1.

$$
\begin{aligned}
&P(T \in (-4,-2] \cup (-3,1) \cup [4,5) \cup [4.5,6)) \\
=&\int_0^1 e^{-t} dt + \int_4^6 e^{-t} dt \\
=&0.647
\end{aligned}
$$

2.

a.

Let Y be the total time and X be the amount of times we have to wait. Then we have that

$$E[Y] = E[E[Y|X]] = E\left[x \cdot \frac{1-0}{2}\right] = \frac{1}{2}E[X] = \frac{1}{2} \cdot \frac{1}{\frac{1}{6}} = 3$$

b.

$$V[Y|X] = E[Y^2|X] - (E[Y|X])^2 = x \cdot E[Y_i^2] + x \cdot (x-1)E[Y_iY_j] = \frac{x}{12}$$

since

$$E[Y_i^2] = \int_0^1 y^2 dy = \frac{1}{3}$$

and

$$E[Y_iY_j] = \left(\int_0^1 ydy\right)^2 = \frac{1}{4}$$

$$V[X] = \frac{1-\frac{1}{6}}{\frac{1}{6^2}} = 30$$

$$V[Y] = E[V[Y|X]] + V[E[Y|X]] = E[\frac{x}{12}] + V[0.5x] = \frac{1}{12} \cdot 6 + 0.25 \cdot 30 = 8$$

3.

a.

As $\int 2z - 2dz = z^2 - 2z + C$, $C = 1$ so that $F_Z(1) = 0$ and $F_Z(2) = 1$. Hence, we have

$$F_Z(z) = \begin{cases} 0 & z < 1 \\ z^2 - 2z + 1 & 1 \leq z \leq 2 \\ 1 & z > 2 \end{cases}$$

b.

Let $U(s)$ be the uniform random variable between 0 and 1, then

$$V(s) = F_X^{-1}(U(s))$$

simulates the random variable.

4.

The inverse function that maps $\frac{e^{x^3}}{e^{x^3}+e^{-x^3}}$ to x.
We also have that

$$\frac{e^{x^3}}{e^{x^3}+e^{x^{-3}}} = \frac{1}{1+e^{-2x^3}}$$

and hence

$$\sqrt[3]{-\frac{1}{2}\ln\left(\frac{1}{\frac{e^{x^3}}{e^{x^3}+e^{-x^3}}}-1\right)} = x$$

which means that F_X^{-1} is the function that maps x to $\sqrt[3]{-\frac{1}{2}ln(\frac{1}{x}-1)}$

b.

Let $U(s)$ be the uniform random variable between 0 and 1, then

$$V(s) = F_X^{-1}(U(s))$$

simulates the random variable.