

Mu Team Test

Haynes Mu Alpha Theta 2019

Instructions

1. You have 50 minutes for this test.
2. No calculators allowed on this test.
3. Units are not required unless problem specifically says [units required]
4. Provide exact answers unless otherwise stated.
5. Put team name and school code on answer sheet.
6. Good luck and have fun!

School _____

Team Name (math puns encouraged) _____

Team Members _____

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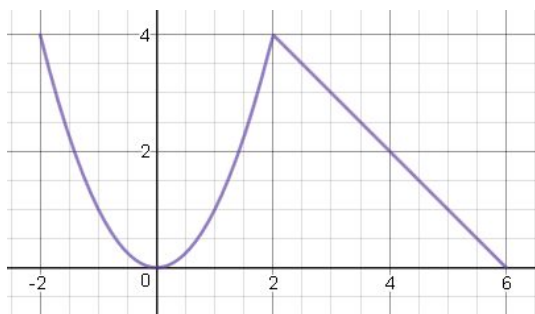
23. _____

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25. _____

1) Hey, you! Yes, you! Evaluate $\lim_{x \rightarrow 2} \frac{\sin(2-x)}{x-2}$.

- 2) Using the graph of the function f on the right, find the intervals for which f' is negative.



3) Please calculate the following definite integral: $\int_{-2}^3 (3x^2 + 2x + 1) dx$. Please.

4) Absolute value functions are cool. Evaluate $\int_{-1}^2 (|2x - 3| - 2|x| + 3) dx$.

- 5) Suppose there is a function, such that:

$$f(x) = \begin{cases} \sin x + \alpha & x < 0 \\ e^{2x} + \beta x + \beta & x \geq 0 \end{cases}$$

What values of α and β would make this function continuous and differentiable?

6) Dr. Bouchon gives you a challenge! Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos^2 2x}{x^2}$

- 7) Are you normal? Because I am! Calculate the slope of the line normal to the graph of $y = \sqrt{13 - x^2}$ at the point where $x = 2$.
- 8) Attention all math competitors! Find the average rate of change of $g(x) = x^3 - 9x$ over the interval $1 \leq x \leq 6$.
- 9) Kevin has two positive numbers whose product is 686 and for which the sum of one and half the other is the minimum possible value. What are Kevin's numbers?
- 10) Jeffery would like to make a request. Differentiate $f(x) = xe^x \sec x$.
- 11) Incoming limit problem! What is $\lim_{x \rightarrow -\infty} (\sqrt{x^2 - \frac{4}{3}x} + 2 + x)$
- 12) Determine $f(5)$ if f is a function that satisfies $\int_0^{f(x)} t^2 dt = 4x + \sin(\pi x)$ for all x .
- 13) You enter a log cabin. Inside lies the function $f(x) = \log_{\ln(x)} x$. What is the derivative of this function? (Note: $\log_a b = \frac{\ln(b)}{\ln(a)}$)

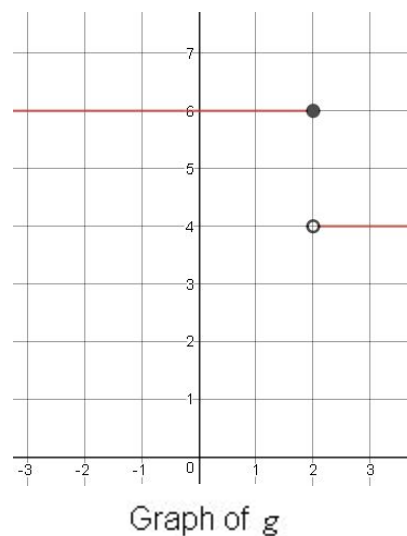
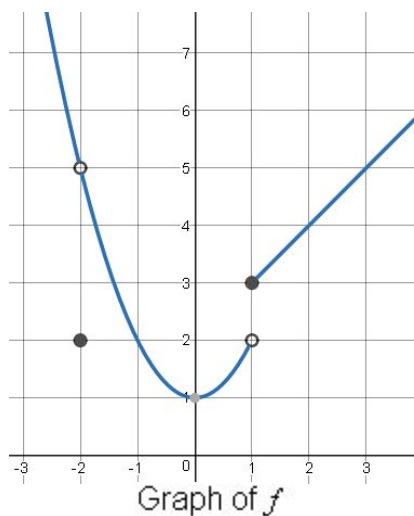
14) You will be evaluated on your ability to evaluate. Evaluate: $\lim_{h \rightarrow 0} \left(\frac{e^{2x+2h} - e^{2x}}{h} \right)$.

