1.

Since the set is compact with content zero, we have that there exists $I_1, I_2, \ldots, I_n$ such that

$$I_1 \cup I_2 \cup \ldots \cup I_n \subset U \text{ and } \sum_{j=1}^{n} \mu(I_j) < \frac{\epsilon}{2}$$

Consider the interval $I_i = (a_{i,1}, b_{i,1}) \times \ldots \times (a_{i,n}, b_{i,n})$. Since the set of rational number is dense, we can find b'_1 such that

$$\mathbb{Q} \ni b'_{i,1} - a_{i,1} < \frac{\epsilon(b_{i,1} - a_{i,1})}{2nN\mu(A_i)}$$

Therefore, $O_{i,1} = [a_{i,1}, b'_{i,1}] \times \ldots \times [a_{i,n}, b_{i,n}]$ satisfies

$$\mu(O_{i,1}) - \mu(I_i) = \mu(I_i)\frac{b'_{i_1} - a_{i,1}}{b_{i,1} - a_{i,1}} - \mu(I_i) = \mu(I_i)\frac{b'_{i_1} - b_{i,1}}{b_{i,1} - a_{i,1}} < \frac{\mu(I_i)}{b_{i,1} - a_{i,1}} \cdot \frac{\epsilon(b_{i,1} - a_{i,1})}{2nN\mu(A_i)} = \frac{\epsilon}{2nN}$$

Then using $O_{i,1}$, we can find $O_{i,2}$ such that $\mu(O_{i,2}) - \mu(O_{i,1}) < \dfrac{\epsilon}{2nN}$ using the same process.

Do this process for the rest $n-1$ subintervals, we have that $O_i = O_{i,n} = [a_{i,1}, b'_{i,1}] \times \ldots \times [a_{i,n}, b'_{i,n}]$ satisfies $I_i \subset O_i$ and

$$\mu(O_i) - \mu(I_i) < \frac{\epsilon}{2nN} \cdot N = \frac{\epsilon}{2n}$$

and hence

$$\sum_{j=1}^{n} \mu(O_i) - \sum_{j=1}^{n} \mu(I_i) < \frac{\epsilon}{2n} \cdot n = \frac{\epsilon}{2}$$

Therefore,

$$\sum_{j=1}^{n} \mu(O_i) < \sum_{j=1}^{n} \mu(I_i) + \frac{\epsilon}{2} < \epsilon$$

Since each intervals in O_i has a rational length, we can split it into cubes and thus got the desired results