

**1.**

The amount of grams needed is

$$\frac{1000}{2 \cdot 10} = 50$$

## 2.

Since the number of customers between Dog-buyers follows the geometric distribution, we know that the probability any of the customers buying will be  $\frac{1}{3}$ . Hence, the probability that any of the customers buying Cockapoos will be  $\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{12}$ . Hence, we have the probability that 5 Cockapoos will be sold in a 12 hours day is

$$\begin{aligned}
 & \sum_{i=0}^{\infty} P(\text{number of customers} = i) \cdot \binom{i}{5} \cdot \left(\frac{1}{12}\right)^5 \cdot \left(\frac{11}{12}\right)^{i-5} \\
 &= \frac{36^i e^{-36}}{i!} \cdot \frac{i!}{5!(i-5)!} \cdot \left(\frac{1}{12}\right)^5 \cdot \left(\frac{11}{12}\right)^{i-5} \\
 &= \frac{3^5 e^{-3}}{5!} \underbrace{\sum_{i=0}^{\infty} \frac{33^i e^{-33}}{i!}}_1
 \end{aligned}$$

### 3.

Using polar coordinates, we have

$$E[X] = \int_G x = \int_0^1 \int_0^{2\pi} r \cos \theta r d\theta dr = \int_0^1 r^2 \sin \theta \Big|_0^{2\pi} = 0$$

$$E[Y] = \int_G y = \int_0^1 \int_0^{2\pi} r \sin \theta r d\theta dr = - \int_0^1 r^2 \cos \theta \Big|_0^{2\pi} = 0$$

$$E[XY] = \int_G xy = \int_0^1 \int_0^{2\pi} r \cos \theta r \sin \theta r d\theta dr = \int_0^1 r^3 \left( -\frac{\cos(2\theta)}{2} \right) \Big|_0^{2\pi} dr = 0$$

Hence,

$$\text{Cov}(X, Y) = E[XY] - E[X]E[Y] = 0$$

$$f_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{1}{\pi R^2} dy = \frac{2\sqrt{1-x^2}}{\pi R^2}$$

$$f_Y(y) = \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{\pi R^2} dx = \frac{2\sqrt{1-y^2}}{\pi R^2}$$

and hence

$$f_X(x) \cdot f_Y(y) \neq f_{X,Y}(x, y)$$

Hence, it is not independent

#### 4.

Let  $t$  be the time of the first event of both distributions. Then since it is poisson, we know that the probability that the event is from the first distribution is

$$\frac{(t-0) \left(\frac{1}{2}\right)^3}{\frac{t-0}{32} C_3^5 + (t-0) \left(\frac{1}{2}\right)^3} = \frac{2}{7}$$

At time 8, the number of species 1 in sites  $i$  follows the poisson distribution with mean

$$8 \left(\frac{1}{2}\right)^i = 2^{3-i}$$

The number of species 2 in sites  $i$  follows the poisson distribution with mean

$$\frac{1}{4} \frac{5!}{i!(5-i)!}$$

Using the mean and the formula for the poisson distribution, we can calculate the likeliest number of both species on all sites is

Species 1: 4,2,1,0,0 respectively in 5 sites

Species 2: 1,2,2,1,0 respectively in 5 sites

We have at time  $t$ , the probability that there is no animals in sites 2 and 3 are

$$P(\text{no animals in site 2}) = e^{-t\left(\frac{t}{4} + \frac{10t}{32}\right)} = e^{-\frac{9t}{16}}$$

$$P(\text{no animals in site 3}) = e^{-t\left(\frac{t}{8} + \frac{10t}{32}\right)} = e^{-\frac{7t}{16}}$$

Hence, the probability that there is no animals in both sites are

$$P(\text{no animals in sites 2 and 3}) = e^{-t}$$

Then let  $T$  be the distribution of the time the first animal appear in any site 2 or 3. We have that

$$P(T > t) = e^{-t}$$

which means  $T$  is an exponential distribution with mean 1. For the animal appear, we know that there is a  $\frac{9}{16}$  chance it is from site 2 and  $\frac{7}{16}$  chance it is from site 3. Hence, the expected time for 2 animals to appear in sites 2 and 3 are

$$\left(\frac{7}{16}\right)^2 \cdot 2 + \left(\frac{9}{16}\right)^2 \cdot 2 + \left(\frac{7}{16}\right)^2 \left(\frac{9}{16}\right) \cdot 3 \cdot \binom{2}{1} + \left(\frac{9}{16}\right)^2 \left(\frac{7}{16}\right) \cdot 3 \cdot \binom{2}{1} = 2.4921875$$