3 p.77

$$(3x^2 - 2xy + 2) + (6y^2 - x^2 + 3)y' = 0$$
$$\implies (6y^2 - x^2 + 3)dy + (3x^2 - 2xy + 2)dx = 0$$

The DE is exact since

$$\frac{\partial}{\partial x}(6y^2 - x^2 + 3) = -2x = \frac{\partial}{\partial y}(3x^2 - 2xy + 2)$$

Then we have that

$$\varphi(x, y) = \int (3x^2 - 2xy + 2)dx = x^3 - x^2y + 2x + c(y)$$

$$\frac{\partial \varphi}{\partial y} = \frac{\partial}{\partial y}(x^3 - x^2y + 2x + c(y)) = 6y^2 - x^2 + 3$$

Hence,

$$c(y) = 2y^3 + 3y$$

and therefore

$$\varphi(x, y) = x^3 - x^2y + 2x + 2y^3 + 3y = c$$

is the solution.

4 p.77

$$\frac{dy}{dx} = -\frac{ax + by}{bx + cy}$$
$$\implies (bx + cy)dy + (ax + by)dx = 0$$

The DE is exact as

$$\frac{\partial}{\partial x}(bx + cy) = b = \frac{\partial}{\partial y}(ax + by)$$

Then we have that

$$\varphi(x, y) = \int (ax + by)dx = \frac{ax^2}{2} + bxy + c(y)$$

$$\frac{\partial \varphi}{\partial y} = \frac{\partial}{\partial y}(\frac{ax^2}{2} + bxy + c(y)) = bx + cy$$

Hence,

$$c(y) = \frac{cy^2}{2}$$

and therefore

$$\varphi(x, y) = \frac{ax^2}{2} + bxy + \frac{cy^2}{2} = c$$

is the solution to the DE.

7 p.77

$$\left(\frac{y}{x} + 6x\right) + (\ln x - 2)\frac{dy}{dx} = 0$$
$$\implies \left(\frac{y}{x} + 6x\right) dx + (\ln x - 2)dy = 0$$

The DE is exact as

$$\frac{\partial}{\partial x}(\ln x - 2) = \frac{1}{x} = \frac{\partial}{\partial y}\left(\frac{y}{x} + 6x\right)$$

Then we have that

$$\varphi(x, y) = \int \left(\frac{y}{x} + 6x\right) dx = y \ln x + 3x^2 + c(y)$$

$$\frac{\partial \varphi}{\partial y} = \frac{\partial}{\partial y}\left(y \ln x + 3x^2 + c(y)\right) = \ln x - 2$$

Hence,

$$c(x) = -2y$$

and therefore,

$$\varphi(x, y) = y \ln x + 3x^2 - 2y = c$$

is the solution to the DE.

10 p.77

$$\left(9x^2+y-1\right)+(x-4y)\frac{dy}{dx}=0$$
$$\implies \left(9x^2+y-1\right)dx+(x-4y)dy=0$$

The DE is exact as

$$\frac{\partial}{\partial x}(x-4y)=1=\frac{\partial}{\partial y}\left(9x^2+y-1\right)$$

Then we have that

$$\varphi(x,y)=\int\left(9x^2+y-1\right)dx=3x^3+yx-x+c(y)$$

$$\frac{\partial\varphi}{\partial y}=\frac{\partial}{\partial y}\left(3x^3+yx-x+c(y)\right)=x-4y$$

Hence,

$$c(x)=-2y^2$$

and therefore,

$$\varphi(x,y)=3x^3+yx-x-2y^2=c$$

is the solution to the DE. Since $y(1)=0, c=2$. Therefore,

$$3x^3+yx-x-2y^2-2=0$$
$$\implies -2\left(y-\frac{x}{4}\right)^2+\frac{x^2}{8}+3x^3-2=0$$
$$\implies \left(y-\frac{x}{4}\right)^2=\frac{x^2}{16}+\frac{3x^3}{2}-1$$

Hence,

$$\frac{x^2}{16}+\frac{3x^3}{2}-1>0$$
$$24x^3+x^2-16>0$$

need to be true so that the solution is valid.

1 p.112

$$\begin{aligned} & y'' + 2y' - 3y = 0 \\ \Longrightarrow & \; r^2 e^{rt} + 2re^{rt} - 3e^{rt} = 0 \\ \Longrightarrow & \; r \in \{1, -3\} \end{aligned}$$

Therefore,

$$y(t) = c_1 e^t + c_2 e^{-3t}$$

3 p.112

$$\begin{aligned} & 6y'' - y' - y = 0 \\ \implies & 6r^2 e^{rt} - r e^{rt} - e^{rt} = 0 \\ \implies & r \in \left\{ \frac{1}{2}, \frac{-1}{3} \right\} \end{aligned}$$

Therefore,

$$y(t) = c_1 e^{t/2} + c_2 e^{-t/3}$$

10 p.112

$$
\begin{aligned}
&2y'' + y' - 4y = 0 \\
\implies &2r^2 e^{rt} + 1re^{rt} - 4e^{rt} = 0 \\
\implies &r \in \left\{ \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4} \right\}
\end{aligned}
$$

Therefore,

$$y(t) = c_1 e^{\frac{-1+\sqrt{33}}{4}t} + c_2 e^{\frac{-1-\sqrt{33}}{4}t}$$

as $t \to \infty$, $y(t) \to c_1^{\frac{-1+\sqrt{33}}{4}t}$, which diverges to ∞ if $c_1 > 0$, diverges to $-\infty$ if $c_1 < 0$, and converges to 0 if $c_1 = 0$

12 p.112

$$
\begin{aligned}
&4y'' - y = 0 \\
\implies &4r^2 e^{rt} - e^{rt} = 0 \\
\implies &r \in \left\{ \frac{1}{2}, -\frac{1}{2} \right\}
\end{aligned}
$$

Therefore,

$$y(t) = c_1 e^{t/2} + c_2 e^{-t/2}$$

as $t \to \infty$, $y(t) \to c_1 e^{t/2}$, which diverges to ∞ if $c_1 > 0$, diverges to $-\infty$ if $c_1 < 0$, and converges to 0 if $c_1 = 0$