Course 2: Supervised Learning

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Bretagne-Pays de la Loire
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Summary

Last session

1. AI definition
2. Applications
3. Deep learning
4. Open issues

Today's session

- Learning from labeled examples
- Challenges of supervised learning

Notations

Vector space ($\mathbb{R}^d$)

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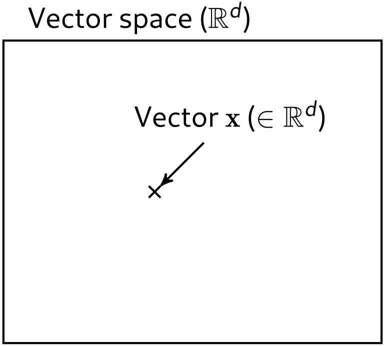

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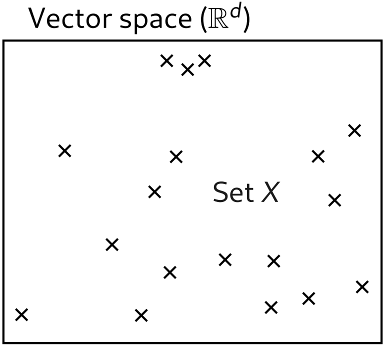

Definition

Supervised learning methods use **labels** $\hat{\mathbf{y}}$ associated with examples $\mathbf{x}$ to learn a function f such as $\hat{\mathbf{y}} \approx f(\mathbf{x})$, with the aim of **generalizing** ($\neq$ memorizing) to unlabeled examples.

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Goal of supervised learning:

learning the mapping function $f()$

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$\xrightarrow{f()}$ $\hat{\mathbf{y}}$: "cat"

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- **Regression** ($\mathbf{y}$ is scalar)
- Tons of applications:
  - Pattern recognition,
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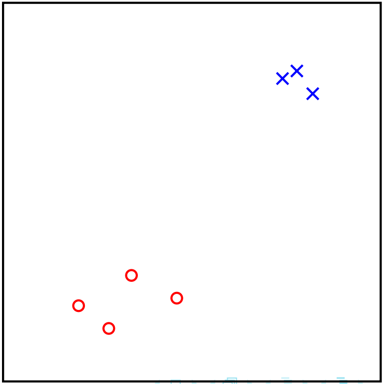

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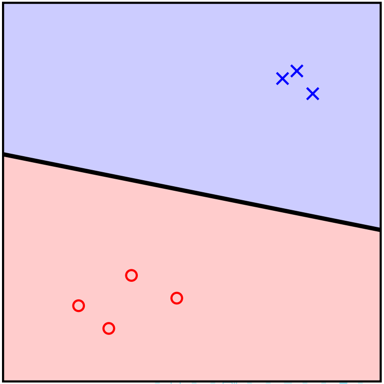

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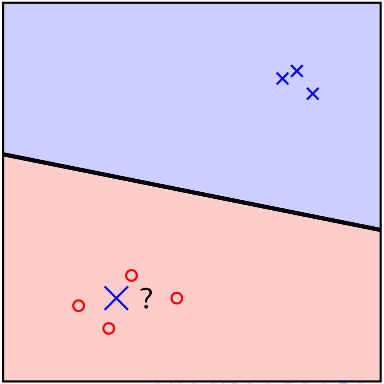

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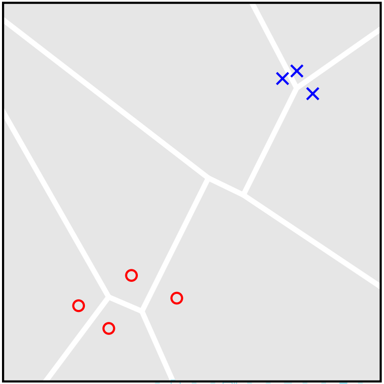
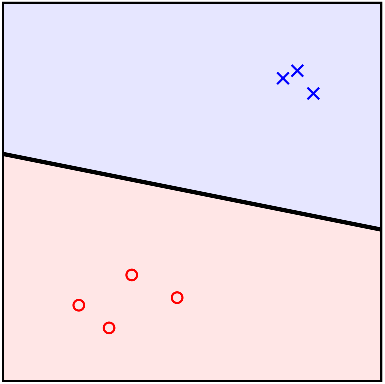


Challenges of supervised learning (1/5)

An ill-defined problem

- An infinity of potential solutions, one must be the “best one” but is unreachable,
- $\Rightarrow$ requires **priors or constraints**.



