MATH 217 (Fall 2022)
Honors Advanced Calculus, I

Assignment #10

1. Let $a, b > 0$. Determine the area of the ellipse
$$E := \left\{(x,y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\right\}.$$

2. Let
$$D := \left\{(x,y,z) \in \mathbb{R}^3 : 1 \leq \sqrt{x^2+y^2} \leq z \leq 2\right\}.$$
Compute $\int_D \frac{z}{\sqrt{x^2+y^2}}$.

3. Let D in spherical coordinates be given as the solid lying between the spheres given by $r = 2$ and $r = 4$, above the xy-plane and below the cone given by the angle $\theta = \frac{\pi}{3}$. Evaluate the integral $\int_D xyz$.

4. Let $D := \{(x,y,z) \in \mathbb{R}^2 : x^2 + y^2 + z^2 \leq 4,\ x^2 + y^2 \geq 1,\ z \geq 0\}$. Determine $\mu_3(D)$.

5. Let $\varnothing \neq U \subset \mathbb{R}^3$ be open, and let $f, g : U \to \mathbb{R}$ be twice continuously partially differentiable. Show that $\operatorname{div}(\nabla f \times \nabla g) = 0$ on U, where $\times$ denotes the cross product in $\mathbb{R}^3$.

6*. Let $D \subset \mathbb{R}^2$ be the trapeze with vertices $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$. Evaluate $\int_D \exp\left(\frac{x+y}{x-y}\right)$. (*Hint*: Consider
$$\phi \colon \mathbb{R}^2 \to \mathbb{R}^2, \quad (u,v) \mapsto \left(\frac{1}{2}(u+v), \frac{1}{2}(u-v)\right)$$
and apply Change of Variables.)

Due Monday, December 5, 2022, at 10:00 a.m.; no late assignments.