2018 MU A INDIVIDUAL

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1. Find $\lim_{x\to 0}\left(\frac{2e^x-2}{x}\right)$

$2e^x =$ 2

2. Find $\lim_{x\to\infty}\left(\frac{ax^2-bc+c}{(ax-b)(bx-c)}\right)$

ax^2

$\frac{ax^2}{abx^2} =$ $\boxed{\frac{1}{b}}$

abx^2

3. Find $\frac{d}{dx}\left(\sin^3(4x)\right)$

$(\sin(4x))^3$

$\boxed{3\sin(4x)^2 \cdot \cos(4x) \cdot 4}$

4. Values of $f(x)$ and $f'(x)$ are represented in the table below.
If $k(x)=e^x f(x)$, find $k'(5)$

x	$f(x)$	$f'(x)$
5	7	−3

$k'(x) = (e^x)(f'(x)) + (f(x))(e^x)$

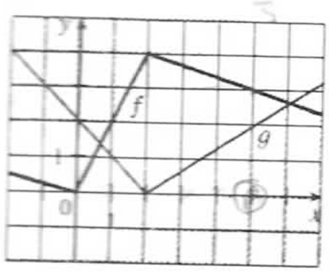
$k'(5) = (e^5)(f'(5) + f(5))$

$= e^5(-3+7)$

$\boxed{4e^5}$

$4e^5$

5. If $f(x)$ and $g(x)$ are the functions whose graphs are shown and $k(x)=\frac{f(x)}{g(x)}$, find $k'(5)$



$k'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{(g(5))^2}$

$\frac{(2)(-\frac{1}{3}) - (3)(\frac{2}{3})}{2^2}$

$-\frac{2}{3} - \frac{6}{3}$

$-\frac{8}{3}$

$\boxed{-\frac{2}{3}}$

$-\frac{8}{3} \div 4$

$-\frac{2}{3} - 2$

6. Find the slope of the tangent line to $f(x)=A-\frac{B}{x}+\frac{C}{\sqrt{x}}$ when $x=4$

$A - Bx^{-1} + Cx^{-1/2}$

$f'(x) = Bx^{-2} - \frac{1}{2}Cx^{-3/2}$

$f'(4) = \frac{B}{16} - \frac{1}{2}C(4)^{-3/2}$

$\frac{B}{16} - \frac{C}{\ }$

7. Find the point(s) where $y = x + \sin x$ has a horizontal tangent on $\left[0, 2\pi\right)$ (answer as an exact ordered pair)

$y' = 1 + \cos x$ $\quad$ $\pi + \sin\pi$

$1 + \cos x = 0$ $\quad$ π

$\cos x = -1$

$x = \pi$

$\boxed{(\pi, \pi)}$

8. If $y^3 + y^2 - 5y - x^2 = -4$, find $\dfrac{dy}{dx}$

$3y^2 \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} - 5 \cdot \frac{dy}{dx} - 2x = 0$

$\frac{dy}{dx}(3y^2 + 2y - 5) = 2x$

$\boxed{\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}}$

9. Estimate the value of $f(x) = \int_0^6 x^2 dx$ using a midpoint sum with 3 subintervals

$(2)(1^2) + (2)(3^2) + 2(5)^2$ $\quad$ 1 3 5

$2 + 18 + 50$

$20 + 50$

$\boxed{70}$

10. If $\int_{-1}^{1} f(x)dx = 3$, $\int_{2}^{3} f(x)dx = -2$ and $\int_{1}^{3} f(x)dx = 5$, evaluate $\int_{-1}^{2} f(x)dx$

