1.

1.

Let $u = \sin t$, then $du = \cos t dt$, hence the integral becomes

$$\int \frac{du}{(1+u)^{1/2}} = 2\sqrt{1+u} = 2\sqrt{(1+\sin t)}$$

2.

Let $u = \ln x$, then $du = \frac{dx}{x}$, hence the integral becomes

$$\int \frac{dx}{x \ln x} = \int \frac{du}{u} = \ln u = \ln \ln x$$

3.

With $t = y - 2$ hence $dt = dy$, we have

$$\int \frac{dy}{y^2 - 4y + 8} = \int \frac{dy}{(y-2)^2 + 4} = \int \frac{dt}{t^2 + 4} = 2 \cdot \frac{1}{4} \int \frac{d(t/2)}{\left(\frac{t}{2}\right)^2 + 1} = \frac{\arctan(\frac{y-2}{2})}{2}$$

4.

Let $u = \sqrt{x-1}$, then $du = \frac{1}{2\sqrt{x-1}} dx$

$$\int \frac{dx}{(x-1)\sqrt{x^2 - x}} = \int \frac{2du}{u^2 \cdot \sqrt{u^2+1}}$$

Let $u = \tan(t)$, then $du = \dfrac{1}{\cos^2(t)} dt$, the integral becomes

$$\int \frac{2dt}{\tan^2(t) \cdot \sqrt{\tan^2(t) + 1}} \cdot \frac{1}{\cos^2(t)} = \int \frac{\cos(t)}{\sin^2(t)} dt$$

Let $y = \sin(t)$, $dy = \cos(t)dt$

$$\int \frac{1}{y^2} dy = -\frac{1}{y} = -\frac{1}{\sin(t)} = -\frac{1}{\sin(\arctan(u))} = -\frac{1}{\sin(\arctan(\sqrt{x-1}))} = -\frac{\sqrt{x}}{\sqrt{x-1}}$$

5.

$$\int \sin^2(x) dx = \int \frac{1 - \cos(2x)}{2} dx = \frac{x}{2} - \frac{\sin(2x)}{4}$$

6.

$$\int x^2 \ln x dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9}$$

7.

Let $I = \int e^{2x} \sin x dx$

$$\begin{aligned} I = \int e^{2x} \sin x dx &= \frac{e^{2x} \sin x}{2} - \int \frac{e^{2x} \cos x}{2} dx \\ &= \frac{e^{2x} \sin x}{2} - \frac{e^{2x} \cos x}{4} + \frac{I}{4} \\ \implies I &= \frac{4}{3} \left(\frac{2e^{2x} \sin x - e^{2x} \cos x}{4} \right) \end{aligned}$$

8.

$$\begin{aligned} \int \frac{3x^2 + 4x + 4}{x^3 + x} dx &= \int \left(\frac{-x + 4}{x^2 + 1} + \frac{4}{x} \right) dx \\ &= \int \left(-\frac{x}{x^2 + 1} + \frac{4}{x^2 + 1} + \frac{4}{x} \right) dx \\ &= -\frac{\ln(x^2 + 1)}{2} + 4 \ln(|x|) + 4 \arctan(x) + C \end{aligned}$$

9.

Let $u = \cos x$, then $du = -\sin x dx$

$$\begin{aligned} \int \sin^3 x \cos^4 x dx &= \int (1 - \cos^2 x) \cos^4 x \sin x dx \\ &= -\int (1 - u^2) u^4 du \\ &= \frac{u^7}{7} - \frac{u^5}{5} \\ &= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{x} \end{aligned}$$

10.

Let $u = \sqrt{e^{2x} - 1}$, hence $du = \dfrac{e^{2x} dx}{\sqrt{e^{2x} - 1}}$ and $u^2 + 1 = e^{2x}$

$$\int \frac{dx}{\sqrt{e^{2x} - 1}} = \int \frac{1}{u^2 + 1} du = \arctan(u^2 + 1) = \arctan(e^{2x})$$

2.

A

Let λ be an eigenvalue, then

$$\begin{aligned}&\det(A-\lambda I)=0\\&(2-\lambda)^2+1=0\\&\lambda^2-4\lambda+5=0\\&\lambda=2\pm i\end{aligned}$$

For $\lambda = 2+i$, let $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ be its eigenvector, then

$$\begin{aligned}A\cdot v&=\lambda\cdot v\\ \begin{pmatrix}-i & -1\\ 1 & -i\end{pmatrix}\cdot v&=0\\ \begin{pmatrix}-i\cdot v_1-1\cdot v_2\\ v_1-i\cdot v_2\end{pmatrix}\cdot v&=0\end{aligned}$$

Hence, $v_1 = i\cdot v_2$, thus $v = \lambda_1 \begin{pmatrix} i \\ 1 \end{pmatrix}$ is an eigenvector for all $\lambda_1 \in \mathbb{R}$ For $\lambda = 2-i$, let $v = \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$ be its eigenvector, then

$$\begin{aligned}A\cdot v&=\lambda\cdot v\\ \begin{pmatrix}i & -1\\ 1 & i\end{pmatrix}\cdot v&=0\\ \begin{pmatrix}i\cdot v_1-1\cdot v_2\\ v_1+i\cdot v_2\end{pmatrix}\cdot v&=0\end{aligned}$$

Hence, $v_2 = i\cdot v_1$, thus $v = \lambda_2 \begin{pmatrix} 1 \\ i \end{pmatrix}$ is an eigenvector for all $\lambda_2 \in \mathbb{R}$ Clearly, The geometric multiplicity of both eigenvalue is 1 since the geometric multiplicity cannot be 0 and cannot exceed the algebraic multiplity of the eigenvalue , which is 1 . The geometric multiplicity of both eigenvalue are 1 as the dimension of both eigenspace are 1. Hence, it is diagonalizable as the algebraic multiplicity of both eigenvalue is equal to the geometric multiplicity of itself .

B

Let λ be an eigenvalue, then

$$\begin{aligned}&\det(B-\lambda I)=0\\&(-1-\lambda)^3=0\\&\lambda=-1\end{aligned}$$

Let $v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$ be an eigenvector. Then

$$(A+1) \cdot v = 0$$
$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \cdot v = 0$$

Hence, $v_1 = v_3 = 0$, which means that $v = \lambda^* \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector for all $\lambda^* \in \mathbb{R}$. Notice that $A - \lambda$ has nullity 2 hence the geometric multiplicity is 2 but the algebraic multiplicity is 3. Therefore, b is not diagonalizable.

3.

11.j

12.c

13.g

14.b

15.h

16.e