Name:_____
School:_____

# Mu A Integration Area Test 2010

Louisiana State Competition

1. Evaluate $\int_0^4 \sqrt{16-x^2}\,\mathrm{d}x$.

   **A**. $\frac{\pi}{2}$ **B**. $\pi$ **C**. $2\pi$ **D**. $4\pi$ **E**. NOTA

2. Using four rectangles on a regular partition of $[0,2]$, calculate the **lower sum** approximation of $\int_0^2 (x^2+1)\,\mathrm{d}x$.

   **A**. 1 **B**. $\frac{14}{3}$ **C**. $\frac{9}{4}$ **D**. $\frac{15}{4}$ **E**. NOTA

3. If $\int_a^b f(x) = 0$, then which of the following must be true?

   **A**. $f(x) = 0$ **B**. $a = b$ **C**. $f(-x) = -f(x)$

   **D**. At least one of the choices A, B, or C **E**. NOTA

4. Evaluate $\int \frac{x+e^x}{xe^x}\,\mathrm{d}x$.

   **A**. $-e^{-x} - \frac{1}{x^2} + C$ **B**. $e^{-x} - \ln|x| + C$ **C**. $e^{-x} + \ln|x| + C$

   **D**. $-e^{-x} + \ln|x| + C$ **E**. NOTA

5. Evaluate $\int \frac{2}{\sqrt{9-4x^2}}\,\mathrm{d}x$.

   **A**. $\sin^{-1}(\frac{2x}{3}) + C$ **B**. $\frac{1}{3}\sin^{-1}(\frac{2x}{3}) + C$ **C**. $\frac{1}{3}\tan^{-1}(\frac{2x}{3}) + C$

   **D**. $\sec^{-1}(\frac{2x}{3}) + C$ **E**. NOTA

6. If $f(x)$ is a continuous function such that $\int_9^1 f(x)\,\mathrm{d}x = -4$ and $\int_1^6 f(x)\,\mathrm{d}x = 12$, then evaluate $\int_5^9 (3f(x)+6)\,\mathrm{d}x$.

   **A**. -12 **B**. 0 **C**. 48 **D**. 72 **E**. NOTA

7. Find the volume of a solid given that its base is an isosceles right triangle with legs of length four and cross sections perpendicular to one of its legs are semicircles.

   **A**. $\pi/2$ **B**. $\pi$ **C**. $8\pi/3$ **D**. $16\pi/3$ **E**. NOTA

8. The velocity of a particle moving on a line at time $t$ is $v(t) = 3t^{\frac{1}{2}} + 5t^{\frac{3}{2}}$ feet per second. How many feet did the particle travel from $t = 1$ to $t = 9$ seconds?

**A**. 536 **B**. 496 **C**. 492 **D**. 248 **E**. NOTA

9. What is the value of $c$ guaranteed by the mean value theorem for integrals for the function $f(x) = \frac{6}{x^2}$ on the interval $[1, 2]$?

**A**. $\sqrt{2}$ **B**. $2\sqrt{2}$ **C**. $\frac{3}{2}$ **D**. $\frac{\sqrt{2}}{2}$ **E**. NOTA

10. A particle moves along the $x$-axis. Find the average value of the velocity on the closed interval $[1, 4]$ when $v(t) = 2t^3 - 4t^2 + 3t + 2$.

**A**. 22 **B**. 24 **C**. 33 **D**. 67 **E**. NOTA

11. If $F(x) = \int_0^{x^2} \frac{\tan(\pi t)}{(1+t)}\,\mathrm{d}t$, find $F'(\frac{3}{2})$.

**A**. $\frac{12}{13}$ **B**. $\frac{8}{13}$ **C**. $\frac{4}{13}$ **D**. $\frac{3}{13}$ **E**. NOTA

12. What is the area of the region between the curves $y = 4\sin(\frac{x}{2})$ and $y = 2\sin x$ on the interval $[0, 2\pi]$?

