1.

Let T be the time expected for the next first call. Then, since we have that

$$P(T > t) = P(\text{no call occurs before t seconds}) = e^{-10t/60} = e^{-t/6}$$

which means that T is an exponential distribution with $\lambda = 1/6$. Hence, we can calculate

$$E[T|T > 10] = \frac{\int_{10}^{\infty} \frac{t}{6} e^{-t/6} dt}{\int_{10}^{\infty} \frac{1}{6} e^{-t/6} dt} = \frac{16e^{-5/3}}{e^{-5/3}} = 16$$

Hence, the expected time for the 90-th call to be processed is $16 \cdot 90/60 = 24$ minutes.

2.

Let T_1 be the time she has to wait to pass the entrance and T_2 be the time she passes the bar line.
For the first process:
We have that $T_{1_C}^S S$ is an exponential distribution with $\lambda = \lambda_S - \lambda_A = 1$.
For the second process:
We have that $r = \frac{\lambda_A}{\lambda_S} = 2.5 \in (2,3) = (k-1,k)$, hence

$$\pi(3) = \pi(0) \cdot \frac{r^3}{3!} = \left(\sum_{j=0}^{1} \frac{2.5^j}{j!} + \frac{3 \cdot 2.5^{3-1}}{(3-1)!(3-2.5)} \right)^{-1} \cdot \frac{2.5^3}{3!} = \frac{125}{1068}$$

$$B_K = -\pi(k) \left(\frac{k\lambda_S^2}{((k\lambda_S - \lambda_A - \lambda_S)(k\lambda_S - \lambda_A)} \right) = -\frac{125}{1068} \cdot (-12) = \frac{125}{89}$$

$$f_{T_C^S S}(t) = -\frac{36}{89} \cdot 2e^{-2t} + \frac{125}{89} e^{-t}$$

Then $T = T_1 + T_2$. Applying convolution twice, we have that

$$\begin{aligned} f_T(t) &= \frac{-36}{89} \int_0^t 2e^{-2\tau} e^{-(t-\tau)} d\tau + \frac{125}{89} \int_0^t e^{-\tau} e^{-(t-\tau)} d\tau \\ &= \frac{-72}{89} (e^{-t} - e^{-2t}) + \frac{125}{89} t e^{-t} \end{aligned}$$

3.

$$
\begin{aligned}
E[Q(\infty)^2] &= \sum_{i=0}^{\infty} i^2 \cdot \pi(i) \\
&= \sum_{i=0}^{\infty} i^2 \cdot \frac{p_S - p_A}{p_S(1-p_S)} \left(\frac{p_A(1-p_S)}{p_S(1-p_A)}\right) \\
&= \frac{0.1}{0.2 \cdot 0.8} \sum_{i=0}^{\infty} i^2 \left(\frac{0.1 \cdot 0.8}{0.2 \cdot 0.9}\right)^i \\
&= \frac{5}{8} \sum_{i=0}^{\infty} i^2 \cdot \left(\frac{4}{9}\right)^i \\
&= \frac{5}{8} \frac{\left(\frac{4}{9}\right)^2 + \frac{4}{9}}{\left(1 - \frac{4}{9}\right)^3} \\
&= \frac{117}{50}
\end{aligned}
$$

4.

We have that

$$\pi(i) = \binom{n}{i} p^i (1-p)^{n-i} = \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i}$$

We also have

$$\pi(i) = \frac{\prod_{j=0}^{i-1} a_j}{\prod_{j=1}^{i} s_j} \pi(0)$$

Hence,

$$\begin{aligned}
\prod_{j=0}^{i-1} a_j &= \frac{n!}{\pi(0) i!(n-i)!} p^i (1-p)^{n-i} \cdot \prod_{j=1}^{i} j(1-p) \\
&= \frac{n!}{\pi(0) i!(n-i)!} p^i (1-p)^{n-i} \cdot i!(1-p)^i \\
&= \frac{n! p^i (1-p)^n}{\pi(0)(n-i)!}
\end{aligned}$$

and thus,

$$\begin{aligned}
a_i &= \frac{\prod_{j=0}^{i} a_j}{\prod_{j=0}^{i-1} a_j} \\
&= \frac{\frac{n! p^{i+1} (1-p)^n}{\pi(0)(n-(i+1))!}}{\frac{n! p^i (1-p)^n}{\pi(0)(n-i)!}} \\
&= \frac{p(n-i)!}{(n-i-1)!} \\
&= p(n-i)
\end{aligned}$$