Stat 265 – Homework Assignment 3 – Fall 2022

**Assignment #3 (75 marks): Due Thursday, November 17 by 9:00pm**

1. (7.5 marks) Lottery winnings (in millions of dollars) for a lottery are modeled by a continuous random variable, *Y*, with density function given by

$$f_Y(y) = \begin{cases} c(8-y) & , \quad 0<y<6 \\ 0 & , \quad else \end{cases}$$ , where $c$ is a constant.

   a. (2.5 marks) What is the value of the constant $c$?
   b. (2.5 marks) Find $V[Y]$.
   c. (2.5 marks) Find $F(y)$.

2. (10 marks) Consider a random variable, $Y$, with probability density function given by

$$f_Y(y) = \begin{cases} \frac{1}{16} & , -1<y<0 \\ \frac{1}{16}+y^3 & , \quad 0<y<1 \\ \frac{5}{32}y & , 1<y<3 \\ 0 & , \quad else \end{cases}$$

   a. (2.5 marks) Find $E[Y]$.
   b. (2.5 marks) Find $P(-0.75<Y<1.25)$.
   c. (2.5 marks) Find $F(y)$.
   d. (2.5 marks) What is the 80th percentile of $Y$?

3. (5 marks) Consider a random variable, $Y$, with density function:

$$f_Y(y) = \begin{cases} ay^b & , \quad 0<y<4 \\ 0 & , \quad else \end{cases}$$

   If $E[Y]=3.0$, what are the constants $a$ and $b$?

4. (5 marks) The skewness coefficient of a random variable is defined as $\gamma = \frac{E\left[(Y-\mu_Y)^3\right]}{\sigma_Y^3}$. If the moment generating function of $Y$ is $m_Y(t) = e^t(1+12t)^{-2}$ , find the skewness coefficient for $Y$.

5. (5 marks) Consider a continuous random variable, $Y$, with moment-generating function given as

$$m_Y(t) = e^{2t}(1-3t)^{-1}.$$

   Consider another random variable, $X=3Y-6$. What is the moment-generating-function of $X$? What is the distribution of $X$?

6. (10 marks) Consider a *Uniform* random variable, $Y$, over the interval (0, 10). Consider another random variable, $X=10\sqrt{2}(2^{-Y/2})$.
   a. (5 marks) Find the variance of $X$.
   b. (5 marks) What is $P(X>4 \mid X<10)$?

7. (5 marks) Suppose $Y$ is a *Normal* random variable. If $P(Y < 1.25) = 0.8508$ and $P(Y > -0.50) = 0.9406$, what are $\mu_Y$ and $\sigma_Y^2$?

8. (7.5 marks) The grade point average (GPA) of a large population of university students is approximately *Normally* distributed with mean 2.25 and standard deviation 0.49. Assume all students are independent.
   a. (2.5 marks) If you need to finish in the top 12% of your class to earn a certain scholarship, what must your GPA be?
   b. (5 marks) Consider two randomly selected students named Fred and Wilma. What is the probability Wilma's GPA is **<u>at least</u>** twice Fred's GPA?

9. (5 marks) Percentages on a professional entrance exam are described by the probability density function
$$f_Y(y) = \begin{cases} 12y^2(1-y) & , \ 0 < y < 1 \\ 0 & , \ else \end{cases}.$$
Note: A value of $y$ = 0.80 means their percentage is 80%.
Suppose you have a sample of 10 people who are going to write the exam. Assume all their scores are independent. What is the probability at least 3 of them achieve scores over 80%?

10. (5 marks) Suppose $Y$ is an *Exponential* random variable. If $P(a < Y < 2a) = 0.16$ and the median of $Y$ is $8.00$, what is $a$? Note: There may be more than one solution. Report all.

11. (5 marks) The length of time $Y$ (in hours) for a contractor to complete a project in the construction of a house has an *Exponential* distribution with mean of **6 hours**. The contractor is given a monetary bonus depending on the time they complete the job according to the following scheme:

| Time (Y) | Bonus (B) | Hour Segment (H) |
|---|---|---|
| 0 to 1 hour | 1000 | 0 |
| 1 to 2 hours | 900 | 1 |
| 2 to 3 hours | 800 | 2 |
| 3 to 4 hours | 700 | 3 |
| And so on… | | |

**Note: The bonus amount CAN be negative.**
Find the expected value and variance of *B*. Note: *B* is a discrete random variable.
Hint 1: I've included a variable called "hour segment" in the table above. You can then define *B* as a function of *H*.
Hint 2: This one is optional. Consider finding the moment-generating function of *H*. You can find the solution without this hint.

12. (5 marks) Claim amounts following car accidents are *Exponentially* distributed with a mean of 5 (in thousands of dollars). A randomly selected policyholder has a probability of 0.65 of having no claims in a year, a probability of 0.25 of having exactly 1 claim in a year, and a probability of 0.10 of having exactly 2 claims in a year. Calculate the probability the total claim amount for this policyholder over the year is less than $4,000.