-1 $\quad$ 1 $\quad$ 2 $\quad$ 3

$3 + 5 - (-2)$

$8 + 2 = 10$

$\boxed{10}$

11. Find $\int \left(\dfrac{(\ln x)^2}{x} \right) dx$

$u = \ln x$

$du = \frac{1}{x} dx$

$\int u^2 du$

$\frac{1}{3}u^3 + C$

$\boxed{\frac{1}{3}(\ln x)^3 + C}$

x^{-1}

12. Find the average value of $f(x)=\frac{1}{x}$ on $[1,3]$ (answer exactly)

$f'(x) = -x^{-2}$

$\frac{\int_1^3 f(x)}{3-1} = \frac{\ln 3 - \ln 1}{2} = \boxed{\frac{\ln 3}{2}}$

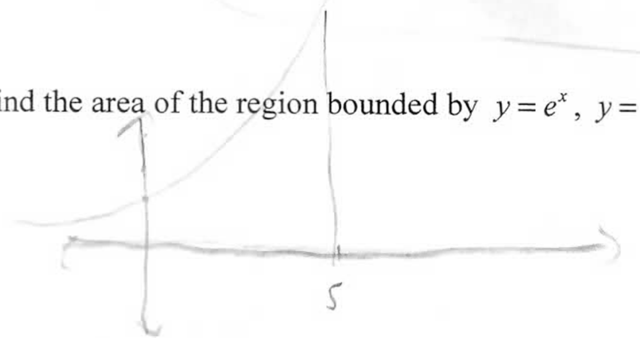
13. Find $f''(x)$ if $f(x)=\int_6^x (4t-3)^{3/2}\,dt$

6

$f'(x) = (4x-3)^{3/2}$

$f''(x) = \frac{3}{2}(4x-3)^{1/2} \cdot 4$

14. Find the area of the region bounded by $y=e^x$, $y=0$, $x=0$ and $x=5$ (answer exactly)



$\int_0^5 e^x\,dx$

$e^x \Big|_0^5$ $\boxed{e^5}$

15. If the curve $y=x^2+c$ is tangent to the line $y=x$, find the value of c

$y' = 2x$

$2x = 1$

$x = \frac{1}{2}$

$y = \left(\frac{1}{2}\right)^2$

$\frac{1}{4} + c = \frac{1}{2}$

$c = \frac{2}{4} - \frac{1}{4}$

$c = \frac{1}{4}$

16. Find an equation of the parabola $y=ax^2+bx+c$ that passes through $(0,1)$ and is tangent to the line $y=x-1$ at $(1,0)$

$y' = 2ax + b$ $y = ax^2 + bx + 1$

$y = \cancel{2a}\ 2x^2 - 3x + 1$

$2ax + b =$

$a + b + 1 = 0$

$a + 1 - 2a + 1 = 0$

$-a + 2 = 0$

$2a + b = 1$

$3(64)^{2/3} + 8$ $\boxed{\$56}$

17. The marginal cost of producing x units of a product is given by $\frac{2}{x^{1/3}}$. The cost of producing 8 units is \$20. Find the cost of producing 64 units.

$2\int x^{-1/3}\,dx$

$2 \cdot \frac{3}{2} x^{2/3} + C$

$3x^{2/3} + C$

$3(8)^{2/3} + C = 20$

$C = 8$

18. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If $h = 3r$ and the radius is increasing at 2 inches per minute when the radius is 6 inches, find the rate of change of volume at that time. Answer exactly including units.

$V = \frac{1}{3}\pi r^2 \cdot 3r$

$V = \pi r^3$

$\frac{dV}{dt} = 3\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 3\pi(6)^2 \cdot (2)$

$\boxed{216\pi \text{ in}^3/\text{min.}}$

19. The measurement of the side of a square is found to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible propagated error in computing the area of the square (include units)

$y = s^2$

$y = x^2$

$dy = 2x \cdot dx$

$dy = 2(12) \cdot \left(\frac{1}{64}\right)$

$\frac{24}{64} = \frac{12}{32} = \frac{6}{16} = \frac{3}{8}$

$\boxed{\pm \frac{3}{8} \text{ in.}^2}$

20. Suppose the distance d in kilometers that a certain car can travel on one tank of gasoline at a speed of v kilometers per hour is given by $d(v) = 8v - \left(\frac{v}{4}\right)^2$. What speed maximizes the distance and therefore minimizes the fuel consumption.

$8v - \frac{v^2}{16}$

$d'(v) = 8 - \frac{2v}{16}$

$d'(v) = 8 - \frac{1}{8}v$

$8 - \frac{1}{8}v = 0$

$8 = \frac{1}{8}v$

$\boxed{v = 64}$