## 1.

As the convention is reversed

$$a_0 = P(0 \to 1) = p_A(1 - p_S)$$

Hence,

$$\begin{aligned}
\pi(0) &= \left(1 + \frac{a_0}{s_1} + \frac{a_0 a_1}{s_1 s_2} + \dots\right)^{-1} \\
&= \left(\sum_{i=0}^{\infty} \left(\frac{1/9}{4/9}\right)^i\right)^{-1} \\
&= \frac{3}{4}
\end{aligned}$$

$$\pi(4) = \pi(0) \cdot \left(\frac{1}{4}\right)^4 = \frac{3}{1024}$$

## 2.

If there is 9 people in the line then

$$P(T = 2n) = \left(\frac{1}{2}\right)^n \frac{(n-1)!}{9!(n-10)!}$$

If the line is operating in steady state, then the formula gives us

$$P(T = 2n) = a \cdot q(1-q)^{n-1} - (a-1) \cdot p_S(1-p_S)^{n-1} = \frac{1}{5}\left(\frac{5}{6}\right)^{n-1} - \frac{1}{5} \cdot \left(\frac{1}{2}\right)^n$$

as $a = \dfrac{1-p_A}{1-p_S} = \dfrac{6}{5}$ and $q = \dfrac{p_S - p_A}{1-p_A} = \dfrac{1}{6}$

**3.**

$$\lim_{t\to\infty} E(Q(t)) = \frac{r}{1-r} = \frac{3}{2} \implies \frac{\lambda_A}{\lambda_S} = r = \frac{3}{5} \implies \lambda_A = \frac{1}{5}$$

Hence,

$$\pi(i) = \frac{2}{5}\left(\frac{3}{5}\right)^i$$

and $f_{T_C^{SS}}$ is a exponential distribution with mean $\lambda_S - \lambda_A = \dfrac{2}{15}$

## 4.

We have that

$$\pi(0) = 1 - r = 1 - \frac{10}{30} = \frac{2}{3}$$

which means that $\frac{2}{3}$ of the time, there is no job left in the buffer. And hence the printer only work $\frac{1}{3}$ of the time. Hence, the expected total time left is $500 \cdot 3 = 1500$