

Special Polar Graphs

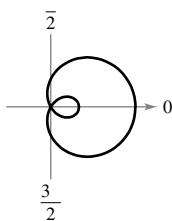
Several important types of graphs have equations that are simpler in polar form than in rectangular form. For example, the polar equation of a circle having a radius of a and centered at the origin is simply $r = a$. Later in the text, you will come to appreciate this benefit. For now, several other types of graphs that have simpler equations in polar form are shown below. (Conics are considered in Section 10.6.)

Limaçons

$$r = a \pm b \cos$$

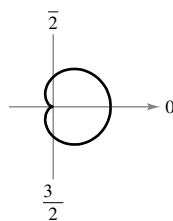
$$r = a \pm b \sin$$

$$(a > 0, b > 0)$$



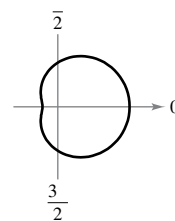
$$\frac{a}{b} < 1$$

Limaçon with
inner loop



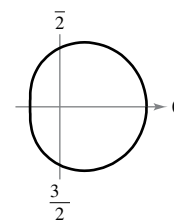
$$\frac{a}{b} = 1$$

Cardioid
(heart-shaped)



$$1 < \frac{a}{b} < 2$$

Dimpled limaçon



$$\frac{a}{b} \geq 2$$

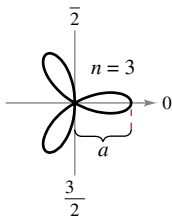
Convex limaçon

Rose Curves

n petals when n is odd

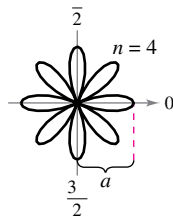
$2n$ petals when n is
even

$$(n \geq 2)$$



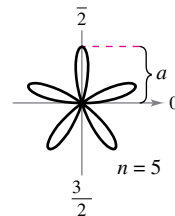
$$r = a \cos n$$

Rose curve



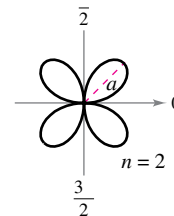
$$r = a \cos n$$

Rose curve



$$r = a \sin n$$

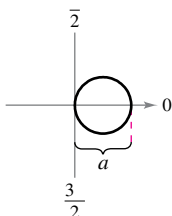
Rose curve



$$r = a \sin n$$

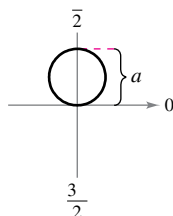
Rose curve

Circles and Lemniscates



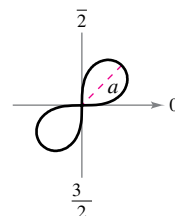
$$r = a \cos$$

Circle



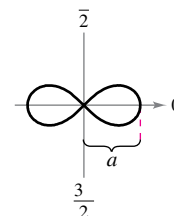
$$r = a \sin$$

Circle



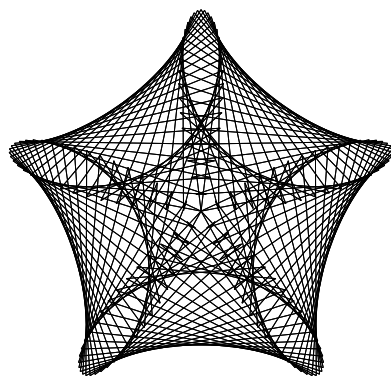
$$r^2 = a^2 \sin 2$$

Lemniscate



$$r^2 = a^2 \cos 2$$

Lemniscate



Generated by Maple

TECHNOLOGY The rose curves described above are of the form $r = a \cos n$ or $r = a \sin n$, where n is a positive integer that is greater than or equal to 2. Use a graphing utility to graph

$$r = a \cos n \quad \text{or} \quad r = a \sin n$$

for some noninteger values of n . Are these graphs also rose curves? For example, try sketching the graph of

$$r = \cos \frac{2}{3}, \quad 0 \leq \theta \leq 6.$$

FOR FURTHER INFORMATION For more information on rose curves and related curves, see the article “A Rose is a Rose . . .” by Peter M. Maurer in *The American Mathematical Monthly*. The computer-generated graph at the left is the result of an algorithm that Maurer calls “The Rose.” To view this article, go to MathArticles.com.