1.

Let A be random value picked between 0 and 2 and B be the random value picked between 1 and 3.

$$\begin{aligned} F_X(x) &= P(\max(A,B) \le x) = P(A \le x, B \le x) \\ &= P(A \le x) \cdot P(B \le x) \end{aligned}$$

If $0 < x < 1$ then $P(B \le x) = 0$ and hence $F_X(x) = 0$.
If $1 < x < 2$ then

$$F_X(x) = \int_0^x \frac{1}{2} da + \int_1^x \frac{1}{2} db = \frac{x}{2} \cdot \left(\frac{x-1}{2}\right)$$

$$\implies f_X(x) = \frac{x}{2} - \frac{1}{4}$$

If $2 < x < 3$ then $P(A \le x) = 1$

$$F_X(x) = \int_1^x \frac{1}{2} db = \frac{x-1}{2}$$

$$\implies f_X(x) = \frac{1}{2}$$

$$E[X] = \int_1^2 \frac{x^2}{2} - \frac{x}{4} dx + \int_2^3 \frac{x}{2} dx = \frac{49}{24}$$

2.

Let A be the minute that the man arrives, B be the minute that the woman arrives. Then since A and B is independent, we can find the joint probability

$$f_{A,B}(a,b) = \frac{1}{60} \cdot \frac{1}{30} = \frac{1}{1800}$$

the probability that the first to arrive waits no longer than 5 minutes is

$$P(|A-B| \leq 5) = P(B-5 \leq A \leq B+5) = \int_{15}^{45} \int_{a-5}^{a+5} \frac{1}{1800} dbda = \frac{1}{6}$$

$$P(A < B) = \int_{15}^{45} \int_{0}^{a} \frac{1}{1800} dbda = \frac{1}{2}$$

3.

Gamma(3,2)

$$1 - \int_0^8 \frac{x^2 e^{-x/2}}{2^3 \cdot 2!} dx = 0.238$$

4.

$$P(Y \in (60,75] \cup [70,90) \cup [120,240) \cup 59) = P(Y \in (60,90) \cup (120,240))$$
$$= \int_{60}^{90} \frac{y^2 e^{-y/60}}{60^3 2!} dy + \int_{120}^{240} \frac{y^2 e^{-y/60}}{60^3 2!} dy = 0.54942$$