Calculus Team Test

1) $\lim_{x \to -1} \frac{x^2 - 3x - 4}{x + 1}$ $\frac{2x-3}{1} = \boxed{-5}$

Use a left-hand Riemann sum to approximate the integral based off of the values in the table.

2) $\int_0^{10} f(x)\,dx$

x	0	2	5	7	10
$f(x)$	3	2	4	5	3

$12 + 23 = \boxed{35}$

$2(3) + 3(2) + 2(4) + 3(5) =$

$6 + 6 + 8 + 15 =$

3) Find the slope of the line tangent to $y = (2x^2 - 1)(x^2 + 3)$ when $x = -1$

$y = 2x^4 - x^2 + 6x^2 - 3$

$y = 2x^4 + 5x^2 - 3$

$y' = 8x^3 + 10x$ $-8 - 10 = \boxed{-18}$

4) Find the general solution to the following differential equation. Be sure to isolate y in your answer.

$\frac{dy}{dx} = \frac{2x}{y^2}$ $\frac{2y^2 - 2x\,2yy'}{(y^2)^2} = \frac{2y^2 - 4xyy'}{y^4} =$

$-4xyy' = \frac{y^4 - 2y^2}{-4xy}$ $y' = \frac{y(y^2-2)}{-4x} = \boxed{\frac{y(y^2-2)}{-4x}}$

5) $y = 3x^5 - 4x^3 - 5x$ Find $\frac{d^4y}{dx^4}$

$y' = 15x^4 - 12x^2 - 5$

$y'' = 60x^3 - 24x$ $y'''' = 360x$

$y''' = 180x^2 - 24$

6) Find $\lim_{x \to \infty} \frac{\ln (x + 2)^4}{\ln x^5}$ $\frac{\ln(x+2)^4}{\ln x^5}$ $\frac{\ln x}{\ln y} = \ln(x - y)$

$\frac{4}{5} \ln \frac{\ln(x+2)}{\ln x}$ $\ln((x+2)^4 - x^5)$ $\boxed{\frac{4}{5}}$ $\frac{\frac{1}{x+2}}{\frac{1}{x}}$ $\frac{x}{x+2}$

7) Find where the function $y = \frac{1}{12}x^4 + \frac{1}{6}x^3 - 6x^2 + 4x - 3$ is Concave Down

$y' = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x + 4$

+ | − | +

-4 3

$y'' = x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$x = -4$ $x = 3$

8) Find $F'(x)$ if $F(x) = \int_{4}^{2x} (t^2 + 8t - 13)\, dt$. Simplify your answer.

$2\left((2x)^2 + 8(2x) - 13\right)$ $\quad 8x^2 + 32x - 26$

$2(4x^2 + 16x - 13)$

You are given a table containing some values of differentiable functions $f(x)$, $g(x)$ and their derivatives. Use the table data and the rules of differentiation to solve the problem.

9)

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	−1	2	2
2	2	−1	4	0
3	1	0	2	$-\frac{3}{2}$
4	2	1	1	−1

Given $h(x) = f(g(x))$, find $h'(3)$

$d(f(g(x))$

$g'(x) \cdot f'(g(x))$

$g'(3) \quad f'(g(3))$

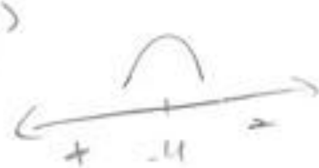
$-\frac{3}{2} \cdot f'(2)$

-1

$\boxed{\frac{3}{2}}$

10) $\lim_{x \to -\infty} -\frac{3x^2}{3x - 1}$ $\quad = \infty$

$-\frac{3x}{3 - \frac{1}{x}}$ $\quad \boxed{\infty}$



11) Find the x-value(s) where $y = -(-5x - 20)^{\frac{2}{3}}$ has a relative maximum.

