1 y' + P(t)y = const a

Multiply both sides by $e^{\int P(t)dt}$ so that

$$y'e^{\int P(t)dt} + P(t)ye^{\int P(t)dt} = ae^{\int P(t)dt}$$

and hence

$$\frac{d}{dt}\left(ye^{\int P(t)dt}\right) = ae^{\int P(t)dt}$$

Integrate both sides

$$ye^{\int P(t)dt} = aP(t)e^{\int P(t)dt} + C$$

2 $\frac{dy}{dx} = f(x, y)$, f is seperable

$$\frac{dy}{dx} = g(x)h(y)$$

$$\int g(x)dx = \int \frac{1}{h(y)}dy$$

3 $\frac{dy}{dx} = f(x, y)$, f is homogeneous of degree 0

Let $y = xu$, then

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

Then we can find

$$x\frac{du}{dx} = P(u)$$

$$\int \frac{1}{P(u)}du = \int \frac{1}{x}dx$$

4 $\frac{dy}{dt} = f(y)$

$\overline{y}$ is an equilibrium if $f(\overline{y}) = 0$ and is a constant solution to the DE.

All solutions with initial points close to $\overline{y} = 0$ eventually move away from $\overline{y} = 0$ when t is suciently large. In this case, equilibrium $\overline{y}$ is unstable.

All solutions with initial points close to $\overline{y} = 0$ will stay close to $\overline{y} = 0$ for all $t > 0$. In such a case, we say the equilibrium $\overline{y} = 0$ is stable

All solution with initial points close to $\overline{y} = 0$ not only stay close to $\overline{y} = 0$ but also eventually converge to $\overline{y} = 0$ as $t \to \infty$.In this case, equilibrium $\overline{y} = 0$ is asymptotically stable.

5 $f(x,y) + g(x,y)\frac{dy}{dx}$

$$M(x,y)dx = N(x,y)dy$$

is exact iff

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

if exact, there is φ

$$\frac{\partial \varphi}{\partial x} = M \text{ and}$$

Hence, integrate that wrt to x, We have

$$\varphi = \frac{dM}{dx} + c(y)$$

and integrate that wrt to y

$$N = \frac{\partial \varphi}{\partial y} = ...$$

6 y" + P(t)y' + Q(t)y = 0

Let $y = e^{rt}$. Solve quadratic. In case of 1 solution, $y_1 = e^{rt}, y_2 = te^{rt}$. In case $r_1 = a+ib, r_2 = a-ib$, $y_1(t) = e^{at}(\cos(bt)+i\sin(bt))$ and similarly for y_2, taking real and imaginary part, we obtain $y_1 = e^{at}\cos(bt), y_2 = e^{at}\sin(bt)$

7 Method of reduction order if one solution is known

Let $y(t) = v(t)y_1(t)$ and substitute in. Let $w = v'$ then solve for first order DE w

8 y" + P(t)y' + Q(t)y = 0