BMHS MA0 Tournament - 2020

Calculus AB Individual Test

1) Find $\lim_{x \to -3} \frac{x^2 - 9}{x + 3}$ $= \frac{2x}{1} = -6$

2) Find $\lim_{x \to \infty} \frac{x}{\sin(-3x)}$ $= \frac{x}{-1} = \infty$

$\sin(-\infty)$

3) $y = 2x^5 + 5x^2 - x$ Find $\frac{d^4y}{dx^4}$

$y' = 10x^4 + 10x - 1$ $y''' = 120x^2$

$y'' = 40x^3 + 10$ $y'''' = 240x$

Below is a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

4)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	-1	3	-1
2	3	-1	2	-1
3	2	-1	1	$\frac{1}{2}$
4	1	-1	3	2

Given $h(x) = (f(x))^2$, find $h'(2)$

$2\, f(x)\, f'(x)$

$2 \cdot 3 \cdot -1 = -6$

5) If $y = 5^{2x^4}$ find the instantaneous rate of change when $x = -1$

$5^{2x^4} \cdot 8x^3 \cdot \ln 5 =$ $e^{\ln 5}$

$25 \cdot -8 \cdot \ln 5 = \boxed{-200 \ln 5}$

25
× 8
200

6) A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. Find the intervals of time when the particle is speeding up.

$s(t) = t^3 - 15t^2$ $v'(t) = 3t^2 - 30t$ $a'(t) = 6t - 30$

$0 = 3t(t - 10)$ $30 = 6t$

$t = 0$ $t = 10$ $t = 5$

\+ — +

0 10

— +

5

7) Find $\int \frac{5}{x}\, dx$ $5\ln|x| + C$

8) Differentiate the function $y = \sin 2x^5$

$\cos 2x^5 \cdot 10x^4$

9) Find $(f^{-1})'(10)$ for the function

inverse

$f'(x) = 20x^4 + 5$

$f(x) = 4x^5 + 5x + 1$ $\frac{1}{4x^5+5x+1}$ $\frac{1}{\quad}$

$x = 4y^5 + 5y + 1$

$x - 1 = y(4y^4 + 5)$

$x - 1 = 4y^5 + 5y$

$1 = 20y^4 + 5$

$-4 = 20y^4$

$-\frac{1}{5} = y^4$

10) A supermarket employee wants to construct an open-top box from a 10 by 16 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

$1 \cdot 8 \cdot 12 = 96$

$2)\ 6 \cdot 12 = 144$

$3 \cdot 4 \cdot 12 = 120$

11) Find the y-value of the ordered pair where the absolute maximum occurs for the function $y = x^3 - 3x^2 + 3$ on the interval $[-1, 1]$

$y' = 3x^2 - 6x$

$-1 = -1$

$0 = 3$

$\boxed{3}$

$1 = 1$

12) Find the equation of the line tangent to the graph of $y = \frac{5x^4}{2x^3 - 4}$ at the point $\left(1, -\frac{5}{2}\right)$.

Answer in point-slope form.

$-\frac{5}{2} = -\frac{70}{4} + b$

$b = 15$

$\frac{(2x^3-4)(20x^3) - (5x^4)(6x^2)}{(2x^3-4)^2} = \frac{40x^6 - 80x^3 - 30x^6}{(2x^3-4)^2} = \frac{10x^6 - 80x^3}{(2x^3-4)^2} = \frac{10-80}{4} = -\frac{70}{4}$

13) Use implicit differentiation to find $\frac{dy}{dx}$ at $(1, -1)$.

$\boxed{y = -\frac{70}{4}x + 15}$

$\frac{70}{4} - \frac{5}{2} = \frac{70-10}{4} = \frac{60}{4} = 15$

$4x^2 + xy^2 = -2y^3 + 3$

$8x + x2yy' + y^2 = -6y^2y'$

$1 - y = -\frac{70}{4}\left(x - \frac{5}{2}\right)$

$x2yy' + 6y^2y' = \frac{-8x - y^2}{2xy + 6y^2} = \frac{-8-1}{-2+6} = \frac{-9}{4}$

$y'(x2y + 6y^2)$

$-y = -\frac{70}{4}x - \frac{350}{8} + \frac{8}{8}$

$-y = -\frac{70}{4}x - \frac{342}{8}$

14) Find $\int_{-2}^{1} 6x(x^2 - 4)^2\, dx$

$u = x^2 - 4$

$du = 2x\, dx$

$dx = \frac{du}{2x}$

$3\int_{-2}^{1} \frac{(u)^2}{1} \cdot du$

$3\int_{-2}^{1} u^2\, du =$

$\Big]_{-2}^{1} = 1 + 8 =$

u^3

$(x^2-4)^3 \Big]_{-2}^{1} =$

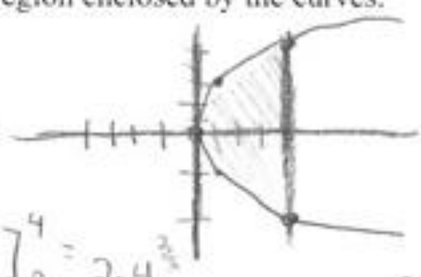
$-27 - (0)$

15) Find the area of the region enclosed by the curves.

$\sqrt{x} = x^{\frac{1}{2}}$

$\sqrt[2]{x^3} = x^{\frac{3}{2}}$

$y = 2\sqrt{x},\ y = -\sqrt{x},$
$x = 0,\ x = 4$



$\boxed{8}$

$\int_0^4 (2\sqrt{x} + \sqrt{x})\, dx =$

$3\int_0^4 x^{\frac{1}{2}}\, dx = 2x^{\frac{3}{2}} \Big]_0^4 = 2 \cdot 4^{\frac{3}{2}}$

$2\overline{)342}$
-32
22