1.

a.

Since the number of customers arriving every minutes follows a poisson distribution with mean 2, the number of customers arriving every 5 minutes follows a poisson distribution with mean 10.

$$\frac{10^6 e^{-10}}{6!} \cdot \frac{10^8 e^{-10}}{8!} = 0.0071$$

b.

Since the number of customers arriving every minutes follows a poisson distribution with mean 2, the number of customers arriving every 20 seconds follows a poisson distribution with mean 2/3.

$$C_6^{15} \cdot C_8^{15} \cdot \left(\frac{2}{3}\right)^{14} \left(1 - \frac{2}{3}\right)^{16} = 0.00256$$

c.

$$C_6^{60} \cdot C_8^{60} \cdot \left(\frac{1}{6}\right)^{14} \left(1 - \frac{1}{6}\right)^{106} = 0.0066$$

d.

We can see that the answers are close to each other, and since $5 < 20$, the answer in part c is closer to the true answers, which is the answers in part a.

2.

$$\lambda = \frac{p}{\Delta} = 3$$

$$\begin{aligned}
P(X(1)X(2)X(3) = 4) &= P(X(1) = 1, X(2) = 1, X(3) = 4) + P(X(1) = 1, X(2) = 2, X(3) = 3) \\
&= e^{-3\cdot 3}\frac{(3\cdot 3)^4}{4!}\begin{pmatrix} & 4 & \\ 1 & 0 & 3\end{pmatrix}\left(\frac{1}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^0 \cdot \left(\frac{1}{3}\right)^3 \\
&+ e^{-3\cdot 3}\frac{(3\cdot 3)^2}{2!}\begin{pmatrix} & 2 & \\ 1 & 1 & 0\end{pmatrix}\left(\frac{1}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^1 \cdot \left(\frac{1}{3}\right)^0 \\
&= 0.00277672
\end{aligned}$$

3.

$$
\begin{aligned}
P(X=k) &= \sum_{i=k}^{\infty} P(Y=i) \cdot \left(\frac{3}{4}\right)^{k} \cdot \left(\frac{1}{4}\right)^{i-k} \cdot \binom{i}{k} \\
&= \sum_{i=k}^{\infty} \frac{8^i e^{-8}}{i!} \cdot 3^k \cdot \left(\frac{1}{4}\right)^{i} \cdot \frac{i!}{k!(i-k)!} \\
&= \frac{3^k}{k!} \sum_{i=k}^{\infty} \frac{2^i e^{-8}}{(i-k)!} \\
&= \frac{3^k}{k!} \sum_{i=0}^{\infty} \frac{2^{i+k} e^{-8}}{i!} \\
&= \frac{6^k e^{-6}}{k!} \sum_{i=0}^{\infty} \frac{2^i e^{-2}}{i!} \\
&= \frac{6^k e^{-6}}{k!}
\end{aligned}
$$

4.

a.

$$P(N(T_1+1) \geq 4) = P(N(T_1+1) = 4|N(T_1) = 1) = 1-\sum_{i=0}^{2} \frac{5^i e^{-5}}{i!} = 0.875348$$

b.

$$\hat{\lambda} = \frac{1}{1/2} = 2$$

$$P(\hat{N}(T_1+1) \geq 4) = P(\hat{N}(T_1+1) = 4|\hat{N}(T_1) = 1) = 1-\sum_{i=0}^{2} \frac{2^i e^{-2}}{i!} = 0.3233$$