Name:_____
School:_____

Calculus BC Individual 2019

Catholic High Tournament

1. Find $\lim_{x \to \pi/4} \left(\frac{\sin(x) - \cos(x)}{\tan(x) - 1} \right)$

2. Find $\lim_{h \to 0} \left(\frac{7^{x+h} - 7^x}{h} \right)$

3. Find the slope of the tangent to $xy + 2x - 5y = 2$ at $(3, 2)$

4. If $f(t) = e^{2t} \sin(3t)$, find $f'(0)$

5. If $u = \ln \sqrt{v^2 + 2v - 1}$, find and simplify $\dfrac{\mathrm{d}u}{\mathrm{d}v}$

6. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $y = \dfrac{1 + \sin x}{1 - \sin x}$

7. Find $\dfrac{\mathrm{d}^3y}{\mathrm{d}x^3}$ if $y = \ln(5x)$

8. Find all critical point(s) for $f(x) = (x - 2)(x - 3)^4$

9. If $f(x) = x - 2\sin x$ on $[0, 2\pi]$, give the x coordinate(s) of the relative maximum point(s)

10. At what point on $y = (\ln(x + 4))^2$ is the tangent line horizontal?

11. As a spherical balloon is being inflated, its radius r (in cm) is given by $r = 3\sqrt[3]{t}$ for $0 \leq t \leq 10$. Find the rate of change after 8 seconds for the volume of the balloon (include units)

12. Find the area of the region bounded by $y = \dfrac{1}{\sqrt{1-x^2}}$ and $y = -\dfrac{1}{\sqrt{1-x^2}}$ on $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

13. Find y if $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 24x(3x^2-1)^3$ and $y(0) = -3$

14. $\displaystyle\int_{\sqrt{e}}^{e} \left(\frac{\ln x}{x}\right) \mathrm{d}x$

15. Find $\displaystyle\int \sin^2(x)\cos^3(x)\,\mathrm{d}x$

Answers

1. $\sqrt{2}/2$
2. $7^x \cdot \ln 7$
3. 2
4. 3
5. $\frac{v+1}{v^2+2v-1}$
6. $\frac{2\cos x}{(1-\sin x)^2}$
7. $2x^{-3}$
8. $(3,0)$, $(\frac{11}{5}, \frac{4^4}{5^5})$
9. $x = 5\pi/3$
10. $(-3,0)$
11. 36π cm^3/sec
12. $2\pi/3$
13. $y = (3x^2-1)^4 - 4$
14. 3/8
15. $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$