2018 BMHS Mu Alpha Theta Tournament

Calculus AB Individual Test

1) $\lim_{x \to -2} \dfrac{x^2 + x - 2}{x + 2}$

$\dfrac{\cancel{(x+2)}(x-1)}{\cancel{(x+2)}} = (x-1) = -2-1 = -3$

2) $\lim_{x \to \infty} \dfrac{\sin x}{4x}$ $= 0$

3) If $y = -4x^5 + 4x^3 - 4x$ Find $\dfrac{d^4y}{dx^4}$

$y' = -20x^4 + 12x^2 - 4$

$y'' = -80x^3 + 24x$

$y''' = -240x^2 + 24$

$\boxed{y'''' = -480x}$

Below is a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve each problem.

4)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−1	1	1
2	2	−1	2	$\frac{3}{2}$
3	1	0	4	$\frac{1}{2}$
4	2	1	3	−1

Given $h(x) = (f(x))^2$, find $h'(2)$

$h'(x) = 2f(x) \cdot f'(x)$

$2f(2) \cdot f'(2)$

$2(2) \cdot (-1) = -4$

$2x^3 (e^{\ln(2)})^{2x^3 \cdot 1}$

$e^{\ln 2} \cdot e^{2x^3}$

5) If $y = 2^{2x^3}$ find the instantaneous rate of change when $x = 1$

$e^{\ln(2) \cdot 2x^3}$ $(e^{\ln(2)})^{2x^3}$ $2^{2x^3} \cdot \ln(2) \cdot 6x^2$

6) Use implicit differentiation to find $\dfrac{dy}{dx}$ at $(-1, 1)$.

$4x^2 + 2x^3y + y = 3$

$8x + 2\left[(x^3)\left(\frac{dy}{dx}\right) + y\right] + \frac{dy}{dx} = 0$

$8 + 2\left[(1)\left(\frac{dy}{dx}\right) - 1\right] + \frac{dy}{dx} = 0$

$8 + \frac{dy}{dx}(2) - 2 + \frac{dy}{dx} = 0$

$\frac{dy}{dx}(3) = -6$

$\frac{dy}{dx} = \boxed{-2}$

~~7)~~ Find $(f^{-1})'(-4)$ for the function

$-3t(t-10) = 0$

8) A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. Find the intervals of time when the particle is speeding up.

$s(t) = -t^3 + 15t^2$

$s'(t) = -3t^2 + 30t$

$-3t^2 + 30t = 0$

$3t(-t + 10) = 0$

$s'(t) = -3t^2 + 30t$

$s''(t) = -6t + 30$

$-3t^2 + 30t = 0$

$t^2 - 10t = 0$

$t(t - 10) = 0$

$-6t + 30 = 0$

$t - 5 = 0$ $t = 5$

+ + −

0 5 10

+ −

5

$(0, 5) \cup (10, \infty)$

9) Find the ordered pair where the absolute minimum occurs for the function $y = (5x - 15)^{\frac{2}{3}}$ on the interval $[0, 4]$

$-15^{2/3}$

$y = \frac{2}{3}(5x - 15)^{-1/3}(5)$

$0^{\frac{2}{3}}$

$5^{\frac{2}{3}}$

$5x - 15 = 0$

$5x = 15$

$x = 3$

$[3, 0]$

$\frac{26 \pm 2\sqrt{69}}{6} = \frac{13 \pm \sqrt{69}}{3}$ $26 \cdot 26 = 436$ → $676 - 480 = 276$ $\frac{26 \pm \sqrt{276}}{6}$

10) A supermarket employee wants to construct an open-top box from a 10 by 16 in. piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What should the size length of each square be in order to create a box with the largest possible volume?

$\frac{26 \pm \sqrt{26^2 - 4(3)(40)}}{2(3)}$

$\sqrt{276}$ 69 4 30

$V = lwh$

$V = (16-2x)(10-2x)(x)$

$(16-2x)(10-2x)$

$4x^2 - 20x - 32x + 160$

$4x^2 - 52x + 160$

lwh

11) Find the slope of the line tangent to the graph of $y = \frac{3}{x^3+4}$ at the point $\left(1, \frac{3}{5}\right)$

$y = 3(x^3+4)^{-1}$

$y' = -3(x^3+4)^{-2}(3x^2)$ $-3(5)^{-2}(3)$ $-9\left(\frac{1}{25}\right)$ $-\frac{9}{25}$

