Analyse des données européennes

FALL

2024-12-26

Contents

1 Contexte général

Les données analysées dans ce projet concernent différents indicateurs socio-économiques et environnementaux pour un ensemble de pays européens. Ces indicateurs offrent une vision globale de divers aspects comme la démographie, l'économie, l'énergie, l'emploi et l'environnement. Les analyses visent à explorer les structures sous-jacentes et les relations entre ces variables, ainsi qu'à regrouper les pays selon des caractéristiques communes.

Description des variables

Les données contiennent 16 variables descriptives, reflétant des dimensions clés :

```
Population au 1er janvier : Nombre absolu d'habitants.
Population jeune (15-29 ans) : Pourcentage de jeunes dans la population totale.
Premières demandes d'asile : Nombre absolu de demandes.
Écart de rémunération entre les sexes : Pourcentage de différence de salaire horaire brut moyen entre ho
Salaire minimum : Montant en euros par mois.
Décrocheurs scolaires précoces : Pourcentage de la population âgée de 18 à 24 ans quittant prématurément
Taux d'inflation : Variation en pourcentage par rapport à l'année précédente.
Taux de chômage : Pourcentage de la population active âgée de 15 à 74 ans.
Taux de chômage des jeunes : Pourcentage de la population active de moins de 25 ans.
PIB par habitant : Produit intérieur brut en euros par habitant.
Dette brute du gouvernement : Pourcentage de la dette brute par rapport au PIB.
Émissions de gaz à effet de serre : Quantité moyenne en tonnes par habitant.
Énergies renouvelables : Pourcentage dans la consommation finale brute d'énergie.
Prix de l'électricité : Montant en euros par MWh, incluant les taxes.
Dépendance aux importations d'énergie : Pourcentage de dépendance à l'énergie importée.
Taux de risque de pauvreté ou d'exclusion sociale : Pourcentage de la population à risque de pauvreté ou
```

L'objectif de cette analyse est d'explorer et de réduire la dimensionnalité des données grâce à une analyse en composantes principales (ACP), puis de grouper les pays selon leurs caractéristiques à l'aide de méthodes de classification. L'ACP permet de visualiser les similitudes entre pays et d'identifier les variables les plus importantes. Les méthodes de classification, notamment la classification ascendante hiérarchique (CAH) et l'algorithme des centres mobiles (k-means), permettent d'interpréter les regroupements obtenus. Les résultats des classifications seront comparés afin de comprendre les proximités entre pays et leur cohérence.

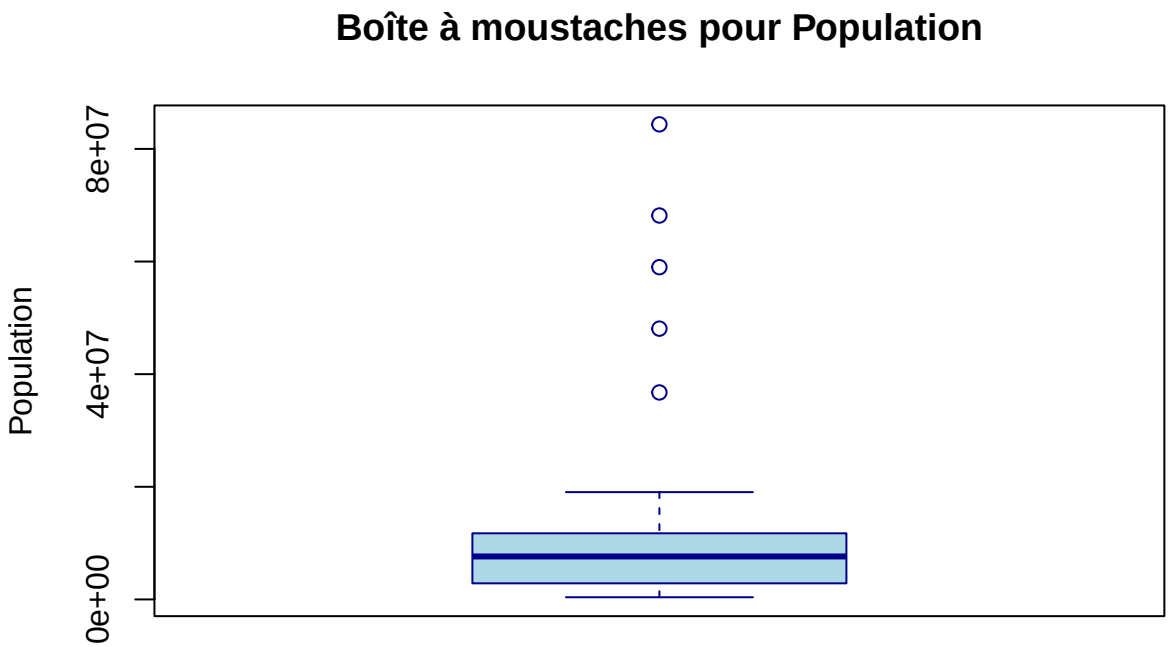
Pour la préparation des données, une normalisation a été appliquée pour rendre les variables comparables. L'analyse inclut la création de matrices de dissimilarité, l'utilisation de la décomposition en valeurs propres pour l'ACP, et l'application des méthodes de classification sur les données normalisées. Les regroupements obtenus seront interprétés à travers l'étude des centres de gravité, des inerties et des plans factoriels. Enfin, une attention particulière sera portée à l'analyse des proximités des pays dans des zones spécifiques de l'espace factoriel, afin de mieux comprendre les similarités entre pays.

```r
# Vérification et chargement des bibliothèques nécessaires
if (!require(ggplot2)) install.packages("ggplot2", dependencies = TRUE)
library(ggplot2)

# Chargement de TinyTeX si nécessaire (une seule fois)
if (!tinytex::is_tinytex()) {
  tinytex::install_tinytex()
}
```

