9 p.33

$$\begin{aligned}
&y'(t) - y(t) = 2te^{2t} \\
\implies &y'(t)e^{-t} - y(t)e^{-t} = 2te^{t} \\
\implies &\frac{d}{dt}(y(t)e^{-t}) = 2te^{t} \\
\implies &y(t)e^{-t} = 2(t+1)e^{-t} + C \\
\implies &y(t) = -2(t+1) + Ce^{t} \\
\implies &y(t) = -2(t+1) + 3e^{t}
\end{aligned}$$

as $y(0) = 1$.

12 p.33

Let $\mu(t) = e^{\int \frac{t+1}{t} dt} = e^{t+\ln(t)}$

$$
\begin{aligned}
& ty'(t) + (t+1)y(t) = t \\
\implies & y'(t) + \frac{t+1}{t} y(t) = 1 \\
\implies & \frac{d}{dt}(y(t)\mu(t)) = \mu(t) \\
\implies & y(t)\mu(t) = \int \mu(t) dt \\
\implies & y(t) = \int e^{t+\ln(t)} dt \cdot e^{-t-\ln(t)} \\
\implies & y(t) = (t-1)e^t \cdot e^{-t-\ln(t)} + C \cdot e^{-t-\ln(t)} \\
\implies & y(t) = (t-1)e^{-\ln(t)} + 4\ln(2)e^{-t-\ln(t)}
\end{aligned}
$$

as $y(\ln(2)) = 1$.

21 p.34

$$
\begin{aligned}
& y'(t) - \frac{3}{2}y(t) = 3t + 2e^t \\
\Longrightarrow\ & y'(t)e^{-\frac{3}{2}t} - \frac{3}{2}y(t)e^{-\frac{3}{2}t} = (3t + 2e^t)e^{-\frac{3}{2}t} \\
\Longrightarrow\ & \frac{d}{dt}(y(t)e^{-\frac{3}{2}t}) = -\frac{\mathrm{e}^{-\frac{3t}{2}}\left(12\mathrm{e}^t + 6t + 4\right)}{3} + C \\
\Longrightarrow\ & y(t) = -\frac{12\mathrm{e}^t + 6t + 4}{3} + Ce^{\frac{3}{2}t} \\
\Longrightarrow\ & y(t) = -\frac{12\mathrm{e}^t + 6t + 4}{3} + \left(y_0 + \frac{16}{3}\right)e^{\frac{3}{2}t}
\end{aligned}
$$

as $y(0) = y_0$. Hence, the value of y_0 that seperateds solutions that grow positively as $t \to \infty$ and negatively is $-\frac{16}{3}$. If $y_0 = \frac{-16}{3}$, then

$$
y(t) = -\frac{12e^t + 6t + 4}{3}
$$

Therefore, $y(t) \to -\infty$ as $t \to \infty$.

2 p.40

$$\begin{aligned}
&y'(t) + y^2(t)\sin(x) = 0 \\
\implies & -\frac{y'(t)}{y^2(t)} = \sin(x) \\
\implies & \frac{1}{y} = -\cos(x) + C \\
\implies & y = -\frac{1}{\cos(x) + C}
\end{aligned}$$

6 p.40

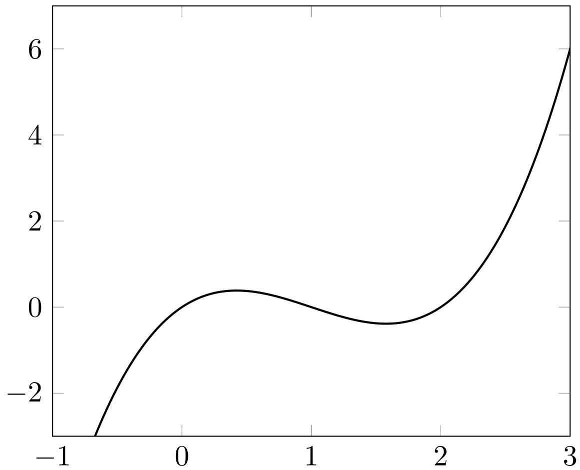
$$\frac{dy}{dx} = \frac{x^2}{1+y}$$
$$\implies (1+y)dy = x^2 dx$$
$$\implies y^2 + y = \frac{x^3}{3} + C$$
$$\implies y^2 + y - \frac{x^3}{3} = C$$

28 p.41

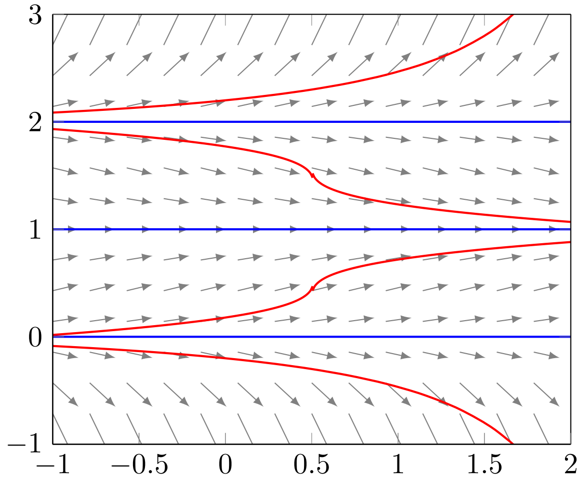
Let $y = xu$, then

$$
\begin{aligned}
&\frac{dy}{dx} = \frac{4y-3x}{2x-y} \\
&\Longrightarrow u + x\frac{du}{dx} = \frac{4ux-3x}{2x-ux} = \frac{4u-3}{2-u} \\
&\Longrightarrow x\frac{du}{dx} = \frac{4u-3-2u+u^2}{2-u} \\
&\Longrightarrow \frac{2-u}{u^2+2u-3}du = \frac{1}{x}dx \\
&\Longrightarrow \int \frac{1/4}{u-1} + \frac{-5/4}{u+3}du = \ln|x| + C \\
&\Longrightarrow \frac{1}{4}\ln|u-1| - \frac{5}{4}\ln|u+3| - \ln|x| = C \\
&\Longrightarrow \ln\left(|u-1|^{-1/4}|u+3|^{5/4}|x|\right) = C \\
&\Longrightarrow |u-1|^{-1/4}|u+3|^{5/4}|x| = e^C > 0 \\
&\Longrightarrow |u-1|^{-1}|u+3|^{5}|x|^4 = e^{4C} > 0 \\
&\Longrightarrow |u-1|^{-1}|u+3|^{5}|x|^4 = C > 0 (\text{change } C = e^{4C}) \\
&\Longrightarrow \left(\frac{y}{x}-1\right)^{-1}\left(\frac{y}{x}+3\right)^5 x^4 = C \\
&\Longrightarrow (y-x)(y+3x) = C
\end{aligned}
$$

2 p.69



The equilibrum points are $0, 1, 2$ as $\frac{dy}{dt} = 0$.



We can see that $2, 0$ is unstable and 1 is asymptotically stable. The red lines are some graphs of solutions, while the blue lines are the phase line.