# Precalculus Math Bowl

*Haynes Mu Alpha Theta 2019*

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## SET 1

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**Toss-up 1:** If $f(x) = 2 - x^2$, what is $f(x-4)$?
**Bonus 1:** If $f(x) = x^3 - 6x^2 + 11x - 6$ and one of the roots of $f(x)$ is $x = 1$, what are the remaining roots?

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**Toss-up 2:** What is the domain of $f(x) = 4x(2x+1)^{-1}$?
**Bonus 2:** Solve the inequality: $\sqrt[3]{x} < x$

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**Toss-up 3:** Let $f(x) = \sqrt{x}$. Find a formula for a function $g$ whose graph is obtained from $f$ by reflecting across the x-axis, and then shifting up 1 unit.
**Bonus 3:** Solve the equation: $log_3(t-4) + log_3(t+4) = 2$

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**Toss-up 4:** Find the center of the following circle:

$$(x+e)^2 + (y-\sqrt{2})^2 = \pi^2$$

**Bonus 4:** Suppose you have the following sequence: $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \ldots$
If $\frac{1}{3}$ was the 1st term in this sequence, what is the 6th term of this sequence?

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**Toss-up 5:** Suppose $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & -2 \\ 4 & 8 \end{bmatrix}$. What is $A - 2B$?
**Bonus 5:** Find the determinant of the given matrix: $\begin{bmatrix} 12 & -7 \\ -5 & 3 \end{bmatrix}$

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**Toss-up 6:** Convert the angle $\pi^o$ from degree measure into radian measure, giving the exact value in terms of $\pi$.
**Bonus 6:** Find two angles that are coterminal to the angle $-\frac{\pi}{4}$ that lie in the interval $[0, 4\pi]$.

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**Toss-up 7:** Compute the area of the circular sector with the given central angle and radius:

$$\theta = 240^o, \; r = 5$$

**Bonus 7:** For how many values of $x$ does $sin(100x) = \frac{3}{7}$ on the interval $[0, 2\pi]$?

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**Toss-up 8:** If $sin(\theta) = -\frac{7}{25}$ with $\theta$ in Quadrant IV, what is $cos(\theta)$?
**Bonus 8:** What is the period of the function $h(t) = sin(2t - \pi)$?

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**Toss-up 9:** Solve $\ln x + \ln(x+2) = \ln(x+6)$.
**Bonus 9:** Solve $6(5^{2x-9}) = 24$ in terms of natural logarithms.

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**Toss-up 10:** Find sin75°
**Bonus 10:** Find the smallest *positive* angle (in degrees) between the lines with equations $y = \frac{3}{2}x + 8$ and $y = \frac{1}{5}x - 6$.

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## SET 2

**Toss-up 1:** Find $\sin(\tan^{-1}(4/3))$.
**Bonus 1:** Find $\sin 2\theta$ if $\sin\theta = 3/5$ and $\theta$ is in Quadrant II.

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**Toss-up 2:** If a given rectangular coordinates are $(5, 5\sqrt{3})$, find the set of polar coordinates for this point.
**Bonus 2:** Find the polar equation for the line with equation $x = 4$ in rectangular coordinates.

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**Toss-up 3:** If an angle $\theta$ in standard position with the point $(6, -8)$ is on its terminal side, find $\cos\theta$.
**Bonus 3:** Bruce Banner was experimenting with radioactive isotopes. A specific radioactive isotope labeled "Isotope Hulk" has a half-life of 450 years. How long will it take for a sample of this isotope to decrease from 6 mg to 2 mg? Leave answers in exact form. (Hint: $y = Ce^{kt}$)

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**Toss-up 4:** Find the maximum value of the function $f(x) = -x^2 - 5x + 9$.
**Bonus 4:** Find an equation of the non-vertical line which intersects the parabola $y = x^2$ only at the point (3,9).

