Stat 265 – Homework Assignment 2 – Fall 2022

Assignment #2 (75 marks): Due Thursday, October 13 by 9:00pm (Edmonton Time)

1. (10 marks) The number workplace injuries in a month from a particular company is modeled by a random variable, $\boldsymbol{X}$, with probability distribution

$$P(\boldsymbol{X}=x)=\begin{cases} \dfrac{c}{x+2} & ,x=0,1,2,3 \\ p & ,x=4,5 \\ 0 & ,else \end{cases} \quad , c \text{ and } p \text{ are constants.}$$

a) (5 marks) If the expected value of $\boldsymbol{X}$ is 1.50, find the values of $\boldsymbol{c}$ and $\boldsymbol{p}$.

b) (5 marks) Given the company had at least 2 workplace injuries in a month, what is the probability they had under 5 workplace injuries in that month?

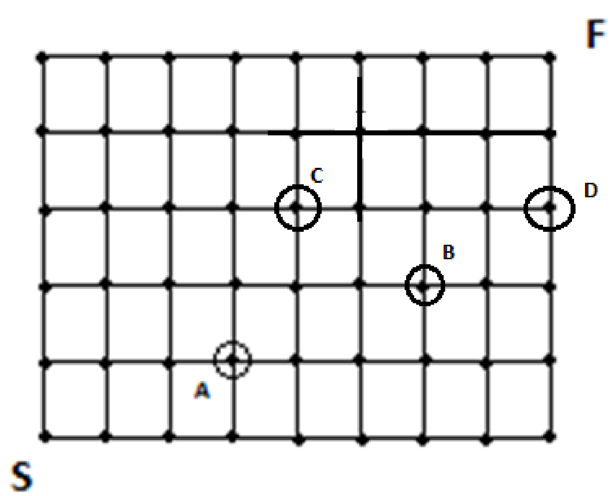
2. (5 marks) Three bowls are labeled 1, 2, and 3. Bowl $\boldsymbol{i}$ contains $\boldsymbol{i}+2$ white and $5-\boldsymbol{i}$ red balls, for $\boldsymbol{i}$ = 1, 2, 3. You will select a bowl and then draw 2 balls from that bowl **without** replacement. Suppose that 20% of the time you will pick bowl 1, 50% of the time you will pick bowl 2, and 30% of the time you will pick bowl 3. Let $\boldsymbol{Y}$ be the random variable denoting the number of red balls selected. Let $\boldsymbol{W}=2^{\boldsymbol{Y}}-1$ be your win amount. What is the expected value of your win amount on any given play?

3. (10 marks) Consider a game that consists of making four selections from a deck of 10 cards. The deck contains one Ace, two Kings, three Queens, and four Jacks. That is, the cards in the deck are (A K K Q Q Q J J J J).

a) (5 marks) For this part you will select cards **with replacement**. You win \$5 for each Ace selected, \$3 for each King selected, \$1 for each Queen selected, and lose \$2 for each Jack. Let the random variable $\boldsymbol{Y}$ denote the total amount you win from the four selections. Find the expected value and variance of $\boldsymbol{Y}$.

b) (5 marks) For this part you will select cards **without replacement**. You win $W=9-5X$, where the random variable $\boldsymbol{X}$ denotes the **total** number of Kings and Queens selected. Find the expected value and variance of $\boldsymbol{W}$.

4. (5 marks) Consider the grid below. Suppose that starting at point S you can go one step up or one step to the right at each move. This is continued until the point labeled F is reached. All paths from S to F are equally likely. On your random journey you must pay \$10 if you pass through point A, \$5 if you pass through B, \$3 if you pass through point C, and \$6 if you pass through point D. Let the random variable $\boldsymbol{Y}$ be the total cost to travel from S to F. What is the expected value of $\boldsymbol{Y}$.



5. (5 marks) Consider randomly selecting 5-cards **without replacement** from a standard 52-card deck. Let the random variable $\boldsymbol{X}$ denote the number of Hearts in your hand (there are 13 hearts in the deck). Given you have at least two Hearts in your hand, what is the probability you have exactly three Hearts in your hand?

6. (5 marks) A hockey team will win a game with a probability of 0.60 (and lose otherwise) and all games are independent and identically distributed. What is the probability this team will win their 4^{th} game before having their 4^{th} loss?

7. (10 marks) Job applicants for Out-of-The-Box Inc. are given a series of independent tasks. All applicants will successfully complete any task with a probability of 0.19.

a) (5 marks) If they have 5 attempts to successfully complete a task to receive a job, what is the probability a randomly selected applicant gets a job?

b) (5 marks) Applicant's salaries are based on the random variable $\boldsymbol{Y}$ described below, where $\boldsymbol{X}$ is the random variable representing the number of attempts an applicant takes until they successfully complete their first task. What is the standard deviation of a randomly selected applicants salary? **Note:** The salary distribution below includes salaries of 0 if they do not successfully complete a task in the first 5 tries. Zero salaries should not be ignored.

$$\boldsymbol{Y} = \begin{cases} 1,000,000 - 175,000\boldsymbol{X} & , \boldsymbol{X} = 1,2,3,4,5 \\ 0 & , \quad else \end{cases}$$

8. (5 marks) In any given day, my computer will either crash once or it won't crash. The probability my computer will crash on any given day is 0.42 (I need a new computer). Assume all days are independent and identically distributed. In the next week, what is the probability my computer has its third crash on the seventh day, given it did not crash in the first two days of the week?

9. (5 marks) In the daily production of a certain kind of metal beam, the number of defects per foot on each beam, $\boldsymbol{Y}$, follows a Poisson distribution with mean 0.52. The profit per foot on a beam when it is sold is given by $\boldsymbol{X}$, where $\boldsymbol{X} = 30 - 2\boldsymbol{Y} - 3\boldsymbol{Y}^2$. What is the expected value for the **total** profit on three randomly selected independent 1-foot beams?

10. (5 marks) You have an option to purchase a yearly insurance policy to protect from hail damage to your vehicle for a yearly cost of \$750. The policy will pay nothing for the first incident of the year and will pay you a fixed amount of \$1,000 for each incident thereafter. Suppose the number of times per year you will have hail damage to your car (call this the random variable $\boldsymbol{Y}$) follows a Poisson distribution with a mean of 1.45. What is the expected value of the difference between the amount you would be reimbursed in the case of hail damage to your car and the amount you paid for this policy. Would you purchase this insurance?

11. (10 marks) Future Hall of Fame Edmonton Spoilers hockey player Donnor McCavid has an average of 1.43 points per game in his professional hockey career thus far. Suppose his points per game (call this the random variable $\boldsymbol{Y}$) can be modeled according to a Poisson distribution with a mean of 1.43 points per game. Assume all games are independent and identical.

a) (5 marks) In his first 10 games of the upcoming season, what is the probability he will have at least 2 games with exactly 3 points in each of those games?

b) (5 marks) Consider the random variable $\boldsymbol{U}$ to be the number of games it takes Donnor to have his first game with at least 3 points. What is the expected value and variance of $\boldsymbol{U}$?