2020 CATHOLIC HIGH SCHOOL CALCULUS A INDIVIDUAL

1. Find $\lim_{x \to 2}\left(\frac{x-2}{x^3-8}\right)$ $\frac{1}{3x^2} = \boxed{\frac{1}{12}}$

$\frac{1}{2}a+1$

$\frac{1}{4}a-1$

2. Find a for which $f(x)=\begin{cases} ax+1 & if \quad x \le -3 \\ ax^2-1 & if \quad x > -3 \end{cases}$ is continuous

$f'(x) = \begin{cases} a \\ 2ax \end{cases}$ $\cancel{1} = \frac{2ax}{}$ $x = \frac{1}{2}$

$1 = 2x$

$\frac{1}{2}a + 1 = \frac{1}{4}a - 1$

3. Find and simplify the derivative of $e^{\ln x} + x^{\ln e} + \ln(e^x)$

$\frac{e^{\ln x}}{x} +$

$\frac{1}{2}a + 2 = \frac{1}{4}a$

$2 = -\frac{1}{2}a$

$a = -4$

4. If $f(x)$ has values given in the table below, and $g(x) = \frac{3}{f(x)}$, find $g'(5)$

$f(5)$	$f'(5)$
-3	6

$\frac{-3f'(x)}{[f(x)]^2} = \frac{-3(6)}{9} = \frac{-18}{9} = \boxed{-2}$

$\frac{\frac{\sqrt{3}}{2}}{2} = \frac{2}{\sqrt{3}}$

5. Find the slope of the tangent line to $y = \csc x$ when $x = \frac{\pi}{6}$

$y' = -\csc x \cot x$

$-\csc\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)$ $\left(\frac{2}{\sqrt{3}}\right)\sqrt{3} = \boxed{2}$

6. The cost in dollars of manufacturing q units of a product is $C(q) = 5q^2 + 2q + 300$. Use marginal cost to estimate the cost of the manufacturing the 31st unit.

$C'(q) = 10q + 2 =$

$\boxed{312}$ $= 10(31) + 2 =$

$31 \times 10 = 310$, $+2 = 312$

7. If $f(x) = \frac{x}{2x+1}$, find $f''(x)$

$f'(x) = \frac{\cancel{2x} + 1 - \cancel{2x}}{(2x+1)^2} = \frac{1}{(2x+1)^2}$

$f''(x) = \frac{-(1)2(2x+1)}{(2x+1)^4}$ $-4(2x+1)$

$= \frac{-8x-4}{(2x+1)^4} = \frac{-4\cancel{(2x+1)}}{(2x+1)^{\cancel{4}3}}$

$\boxed{\frac{-4}{(2x+1)^3}}$

8. Find $\frac{dy}{dx}$ if $(\sin x)(\cos y) = y$

$\sin x(-\sin y)y' + \cos y\cos x = y'$

$-\sin x\sin y\,y' - y' = -\cos y\cos x$

$y'(-\sin x\sin y - 1) = \boxed{\frac{-\cos x\cos y}{-\sin x\sin y - 1}}$

$-1(\sin x\sin y + 1)$

$\frac{12x}{x^4} = -12$ $\qquad \frac{2x}{4} = \frac{1}{2}$ $\qquad \frac{-6}{x^2}$

9. On what open interval(s) is $f(x) = 6x^2 + \frac{x}{2} + \frac{6}{x} + 3$ concave down?

$12x = -12x^4$ $\quad f'(x) = 12x + \frac{1}{2} - \frac{6}{x^2}$

$0 = 12x + 12x^4$

$12x(1+x^3)$ $\quad f''(x) = 12 + \frac{12x}{x^4} = 0$ $\quad \boxed{x = -1}$

$x = 0 \quad x = -1$

10. If $h(x) = \sin(g(x))$, $g(2) = \frac{\pi}{4}$, $g'(2) = 5$ and $g''(2) = 3$, find $h''(2)$

$h'(x) = \cos(g(x)) \cdot g'(x)$

$h''(x) = -\sin(g(x)) \cdot g'(x) \cdot g'(x) + \cos(g(x)) \cdot g''(x)$

$\left[-\frac{\sqrt{2}}{2} \cdot 25\right] + \frac{\sqrt{2}}{2} \cdot 3$

$= -\frac{25\sqrt{2}}{2} + \frac{3\sqrt{2}}{2} = \boxed{\frac{-22\sqrt{2}}{2}}$

$= -11\sqrt{2}$

$f'(x) = \sec^2(x)$

11. Find $f(x)$ if $\frac{df}{dx} = \sec^2(x)$ and $f\left(\frac{\pi}{4}\right) = 2$

$f(x) = \tan x$ $\quad 2 \cdot \tan\left(\frac{\pi}{4}\right) = 1 \quad 2 = 2$

$2\tan x$

$2\sec^2 x$

12. Find the instantaneous rate of change of $f(\theta) = |\tan\theta|$ when $\theta = \frac{2\pi}{3}$

Khanh Le

$f'(\theta) = |\sec^2\theta|$

$= \left|\sec^2 \frac{2\pi}{3}\right| = \boxed{4}$

13. Find the coordinates of the absolute maximum point of $f(x) = x^{2/3}(5 - 2x)$ on $[-1,2]$

$f'(x) = x^{\frac{2}{3}}(-2) + (5-2x)\frac{2}{3}x^{-\frac{1}{3}} = 0$

$-2x^{\frac{2}{3}} + \frac{10}{3}x^{-\frac{1}{3}} - \frac{4}{3}x^{\frac{2}{3}}$

$-1 \qquad 2$

14. In a right circular cone, the radius is 4 inches and is increasing at 5 in/sec while the height is 3 inches and is decreasing at 6 in/sec. Find the rate of change of volume at that time (include units)

$\frac{4\pi}{3} \cdot \frac{2}{1} = \frac{8\pi}{3}$

$V = \frac{4}{3}\pi r^3$ $\qquad V' = \frac{4\pi}{3} r^2 \cdot h' + h \frac{8\pi}{3} r \cdot r'$

$V = \frac{4}{3}\pi r^2 h$ $\qquad V' = \left[\frac{4\pi}{3} \cdot \frac{16}{1} \cdot -\frac{6}{1}\right] + \left[3 \cdot \frac{8\pi}{3} \cdot \frac{4}{1} \cdot \frac{5}{1}\right]$

$= -128\pi + 160\pi = \boxed{32\pi}$ $\qquad (0,0)$

15. Find the slope intercept equation of the line tangent to $y = e^x$ that passes through the origin.

$y' = e^x$ $\qquad y - y_1 = m(x - x_1)$

$y = mx + b$

$y = e^x x + b$ $\qquad y = e^x(x)$

$\boxed{y = xe^x}$

$160 - 128 = 32$