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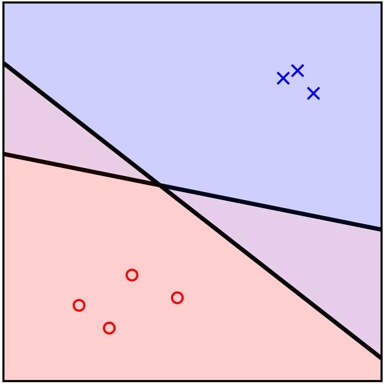

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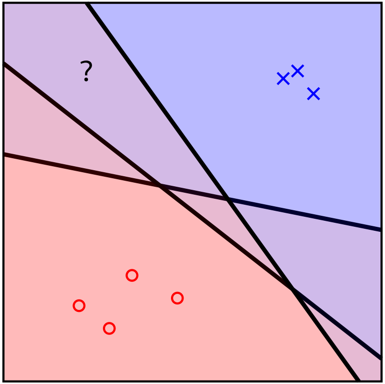

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Bias/variance trade-off

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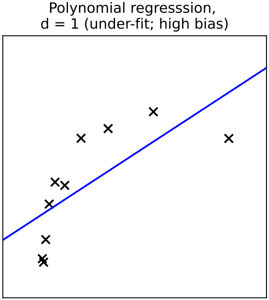

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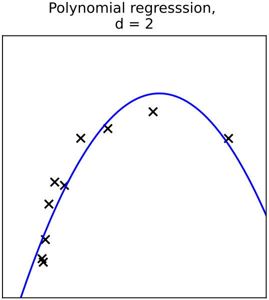

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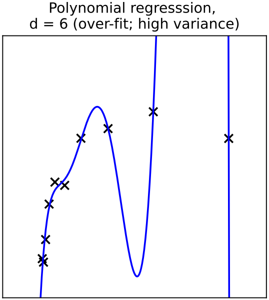

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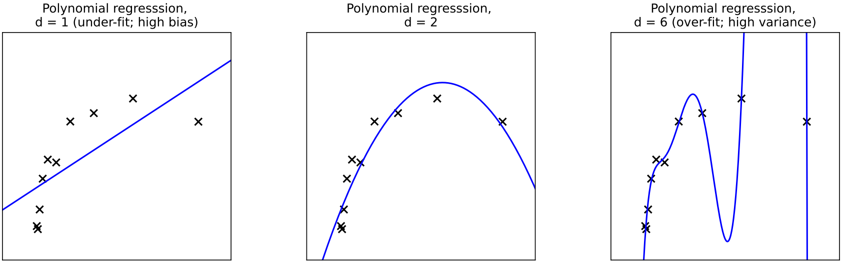

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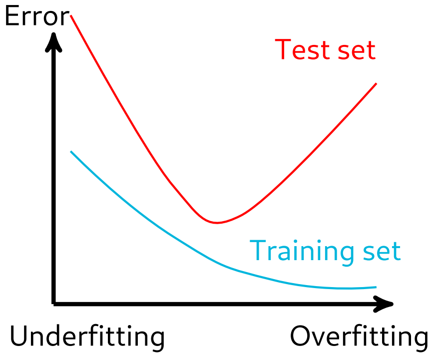

Crossvalidation

- To detect overfitting, split training dataset in two parts:
  1. A first part is used to train,
  2. A second part is used to validate,

Challenges of supervised learning (3/5)

Curse of dimensionality

- Geometry is not intuitive in **high dimension**,
- Efficient methods in 2D are not necessarily still valid.

$$V_d^s = \frac{\pi^{d/2} R^d}{\Gamma(d/2+1)} \text{ versus } V_d^c = (2R)^d$$

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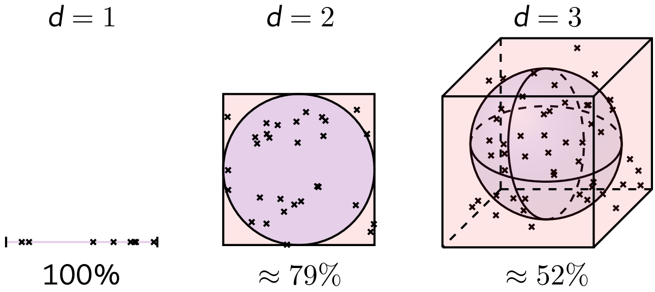

