Advanced Math Individual

Haynes Mu Alpha Theta 2019

Instructions

1. You have 50 minutes for this test.
2. No calculators allowed on this test.
3. Do all scratch work on your test.
4. Provide exact and fully simplified answers unless otherwise stated.
5. Units are not required; if units are given, however, they must be correct.
6. Put name and school code on answer sheet.
7. Good luck and have fun!

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School_____

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Haynes MAO 2019- Advanced Math Individual

1) Haseeb has a DVD rotating at 210 revolutions per minute. Find the angular speed of the DVD in radians per second.

2) From the top of a 75 ft building, Eddie sees a plane at an angle of elevation of 36°. If the distance from Eddie to the plane is 450 ft, how high in the air is the plane? (Note: $sin(36°) \approx 0.6,\ cos(36°) \approx 0.8$)

3) If $csc\,\theta = \frac{17}{8}$ and $0 < \theta < \frac{\pi}{2}$, what is $tan\,\theta$?

4) Determine the constants P and Q given that the parabola $y = Px^2 + Qx - 5$ passes through the points (-2, 0) and (-1, -3).

5) Find the range of the function $f(x) = 5sin[2(x + \frac{\pi}{3})] - 4$

6) Simplify the trig identity: $\frac{sinx}{1-cosx} + \frac{1-cosx}{sinx}$

7) Find the exact value of $cos(\frac{5\pi}{12})$.

8) Convert $(6, \frac{5\pi}{6})$ from polar to rectangular coordinates.

9) Solve the equation $2log(x) + log\ 3 = log\ 75$.

10) Olivia and Elisa leave Haynes Airport on different planes. After 1 hour, Olivia and Elisa are 4 and 5 miles from the airport, respectively. The distance between their planes is 6 miles. If C is the measure of the acute angle between the two planes' paths, find $sin\ C$ (Assume the planes fly in straight lines).

11) James, Anupam, and Kiet take 6 hours to do a job working together. If James can finish the job 7 times faster than Kiet, and Anupam can finish the job 4 times faster than Kiet, how long will it take for Anupam to finish the job?

12) Find the domain of $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$

13) Find the distance between the center of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$ and the origin.

14) The mass of bacterial culture is increasing exponentially, starting at 2 mg at t= 0 hours and reaching 5 mg at t= 30 hours. If the mass of the culture at t=25 hours can be

expressed in the form ab^c where a, b, c are positive rational numbers in simplified form, find abc.

15) Suppose that real number x satisfies:

$$\sqrt{98-x^2}+\sqrt{42-x^2}=8$$

What is the value of $\sqrt{98-x^2}-\sqrt{42-x^2}$?

16) Given that complex numbers x, y satisfy $x^3-y^3=98i$ and $x-y=7i$, if $xy=a+bi$ where a, b are real numbers, find the value of $\frac{a+b}{3}$.

17) For what values of x in the interval $[0, \pi]$ does the graph of $y=cot(2x)$ have a vertical asymptote?

18) If $(-1+i)^{10}+(\sqrt{3}+i)^6$ is expressed in the form $a+bi$, find $a+b$.

19) Aakash and Kevin are standing at each of the foci of the ellipse with equation $\frac{x^2}{80} + \frac{y^2}{60} = 1$. If d is the distance between them, find d^2.

20) Solve for the sum of the possible values of x:

$$\begin{vmatrix} x & 2 & 1 \\ 3 & x & 0 \\ 1 & 0 & 1 \end{vmatrix} = 6$$

Answer Key

(Note: units are not required if they are in parentheses; if provided, however, units must be correct)

1. 7π (rad/s)
2. 345 (ft)
3. 8/15
4. $P = 1/2,\ Q = -3/2$
5. $[-9,\ 1]$ (accept $-9 \le f(x) \le 1$ or $-9 \le y \le 1$)
6. $2csc(x)$ (accept $\frac{2}{sin(x)}$)
7. $\frac{\sqrt{6}-\sqrt{2}}{4}$
8. $(-3\sqrt{3},\ 3)$ (accept (-√(27), 3)
9. $x = 5$
10. $\frac{3\sqrt{7}}{8}$ (accept $\sqrt{63}/8$)
11. 18 (hrs)
12. [-1, 2] (accept $-1 \le x \le 2$)
13. $\sqrt{13}$
14. 25/6
15. 7
16. 7
17. $x = 0,\ \pi/2,\ \pi$
18. -96
19. 80
20. 1