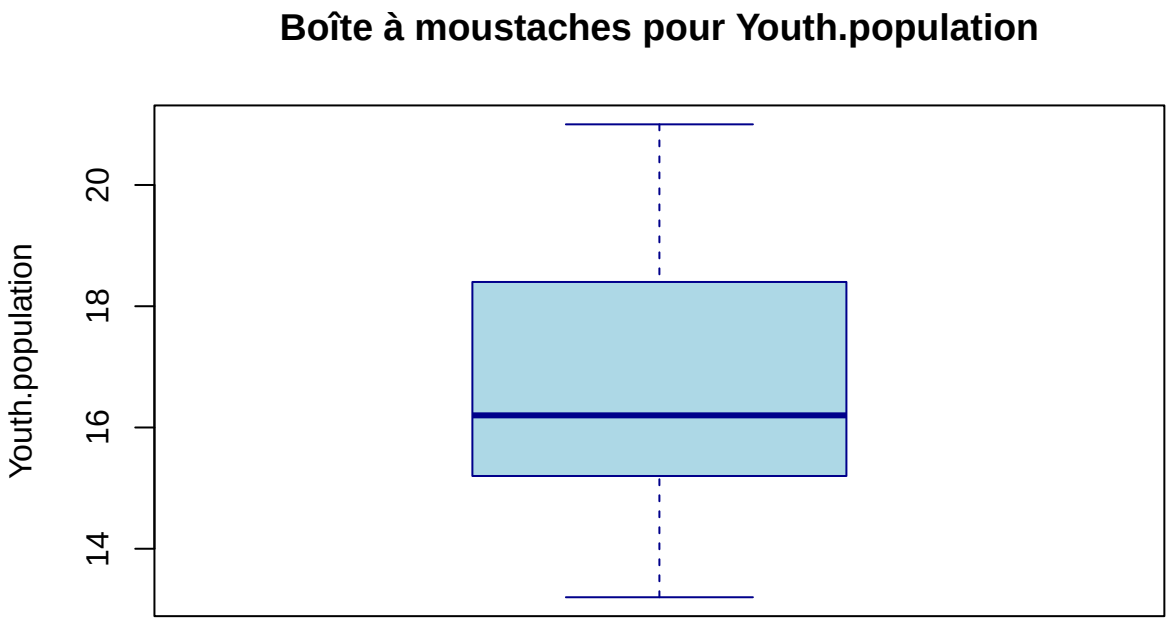
```r
euro_data <- read.csv("data/euro.csv", header = TRUE, sep = ";")

# Normalisation des données (Min-Max Scaling)
euro_data_normalized <- as.data.frame(lapply(euro_data[, -1], function(x) {
  (x - min(x, na.rm = TRUE)) / (max(x, na.rm = TRUE) - min(x, na.rm = TRUE))
}))
```


```r
euro_data <- read.csv("data/euro.csv", header = TRUE, sep = ";")

# Normalisation des données (Min-Max Scaling)
euro_data_normalized <- as.data.frame(lapply(euro_data[, -1], function(x) {
  (x - min(x, na.rm = TRUE)) / (max(x, na.rm = TRUE) - min(x, na.rm = TRUE))
}))
```


```r
# Supprimer la première colonne si elle contient des noms (optionnel)
euro_data <- euro_data[, -1]

# Convertir les colonnes en numériques si nécessaire
euro_data <- data.frame(lapply(euro_data, function(x) as.numeric(as.character(x))))

selected_columns <- colnames(euro_data)[1:2]  # Diagrammes 1 à 2
for (col_name in selected_columns) {
  boxplot(euro_data[[col_name]],
          main = paste("Boîte à moustaches pour", col_name),
          ylab = col_name,
          col = "lightblue",
          border = "darkblue")

  # Interprétation
  cat(paste("\n**Interprétation pour", col_name, ":**\n"))
}
```