**A**. 12 **B**. $12\sqrt{2}$ **C**. $12\pi$ **D**. $6\sqrt{3}$ **E**. NOTA

13. Solve for $a$: $\int_1^4 (4ax^2 + 2x + 3a)\,\mathrm{d}x = \int_{-2}^1 (2ax^2 - 3ax + 2)\,\mathrm{d}x$

**A**. $\frac{3}{19}$ **B**. $\frac{5}{41}$ **C**. $-\frac{6}{55}$ **D**. $-\frac{7}{65}$ **E**. NOTA

14. What is the area of the region bounded by $f(x) = 10 - 2^x$ and $g(x) = 10$ on the interval $[0, 3]$?

**A**. $7\ln 2$ **B**. $\frac{8}{\ln 2}$ **C**. $8\ln 2$ **D**. $\frac{7}{\ln 2}$ **E**. NOTA

15. Which of the following are antiderivatives of $\frac{\ln^2 x}{x}$?

I. $\frac{\ln^3 x}{3}$ II. $\frac{\ln^3 x}{3} + 6$ III. $\frac{2\ln x - \ln^2 x}{x^2}$

**A**. I only **B**. III only **C**. I and II only **D**. I and III only **E**. NOTA

16. The region in the first quadrant bounded by the axes and the graphs of $\sqrt{x}+\sqrt{y}=4$ and $x=4$ is revolved about the $x$-axis. Find the volume of the solid generated (to the nearest hundredth).

**A**. 764.04 **B**. 243.20 **C**. 92.15 **D**. 33.51 **E**. NOTA

17. If $\int_1^{12} |cx-6|\,\mathrm{d}x = \frac{73}{3}$ where $1 < \frac{6}{c} < 12$, find c.

**A**. $\frac{10}{3}$ **B**. $\frac{4}{3}$ **C**. $\frac{3}{2}$ **D**. $\frac{2}{3}$ **E**. NOTA

18. Find the general solution for the differential equation $(3x^2+9)\frac{\mathrm{d}y}{\mathrm{d}x} = xy$

**A**. $y = C(3x^2+9)^{\frac{1}{6}}$ **B**. $y = C(x^2+3)^{\frac{1}{2}}$ **C**. $y = C(x^2+3)^{\frac{1}{3}}$

**D**. $y = C(x^2+3)^{\frac{1}{6}}$ **E**. NOTA

19. Evaluate: $\int_1^e [\frac{1}{x} - \frac{1}{x}\cdot\ln(\frac{1}{x})]\,\mathrm{d}x$

**A**. $-\frac{1}{2}$ **B**. 0 **C**. $\frac{1}{2}$ **D**. $\frac{3}{2}$ **E**. NOTA

20. Solve the differential equation, $x^2y' - x = 1$, satisfying the condition $y(1) = 2$.

**A**. $y = 2 - \ln(x)$ **B**. $y = 2 - \ln(x^2)$ **C**. $y = 3 - \frac{1}{x} + \ln(x)$

**D**. $y = 3 + \frac{1}{x} + \ln(x)$ **E**. NOTA

21. Semi-circular cross sections parallel to the $y$-axis are taken around the graph of $y = 3x^2 - 6x$ on the interval $[2, 4]$. Find the volume of the surface formed.

**A**. $\frac{186\pi}{5}$ **B**. $\frac{744\pi}{5}$ **C**. $\frac{1488\pi}{5}$ **D**. $\frac{372\pi}{5}$ **E**. NOTA

22. If $f(x)$ is a continuous function such that $\int_1^9 f(x)\,\mathrm{d}x = 12$ and $\int_1^6 f(x)\,\mathrm{d}x = 15$, then evaluate $\int_6^9 4(f(x)+2)\,\mathrm{d}x$.

**A**. -12 **B**. -6 **C**. 6 **D**. 12 **E**. NOTA

23. A particle moves along the $x$-axis so that its acceleration at any time $t$ is given by $a(t) = 6t - 18$. At time $t = 0$, the velocity of the particle is 24. At $t = 1$, its position is 20. What is the total distance traveled by the particle from $t = 0$ to $t = 4$?

**A**. 4 **B**. 16 **C**. 20 **D**. 24 **E**. NOTA

24. For a certain curve, $\frac{dy}{dx} = \sqrt{3 + xy + 3x + y}$. The curve passes through the points $(-1,\ 1)$ and $(8,\ b)$. Find the value of $b$.

**A**. 8 **B**. 22 **C**. 118 **D**. 121 **E**. NOTA

25. Evaluate the indefinite integral: $\int \frac{2x}{x^2+6x+10}\,dx$

**A**. $\ln|x^2 + 6x + 10| + C$ **B**. $2\ln|x^2 + 6x + 10| + \arctan(x + 3) + C$

**C**. $\ln|x^2+6x+10|+6\arctan(x+3)+C$ **D**. $\ln|x^2+6x+10|-6\arctan(x+3)+C$ **E**. NOTA

26. $F(x) = \int_0^{2\sin(x)} \sqrt{1 + t^3}\,dt$. Find $F'(\pi)$.

**A**. -6 **B**. -2 **C**. 3 **D**. 6 **E**. NOTA

**Tiebreaker:** The area enclosed by the graphs of $y = x^2 - 4x$ and $y = x + 6$ is rotated about the line $x = 8$. Find the volume of the solid formed.