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**Toss-up 5:** Find $\sec\theta$ if $\tan\theta = ¾$ and $\pi < \theta < 3\pi/2$
**Bonus 5:** Solve the equation $x - 5\sqrt{x} = -6$

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**Toss-up 6:** Solve the inequality $\frac{x^2 - 2x - 24}{x^2 - 8x - 20} \geq 0$.
**Bonus 6:** Simplify the expression $\ln\left(\frac{x^4}{(2x-1)^3(7x-5)^8}\right)$

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**Toss-up 7:** Solve for $x$ in the equation $(\sin x + \cos x)^2 = 1$ over the interval $[0, 2\pi]$.
**Bonus 7:** Find the area of a right triangle with integer side lengths and hypotenuse 17.

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**Toss-up 8:** If $b \oplus a = \ln(ab)$ and $c \star d = e^{cd}$ then what is the value of $(e \oplus e) \star (1 \oplus \sqrt{e})$?
**Bonus 8:** Solve for x: $\log_2 \frac{5x+7}{x} = 4$

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**Toss-up 9:** Find the maximum value of $f(x) = 2019cos(2019x + 2019) + 2019$.
**Bonus 9:** Compute the value of $\sqrt{10201}$.

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**Toss-up 10:** Given that $\tan x = 2018/2019$, compute the value of $\cos x \csc x$.
**Bonus 10:** What polar curve represents a symbol of an important day this past week?

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**Tiebreaker Toss-up:** Chuck the woodchuck devours wood at a rate of 42 pieces per minute. In one hour, how much wood could Chuck the woodchuck chuck if a woodchuck could chuck wood?
**Tiebreaker Bonus:** Compute $i^{2019} + i^{2018} + i^{2017} + i^{2016}$.

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## Answer Key

SET 1

**T1:** $-x^2 + 8x - 14$
**B1:** $x = 2$ and $x = 3$

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**T2:** $(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)$
**B2:** $[-1, 0] \cup [1, \infty)$

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**T3:** $g(x) = -\sqrt{x} + 1$
**B3:** $t = 5$

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**T4:** $(-e, \sqrt{2})$
**B4:** $\frac{1}{720}$

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**T5:** $\begin{bmatrix} -8 & 1 \\ -7 & -12 \end{bmatrix}$
**B5:** 1

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**T6:** $\frac{\pi^2}{180}$
**B6:** $\frac{7\pi}{4}, \frac{15\pi}{4}$ or $315^o, 675^o$

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**T7:** $\frac{50\pi}{3}$ square units
**B7:** 200

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**T8:** $\frac{24}{25}$ ( $\sqrt{\frac{576}{625}}$ is acceptable)

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**B8:** $\pi$

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**T9:** $x = 2$
**B9:** $x = \frac{9\ln 5 + \ln 4}{2\ln 5}$ ($x = \frac{9}{2} + \frac{\ln 2}{\ln 5}$ is acceptable)

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**T10:** $\frac{\sqrt{5}+\sqrt{2}}{4}$
**B10:** 45° (do not accept $\pi/4$; form was specified)

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## SET 2

**T1:** 4/5
**B1:** -24/25

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**T2:** $(10, \frac{\pi}{3})$
**B2:** $r = 4\sec\theta$ (accept $r = \frac{4}{\cos\theta}$)

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**T3:** 3/5
**B3:** $\frac{450\ln 3}{\ln 2}$ years

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**T4:** 61/4
**B4:** $y = 6x - 9$ (accept $y - 9 = 6(x - 3)$)

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**T5:** $\sec\theta = -5/4$
**B5:** $x = 9, x = 4$

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**T6:** $(-\infty, -4] \cup (-2, 6] \cup (10, \infty)$
**B6:** $4\ln x - 3\ln(2x - 1) - 8\ln(7x - 5)$

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**T7:** $0, \pi, 2\pi$
**B7:** 60

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**T8:** e
**B8:** $x = 7/11$

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**T9:** 4038
**B9:** 101

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**T10:** 2019/2018

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**B10:** Cardioid (Valentine's Day)

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**TXX:** 2520 pieces
**TXX:** 0