2018 CATHOLIC HIGH SCHOOL CALCULUS A INDIVIDUAL

$\cos^2 x = 1 - \sin^2 x$

1. Find $\lim_{x\to\pi/2}\left(\dfrac{\cos^2(x)}{1-\sin x}\right)$

$\dfrac{(\cos x)^2 = -2\cos x \sin x}{-\cos x}$ $\quad \dfrac{-2(0)(1)}{0}$ $\quad \dfrac{(1+\sin x)(1-\cancel{\sin x})}{\cancel{1-\sin x}}$

$1+\sin x$

$1+1$

$= \boxed{2}$

2. Find $\lim_{x\to 2}\left(\dfrac{e^x - e^2}{x-2}\right)$

$\dfrac{e^x}{1} = e^x \quad \boxed{e^2}$

3. Find the equation of the line tangent to $y = \ln x$ when $x = e$ (answer in slope intercept form)

$\dfrac{dy}{dx} = \dfrac{1}{x} \quad \dfrac{1}{e} \quad \ln(e) = 1 \quad (e,1)$

$\dfrac{1}{e} \quad y - 1 = \dfrac{1}{e}(x - e)$

$y - 1 = \dfrac{1}{e}x - 1$

$\boxed{y = \dfrac{1}{e}x}$

4. Find the coordinates of all points on $y = x^3 - x$ that have tangent lines parallel to $y = 2x$

$-1+1=0$

$\dfrac{dy}{dx} = 3x^2 - 1$

$3x^2 - 1 = 2$

$3x^2 = 3$

$x^2 = 1 \quad x = \pm 1$

$\boxed{(1,0) \quad (-1,0)}$

5. If $j(x) = \dfrac{f(x)}{x}$ and $f(x)$ has values given in the table below find $j'(3)$

x	$f(x)$	$f'(x)$
3	2	-2

~~$j(x) = f(x)$~~

$j'(x) = \dfrac{x(f'(x)) - f(x)}{x^2} \quad \dfrac{3 \cdot -2 - 2}{9} = \dfrac{-6-2}{9} = \boxed{-\dfrac{8}{9}}$

6. Find the value of the derivative for $x = \tan y$ at $\left(1, \dfrac{\pi}{4}\right)$

~~$y = \tan x$~~

$\tan y = x$

$\tan^2 y = x^2$

$\sec^2 y = x^2 + 1$

$\sec^2 y \cdot \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y} \quad \dfrac{1}{x^2+1} \quad \dfrac{1}{1+1} = \dfrac{1}{2} \quad \boxed{\dfrac{1}{2}}$

7. Find the slope of the normal to $x^3 + 2xy = 5$ at $(1,2)$

$3x^2 + 2\left(\dfrac{dy}{dx}\cdot x + y\right) = 0$

$3x^2 + 2\dfrac{dy}{dx}\cdot x + 2y = 0$

$\dfrac{dy}{dx}(2x) = -3x^2 - 2y$

$\dfrac{dy}{dx} = \dfrac{-3x^2 - 2y}{2x} \quad \dfrac{-3(1)^2 - 2(2)}{2(1)} \quad \dfrac{-3-4}{2} = -\dfrac{7}{2}$

$\boxed{\dfrac{2}{7}}$

8. If $y = \ln(5x)$, find $y'''(3)$

$y' = \dfrac{5}{5x} = \dfrac{1}{x} \quad x^{-1}$

$y'' = -x^{-2}$

$y''' = 2x^{-3} \quad 2(3)^{-3} \quad 2\left(\dfrac{1}{27}\right) = \boxed{\dfrac{2}{27}}$

9. The tangent line to $f(x)$ at $(2,7)$ also passes through $(4,-9)$.
Find $f(2)+f'(2)$

$7-8=\boxed{-1}$ $\qquad \frac{-9-7}{4-2}=\frac{-16}{2}=-8$

10. If $f(x)=x-2\sin x$ on $[0,2\pi]$, give the x coordinate of the relative maximum point

$\boxed{\frac{5\pi}{3}}$

$f'(x)=1-2\cos x$

$1-2\cos x=0$

$2\cos x=1$

$\cos x=\frac{1}{2}$

$\frac{5\pi}{3}$ 300

$\boxed{x=\frac{\pi}{3}, \frac{5\pi}{3}}$

$-\quad+\quad-$

$\frac{\pi}{3} \quad \frac{5\pi}{3}$

11. On what open interval(s) is $f(x)=x^3-6x^2+15$ decreasing?

$\boxed{(0,4)}$

$3x^2-12x=0$

$3x(x-4)=0$

$x=0,4$

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$0 \quad 4$

12. Find the maximum value of $f(\theta)=\theta+\cos\theta$ on $[0,\pi]$

$(0,1)$

$(\frac{\pi}{2},\frac{\pi}{2})$

$(\pi,\pi-1)$

$3.14-1$

$\frac{3.14}{2}$

$f(x)=x+\cos x$

$f'(x)=1-\sin x$

$1-\sin x=0 \qquad \sin x=1 \qquad x=\frac{\pi}{2}$

13. If the position of a particle moving along a horizontal number line is given by $x=2t^3+3t^2-36t+40$ (x in cm, t in seconds), find the acceleration (in cm/sec^2) when the velocity is 0.

$x'=6t^2+6t-36$

$x''=6t^2+6t-36$

$12t+6$

$6t^2+6t-36=0$

$t^2+t-6=0$

$(t+3)(t-2)=0$

$t=-3,2$

$12(2)+6$

$24+6$

$\boxed{30}$ cm/sec^2

14. The area of a rectangle is increasing at 14 cm^2/sec and the length of the rectangle is decreasing at 2 cm/sec. How fast is the width changing when the length is 12 cm and the area is 60 cm^2? (include units)

$A=lw$

$\frac{dA}{dt}=l\cdot\frac{dw}{dt}+w\cdot\frac{dl}{dt}$

$14=12\cdot\left(\frac{dw}{dt}\right)+18\cdot(-2)$

$2(12)+2(x)=60$

$2x=60-24$

$2x=36$

$x=18$

$14=12\cdot\frac{dw}{dt}-36$

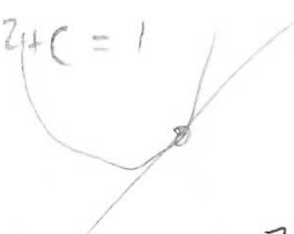
$12\frac{dw}{dt}=50$

$\frac{dw}{dt}=\frac{50}{12}=\boxed{\frac{25}{6}}$

cm/sec

15. Find c for which $y=x^2+c$ is tangent to the line $y=x$.

$x^2+c=1$



$y=1$

(c,c)

$y-c=x-c$

$2x=1$

$\frac{1}{4}+c=\frac{1}{2}$