BMHS MA0 Tournament - 2018

Calculus Individual Test - BC

$(x+6)=0$

$x=-6$

1) Find the y-value of the absolute maximum for the function $y=-(x+6)^{\frac{2}{3}}$ on the interval $[-4, 1]$

$y' = -\frac{2}{3}(x+6)^{-1/3}$ $\quad \frac{-2}{3(x+6)^{1/3}} = 0$ $\quad y = -(2)^{2/3}$ $\quad y = -(7)^{2/3}$ $\quad \sqrt[3]{2^2} = \sqrt[3]{4}$

2) Find the equation of the line tangent to the graph of $y=\frac{1}{5x^4+5}$ at the point $\left(1, \frac{1}{10}\right)$.

$y = -(5x^4+5)^{-1}$

$y' = (5x^4+5)^{-2}(20x^3)$

$y'(1) = (5+5)^{-2}(20)$

$\frac{1}{10^2}\,20$ $\quad \frac{1}{100}\,20 = \frac{1}{5}$

$y = \frac{1}{5}(x-1)+\frac{1}{10}$

$y = \frac{1}{5}x - \frac{1}{5} + \frac{1}{10}$

$y = \frac{1}{5}x - \frac{1}{10}$

$-\frac{2}{10}+\frac{1}{10} = -\frac{1}{10}$

3) Use implicit differentiation to find $\frac{dy}{dx}$ at $(2, -1)$.

$\frac{d}{dx}\left(5 = 5x^2 + 3x^2y + 3y\right)$

$0 = 10x + 3\left(x^2\frac{dy}{dx} + 2xy\right) + 3\frac{dy}{dx}$

$-10x = 3x^2\frac{dy}{dx} + 6xy + 3\frac{dy}{dx}$

$\frac{dy}{dx}(3x^2+3) = -10x - 6xy$

$\frac{dy}{dx} = \frac{-10x-6xy}{3x^2+3}$ $\quad (2,-1)$

$\frac{dy}{dx} = \frac{-20+6(2)(+1)}{3(4)+3} = \frac{-8}{15}$

4) Find $\displaystyle\int_0^2 \frac{24x}{(4x^2+4)^2}\,dx$ $\quad 3\int_0^2 \frac{1}{u^2}$

$u = 4x^2+4$

$du = 8x\,dx$

$3\left[-u^{-1}\right]_0^2$

$3\left[-(4x^2+4)^{-1}\right]_0^2$

$3\left[-(4(2)^2+4)^{-1} - (-(4(0)^2+4)^{-1})\right]$

$3\left[-(16+4)^{-1} + 4^{-1}\right]$

$3\left(-\frac{1}{12}+\frac{1}{4}\right)$

$-\frac{3}{12}+\frac{3}{4} = -\frac{1}{4}+\frac{3}{4} = \frac{1}{2}$

5) Find the area of the region enclosed by the curves.

$y=3\sqrt{x},\ y=-3\sqrt{x},$
$x=0,\ x=4$

$\int_0^4 \left(3\sqrt{x}+3\sqrt{x}\right)dx$

$6\int_0^4 x^{1/2}\,dx$

$6\left(\frac{2}{3}x^{3/2}\right)_0^4 = 6\left(\frac{2}{3}(4)^{3/2}-0\right)$

$= 4(\sqrt{64}) = 4(8) = 32$

6) A spherical snowball melts at a rate of $\frac{32\pi}{3}$ in³/sec. At what rate is the radius of the snowball changing when the radius is 4 in? Include units.

$V = \frac{4}{3}\pi r^3$ $\quad dV = 4\pi r^2\,dr$

$-\frac{32\pi}{3} = 4\pi(4)^2\,dr$

$-\frac{32\pi}{3} = 64\pi\,dr$

$-\frac{32\pi}{3}\cdot\frac{1}{64\pi} = -\frac{1}{6} = dr$ $\quad$ in/s

7) Approximate the area under the curve over the interval $[-2, 2]$ using 2 midpoint rectangles.