15) Are you average? Because I am! Find the average value of $f(x) = \frac{1}{x}$ on $[1, 7]$. (exact answer)

16) Question. What is the area between the curve $y = x^2 + x$ and $y = -1$ from $x = 0$ to $x = 3$?

17) The base of a solid is the region enclosed by the graphs of $y = x$, $y = 2x$, and $x = 4$. Cross sections of the solid perpendicular to the x -axis are equilateral hexagons. Find the volume of the solid. (Hint: The area of a regular hexagon is $\frac{3\sqrt{3}}{2}s^2$, where s is a side).

18) The graphs of f and g are shown in the figures below. If possible, evaluate $\lim_{x \rightarrow 1} (f(x)g(x+1))$.



19) Lines! Lines! Lines! Find the equation of the line tangent to the graph of $f(x) = 7xe^{3x-9}$ at $x = 3$ in slope-intercept form.

20) Max Pan wants to find a max! Identify the maximum value of the function $f(x) = 2xe^{-x}$.

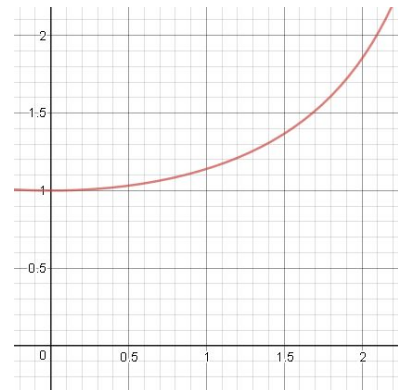
21) Johnny Knotubrite was diligently working on his homework, when all of the sudden he comes across a mysterious equation: $\pi x + ey = c$, where c is a constant. Find $\frac{dy}{dx}$ for this mysterious equation.

22) Satisfaction! Find the values of x that satisfy the Mean Value Theorem for derivatives for the function $f(x) = 4x^3 + 2$ on the interval $[1, 4]$.

23) You can definitely calculate this. Calculate the definite integral:

$$\int_0^1 (x+1)^7 x^{28} (5x^4 + 4x^3) dx \quad (\text{Hint: u-substitution})$$

24) What is the volume of the solid of revolution about the x-axis bound by $y = \sec(\frac{x}{2})$ and $y = 0$ between $x = 0$ and $x = \pi/2$? (Graph is provided at the right).



25) When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, what is the radius?

Answer Key

1. -1
2. $(-2, 0) \cup (2, 6)$ or $[-2, 0) \cup (2, 6]$
3. 45
4. 10.5
5. $\alpha = 0, \beta = -1$
6. 4
7. $3/2$
8. 34
9. $7\sqrt{7}$ and $14\sqrt{7}$
10. $xe^x \sec(x) \tan(x) + xe^x \sec(x) + e^x \sec(x)$
11. $8/3$
12. $\sqrt[3]{60}$
13. $\frac{\ln(\ln x) - 1}{x \ln^2(\ln x)}$
14. $2e^{2x}$
15. $\frac{\ln 7}{6}$
16. 16.5
17. $32\sqrt{3}$
18. 12
19. $y = 70x - 189$
20. $2/e$
21. $-\frac{\pi}{e}$
22. $x = \sqrt{7}$
23. 32
24. 2π
25. $1/\pi$