Kha's Mock Ciphering Questions

Calculus AB Set 2
LaMAΘ State Convention
Thursday to Saturday
March 26 - 28, 2020

Rules

- Two minutes are allotted for each question.
- All answers must be in exact, simplified form unless otherwise requested.
- Four points are awarded for answering the question correctly within one minute.
- Two points are awarded for answering correctly within two minutes.
- Good luck and have fun!

1. Find $\frac{dy}{dx}$ given $y = 3^{x \ln x}$

2. Evaluate $\int \theta e^{\theta}\, dx$

3. Find $\frac{dy}{dx}$ given $y = (x^2 + x^3)^4$

4. Find f' in terms of g' if $f(x) = g(\ln x)$

5. Find the area bound by the given curves:

$$y = x^2, \quad y = 4x - x^2$$

6. Find the volume created by rotating the region bounded by $y = 2x$ and $y = x^2$ about the x-axis.

7. Solve the following differential equation by finding y:

$$\frac{dy}{dt} = 1 - t + y - ty$$

8. Evaluate $\lim_{x \to 1} \dfrac{\sqrt[3]{x} - 1}{\sqrt{x} - 1}$

9. Evaluate $\displaystyle\int \frac{1-x^2}{1-x}\,dx$

10. Suppose there is a number $A = 100999897\ldots4321$, which is composed by concatenating numbers 100 to 1 in descending order. What is the Ath derivative of $-\cos(x)$?

Answers

1. $3^{x \ln x}(\ln 3)(1 + \ln x)$
2. $\theta e^{\theta} x + C$
3. $4(x^2 + x^3)^3(2x + 3x^2)$
4. $g'(\ln x) \cdot \frac{1}{x}$
5. $\frac{8}{3}$
6. $\frac{64\pi}{15}$
7. $y = Ce^{t - \frac{t^2}{2}} - 1$
8. $\frac{2}{3}$
9. $x + \frac{x^2}{2} + C$
10. $\sin(x)$

Hints

1. Rewrite $3^{x \ln x}$ as $e^{\ln 3 \cdot x \ln x}$.
2. Since you're integrating with respect to x, you can treat θ as a constant.
3. Chain rule.
4. Chain rule.
5. Find the bounds by setting the two equations equal. Then subtract the area of the bottom curve from the top curve.
6. Same as above, but instead subtract volume of the bottom curve from the top curve.
7. Rewrite $\frac{dy}{dx} = 1 - t + y(1 - t) = (1 - t)(1 + y)$.
8. L'Hôpital's rule.
9. Rewrite $\displaystyle\int \frac{(1-x)(1+x)}{1-x}\,dx = \int (1+x)\,dx$.
10. $A \bmod 4 = 1$, therefore the Ath derivative of $-\cos(x)$ is equivalent to the 1st derivative of $-\cos(x)$, which is $\sin(x)$.