1.

From the Classical Girsanov Theorem, we have that

$$dA_t = -X_t A_t dB_t$$

with $A_0 = 1$ is the likelihood martingale that change the standard brownian motion into the Ornstein-Uhlenbeck process.

2.

We have that

$$\pi(i) = \binom{n}{i} p^i (1-p)^{n-i} = \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i}$$

Also

$$\frac{\pi(i)}{\pi(0)} = \frac{a_{i-1}a_{i-2}\dots a_0}{s_i s_{i-1}\dots s_1} = \frac{n!}{i!(n-i)!} p^i (1-p)^{-i}$$

However,

$$s_i s_{i-1}\dots s_1 = i!(1-p)^i$$

Hence,

$$a_{i-1}a_{i-2}\dots a_0 = \frac{n!}{(n-i)!} p^i$$

Hence, we can speculate

$$a_i = p(n-i)$$

Since $\pi(i) = 0$ for $i > n$, $a(i) = 0$ for $i >= n$.

3.

We have that

$$S_{100} - 30 \to \sqrt{0.3 \cdot 0.7} \cdot 10B^1$$

Also

$$S_{200} - S_{100} - 30 \to \sqrt{0.3 \cdot 0.7} \cdot 10(B^2 - B^1)$$

We also have that Thus

$$P((S_{100} - 30)^2 + (S_{200} - S_{100} - 30)^2) = P((B^1)^2 + (B^2 - B^1)^2 > 20/21)$$

$B^1, B^2 - B^1$ are the standard normal distributions. Hence, the sum of those squared is the exponential distribution with mean 2. Hence, the probability is

$$1 - e^{-2\frac{20}{21}} = 1 - e^{-40/21}$$

4.