

Name: _____
School: _____

Interschool 2018

St. Paul's Tournament

1. How many 3 digit integers have the property that their digits, read left to right, are in strictly increasing order?
2. Leap Day on February 29, 2004 occurred on a Sunday. On what day of the week will Leap Day February 29, 2020 occur?
3. Calvin leaves Utah (on his way back to Hawaii), driving at a constant speed. After a while, he passes a mile marker with a two-digit number on it. An hour later, he passes another mile marker with the same two digits on it, but in reverse. In another hour, he passes a 3rd mile marker again with the same 2 digits, but separated by a zero. What is the rate of Calvin's car in miles per hour?
4. What is the largest 7-digit number that contains each of the digits 1 through 7 and has the property that the sum of any 2 consecutive digits is a prime number?
5. You have 63 one dollar bills and 6 envelopes. You want to place the bills in the envelopes in such a way that any amount from \$1 to \$63 could be obtained by selecting a combination of envelopes. If each envelope must have at least \$1, how many dollars would you place in the envelope with the greatest number of bills?
6. Between noon and the following midnight, how many times do the hands of a regular clock form a right angle?
7. This important mathematical concept was invented around the 9th century in India and called Sunya in Sanskrit, and later called zephirum in Medieval Latin, but today what do we call it?
8. Two students play a game based on the total roll of two standard dice. Student A says that a 12 will be rolled first. Student B says that two consecutive 7's will be rolled first. The students keep rolling until one of them wins. What is the probability that A will win?

9. Using only the numbers 1, 3, 4, and 6, together with $+$, $-$, \times , \div , and unlimited use of brackets, make the number 24. Each number must be used precisely once. Each operation may be used zero or more times. Decimal points are not allowed, nor can you combine 2 or more digits to make a number greater than 10.
10. A car travels downhill at 72 mph, on the level at 63 mph, and uphill at only 56 mph. The car takes 4 hours to travel from town A to town B. The return trip takes 4 hours and 40 minutes. Find the distance between the two towns.
11. The towns of Alpha, Beta, and Gamma are equidistant from each other. If a car is three miles from Alpha and four miles from Beta, what is the maximum possible distance of the car from Gamma? Assume the land is flat.
12. In triangle ABC, sides $AB=20$, $AC=11$, and $BC=13$. Find the diameter of the semicircle inscribed in ABC, whose diameter lies on side AB, and that is tangent to sides AC and BC.
13. Each letter in the long division problem below stands for a single digit of a decimal number. The letter M is not zero and no leading digit is zero. Different letters may be used for the same digit. What is the value of B?

$$\begin{array}{r}
 \\
 \\
 \\
 \hline
 \\
 \\
 \hline
 \\
 \\
 \hline

 \end{array}$$

14. If $4^x + 4^{-x} = 7$, then what is $8^x + 8^{-x}$?

15. Suppose that A, B, C, and D are real numbers such that the following matrix equation holds true:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} B+C & -A \\ B-D & A \end{bmatrix}$$

Determine the value of the following determinant: $\begin{vmatrix} A & B \\ C & D-B \end{vmatrix}$

16. The operation \square is defined by $x\square y = 4x - 5y + xy$ for all real numbers x and y . For how many real numbers y does $5\square y = 20$? Pick one of the following options:

- (a) 1
- (b) 2
- (c) 4
- (d) infinitely many
- (e) none of (a) through (d) is correct

17. Line segments drawn from the vertex opposite the hypotenuse of a right triangle to the points trisecting the hypotenuse have lengths $\sin x$ and $\cos x$, where x is a real number such that $0 < x < \frac{\pi}{2}$. Find the length of the hypotenuse.

18. List all integer values of x such that $x^2 - 5x - 1$ is a perfect square.

6. 22

19. 2^{29} is a 9-digit integer with distinct digits. Which digit (from 0 to 9) does it not contain?

20. A magic wish-granting rectangular belt always shrinks to $1/2$ its length and $1/3$ its width whenever its owner makes a wish. After three wishes, the surface area of the belt's front side was 4 cm^2 . What was the original length, if the original width was 9 cm?

21. Suppose that positive integers a , b , and c satisfy $ab + c = 242$ and $ac + b = 370$. What is the maximum possible value of c ?

Answers

1. 34
Source: [AMC 12B 2006 Problem #9](#)
2. Saturday
Source: [AMC 10B 2006 Problem #16](#)
3. 45 mph.
Source: [MAO Nationals Interschool Test 2015 Problem #5](#)
4. 7652341
Source: [MAO Nationals Interschool Test 2015 Problem #9](#)
5. 32 dollars
Source: [MAO Nationals Interschool Test 2015 Problem #10](#)
6. 22
7. 0
8. $7/13$
Source: <http://www.qbyte.org/puzzles/p011s.html>
9. $6 \div (1 - 3 \div 4)$
Source: <http://www.qbyte.org/puzzles/p028a.html>
10. 273
Source: <http://www.qbyte.org/puzzles/p075s.html>
11. 7 miles
Source: <http://www.qbyte.org/puzzles/p139s.html>
12. 11
Source: <http://www.qbyte.org/puzzles/p143s.html>
13. 2
Source: [MAO Nationals Interschool Test 2015 Problem #32](#)
14. 18
Source: [MAO Nationals Interschool Test 2015 Problem #33](#)
15. 0
Source: [MAO Nationals Interschool Test 2015 Problem #35](#)
16. (d) infinitely many
Source: [Spin on AHSME 1993 Problem #4](#)
17. $3/\sqrt{5}$
Source: [AHSME 1980 Problem #23](#)
18. $-5, 10$
19. 4
20. 96 cm
21. 166
Source: [Florida MAO State Interschool Test 2015 Problem #7](#)