1 p.122

For $t \in (-\infty, \infty)$

$$W(t) = \begin{vmatrix} e^{2t} & e^{-3t/2} \\ 2e^{2t} & -\frac{3}{2}e^{-3t/2} \end{vmatrix} = -\frac{3}{2}e^{t/2} - 2e^{t/2} \neq 0$$

Hence, they are linearly independent on $(-\infty, \infty)$.

2 p.122

For $t \in (-\infty, \infty)$

$$W(t) = \begin{vmatrix} \cos(t) & \sin(t) \\ -\sin(t) & \cos(t) \end{vmatrix} = 1$$

Hence, they are linearly independent on $(-\infty, \infty)$.

3 p.122

For $t \in (-\infty, \infty)$

$$W(t) = \begin{vmatrix} e^{-2t} & te^{-2t} \\ -2e^{-2t} & -(2t-1)\,\mathrm{e}^{-2t} \end{vmatrix} = e^{-4t}(-2t-1) + 2te^{-4t} = -e^{-4t} \neq 0$$

Hence, they are linearly independent on $(-\infty, \infty)$.

4 p.122

For $t \in (-\infty, \infty)$

$$W(t) = \begin{vmatrix} e^t \sin(t) & e^t \cos(t) \\ e^t(\sin(t) + \cos(t)) & e^t(\cos(t) - \sin(t)) \end{vmatrix} = -e^{2t} \neq 0$$

Hence, they are linearly independent on $(-\infty, \infty)$.

1 p.128

$$\exp(2-3i) = e^2\cos(-3) + ie^2\sin(-3)$$

2 p.128

$$e^{i\pi} = \cos(\pi) + i\sin(\pi) = -1$$

3 p.128

$$e^{2-\pi i/2} = e^2 \cos(-\pi/2) + ie^2 \sin(-\pi/2) = -ie^2$$

4 p.128

$$2^{1-i} = 2\cos(-1) + 2i\sin(-1)$$

12 p.128

$$\begin{aligned} y'' - 4y &= 0 \\ \implies r^2 e^{rt} - 4e^{rt} &= 0 \\ \implies r &\in \{2, -2\} \end{aligned}$$

Hence, two exponential solutions are

$$y_1 = e^{2t} \text{ and } y_2 = e^{-2t}$$

As

$$W(t) = \begin{vmatrix} e^{2t} & e^{-2t} \\ 2e^{2t} & -2e^{-2t} \end{vmatrix} = -2 - 2 = -4$$

y_1, y_2 are linearly independent, and hence are a fundamental set of solutions.
We have that

$$y(0) = c_1 + c_2 = 0 \text{ and } y'(0) = 2c_1 - 2c_2 = 1$$

Therefore, we can find c_1, c_2 and

$$y(t) = \frac{1}{4}e^{2t} - \frac{1}{4}e^{-2t}$$

Therefore, as $t \to \infty$, $y(t) \to \infty$

13 p.128

$$
\begin{aligned}
&y'' - 2y' + 5y = 0 \\
\implies &r^2 e^{rt} - 2re^{rt} + 5e^{rt} = 0 \\
\implies &r \in \{1-2i, 1+2i\}
\end{aligned}
$$

Hence, two exponential solutions are

$$y_1 = e^{(1-2i)t} \text{ and } y_2 = e^{(1+2i)t}$$

As

$$W(t) = \begin{vmatrix} e^{(1-2i)t} & e^{(1+2i)t} \\ (1-2i)e^{(1-2i)t} & (1+2i)e^{(1+2i)t} \end{vmatrix} = e^{2t}(1+2i-1+2i) \neq 0$$

y_1, y_2 are linearly independent, and hence are a fundamental set of solutions.
We have that

$$y(\pi/2) = c_1 e^{(1-2i)\pi/2} + c_2 e^{(1+2i)\pi/2} = 0 \text{ and } y'(\pi/2) = (1-2i)c_1 e^{(1-2i)\pi/2} + (1+2i)c_2 e^{(1+2i)\pi/2} = 2$$

Therefore, we can find c_1, c_2 and

$$y(t) = \frac{-1}{2} i e^{-\pi/2} e^{(1-2i)t} + \frac{1}{2} i e^{-\pi/2} e^{(1+2i)t}$$

1 p.135

$$
\begin{aligned}
& y'' - 2y' + y = 0 \\
\implies & r^2 e^{rt} - 2re^{rt} + e^{rt} = 0 \\
\implies & r = 1
\end{aligned}
$$

Therefore,

$$y(t) = ce^t$$

is a solution to the DE.

3 p.135

$$
\begin{aligned}
&4y'' - 4y' - 3 = 0 \\
\implies &4r^2 e^{rt} - 4re^{rt} - 3e^{rt} = 0 \\
\implies &r \in \left\{\frac{3}{2}, -\frac{1}{2}\right\}
\end{aligned}
$$

Hence, two exponential solutions are

$$y_1 = e^{\frac{3t}{2}} \text{ and } y_2 = e^{-\frac{t}{2}}$$

As

$$W(t) = \begin{vmatrix} e^{3t/2} & e^{-t/2} \\ 3/2e^{3t/2} & -1/2e^{-t/2} \end{vmatrix} = -1/2e^t - 3/2e^t = -2e^t \neq 0$$

y_1, y_2 are linearly independent, and hence are a fundamental set of solutions. Therefore,

$$y(t) = c_1 e^{3t/2} + c_2 e^{-t/2}$$

10 p.135

$$
\begin{aligned}
& y'' - 6y' + 9 = 0 \\
\Longrightarrow\ & r^2 e^{rt} - 6re^{rt} + 9e^{rt} = 0 \\
\Longrightarrow\ & r = 3
\end{aligned}
$$

Hence, the solution is

$$y(t) = c_1 e^{3t} + c_2 t e^{3t}$$

$$y(0) = c_1 = 0 \text{ and } y'(0) = 3c_1 + c_2 = 2$$

Hence, the solution is

$$y(t) = 2te^{3t}$$

11 p.135

$$\begin{aligned} &y'' + 4y' + 4y = 0 \\ \implies &r^2 e^{rt} + 4re^{rt} + 4e^{rt} = 0 \\ \implies &r = -2 \end{aligned}$$

Hence, the solution is

$$y(t) = c_1 e^{-2t} + c_2 t e^{-2t}$$

$$y(-1) = c_1 e^2 - c_2 e^2 = 2 \text{ and } y'(-1) = -2c_1 e^2 + 3c_2 e^2 = 1$$

Hence, the solution is

$$y(t) = \frac{7}{e^2} e^{-2t} + \frac{5}{e^2} t e^{-2t}$$