

2019 CATHOLIC HIGH SCHOOL CALCULUS A INDIVIDUAL

1. Find  $\lim_{x \rightarrow 0} \left( \frac{e^x - e^{2x}}{1 - e^x} \right)$   $\frac{e^x - 2e^{2x}}{-e^x} \quad \frac{1 - 2(1)}{-1} = \frac{-1}{-1} = 1$

2. Find  $a$  if  $\lim_{x \rightarrow \infty} \left( \frac{ax^2 - 4x + 3}{2x^2 + 5x + 2} \right) = 6$   $a = 12$

3. If  $f(x) = x \ln x$ , find  $f''(e)$   $f'(x) = (x)\left(\frac{1}{x}\right) + (1)(\ln x)$   
 $f''(x) = 0 + \frac{1}{x}$

$\boxed{\frac{1}{e}}$

4. Suppose  $F(x) = f(x^2 + 1)$  and  $f'(5) = 3$ , find  $F'(2)$

$F(2) = f(5)$   $F'(2) = 3$

$F'(2) = f'(5)$

5. Find the coordinates of the points on  $y = x^3 - 3x^2$  where the tangent line is horizontal

$y' = 3x^2 - 6x$   $0 = 3x(x - 2)$   
 $(x = 0, 2)$

6. Find  $\frac{d^2y}{dx^2}$  if  $y = \log_5 x$

$\frac{dy}{dx} = \frac{\ln x}{\ln 5}$   $\frac{d^2y}{dx^2} = \frac{1}{\ln 5} \left( \frac{1}{x} \right)$

$\frac{(+)(\frac{1}{x})}{(\ln 5)^2} = 0$

7. A function has derivative  $f'(x) = x(x-3)^2(x+1)^4$ . What is the total number of local extreme points on  $f(x)$ ?  $x = 0, 3, -1$

$-1 \quad 0 \quad 3$

$-2 \quad -5$   
 $-(+)(+) \quad -(+)(+)$   
 $2$

8. Find  $a$  and  $b$  such that  $p(1) = 0$  and  $p'(1) = 4$  for  $p(x) = x^2 + ax + b$

$p'(x) = 2x + a$   
 $p'(1) = 2(1) + a = 4$   
 $a + 2 = 4$   
 $a = 2$

$p(1) = 1 + 2(1) + b = 0$   
 $b + 3 = 0$   
 $b = -3$

9. Find the slope of the tangent to  $x^2y^2 = 9$  at  $(-1, 3)$

$$(x^2)(2y) \frac{dy}{dx} + (y^2)(2x) = 0$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = \frac{-2(+1)(9)}{2(1)(3)} = \frac{-18}{6} = -3$$

10. Let  $f(x)$  and  $g(x)$  have values given in the table below.

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
2	3	-1	-5	2

If  $j(x) = \frac{f(x)}{5x}$ , find  $j'(2)$

$$j'(x) = \frac{5x(f'(x)) - (f(x))(5)}{25x^2}$$

$$j'(2) = \frac{10(f'(2)) - f(2)(5)}{100}$$

$$= \frac{10(-1) - 3(5)}{100} = \frac{-10 - 15}{100} = \frac{-25}{100} = -\frac{1}{4}$$

11. If  $y = x - \frac{2}{3x^3}$ , find  $\frac{d^2y}{dx^2}$

$$\frac{d}{dx} \left( y = x - \frac{2}{3}x^{-3} \right)$$

$$= 1 - \frac{2}{3}(-3)x^{-4}$$

$$\frac{d}{dx} \left( 1 - \frac{2}{3}(-3)x^{-4} \right)$$

$$= 2(-4)x^{-5} = -8x^{-5}$$

12. The length of a rectangle is decreasing at 2 cm/sec, and the width is increasing at 2 cm/sec. How fast is the area changing when the length is 12 cm and the width is 5 cm? (include units)

$$\frac{dL}{dt} = -2$$

$$\frac{dA}{dt} = (L) \frac{dW}{dt} + (W) \frac{dL}{dt}$$

$$= 12(2) + (5)(-2) = 24 - 10 = 14$$

13. Find the coordinates of the relative maximum point for  $f(x) = x^3 + 3x^2 + 4$

$$f'(x) = 3x^2 + 6x$$

$$3x(x+2) = 0$$

$$x = 0, -2$$



$$x = -3, -1, 1, -2$$

$$3(-3) - 18 = -27$$

$$f(-2) = -8 + 3(4) + 4 = -8 + 12 + 4 = 8$$

14. If  $u = \ln \sqrt{v^2 + 2v - 1}$ , find and simplify  $\frac{du}{dv}$

$$\frac{du}{dv} = \frac{1}{(\sqrt{v^2 + 2v - 1})} \left( \frac{1}{2} \right) (v^2 + 2v - 1)^{-1/2} (2v + 2)$$

$$\ln x^3 = 3 \ln x$$

$$\frac{1}{2}$$

15. If  $R$  is variable and  $r$  is constant, find  $\frac{d}{dR}(r^2 R^3)$ .

$$\frac{v+1}{(\sqrt{v^2 + 2v - 1})}$$

$$3r^2 R^2$$