$y' = \frac{10}{3 \cdot \sqrt[3]{-5x-20}}$ $\quad \frac{10}{3(-5x-20)^{1/3}} = 0$

12) Find $\frac{dy}{dx}$ at $x = \frac{1}{2}$ if $y = \arcsin(x^2)$

$\frac{1}{\sqrt{1-x^2}}$ $\quad \frac{2x}{\sqrt{1-x^4}}$ $\quad \frac{1}{\sqrt{1-\frac{1}{16}}}$ $\quad \frac{1}{\sqrt{\frac{15}{16}}}$ $\quad \left(\sqrt{\frac{15}{16}}\right)$ $\quad \frac{4}{15}$

$\boxed{\frac{4\sqrt{15}}{15}}$

13) Find the average value of $f(x) = -\frac{1}{x^3}$ on the interval $[-2, -1]$

$\frac{1}{b-a} \int_a^b f(x)\, dx$ $\quad \int_{-2}^{-1} -\frac{1}{x^3}\, dx$ $\quad -x^{-3}$

$\int_{-2}^{-1} \frac{1}{2} x^{-2}$ $\quad \frac{1}{2x^2}$ $\quad \frac{1}{2 \cdot 1} - \frac{1}{2 \cdot 4}$

$\frac{1}{2} - \frac{1}{8}$ $\quad \boxed{\frac{3}{8}}$

14) Find $\int (20x^3 + 3x^2 + 4x)\, dx$

$5\frac{20}{4}x^4 + x^3 + 2x^2 + C$ $\quad \boxed{5x^4 + x^3 + 2x^2 + C}$

$\frac{1}{5} - 1 = \boxed{-\frac{4}{5}}$

15) Find $\int_0^1 -\frac{8x}{(4x^2 + 1)^2}\, dx$

$u = 4x^2 + 1$

$du = 8x\, dx$ $\quad \boxed{-\frac{4}{5}}$

$dx = \frac{du}{2x}$

$-\frac{8}{u^2} \cdot \frac{du}{8}$ $\quad \int_1^5 -\frac{1}{u^2}\, du$ $\quad -u^{-2}$ $\quad u^{-1}$

$-u^{-2} = \left.\frac{1}{4x^2+1}\right]_0^1$

16) Let $f(x) = \begin{cases} \sin(x+3) - a, & -5 \le x \le -3 \\ 3x + a, & -3 \le x \le 5 \end{cases}$

Find a such that $f(x)$ is continuous at $x = -3$

$\sin(x+3) - a = 3x + a$

$0 - a$

$-a = -9 + a$

$2a = 9$

$\boxed{a = \frac{9}{2}}$

17) For what value(s) of x does the Mean Value Theorem for Derivatives hold for the function $y = -x^3 + 4x^2 - 2$ on the interval $[1, 4]$

$-3x^2 + 8x = -1$

$-3x^2 + 8x + 1 = 0$

$\frac{-8 \pm \sqrt{64 + 4(3)(1)}}{2(-3)}$

$\frac{-8 \pm \sqrt{76}}{-6}$

18) A farmer wants to construct a rectangular pigpen using 400 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

100 by 200

$l = 100 \quad w = 200$

$2l + w = 400$

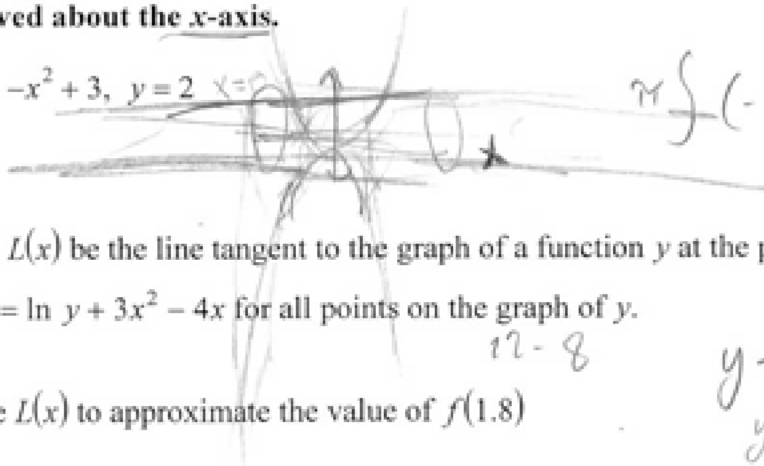
$lw = A$

$l(400 - 2l) = A$

$400 - 4l = 0$

For each problem, find the volume of the solid that results when the region enclosed by the curves is revolved about the *x*-axis.

19) $y = -x^2 + 3, \ y = 2$



$\pi \int (-x^2 + 3$

∞

20) Let $L(x)$ be the line tangent to the graph of a function y at the point $(2, 1)$. It is known that

$\frac{dy}{dx} = \ln y + 3x^2 - 4x$ for all points on the graph of y.