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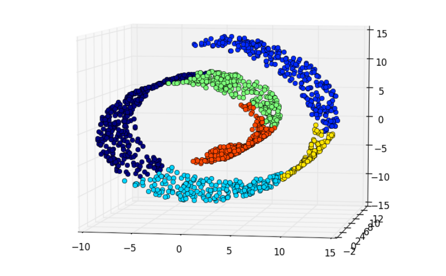
Challenges of supervised learning (4/5)

Riemannian manifolds



Linear separability and need for embedding

Challenges of supervised learning (4/5)

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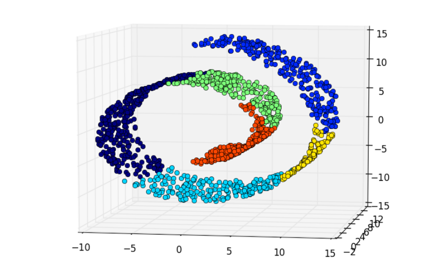

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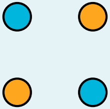

Challenges of supervised learning (5/5)

Computation time

Example on ImageNet, simply going through all images:

- $n = 10.000.000$, $d \approx 1.000.000$,
- $\approx 10^{13}$ elementary operations,
- $\approx$ 2h45 on a modern processor.

Scalability

- Finding the best solution to a problem would be feasible with unlimited computation time,
- But searching through the space of possible functions is often **untractable**,
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Definition

- Let us fix d,
- The **VC dimension** is a measure of the genericity of a method,
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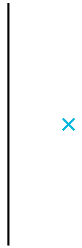

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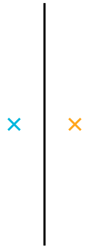

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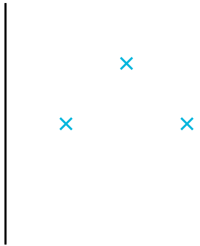

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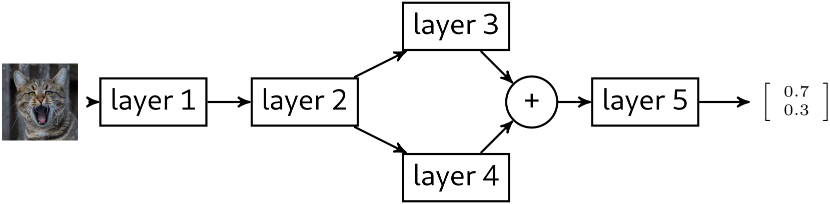
VC is 3.

The case of deep learning in classification

Inputs/outputs

- Often: inputs are **raw signals** or **feature vectors**,
- Often: outputs are vectors which **highest value** indicate the **category of the input**.



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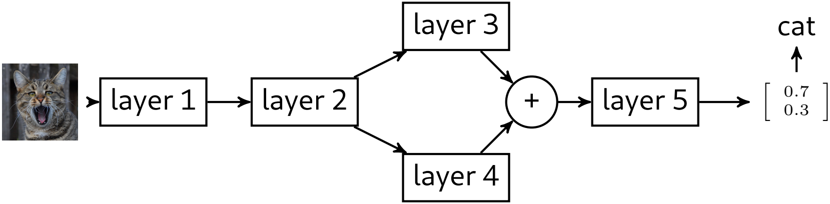

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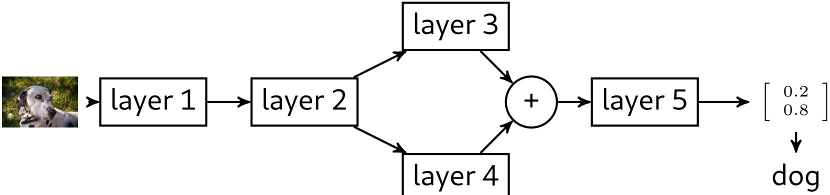

