Stat 265 – Homework Assignment 4 – Fall 2022

Assignment #4 (75 marks): Due Tuesday, December 6 by 9:00pm (Edmonton time)

1. (10 marks) The time to wrap presents at a mall wrapping station follow an Exponential distribution with a mean of 5 minutes. All wrapping times are independent and identically distributed.
   a) (5 marks) What is the probability the total amount of time to wrap four randomly selected presents is under 15 minutes?
   b) (5 marks) Consider three randomly selected wrapping times. What is the probability the quickest (minimum) time is under 3 minutes?

2. (10 marks) Suppose X and Y are jointly continuous random variables with probability density function

$$f(x,y) = \begin{cases} \frac{1}{4}, & 0 < x < 4,\ 0 < y < 1 \\ 0 & , else \end{cases}$$

a) (5 marks) Find $P(X - Y < 1.50)$.

b) (5 marks) Find $P(XY < 2.00)$.

3. (15 marks) Suppose X and Y are jointly continuous random variables with probability density function

$$f(x,y) = \begin{cases} c \ , & x^2 < y < 4 \text{ for } -2 < x < 2 \\ 0 \ , & else \end{cases}.$$

a) (2.5 marks) Find the constant, c, so that this is valid joint density function.

b) (5 marks) Find $P(Y < 2 - X)$.

c) (5 marks) Find $P(X < 1.00 \mid Y = 2.25)$.

d) (2.5 marks) Are X and Y independent? Justify your answer citing an appropriate theorem.

4. (10 marks) Suppose X and Y are jointly continuous random variables with probability density function

$$f(x,y) = \begin{cases} \frac{1}{15}(x+y), & 0 < x < 2,\ 0 < y < 3 \\ 0 & , else \end{cases}$$

a) (5 marks) Find $Cov[X,Y]$.

b) (5 marks) Find $V[Y \mid X = 1.50]$.

5. (5 marks) The joint desity for continuous random variables, X and Y, is

$$f(x,y) = \begin{cases} \frac{\sqrt{y}}{4\pi} e^{-\frac{1}{8}(x^2 - 2x + 4y + 1)} & , -\infty < x < \infty,\ y > 0 \\ 0 & , \text{else} \end{cases}.$$

Let $Z = 3X - 5Y + 100$. Find σ_Z^2.

6. (5 marks) Suppose X and Y are jointly continuous random variables with probability density function

$$f(x,y) = \begin{cases} \frac{3}{128}(8 - x - y), & 0 < x < y < 8 - x,\ 0 < x < 4. \\ 0 & , else \end{cases}$$

Find the marginal density of Y.

7. (10 marks) In the event of a car accident where the policyholder is at fault, their car insurance policy covers damage to their car and the other drivers car. The damage amount to the policyholders car, X, is *Exponentially* distributed with a mean of 1.75 (in thousands of dollars). Given $X = x$, the damage amount to the other drivers car, Y, is *Uniformly* distributed over the range x to 2.25x.
   a) (5 marks) Find the expected total damage amount for the two cars. That is, find $E[X+Y]$.
   b) (5 marks) Find $P(Y > X^2)$.

8. (5 marks) A light system has two bulbs in joint operation. Let X and Y denote the lengths of life of the two bulbs in years (X is th elifetime of bulb 1, Y is the lifetime of bulb 2). The joint density function for X and Y is

$$f(x,y) = \begin{cases} \frac{1}{16}e^{-y/4} & , \quad 0 < x < y \\ 0 & , \quad else \end{cases}$$

   Given that bulb 1 lasted over 3.0 years, what is the probability bulb 2 lasted between 6.0 years and 9.0 years? That is, find $P(6.0 < Y < 9.0 \mid X > 3.0)$.

9. (5 marks) A random number, X, is generated using the probability density function given by

$$f(x) = \begin{cases} \frac{5}{32}x^4 & , \quad 0 < x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}.$$

   Given $X = x$, another random number, Y, is generated from the *Geometric*$(p = x/2)$. Find σ_Y^2.