

2019 CATHOLIC HIGH SCHOOL CALCULUS B INDIVIDUAL

1. Find  $\lim_{x \rightarrow \frac{\pi}{4}} \left( \frac{\sin x - \cos x}{\tan x - 1} \right)$

$\frac{\cos x + \sin x}{\sec^2 x} = \frac{\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}}{2} = \frac{\sqrt{2}}{2}$

2. Find  $\lim_{h \rightarrow 0} \left( \frac{7^{x+h} - 7^x}{h} \right)$

$\frac{f(x+h) - f(x)}{h}$   $f(x) = 7^x$   
 $f'(x) = 7^x \cdot \ln 7$

3. Find the slope of the tangent to  $xy + 2x - 5y = 2$  at  $(3, 2)$

$(x \cdot y' + y) + 2 - 5y' = 0$   
 $(3 \cdot y' + 2) + 2 - 5y' = 0$   $4 = 2y'$   
 $3y' + 2 + 2 - 5y' = 0$   $y' = 2$

4. If  $f(t) = e^{2t} \sin(3t)$ , find  $f'(0)$

$f'(t) = e^{2t} (\cos(3t) \cdot 3 + \sin(3t) \cdot 2)$   
 $f'(0) = 3$

5. If  $u = \ln \sqrt{v^2 + 2v - 1}$ , find and simplify  $\frac{du}{dv}$

$\frac{du}{dv} = \frac{1}{\sqrt{v^2 + 2v - 1}} \cdot \frac{1}{2} (v^2 + 2v - 1)^{-1/2} (2v + 2)$

6. Find  $\frac{dy}{dx}$  if  $y = \frac{1 + \sin x}{1 - \sin x}$

$y' = \frac{(1 - \sin x)(\cos x) - (1 + \sin x)(-\cos x)}{(1 - \sin x)^2}$   
 $y' = \frac{\cos x - \sin x \cos x + \cos x + \sin x \cos x}{(1 - \sin x)^2} = \frac{2 \cos x}{(1 - \sin x)^2}$

7. Find  $\frac{d^3 y}{dx^3}$  if  $y = \ln(5x)$

$y' = \frac{1}{x} \cdot 5 = \frac{1}{x} x^{-1}$   $y'' = -x^{-2}$   $y''' = 2x^{-3} = \frac{2}{x^3}$

8. Find all critical point(s) for  $f(x) = (x-2)(x-3)^4$

$f'(x) = (x-2) \cdot 4(x-3)^3 + (x-3)^4 = 0$   
 $(x-3)^3 (4(x-2) + (x-3)) = 0$   
 $x = 3, \frac{12}{5}$   
 $4x - 8 + x - 3 = 0$   
 $5x - 11 = 0$   
 $x = \frac{11}{5}$

9. If  $f(x) = x - 2\sin x$  on  $[0, 2\pi]$ , give the  $x$  coordinate(s) of the relative maximum point(s)

$$f'(x) = 1 - 2\cos x$$

$$1 - 2\cos x = 0$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$



10. At what point on  $y = (\ln(x+4))^2$  is the tangent line horizontal?

$$y' = 2(\ln(x+4))' \cdot \frac{1}{x+4} = 0$$

$$x = -3$$

$$(-3, 0)$$

11. As a spherical balloon is being inflated, its radius  $r$  (in cm) is given by  $r = 3\sqrt[3]{t}$  for  $0 \leq t \leq 10$ . Find the rate of change after 8 seconds for the volume of the balloon (include units)

$$V = \frac{4}{3}\pi r^3$$

$$r = 3\sqrt[3]{8} = 3 \cdot 2 = 6$$

$$V' = 4\pi r^2 r' = 4\pi (6)^2 \cdot \frac{1}{6}$$

$$(36\pi)$$

12. Find the area of the region bounded by  $y = \frac{1}{\sqrt{1-x^2}}$  and  $y = -\frac{1}{\sqrt{1-x^2}}$  on  $[-\frac{1}{2}, \frac{1}{2}]$



13. Find  $y$  if  $\frac{dy}{dx} = 24x(3x^2-1)^3$  and  $y(0) = -3$

$$u = 3x^2 - 1 \quad y' = 24x(3x^2-1)^3$$

$$4 \int u^3 du$$

$$4 \left( \frac{1}{4} u^4 \right) + C$$

$$2 \left( \arcsin x \right) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$2 \left( \frac{\pi}{6} + \frac{\pi}{6} \right)$$

$$2 \left( \frac{\pi}{3} \right) \quad y = (3x^2-1)^4 - u$$

$$14. \int_e^e \left( \frac{\ln x}{x} \right) dx$$

$$u = \ln x \quad du = \frac{1}{x} dx$$

$$\int_{1/2}^1 u du = \left( \frac{u^2}{2} \right)_{1/2}^1$$

$$\frac{1}{2} - \frac{1/4}{2}$$

$$\left( \frac{3}{8} \right)$$

$$(-1)^4 + C = -3$$

$$1 + C = -3$$

$$C = -4$$

15. Find  $\int \sin^2(x) \cos^3(x) dx$

$$\cos^2 x \cdot \cos x$$

$$1 - \sin^2 x$$

$$\frac{4}{8} - \frac{1}{8}$$

$$\int u^2 (1-u^2) du$$

$$\int u^2 - u^4 du$$

$$\left( \frac{u^3}{3} - \frac{u^5}{5} \right) + C$$

$$\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$$