Contents

1 8/9

1.1 intuitive counting formula

Let N be a λ-Poisson process and f be any continuous function in $\mathbb{R}$.

We will prove that Note: $f(0) = f(N_0)$. We have,

$$f(N_1) - f(0) = \sum_{i=1}^{N_1} (f(i) - f(i-1))$$

By right contiuity of the poisson process

$$\int_0^1 (f(N_{s^-}+1) - f(N_{s^-}))\, dN_s = \lim_{M\to\infty} \left(f\left(N_{\frac{j-1}{M}}+1\right) - f\left(N_{\frac{j-1}{M}}\right)\right)\left(N_{\frac{j}{M}} - N_{\frac{j-1}{M}}\right)$$

For large M, we have that

$$N_{\frac{j}{M}} - N_{\frac{j-1}{M}} = \begin{cases} 1, \text{at the jump where } N \text{ increases} \\ 0, \text{otherwise} \end{cases}$$

Hence,

$$\int_0^1 (f(N_{s^-}+1) - f(N_{s^-}))\, dN_s = \sum_{i=1}^{N_1} (f(i) - f(i-1)) = f(N_1) - f(0)$$

1.2 problem

problem link
Guess

$$a_t = a_0 \exp\left(\int_0^1 f_s ds\right)$$

which starts right. The exponential rule gives

$$\frac{d}{dt} a_t = f_t a_t$$

Unique? Consider second solution satisfies $\frac{d}{dt}\alpha_t = f_t \alpha_t$ such that $\alpha_0 = a_0$. Set $e_t = \frac{\alpha_t}{a_t}$, then $e_0 = \frac{a_0}{\alpha_0} = 1$ hence by product rule

$$\frac{d}{dt} e^t = \frac{f_t \alpha_t}{a_t} - \frac{\alpha_1 f_t \exp\left(-\int_0^1 f_s ds\right)}{a_0} = 0$$

Hence, $e_t = 1$, hence $\alpha_t = a_t$ for all t.