$y=x^2+x+1$

$1-1+1$

$4+2+1$

$x=$	-2	-1	0	1	2
$y=$	3	1	1	3	7

$(2)(1)+(2)(3)$

$2+6=8$

8) Find $\displaystyle\lim_{x\to 0}\frac{2x}{\ln(x+1)}$

$\lim_{x\to 0} \frac{2(0)}{\ln(1)} = \frac{0}{0}$

$\lim_{x\to 0} \frac{2}{\frac{1}{x+1}}$

$\lim_{x\to 0} 2x+2 = 2$

$\frac{d}{dx}\left(7x^3 - 6x^2\right)$

$F'(x) = 21x^2 - 12x$

9) If $F(x) = \int_{x}^{2x} \left(t^3 - 2t^2 + 1\right) dt$ then find and simplify $F'(x)$

$u = 2x$ $\int_x^u (t^3 - 2t^2 + 1)\,dt$

$\frac{d}{dx}\left[F(u) - F(x)\right]$

$(2x)^3 - 2(2x)^2 + 1$

$8x^3 - 8x^2 + 1 - (x^3 + 2x^2 + 1)$

10) For the function $y = 2x^2 - 12x + 17$, where does the instantaneous rate of change equal the average rate of change on the interval [2, 5]

$4x - 12 = 2$ $4x = 14$ $x = \frac{14}{4} = \frac{7}{2}$

$ARoC = 2$ $\frac{7-1}{3} = 2$

$y' = 4x - 12$

$y'(2) = 8 - 12 = -4$

$f(5) = 50 - 60 + 17 = 7$ $f(2) = 8 - 24 + 17 = 1$ 2

11) Find the volume of the solid that results when the region enclosed by the curves is revolved about the x-axis.

$y = \sqrt[3]{x},\ y = 0,\ x = 1$

$\pi \int_0^1 \left(\sqrt{x}\right)^2 dx$

$\int x^{2/3} = \left[\frac{3}{4}x^{4/3}\right]_0^1 = \frac{3\pi}{4}$

$\frac{y^{1/2}}{x} = x^{-1/2}$

12) Find $\int \frac{\ln x}{\sqrt{x}}\, dx$

$u = \ln x$ $dv = (x^{-1/2})$

$du = \frac{1}{x}$ $v = 2x^{1/2}$

$2x^{1/2}\ln x - \int 2x^{1/2}\left(\frac{1}{x}\right)$

$2x^{1/2}\ln x - 2\int \frac{1}{x^{1/2}}$ $\frac{2}{3}x^{3/2}$

$2x^{1/2}\ln x - 2\left(\frac{2}{3}x^{3/2}\right) + C$

13) Consider the curve defined on the interval $[\,0, 2\pi\,)$ by the parametric equations

$x(t) = \cos 3t$
$y(t) = 3\sin 2t$

$x'(t) = -3\sin\left(\frac{3\pi}{4}\right) = \frac{-3\sqrt{2}}{2}$

$y'(t) = 6\cos\left(\frac{\pi}{2}\right) = 0$

Find the slope of the line tangent to the curve when $t = \frac{\pi}{4}$

$y = -\frac{3\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right)$

14) Find the Average Value of the function over the interval [2 , 5]

$f(x) = \frac{4}{x}$

$\frac{\int_2^5 \frac{4}{x}\,dx}{5 - 2}$

$4\int \frac{1}{x}\,dx$

$4\left[\ln x\right]_2^5$

$\frac{4\ln 5 - 4\ln 2}{3}$

15) If a particle's position is defined by the vector valued function $< \cos 3t\,,\ 6t^4 - e^{2t} >$ then find the particle's acceleration vector when $t = 0$

$v(t) = -3\sin 3t\,,\ 24t^3 - 2e^{2t}$

$a(t) = -9\cos 3t\,,\ 72t^2 - 4e^{2t}$

$a(0) = -9\cos 0\,,\ 4$

$= -9\,,\ 4$