MATH 217 (Fall 2022)
Honors Advanced Calculus, I

Assignment #1

1. Let $+$ and $\cdot$ be defined on $\{\spadesuit, \dagger, \bigcirc, \mathrm{A}\}$ through:

$+$	$\spadesuit$	$\dagger$	$\bigcirc$	A
$\spadesuit$	$\spadesuit$	$\dagger$	$\bigcirc$	A
$\dagger$	$\dagger$	$\bigcirc$	A	$\spadesuit$
$\bigcirc$	$\bigcirc$	A	$\spadesuit$	$\dagger$
A	A	$\spadesuit$	$\dagger$	$\bigcirc$

$\cdot$	$\spadesuit$	$\dagger$	$\bigcirc$	A
$\spadesuit$	$\spadesuit$	$\spadesuit$	$\spadesuit$	$\spadesuit$
$\dagger$	$\spadesuit$	$\dagger$	$\bigcirc$	A
$\bigcirc$	$\spadesuit$	$\bigcirc$	$\spadesuit$	$\bigcirc$
A	$\spadesuit$	A	$\bigcirc$	$\dagger$

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Do these turn $\{\spadesuit, \dagger, \bigcirc, \mathrm{A}\}$ into a field?

2. Show that

$$\mathbb{Q}[i] := \{p + i\,q : p, q \in \mathbb{Q}\} \subset \mathbb{C}$$

with $+$ and $\cdot$ inherited from $\mathbb{C}$, is a field. Is there a way to turn $\mathbb{Q}[i]$ into an ordered field?

3. Let $\varnothing \neq S \subset \mathbb{R}$ be bounded below, and let $-S := \{-x : x \in S\}$. Show that:

   (a) $-S$ is bounded above;

   (b) S has an infimum, namely $\inf S = -\sup(-S)$.

4. Find $\sup S$ and $\inf S$ in $\mathbb{R}$ for

$$S := \left\{ (-1)^n \left(1 - \frac{1}{n}\right) : n \in \mathbb{N} \right\}.$$

Justify, i.e., *prove*, your findings.

5. Let $S, T \subset \mathbb{R}$ be non-empty and bounded above. Show that

$$S + T := \{x + y : x \in S,\ y \in T\}$$

is also bounded above with

$$\sup(S + T) = \sup S + \sup T.$$

6*. An ordered field $\mathbb{O}$ is said to have the *nested interval property* if $\bigcap_{n=1}^{\infty} I_n \neq \varnothing$ for each decreasing sequence $I_1 \supset I_2 \supset I_3 \supset \cdots$ of closed intervals in $\mathbb{O}$.

Show that an Archimedean ordered field with the nested interval property is complete.

Due Monday, September 19, 2020, at 10:00 a.m.; no late assignments.

!!! IMPORTANT !!!

1. The completed assignment has to submitted through Assign2.
2. You are allowed to collaborate on homework assignments—in fact, I encourage you to do so. Still, every student must submit their own homework assignment.
3. All problems have equal weight.
4. Problems marked with an * are bonus problems: they allow you to earn extra marks on an assignment. On this assignment, for instance, you can thus get a mark of 120%.
5. Of the n (probably 9 or 10) homework assignments in the course, only the $n-1$ best will count towards your grade.