Benjamin Franklin MAΘ Tournament 2018

$\tan\theta = \frac{x}{6}$

Mu B Individual

$6\tan\theta = x$

$6\sec^2\theta \cdot \frac{d\theta}{dt} = \frac{dx}{dt}$

1. A balloon rises into the air at 2 ft/sec. A girl letting go of the balloon runs 6 feet away. How fast is the angle of observation changing when the balloon is 10 feet from the ground? Give the answer in its most simplified form and with the correct units.

2. Determine a differentiable function $y = f(x)$ which has the properties $f'(x) = \{f(x)\}^2$ and $f(0) = -\frac{1}{2}$.

$\cos(\cos x) \cdot -\sin x$

3. If $f(x) = x^2 + 2$, $g(x) = \sin(\cos(x))$, $h(x) = ln(x+2)$, find the derivative of $f(g(x)) + h(x)$

$f'(g(x)) \cdot g'(x) + h'(x)$

$\boxed{2(\sin(\cos(x))) \cdot (\cos(\cos x) \cdot -\sin x + \frac{1}{x+2}}$

4. $\lim\limits_{x \to 1} \frac{x^2-x-2}{x^2-2x}$

$\frac{(x-2)(x+1)}{x(x-2)}$ $\frac{x+1}{x} = \boxed{2}$

$\frac{2x-1}{2x-2}$

5. Find the derivative of $f(x) = \int_1^{2x} sin(t^2)dt$

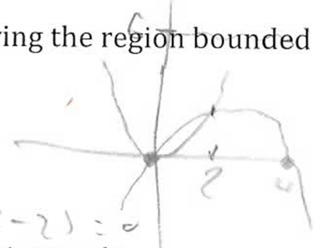
$\boxed{\sin(4x^2) \cdot 2}$

6. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations, $y = x^2$, $y = 4x - x^2$ about y=6.

$x^2 = 4x - x^2$ $x(4-x)$

$2x^2 = 4x$ $2x^2 - 4x = 0$

$2x(x-2) = 0$



$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$\sec^2 x - 1$

For questions 7-9 evaluate the following indefinite integrals:

$\int_0^2 \pi (x^2)^2 dx$ $\int_0^2 \pi (4x - x^2)^2 dx$

7. $\int x\sqrt{1+x^2}dx$

$u = 1 + x^2$

$du = 2x\,dx$

$\frac{1}{2}\int 2x\sqrt{1+x^2}dx$ $\frac{1}{2}\int u^{1/2} du$

$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right] + C$ $\boxed{\frac{1}{3}(1+x^2)^{3/2} + C}$

8. $\int e^x\sqrt{1-e^{2x}}dx$

$u = e^x$ $du = e^x dx$

$u\sqrt{1-u^2}$

$a = 1 - u^2$

$da = -2u\,du$

9. $\int sec^5 x tan^3 x dx$

$\int \sec^5 x \cdot \tan^2 x \cdot \tan x\, dx$

$\int \sec^4 x(\sec^2 x - 1) \sec x \tan x\, dx$

$u = \sec x$

$du = \sec x \tan x\, dx$

$\int u^4(u^2-1)du$

$\int u^6 - u^4 du$

$\frac{u^7}{7} - \frac{u^5}{5} + C$

$\boxed{\frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C}$

10. A wildlife conservation center models the falcon population with the function, $f(t) = t^2 - t + 10$, where t is time in days, and f(t) is the falcon population in falcons. If t=0 represents Monday, what is the rate of change of the falcon population on Friday?

0 1 2 3 4

M T W T F S S

$f'(t) = 2t - 1$

$f'(4) = 8 - 1 = \boxed{7}$

Mu B Individual

$\frac{a^5}{3}-\frac{a^3}{2}-\frac{1}{3}\cdot\frac{1}{2}$

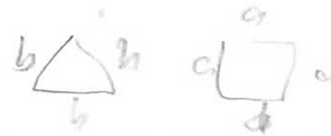
11. Find the limit, if it does not exist justify why it does not exist in the answer,

ex.) $\lim_{t\to\infty} f(t) = \infty$, DNE

$$\lim_{a\to\infty} \frac{\int_1^{a^2} (x^2-x)dx}{a}$$

$\frac{\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^{a^2}}{a}$ $\quad$ $\frac{\left(\frac{a^6}{3}-\frac{a^4}{2}\right)-\left(\frac{1}{3}-\frac{1}{2}\right)}{a}$

DNE, ~~the integral~~ the expression's value diverges!



12. The sum of the perimeters of an equilateral triangle and a square is 10. Find the dimensions of the triangle and the square that produce a minimum total area.

$4a = 10-3b$

$a = \frac{10-3b}{4}$

$3b+4a=10$

$Y = ab$

$y = \left(\frac{10-3b}{4}\right)(b)$

$y = \frac{1}{4}(10b-3b^2)$

$y' = \frac{1}{4}(10-6b)$

$\frac{1}{4}(10-6b)=0$

$-6b=-10$

$b = \frac{10}{6}$ $\boxed{\frac{5}{3}}$

$y'' = \frac{1}{4}(-6)$

$3\left(\frac{5}{3}\right)+4a=10$

$4a=5$

$a = \boxed{\frac{5}{4}}$

13-14. Given $\int_2^4 x^3dx = 60$, $\int_2^4 xdx = 6$, $\int_2^4 dx = 2$, evaluate the following integrals:

$8\int_2^4 xdx$

13) $\int_4^2 xdx$ $\quad$ $\boxed{-6}$

14) $\int_2^4 8xdx$ $\quad$ $8\cdot 6 = 48$

15. The velocity of an automobile starting from rest is $v(t) = 100t/(2t+15)$ where v is measured in feet per second. Find the acceleration at 5 seconds.

$\frac{5-0}{4-7} = \frac{5}{-3}$

16. $tan(x+y^2) = y$ evaluate dy/dx at (0,0)

17. The tangent line to the graph of y=g(x) at the point of (4,5) passes through the point (7,0). Find g(4) and g'(4).

$g(4)=5$ $\quad$ $g'(4) = \frac{5}{3}$

$\frac{0-5}{4-7} = \frac{-5}{-3} = \frac{5}{3}$

$\frac{0-5}{7-4} = \frac{-5}{3}$

18. Evaluate the limit: $\lim_{x\to\infty}\int_0^x te^{-t}dt$

$\frac{t}{e^t}$ $\quad$ $\int_\beta^\alpha xe^{-x}dx$ $\quad$ x $\quad$ e^{-x} $\quad$ 1 $\quad$ $-e^{-x}$

$-xe^{-x} - \int -e^{-x}$ $\quad$ $-xe^{-x}+\int e^{-x}$

$-xe^{-x}\Big|_0^\infty - \left[e^{-x}\right]_0^\infty$ $\quad$ $-xe^{-x}-e^{-x}$

$(0-0)-(0-1)$

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