Name:_____
School:_____

Calculus AB Individual 2018

Brother Martin Tournament

1. $\lim_{x \to -2} \dfrac{x^2 + x - 2}{x + 2}$

2. $\lim_{x \to \infty} \dfrac{\sin x}{4x}$

3. If $y = -4x^5 + 4x^3 - 4x$, find $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4}$

4. Below is a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve the problem below:

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	-1	1	1
2	2	-1	2	$\frac{3}{2}$
3	1	0	4	$\frac{1}{2}$
4	2	1	3	-1

   Given $h(x) = (f(x))^2$, find $h'(2)$

5. If $y = 2^{2x^2}$, find the instantaneous rate of change when $x = 1$

6. Use implicit differentiation to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $(-1, 1)$.

   $4x^2 + 2x^3 y + y = 3$

7. A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. Find the intervals of time when the particle is speeding up.

   $s(t) = -t^3 + 15t^2$

8. Find the ordered pair where the absolute minimum occurs for the function $y = (5x - 15)^{\frac{2}{3}}$ on the interval $[0, 4]$

9. A supermarket employee wants to construct an open-top box from a 10 by 16 inch piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What should the size length of each square be in order to create a box with the largest possible volume?

10. Find the slope of the line tangent to the graph of $y = \dfrac{3}{x^3 + 4}$ at the point $(1, \frac{3}{5})$

11. Differentiate the function: $y = \sin(5x^5)$

12. Find $\displaystyle\int_{-2}^{-1} \frac{24x}{(4x^2 + 2)^2}\,\mathrm{d}x$

13. A spherical snowball is rolled in fresh snow, causing it to grow at a rate of 36π inches^3/sec. How fast is the radius of the snowball increasing when the radius is 8 inches? (Note: For a sphere, $V = \frac{4}{3}\pi r^3$)

14. $\displaystyle\int -4x^{-1}\,\mathrm{d}x$

15. Find $\displaystyle\lim_{x \to 0} \frac{4(e^x - e^{-x})}{\sin(2x)}$

16. If $F(x) = \int_{x}^{x^2} -\frac{1}{t^2}\,\mathrm{d}t$, then find and simplify $F'(x)$

17. For the function $y = x^2 + 2x - 1$, where does the instantaneous rate of change equal the average rate of change on the interval [-4,1]

18. Find the area of the region enclosed by the following curves:

$$y = \frac{4}{x^2}, \quad y = -3,$$
$$x = -2, \quad x = -1$$

Answers

1. -3
2. 0
3. $-480x$
4. -4
5. $16 \ln 2$
6. -2
7. $(0, 5) \cup (10, \infty)$
8. $(3, 0)$
9. 2
10. $-\frac{9}{25}$
11. $25x^4 \cos(5x^5)$
12. $-\frac{1}{3}$
13. $\frac{9}{64}$
14. $-4 \ln |x| + c$
15. 4
16. $F'(x) = x^{-2} - 2x^{-3}$
17. $x = -\frac{3}{2}$
18. 5