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$$\mathcal{L}(\sin(2t)) = \frac{2}{s^2+4}$$

Hence,

$$\mathcal{L}^{-1}\left(\frac{3}{s^2+4}\right) = \frac{3}{2}\sin(2t)$$

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$$\mathcal{L}(\frac{1}{s-a}) = e^{at}$$

Hence,

$$\mathcal{L}^{-1}\left(\frac{2}{s^2+3s-4}\right) = \frac{2}{5}\mathcal{L}^{-1}\left(\frac{1}{s-1} - \frac{1}{s+4}\right) = \frac{2}{5}e^{t} - \frac{2}{5}e^{-4t}$$

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$$\mathcal{L}(\frac{1}{s-a}) = e^{at}$$

Hence,

$$\mathcal{L}^{-1}\left(\frac{2s-3}{s^2-4}\right) = \mathcal{L}^{-1}\left(\frac{1}{4(s-2)} + \frac{7}{4(s+2)}\right) = \frac{1}{4}e^{2t} + \frac{7}{4}e^{-2t}$$

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$$\mathcal{L}^{-1}\left(\frac{1-2s}{s^2+4s+5}\right) = \mathcal{L}^{-1}\left(\frac{-2(s+2)+5}{(s+2)^2+1}\right) = e^{-2t}\mathcal{L}^{-1}\left(\frac{-2s+5}{s^2+1}\right) = e^{-2t}\left(-2\cos(t)+5\sin(t)\right)$$

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Let $Y(s) = \mathcal{L}[y(t)]$. Then

$$\begin{aligned}
&y'' - y' - 6y = 0 \\
\Longrightarrow\ &s^2Y(s) - sy(0) - 6y'(0) - (sY(s) - y(0)) - 6Y(s) = 0 \\
\Longrightarrow\ &s^2Y(s) - s + 6 - sY(s) + 1 - 6Y(s) = 0 \\
\Longrightarrow\ &Y(s) = \frac{s-7}{s^2 - s - 6} = \frac{9}{5(s+2)} - \frac{4}{5(s-3)}
\end{aligned}$$

Hence, the unique solution is

$$y(t) = \frac{9}{5}e^{-2t} - \frac{4}{5}e^{3t}$$

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Let $Y(s) = \mathcal{L}[y(t)]$. Then

$$\begin{aligned}
& y'' - 2y' + 2y = 0 \\
\Longrightarrow\ & s^2Y(s) - sy(0) - 6y'(0) - 2(sY(s) - y(0)) + 2Y(s) = 0 \\
\Longrightarrow\ & s^2Y(s) - 6 - 2sY(s) + 2Y(s) = 0 \\
\Longrightarrow\ & Y(s) = \frac{6}{s^2 - s + 2} = \frac{6}{(s-1/2)^2 + 7/4}
\end{aligned}$$

Hence, the unique solution is

$$y(t) = 6e^{t/2}\sin(\sqrt{7}t/2)\frac{2}{\sqrt{7}} = \frac{12\sqrt{7}}{7}e^{t/2}\sin\left(\frac{\sqrt{7}t}{2}\right)$$

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$$
\begin{aligned}
\mathcal{L}(\sin(t)) &= \mathcal{L}\left(\sum_{n=0}^{\infty} \frac{(-1)^n t^{2n+1}}{(2n+1)!}\right) \\
&= \sum_{n=0}^{\infty} \mathcal{L}\left(\frac{(-1)^n t^{2n+1}}{(2n+1)!}\right) \\
&= \sum_{n=0}^{\infty} \int_0^{\infty} e^{-st} \frac{(-1)^n t^{2n+1}}{(2n+1)!} dt \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int_0^{\infty} e^{-st} t^{2n+1} dt \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \mathcal{L}(t^{2n+1}) \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{(2n+1)!}{s^{2n+2}} \\
&= \sum_{n=0}^{\infty} \frac{(-1)^n}{s^{2n+2}} \\
&= \frac{1}{s^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(s^2)^n} \\
&= \frac{1}{s^2} \frac{1}{1 - \frac{-1}{s^2}} \\
&= \frac{1}{s^2+1}
\end{aligned}
$$

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$$
\begin{aligned}
F(s) &= \int_0^\infty e^{-st} f(t)dt \\
\implies F'(s) &= \int_0^\infty \frac{\partial}{\partial s}(e^{-st} f(t)) \\
&= \int_0^\infty -te^{-st} f(t)dt \\
&= \mathcal{L}(-tf(t))
\end{aligned}
$$

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Let $g(t) = \sin(bt)$

$$F(s) = \mathcal{L}(g(t)) = \frac{b}{s^2 + b^2}$$

Hence,

$$F''(s) = \mathcal{L}(t^2 g(t)) = \mathcal{L}(f(t)) = \frac{\partial^2}{\partial^2 s} \frac{b}{s^2 + b^2} = \frac{2b\left(3s^2 - b^2\right)}{\left(s^2 + b^2\right)^3}$$