# 2019 Scholarship Test
Louisiana State

1. If the diameter of a right cylindrical can with circular bases is increased by 25%, by what percent should the height be increased in order to double the volume of the original can?

2. Let $\downarrow n \downarrow$ be the largest prime number less than n and let $\uparrow n \uparrow$ be the smallest prime number greater than n. Find the value of $41 + (\downarrow 35 \downarrow) - (\uparrow 53 \uparrow) + (\uparrow\downarrow 35 \downarrow\uparrow)$

3. Find the exact value of the sixth term of the sequence $\dfrac{\sqrt{3}}{4}, \sin(\dfrac{\pi}{3}), \cot(\dfrac{\pi}{6}), \ldots$

4. Find the value of $9 - 3 + 1 - \frac{1}{3} + \ldots$ (answer as a fraction)

5. Find the value of x in the triangle below
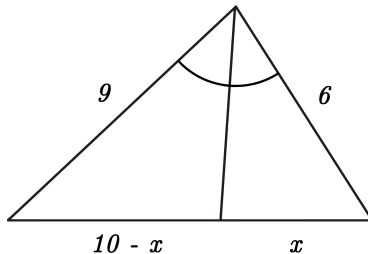
6. Solve for a if $\dfrac{1}{\log_{1024} a} + \dfrac{1}{\log_{729} a} - \dfrac{1}{\log_{36} a} = 4$

7. If $\sec\theta = B$, $\pi < \theta \le 2\pi$, find $\cos(2\theta)\cos(\theta) + \sin(2\theta)\sin(\theta)$

8. Find all numbers x, the sum of whose distances from 1 and from -1 is less than 4

9. After drinking a caffeinated soda, the caffeine level in an adult's bloodstream reaches a peak of about 30 mg. The level then decays exponentially and two hours later there will be 21.675 mg remaining. What percent was lost in the first hour?

10. The quadratic equation $2ax^2 - 4ax + a + 1 = 0$ has two rational roots. If one root is three times the second root, what is the value of a?

11. Solve for x if $\log_2(\log_3(\log_4(x^{3x}))) = 0$

12. Triangle ABC is inscribed in a circle of radius 20 having center Q as shown. Find the measure of $\angle BAC$ to the nearest tenth of a degree
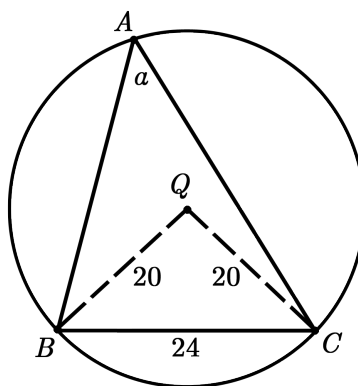
13. Find the term of $(3p + q^3)^7$ that contains q^9

14. In order, the first four terms of a sequence are 2, 6, 12, and 72, where each term, beginning with the third term, is the product of the two preceding terms. If the ninth term is $2^a 3^b$, what is the value of $a + b$?

15. If $\int_{-1}^{1} f(x)dx = 3$, $\int_{2}^{3} f(x)dx = -2$ and $\int_{1}^{3} f(x)dx = 5$, evaluate $\int_{-1}^{2} f(x)dx$

16. Find the rectangular equation of a curve having parametric equations $x = 3\cos t + 7$ and $y = 3\sin t + 7$

17. Trapezoid $ABCD$ has side BC parallel to side AD. $\overline{BE}$, which is parallel to $\overline{CD}$, creates a parallelogram and a triangle of equal area. If $AD = 10$, find the length of $\overline{BC}$.
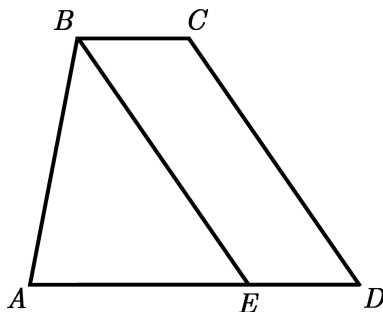
18. An eccentric math teacher presents her problems by matching the base of the number system she uses to the hour of the day. For example, in the hour beginning at 11:00 a.m., the product of 25 and 4 would be written as $23 \cdot 4 = 91$ (because $23_{11} \cdot 4_{11} = 91_{11}$). If the teacher poses the problem $18 \cdot 6$ at 9:30 a.m., what is the answer in the appropriate base?

19. The reciprocal of $2x^2 - 5x + 3$ can be written in the form $\frac{m}{x+p} + \frac{n}{x+q}$. Find the value of $m + n + p + q$

20. Find the absolute maximum value attained by the function $f(x) = A\sin(x) + B\cos(x)$ on $[0, \pi]$, where A and B are positive constants

Answers

1. 28%

2. 50

3. $8\sqrt{3}$

4. $\frac{27}{4}$

5. $x = 4$

6. 12

7. $\frac{1}{B}$

8. $(-2, 2)$

9. 15%

10. $a = 2$

11. $x = 2$

12. 36.9

13. $2835p^4 q^9$

14. 55

15. 10

16. $\frac{(x-7)^2}{9} + \frac{(y-7)^2}{9} = 1$

17. $\frac{10}{3}$

18. 123

19. -2.5

20. $\sqrt{A^2 + B^2}$