1.

$$
\begin{aligned}
\int_0^\infty y f(y) dy &= \int_0^\infty y^2 e^{-y} \\
&= y^2 e^{-y}\Big|_0^\infty + \int_0^\infty 2y e^{y} dy \\
&= 0 - ye^{-y}\Big|_0^\infty + 2\int_0^\infty e^{-y} dy \\
&= 2\left(-e^{-y}|_0^\infty\right) = 2
\end{aligned}
$$

2.

Since g can be any function. Let the function g be the indicator function $1_{(\frac{1}{4},\frac{9}{16})}$. Then we have that

$$\begin{aligned}
P\left(\frac{1}{2} < X < \frac{3}{4}\right) &= E[g(X)] \\
&= \int_0^1 \left(1_{\frac{1}{4}<x^2<\frac{9}{16}}\right) dx \\
&= \int_{\frac{1}{2}}^{\frac{3}{4}} 1dx \\
&= x|_{\frac{1}{2}}^{\frac{3}{4}} \\
&= \frac{1}{4}
\end{aligned}$$

3.

Since the payment is none if it is below the amount D. We have that

$$\begin{aligned} 0.8 \cdot \frac{100000 - 0}{2} &= \int_D^{100000} (x - D) \cdot \frac{1}{100000} dx \\ &= \left(50000 - D + \frac{D^2}{200000} \right) \end{aligned}$$

Hence, we have that $D \simeq 10557.281$

4.

Since $\lim_{y\to 0.5^-} F_Y(y) = 0.5$ and $\lim_{y\to 0.5^+} F_Y(y) = 1$.
We have that $P(Y = 0.5) = 1 - 0.5 = 0.5$ Therefore,

$$E[Y] = \int_0^{0.5} (y)' \cdot y dy + \frac{1}{2} \cdot \frac{1}{2} = \left.\frac{y^2}{2}\right|_0^{0.5} + \frac{1}{4} = \frac{3}{8}$$

It is not continuous at 0.5 and hence is not continuous. It is not discrete because for any open ball around (0.25,f(0.25)) = (0.25,0.25), there is a point in the form $(x, f(x))$ such that it is contained in that ball because f is continuous at 0.25.