Calculus AB Individual 2020
Catholic High Tournament

1. Find $\displaystyle\lim_{x\to 2}\left(\dfrac{x-2}{x^3-8}\right)$

2. Find a for which $f(x) = \begin{cases} ax+1 & \text{if } x \le -3 \\ ax^2 - 1 & \text{if } x > -3 \end{cases}$ is continuous

3. Find and simplify the derivative of $e^{\ln x} + x^{\ln e} + \ln(e^x)$

4. If $f(x)$ has values given in the table below, and $g(x) = \dfrac{3}{f(x)}$, find $g'(5)$

$f(5)$	$f'(5)$
-3	6

5. Find the slope of the tangent line to $y = \csc x$ when $x = \frac{\pi}{6}$

6. The cost in dollars of manufacturing q units of a product is $C(q) = 5q^2 + 2q + 300$. Use marginal cost to estimate the cost of the manufacturing the 31st unit.

7. If $f(x) = \dfrac{x}{2x+1}$, find $f''(x)$

8. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ if $(\sin x)(\cos y) = y$

9. On what open interval(s) is $f(x) = 6x^2 + \dfrac{x}{2} + \dfrac{6}{x} + 3$ concave down?

10. If $h(x) = \sin(g(x))$, $g(2) = \frac{\pi}{4}$, $g'(2) = 5$, and $g''(2) = 3$, find $h''(2)$

11. Find $f(x)$ if $\frac{df}{dx} = \sec^2(x)$ and $f(\frac{\pi}{4}) = 2$

12. Find the instantaneous rate of change of $f(\theta) = |\tan \theta|$ when $\theta = \frac{2\pi}{3}$

13. Find the coordinates of the absolute maximum point of $f(x) = x^{2/3}(5 - 2x)$ on $[-1, 2]$

14. In a right circular cone, the radius is 4 inches and is increasing at 5 in/sec while the height is 3 inches and is decreasing at 6 in/sec. Find the rate of change of volume at that time (include units)

15. Find the slope intercept equation of the line tangent to $y = e^x$ that passes through the origin.

Answers

1. $\frac{1}{12}$

2. $\frac{1}{6}$

3. 3

4. -2

5. $-2\sqrt{3}$

6. $C'(30) = 302$

7. $-\frac{4}{(2x+1)^3}$

8. $\frac{\cos x \cos y}{\sin x \sin y + 1}$

9. $(-1, 0)$

10. $-11\sqrt{2}$

11. $\tan x + 1$

12. -4

13. $(-1, 7)$

14. $8\pi \text{ in}^3/\text{sec}$

15. $y = ex$