

1.

$$\begin{aligned}\int_{-\infty}^{\infty} (x^2 - x)1_{[0,a]}(x)dx &= \int_0^a (x^2 - x)dx \\ &= \left(\frac{x^3}{3} - \frac{x^2}{2} \right) \Big|_0^a \\ &= \frac{a^3}{3} - \frac{a^2}{2} = 0 \\ \implies a &= \frac{3}{2}\end{aligned}$$

2.

$$\begin{aligned}\iint_R xy dx dy &= \int_0^1 \int_0^{x^2} xy dy dx \\&= \int_0^1 x \cdot \frac{y^2}{2} \Big|_{y=0}^{x^2} dx \\&= \int_0^1 \frac{x^5}{2} \\&= \frac{x^6}{12} \Big|_0^1 \\&= \frac{1}{12}\end{aligned}$$

3.

Since $0 \leq 0 < \lambda \leq \lambda$, we have that

$$P([0, \lambda]) = e^\lambda - e^0 = e^\lambda - 1 = 1$$

and hence

$$e^\lambda = 1 \implies \lambda = \ln 2$$

4.

$$\begin{aligned}P((-2, -1) \cup (1, 2]) &= F(-1) - F(-2) + F(2) - F(1) \\&= 0.202\end{aligned}$$