} . \text{ Can you? If so, give the answer.}$$

- 10) At Chili’s, baby back ribs are delicious. What is the volume of the “rib of revolution” formed when the region bounded by the functions $y = \sqrt{x}$ and $y = x^2$ is rotated around the line $y = -1$?

11) Evaluate $\int_1^2 x \ln(x) dx$

12) Evaluate $\int \frac{\sqrt{9-x^2}}{x^2} dx$

13) Jeffery is walking on the parametric curve defined by $x = 3t^2$, $y = \frac{1}{3}t^3 + \frac{3}{2}t^2$. How fast is Jeffery walking at $t = 5$?

14) If $f'(x) = \frac{9}{2x^2+11x+5}$, find $f(x)$ given that $f(1) = 0$.

15) The line tangent to the parametric curve $x = 2e^{5t}$ and $y = 5e^{2t}$ at $t = \ln(2)$ can be written in the form $y = mx + b$. Find $m \cdot b$.

16) Kha has a flower in the shape of $r = 5\cos(3\theta)$. What is the area of one petal of this flower?

17) Determine which of the following series diverges:

a) $\sum_{n=1}^{\infty} (.99)^n$

b) $\sum_{n=1}^{\infty} (4 + (-1)^n)$

c) $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}$

d) $\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{5}}}$

e) $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

18) Mr. Hernandez has a task for you! Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n \cdot n^{-1}}{\ln(n)}$

19) Find the coefficient of the x^5 term in the Taylor series expansion of $f(x) = 3\sin(2x)$ centered at $x = 0$.

20) Congratulations for making it this far! Here's an easy one: Find the x-coordinate of the point of inflection of $f(x)$ if $f'(x) = x^2 + 3x + 4$.

Answer Key

1. $4/7$
2. 0
3. $74^{75} \cos(74x)$
4. $2/7$
5. $y = \frac{5}{6}x + \frac{13}{6}$
6. $(-4/3, -2/3)$ (accept brackets or parentheses)
7. $20 \text{ m}^2/\text{min}$ [units required]
8. \$80
9. $\tan^{-1}(x+1) + C$
10. $29\pi/30$
11. $\ln(4) - 3/4$ or $2\ln(2) - 3/4$
12. $-\frac{\sqrt{9-x^2}}{x} - \sin^{-1}(\frac{x}{3}) + C$ or $-\frac{\sqrt{9-x^2}}{x} + \cos^{-1}(\frac{x}{3}) + C$
13. 50
14. $f(x) = \ln|2x+1| - \ln|x+5| + \ln(2)$
15. $3/2$
16. $25\pi/12$
17. B
18. $[-1, 1)$
19. $4/5$
20. $-3/2$