Name:______________
School:______________

Mu A Individual 2018

Louisiana State Competition

1. Find $\lim_{x\to 0}\left(\frac{2e^x-2}{x}\right)$.

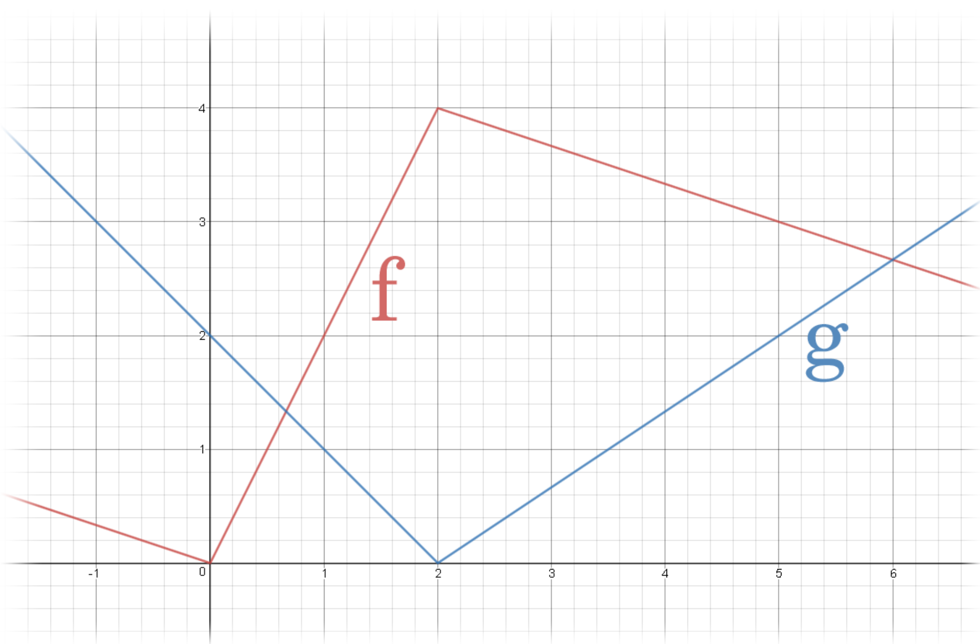
2. Find $\lim_{x\to\infty}\left(\frac{ax^2-bx+c}{(ax-b)(bx-c)}\right)$.

3. Find $\frac{\mathrm{d}}{\mathrm{d}x}(\sin^3(4x))$.

4. Values of $f(x)$ and $f'(x)$ are represented in the table below.
If $k(x)=e^x f(x)$, find $k'(5)$.

x	$f(x)$	$f'(x)$
5	7	-3

5. If $f(x)$ and $g(x)$ are the functions whose graphs are shown and $k(x)=\frac{f(x)}{g(x)}$, find $k'(5)$.



6. Find the slope of the tangent line to $f(x) = A - \frac{B}{x} + \frac{C}{\sqrt{x}}$ when $x = 4$.

7. Find the point(s) where $y = x + \sin x$ has a horizontal tangent on $[0, 2\pi)$ (answer as an exact ordered pair).

8. If $y^3 + y^2 - 5y - x^2 = -4$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

9. Estimate the value of $f(x) = \displaystyle\int_0^6 x^2 \, \mathrm{d}x$ using a midpoint sum with 3 subintervals.

10. If $\int_{-1}^{1} f(x) \, \mathrm{d}x = 3$, $\int_{2}^{3} f(x) \, \mathrm{d}x = -2$, and $\int_{1}^{3} f(x) \, \mathrm{d}x = 5$, evaluate $\int_{-1}^{2} f(x) \, \mathrm{d}x$.

11. Find $\displaystyle\int \left(\frac{(\ln x)^2}{x} \right) \mathrm{d}x$.

12. Find the average value of $f(x) = \frac{1}{3}$ on $[1, 3]$ (answer exactly).

13. Find $f''(x)$ if $f(x) = \displaystyle\int_6^x (4t - 3)^{3/2} \, \mathrm{d}t$.

14. Find the area of the region bounded by $y = e^x$, $y = 0$, $x = 0$, and $x = 5$ (answer exactly).

15. If the curve $y = x^2 + c$ is tangent to the line $y = x$, find the value of c.

16. Find an equation of the parabola $y = ax^2 + bx + c$ that passes through $(0, 1)$ and is tangent to the line $y = x - 1$ at $(1, 0)$.

17. The marginal cost of producing x units of a product is given by $\frac{2}{x^{1/3}}$. The cost of producing 8 units is \$20. Find the cost of producing 64 units.

18. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$. If $h = 3r$ and the radius is increasing at 2 inches per minute when the radius is 6 inches, find the rate of change of volume at that time. Answer exactly including units.

19. The measurement of the side of a square is found to be 12 inches, with a possible error of $\frac{1}{64}$ inches. Use differentials to approximate the possible propagated error in computing the area of the square (include units).

20. Suppose the distance d in kilometers that a certain car can travel on one tank of gasoline at a speed of v kilometers per hour is given by $d(v) = 8v - (\frac{v}{4})^2$. What speed maximizes the distance and therefore minimizes the fuel consumption?

Answers

1. 2
2. $\frac{1}{b}$
3. $12[\sin^2(4x)] \cdot \cos(4x)$
4. $4e^5$
5. $-\frac{2}{3}$
6. $\frac{B}{16} - \frac{C}{16}$
7. (π, π)
8. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{3y^2+2y-5}$
9. 70
10. 10
11. $\frac{1}{3}(\ln x)^3 + c$
12. $\frac{\ln 3}{2}$
13. $6(4x-3)^{1/2}$
14. $e^5 - 1$
15. $c = \frac{1}{4}$
16. $y = 2x^3 - 3x + 1$
17. \$56
18. 216π in.^{3}/min
19. $\pm\frac{3}{8}$ in.^{2}
20. $v = 64$ kilometers per hour