12) Differentiate the function: $y = \sin 5x^5$

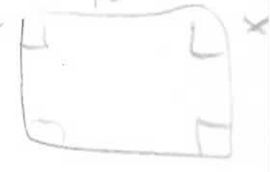
$\sin(5x^5) =$ $\boxed{\cos(5x^5) \cdot 25x^4}$

$V = 4x^3 - 52x^2 + 160x$

$\frac{dv}{dt} = 12x^2 - 104x + 160$

$6x^2 - 52x + 80$

$3x^2 - 26x + 40$



13) Find $\int_{-2}^{-1} \frac{24x}{(4x^2+2)^2}\,dx$

$u = 4x^2 + 2$

$du = 8x\,dx$

$\boxed{\frac{3}{2}}$

14) A spherical snowball is rolled in fresh snow, causing it grow at a rate of 36π in³/sec. How fast is the radius of the snowball increasing when the radius is 8 in? (Note: For a sphere $V = \frac{4}{3}\pi r^3$)

$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$36\pi = 4\pi(8)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{36\pi}{32\pi} = \frac{9}{8}$ in/sec

$3\int_{-2}^{-1} \frac{8x}{(4x^2+2)^2}\,dx$

~~15)~~ Approximate the area under the curve over the interval [0, 4] using 4 left endpoint rectangles.

$3\int_{-2}^{-1} u^{-2}\,du$

16) $\int -4x^{-1}\,dx$

$-4x^{-1}$ $-4\int x^{-1}\,dx$

$\boxed{-4\ln|x| + C}$

$3\left[-u^{-1}\right]_{-2}^{-1}$

17) Find $\lim_{x \to 0} \frac{4(e^x - e^{-x})}{\sin(2x)}$

$\frac{4(e^x + e^{-x})}{2\cos(2x)}$ $\frac{4(2)}{2} = \frac{8}{2} = 4$

$3(-(-1)^{-1} - -(-2)^{-1})$

$3\left(1 + -\frac{1}{2}\right)$

18) If $F(x) = \int_x^{x^2} -\frac{1}{t^2}\,dt$ then find and simplify $F'(x)$

$\int_x^c -t^{-2}\,dt + \int_c^{x^2} -t^{-2}\,dt = -\int_c^x -t^{-2} + \int_c^{x^2} -t^{-2}\,dt$

$3\left(\frac{1}{2}\right)$ $\frac{3}{2}$

19) For the function $y = x^2 + 2x - 1$, where does the instantaneous rate of change equal the average rate of change on the interval [−4, 1]

$\frac{y(1) - y(-4)}{1+4} = \frac{2-7}{5} = -\frac{5}{5} = -1$

$-(-x^{-2}) + (-(x^2)^{-2}) \cdot 2x$

$\boxed{x^{-2} - 2x^{-3}}$

$2x + 2 = -1$

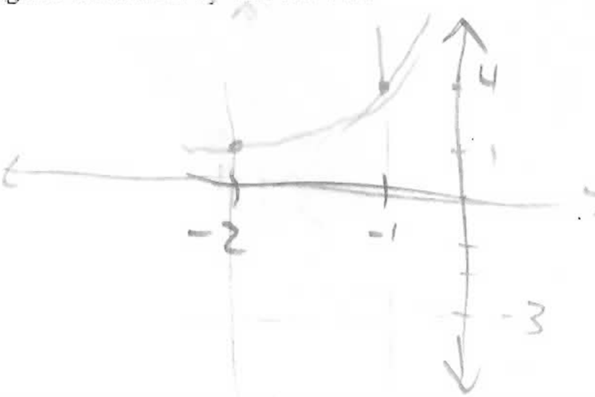
$2x = -3$

$x = -\frac{3}{2}$

20) Find the area of the region enclosed by the curves.

$y = \frac{4}{x^2}$; $y = -3$;

$x = -2$; $x = -1$



$\int_{-2}^{-1} \frac{4}{x^2}\,dx$

$4\int_{-2}^{-1} x^{-2}\,dx$

$4\left[-x^{-1}\right]_{-2}^{-1}$

$16 - 8 - 1$

$8 - 1$ 7

$1 - \frac{1}{2}$

$4\left(\frac{1}{2}\right)$

$= 2$

$2 + (1)(3)$

$2 + 3$

5