MATH 217 (Fall 2022)
Honors Advanced Calculus, I

Solutions #7

1. Let $A = [a_{j,k}]_{j,k=1}^N \in M_N(\mathbb{C})$ be positive definite. Show that, for $K = 1, \ldots, N$, the "upper left corners" $A_K := [a_{j,k}]_{j,k=1,\ldots,K} \in M_K(\mathbb{C})$ of A are also positive definite. (This is the "only if" part of Theorem A.3.8 in the notes.)

2. Let $\begin{bmatrix} a & b \\ b & d \end{bmatrix} \in M_2(\mathbb{R})$ be such that $a > 0$ and $ad - b^2 > 0$. Show that A is positive definite. (*Hint*: Take a look at $\chi_A(\lambda)$ from two different perspectives.)

3. Determine and classify all stationary points of
$$f \colon (-\pi, \pi) \times (-3, 4) \to \mathbb{R}, \quad (x, y) \mapsto (3 + 2\cos x)\cos y.$$
If f attains a local minimum or maximum at one of its stationary points, evaluate it there.

4. Determine and classify all stationary points of
$$f \colon \mathbb{R}^3 \to \mathbb{R}, \quad (x, y, z) \mapsto x^3 - 3x - y^3 + 9y + z^2.$$
If f attains a local minimum or maximum at one of its stationary points, evaluate it there.

5. Determine and classify the stationary points of
$$f \colon \mathbb{R}^2 \to \mathbb{R}, \quad (x, y) \mapsto (x^2 + 2y^2)e^{-(x^2+y^2)}.$$
If f has a local extremum at a stationary point, determine the nature of this extremum and evaluate f there.

6*. Determine the minimum and the maximum of
$$f \colon D \to \mathbb{R}, \quad (x, y) \mapsto \sin x + \sin y + \sin(x + y),$$
where $D := \left\{(x, y) \in \mathbb{R}^2 : 0 \le x, y \le \frac{\pi}{2}\right\}$, and all points of D where they are attained.

Due Monday, November 14, 2020, at 10:00 a.m.; no late assignments.