Stat 265 – Homework Assignment 1 – Fall 2022

Assignment #1 (75 marks): Due Thursday, September 22 at 9:00pm (Edmonton Time)

- 1. (5 marks) From a group of 6 Canadians, 8 Australians and 3 Italians, exactly 3 Canadians, 5 Australians and 2 Italians are to be selected to form a committee. How many different working committees can be formed if one specific Australian (named Charlotte) and one specific Italian (named Leonard) refuse to work together (call this event A)?
- 2. (15 marks) In a game of cards you are dealt 5 cards at random from a standard deck of 52. A standard deck of 52 contains 13 different denominations of cards (Ace, 2, 3, ..., 10, jack, queen, king), each in four different suits (hearts, spades, diamonds and clubs).
 - a) (5 marks) What is the probability of being dealt a hand that includes <u>at least</u> one Ace (call this event *A*)?
 - b) (5 marks) What is the probability of being dealt a hand where all 5 cards are different denominations (call this event *D*)?
 - c) (5 marks) What is the probability of being dealt <u>either</u> of the hands described in parts (a) and (b)?
- 3. (5 marks) Consider a game that consists of dealing out three hands of three cards each from a deck of nine cards. The deck contains the three Aces, three Kings, and three Queens. That is, the cards in the deck are (A A A K K K Q Q Q). What is the probability <u>someone</u> is dealt three of a kind (call this event A)?
- 4. (5 marks) Consider two manufacturing machines, A and B. It is known that machine A has a defect rate 3 times larger than machine B. Further, 65% of all defective items are produced from machine B. What percentage of all items are produced from machine B?
- **5.** (5 marks) From a population of individuals the following statistics are reported:
 - 25% watch football (event F)
 - 40% watch hockey (event H)
 - 30% watch curling (event C)
 - 15% watch football and hockey
 - 20% watch hockey and curling
 - 10% watch football and curling
 - 5% watch all of these sports.

What proportion of individuals watch exactly one of these sports?

- 6. (10 marks) Two players (player A and player B) are playing a game against each other repeatedly until one is bankrupt. When a player wins a game they take \$1 from the other player. All plays of the game are independent and identical.
 - a) (5 marks) Suppose player A starts with \$6 and player B starts with \$6. If player A wins a game with probability 0.50, what is the probability the game ends (someone loses all their money) on **exactly** the 10th play of the game?
 - b) (5 marks) Suppose player A starts with \$2 and player B starts with \$3. If player A wins a game with probability p, what is the probability that player A wins all the money?

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- 7. (5 marks) A personnel director has two lists of applicants for two available jobs. List 1 contains names of 8 qualified applicants and 3 unqualified applicants, whereas list 2 contains the names of 7 qualified applicants and 4 unqualified applicants. A fair die is rolled. If it lands on:
 - 1 or 2: then both individuals are randomly selected from list 1 and given jobs.
 - 3: then both individuals are randomly selected from list 2 and given jobs
 - 4, 5 or 6: then one individual is randomly selected from each list and given jobs. Given two qualified applicants ended up being selected, what is the probability the director rolled a 4, 5, or 6?
- 8. (10 marks) Defective items coming off an assembly line are determined to be caused by exactly one of 3 factors. For any randomly selected defective item, the probability the defect is caused by factor i is $i^3/36$, i=1,2,3. It is also known that for any randomly selected defective item, the

probability of correctly detecting it is caused by factor i upon testing for factor i is $\frac{i}{5}$, i = 1, 2, 3.

Also, there is no possibility of a false positive. For example, if the cause is due to factor 1, there is a zero probability that a test for factors 2 or 3 will result in a positive test result.

- a) (5 marks) For a randomly selected defective item, given a test for factor 1 was negative, what is the probability the defect is caused by factor 1?
- b) (5 marks) For a randomly selected defective item, given a test for factor 2 was negative, what is the probability the defect is caused by factor 3?
- 9. (5 marks) Bert and Earnie are going to play a game where they alternate attempting to complete a task. The first to successfully complete the task wins the game. Suppose there is a 20% chance Bert completes the task on any attempt, there is a 30% chance Earnie completes the task on any attempt, and that all attempts are independent. They will flip a fair (two-sided) coin to see who goes first. What is the probability that Bert wins the game?
- 10. (10 marks) Three soldiers (named A, B, and C) have to decide between themselves, which one goes on a dangerous mission. They decide to take turns drawing straws; there are 4 long straws and 1 short straw. Whoever picks the short straw must go on the mission. They will take turns drawing straws in the following sequence ABCCB ABCCB etc.. For each soldier (A, B, and C), what is the probability that they end up drawing the short straw?
 - a) (5 marks) Suppose they draw straws **without replacement**. Note: This means after a straw is drawn it is not available for the next pick. Note: In this scenario they will determine who goes on the mission after the first 5 draws.
 - b) (5 marks) Suppose they draw straws **with replacement.** Note: This means after a straw is drawn it is replaced and available for the next pick. In this scenario, draws continue indefinitely according to the drawing order described above until someone eventually picks the short straw.