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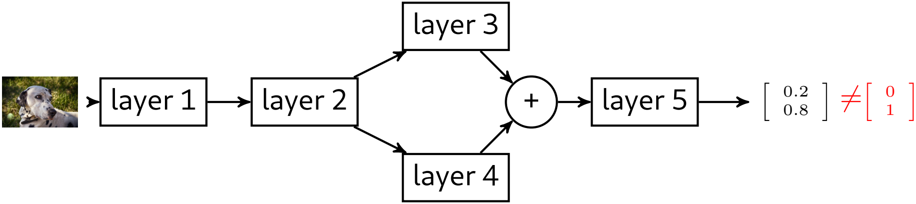

Loss and targets

- Labels are encoded as one-hot-bit vectors and called **targets**,
- Outputs are **softmaxed:** $\mathbf{y}_i \leftarrow \exp(\mathbf{y}_i)/\sum_j \exp(\mathbf{y}_j)$,
- Loss is typically **cross-entropy:** $-\log(\hat{\mathbf{y}}^\top \mathbf{y})$.

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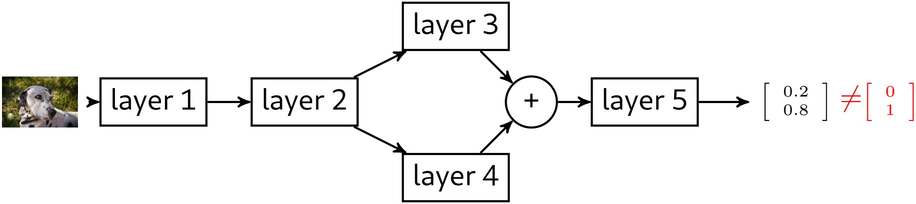

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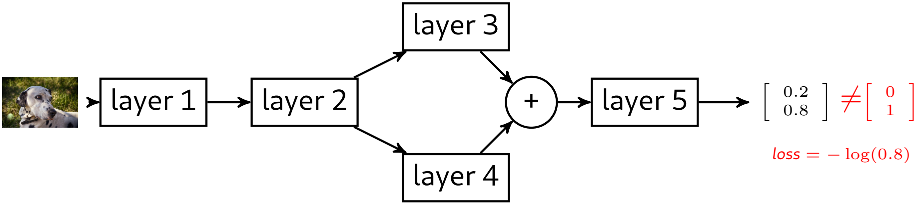

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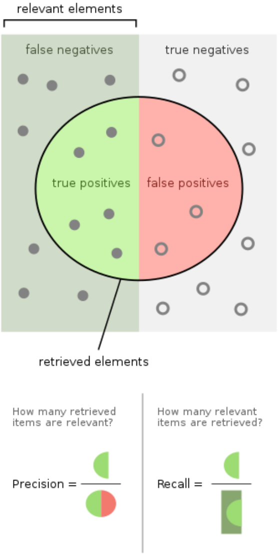
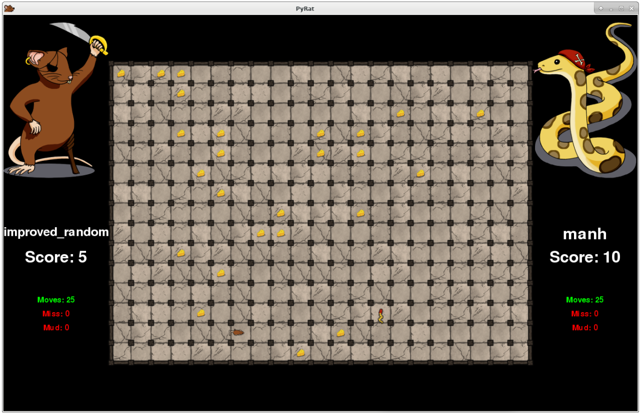
Metrics

In supervised learning : per class metric



Non-symmetric PyRat without walls / mud



Both players follow a deterministic greedy algorithm.
Supervised learning - Two tasks

- Lab 2a - Predict the outcome of a game from the start configuration.
- Lab 2b - Learn the next move using a dataset of winners

Lab Session 2 and assignments for Session 3

Lab Supervised Learning

- Basics of machine learning using sklearn (including new definitions / concepts)
- Tests on PyRat datasets : winner prediction task

Project 1 (P1)

You will choose a supervised learning method. You have to prepare a Jupyter Notebook on this method, including:

- A brief description of the theory behind the method,
- Basic tests on simulated data to show the influence of parameters and hyperparameters
- Tests on PyRat Datasets on the winner prediction task

During Session 3 you will have 7 minutes to present your notebook.