Use $L(x)$ to approximate the value of $f(1.8)$

$\ln 1 + 3(2)^2 - 4 \cdot 2$

$12 - 8$

$y - 1 = 4(x - 2)$

$y - 1 = 4(1.8 - 2) + 1$

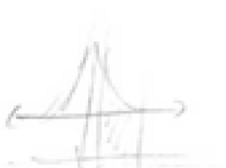
$y = (4 \cdot -0.2) + 1$

$-0.8 + 1$

$y = \boxed{0.2}$

Find the area of the region enclosed by the curves.

21) $y = \frac{3}{x^2}, \ y = -3,$
$x = 1, \ x = 2$



$\int_1^2 \frac{3}{x^2} + 3\,dx$

$-3x^{-1} + 3x \Big|_1^2$

$\frac{-3}{2} + 6 + 3 - 3$

$\frac{9}{2}$

22) Find all intervals where the function $y = -(6x - 24)^{\frac{1}{3}}$ is increasing.

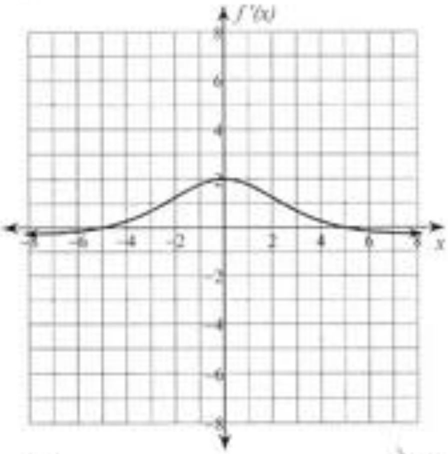
$-\frac{1}{3}(6x - 24)^{-\frac{2}{3}} \cdot 6 = \frac{-2}{(6x-24)^{\frac{2}{3}}} = 0$

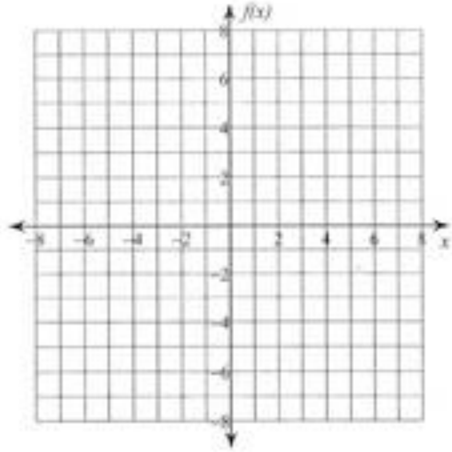
$x = 4$

Always decreasing!

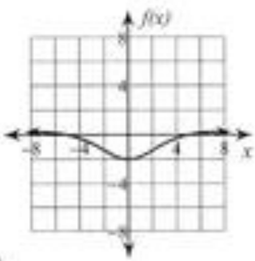
Given the graph of $f'(x)$, choose which of the following sketches could be a possible graph of $f(x)$.

23)





A)



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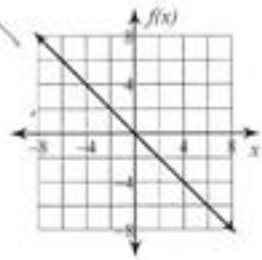
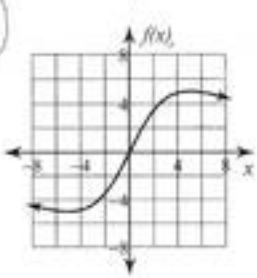


C)



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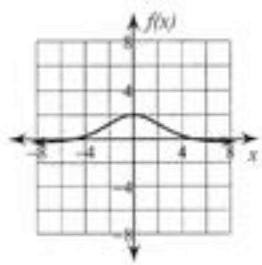

24) If $f(x) = 5x^3 + 2$

$7 = 5x^3 + 2$

Find $(f^{-1})'(7)$

$\frac{1}{f'(f(y))}$

$15x^2$

$f'(1) =$

$\frac{1}{15}$

25) Let a particle's velocity be defined by $v(t) = x^2 - 4x + 3$

What is the total distance traveled on the interval $0 \le t \le 3$

$\frac{1}{3}x^3 - 2x^2 + 3x \Big]_0^3 = \int_0^3$

$\frac{1}{3}x(x^2 - 6x + 9)$

$\frac{1}{3}x(x-3)(x-3)$

$(9 - 6 \cdot 3 + 9$

$(9-18+9)x = \boxed{0}$