# 1 p.188

$$y''' - y'' - y' + y = 2e^{-t} + 3$$

Then as

$$r^3 - r^2 - r + 1 = (r-1)^2(r+1) = 0 \iff r \in \{1, -1\}$$

we have that

$$y_c(t) = c_1 e^t + c_2 t e^t + c_3 e^{-t}$$

as

$$W(0) = \begin{vmatrix} e^t & e^{-t} & te^t \\ e^t & -e^{-t} & e^t + te^t \\ e^t & e^{-t} & 2e^t + te^t \end{vmatrix}_{t=0} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -4 \neq 0$$

Let

$$y_p(t) = Ate^{-t} + B$$

Then

$$\begin{aligned} y' &= Ae^{-t} - Ate^{-t} \\ y'' &= -2Ae^{-t} + Ate^{-t} \\ y''' &= 3Ae^{-t} - Ate^{-t} \end{aligned}$$

Hence,

$$y''' - y'' - y' + y = 4Ae^{-t} + B = 2e^{-t} + 3$$

Hence,

$$y(t) = \frac{1}{2} te^{-t} + 3 + c_1 e^t + c_2 te^t + c_3 e^{-t}$$

## 6 p.188

$$y^{(6)} + y''' = t$$

Then as

$$r^6 + r^3 = r^3(r+1)(r^2 - r + 1)$$

Hence,

$$y_c(t) = c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + c_5 e^{t/2} \sin(\sqrt{3}t/2) + c_6 e^{t/2} \cos(\sqrt{3}t/2)$$

Then let

$$y = At^4$$

We have that

$$y''' = 24At$$

and

$$y^{(6)} = 0$$

which means that $A = 1/24$ and hence

$$y(t) = \frac{1}{24} t^4 c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + c_5 e^{t/2} \sin(\sqrt{3}t/2) + c_6 e^{t/2} \cos(\sqrt{3}t/2)$$

## 2 p.192

$$y''' - y' = t$$

Then as

$$r^3 - r = r(r-1)(r+1)$$

we have that

$$y_c(t) = c_1 + c_2 e^t + c_3 e^{-t}$$

Then we can calculate

$$W(t) = \begin{vmatrix} 1 & e^t & e^{-t} \\ 0 & e^t & -e^{-t} \\ 0 & e^t & e^{-t} \end{vmatrix} = 2 \neq 0$$

$$W^{(1)}(t) = \begin{vmatrix} 0 & e^t & e^{-t} \\ 0 & e^t & -e^{-t} \\ t & e^t & e^{-t} \end{vmatrix} = -2t$$

$$W^{(2)}(t) = \begin{vmatrix} 1 & 0 & e^{-t} \\ 0 & 0 & -e^{-t} \\ 0 & t & e^{-t} \end{vmatrix} = te^{-t}$$

$$W^{(3)}(t) = \begin{vmatrix} 1 & e^t & 0 \\ 0 & e^t & 0 \\ 0 & e^t & t \end{vmatrix} = te^t$$

and then

$$u_1(t) = \int \frac{-2t}{2} dt = -t^2/2$$

$$u_2(t) = \int \frac{te^{-t}}{2} dt = -\frac{te^{-t} + e^{-t}}{2}$$

$$u_3(t) = \int \frac{te^t}{2} dt = \frac{te^t - e^t}{2}$$

and therefore

$$y_p(t) = -\frac{t^2}{2} - \frac{te^{-t} + e^{-t}}{2} e^t + \frac{te^t - e^t}{2} \cdot e^{-t} = \frac{-t^2}{2} + 1$$

and hence

$$y(t) = -t^2/2 + c_1 + c_2 e^t + c_3 e^{-t}$$

## 2 p.192

We have that $a_n = \frac{n}{2^n}$. Then as

$$\left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{2n}{(n+1)}\right|$$

which converges to 2 as $n \to \infty$. Hence, $R = 2$

## 4 p.192

We have that $a_n = 2^n$. Then as

$$\left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{1}{2}\right|$$

which converges to 1/2 as $n \to \infty$. Hence, $R = 1/2$

## 6 p.192

We have that $a_n = \frac{(-1)^n n^2}{3^n}$. Then as

$$\left|\frac{a_n}{a_{n+1}}\right| = \left|\frac{3n^2}{(n+1)^2}\right|$$

which converges to 3 as $n \to \infty$. Hence, $R = 3$