Name:_____
School:_____
Compliments of Mr. Bhakta

Mu A Individual 2019

Benjamin Franklin Tournament

1. Differentiate x^{x^2}.

2. Roy is a slime rancher, and he's just discovered a new slime: the Rad Slime! The rate at which a Rad Slime's radioactive radius increases is 2 inches every hour when fully agitated. Assuming that the Rad Slime's aura is perfectly spherical and that the initial radius is 1 inch, at what rate is the volume of the agitated Rad Slime's aura increasing by in inches^3/hour after 4 hours?

3. If $f(3) = 20$ and $f'(3) = 13$, what is the value of the derivative of $f^{-1}(20)$?

4. Find $\lim_{☺\to 0} \dfrac{(2+☺)^6 - 2^6}{☺}$, or state if it does not exist. (This is cringe!)

5. Find $\left.\frac{dy}{dx}\right|_{x=2}$ $x^4 + 2x^3 - 6$.

6. Generate a point-slope form equation of a tangent line to the function $\frac{60}{x^2}$ at $x = 5$.

7. The position of a particle at time t is modeled by $s(t) = t^3 + 12\ln(t)$. What is its velocity function?

8. If r(x) and g(x) are functions, what is the derivative of r(gr)?

9. Does the Mean Value Theorem apply to $f(x) = \frac{1}{3x-99}$ on $[15, 35]$? If so, for what values?

10. Differentiate $\sin^{-1}(x^2 - 9x + 81)$.

11. If $z(x) = \tan^{-1}(x)$, what is $z''(x)$?

12. Is the function $f(x) = \frac{4x+5}{9-3x}$ continuous at $x = -1$?

13. Find $\lim_{x \to 0} \left(\frac{\sin(3x)}{x} - 3\right)$.

14. Find the equation of the line tangent to $y^2 e^{2x} = 3y + x^2$ at $(0, 3)$.

15. Find the linear approximation of the function $f(x) = \sqrt{1-x}$ about 0 to approximate $\sqrt{0.9}$

Answers

1. $x^{x^2}(2x \ln x + x)$
2. 648π
3. $1/13$
4. 192
5. 56
6. $y - \frac{12}{5} = -\frac{24}{25}(x - 5)$
7. $v(t) = 3t^2 + 12t^{-1}$
8. $r'(gr) \cdot (gr' + rg')$
9. No (because $f(x)$ is not continuous over the interval $[15, 35]$, since it has a discontinuity at $x = 33$)
10. $\frac{2x-9}{\sqrt{1-(x^2-9x+81)^2}}$
11. $\frac{-2x}{(x^2+1)^2}$
12. Yes
13. 0
14. $y = -6x + 3$
15. Linear approximation of $f(x)$: $-\frac{1}{2}x + 1$
    Approximation of $\sqrt{0.9}$ using the linear approximation: 0.95