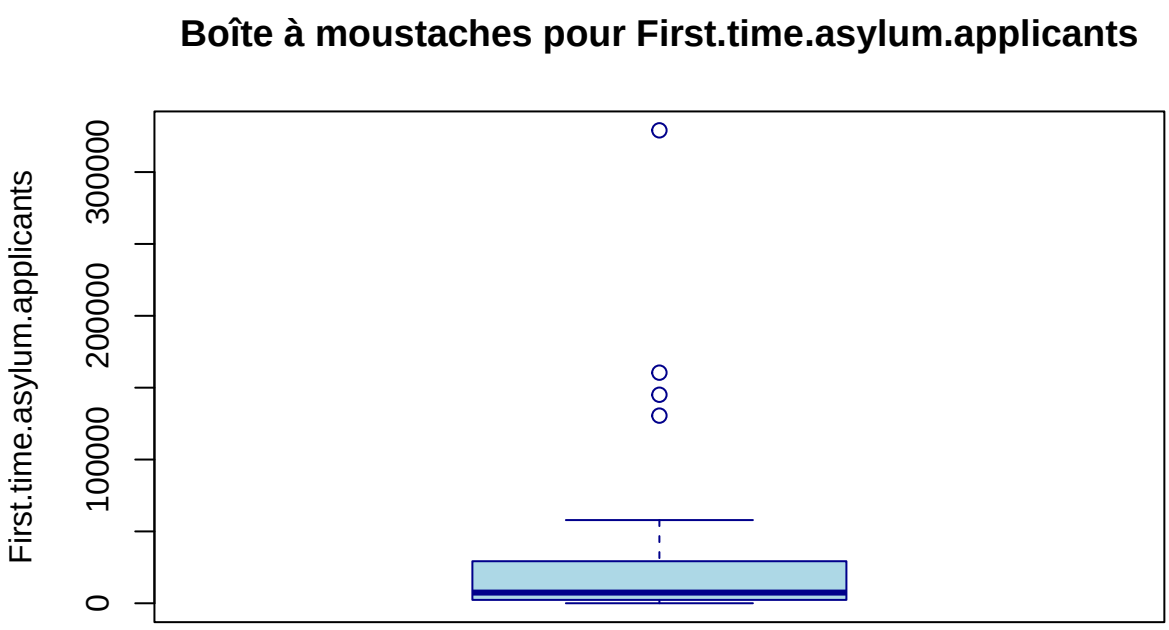


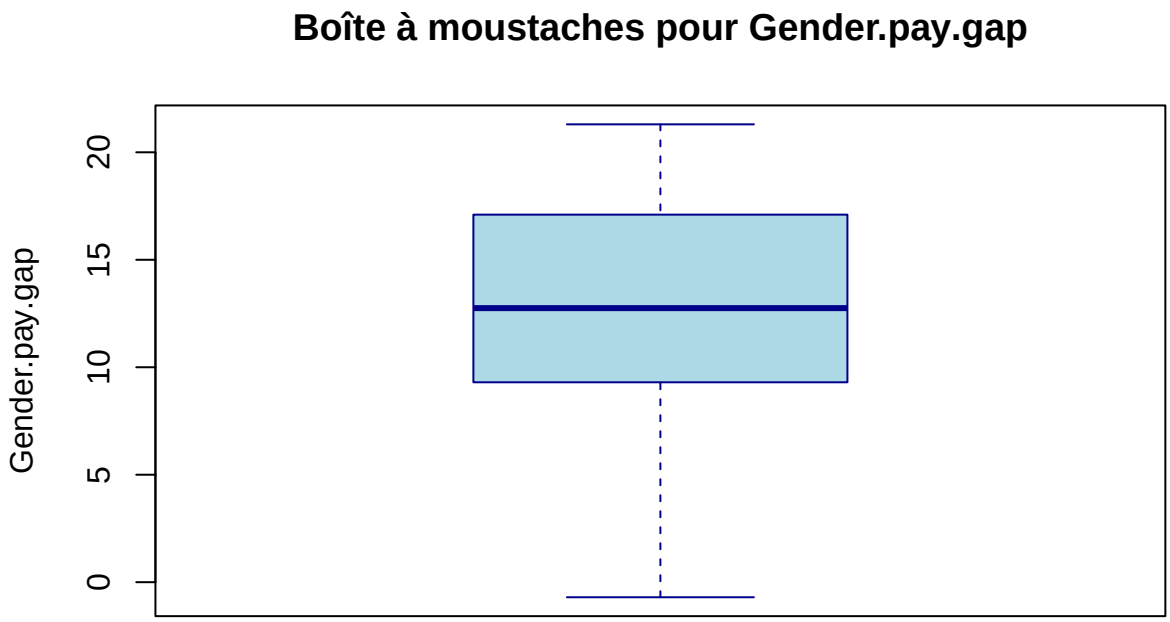

```
##
## **Interprétation pour Population :**
```



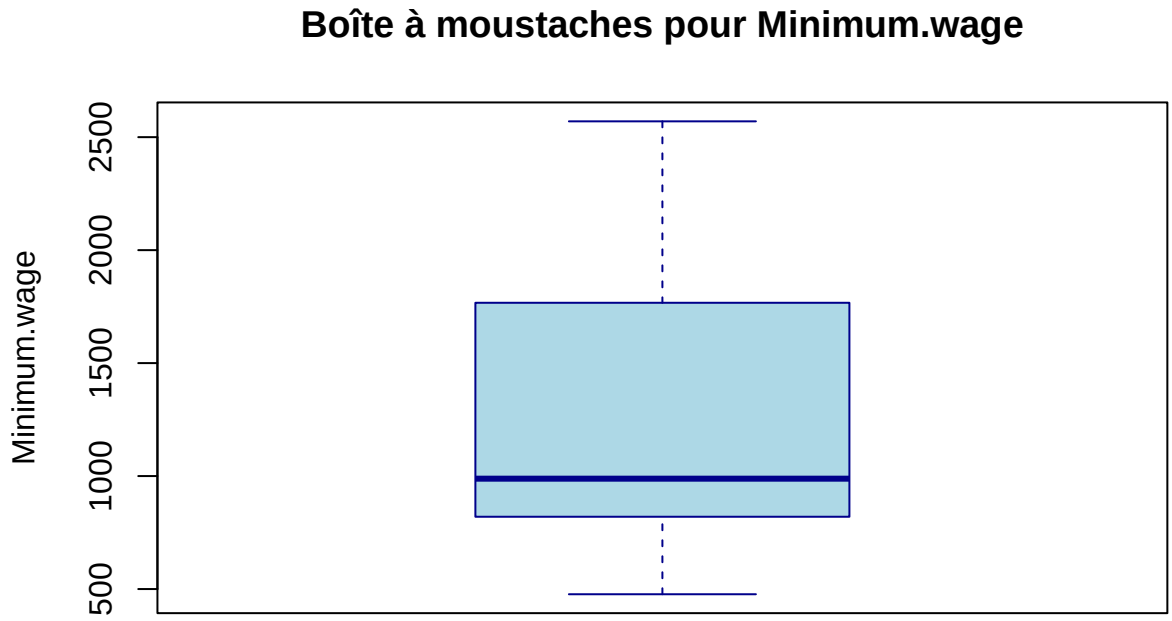

```
##
## **Interprétation pour Youth.population :**
```

```
selected_columns <- colnames(euro_data)[3:4]  # Diagrammes 1 à 2
for (col_name in selected_columns) {
  boxplot(euro_data[[col_name]],
          main = paste("Boîte à moustaches pour", col_name),
          ylab = col_name,
          col = "lightblue",
          border = "darkblue")

  # Interprétation
 # cat(paste("\n**Interprétation pour", col_name, ":**\n"))
}
```

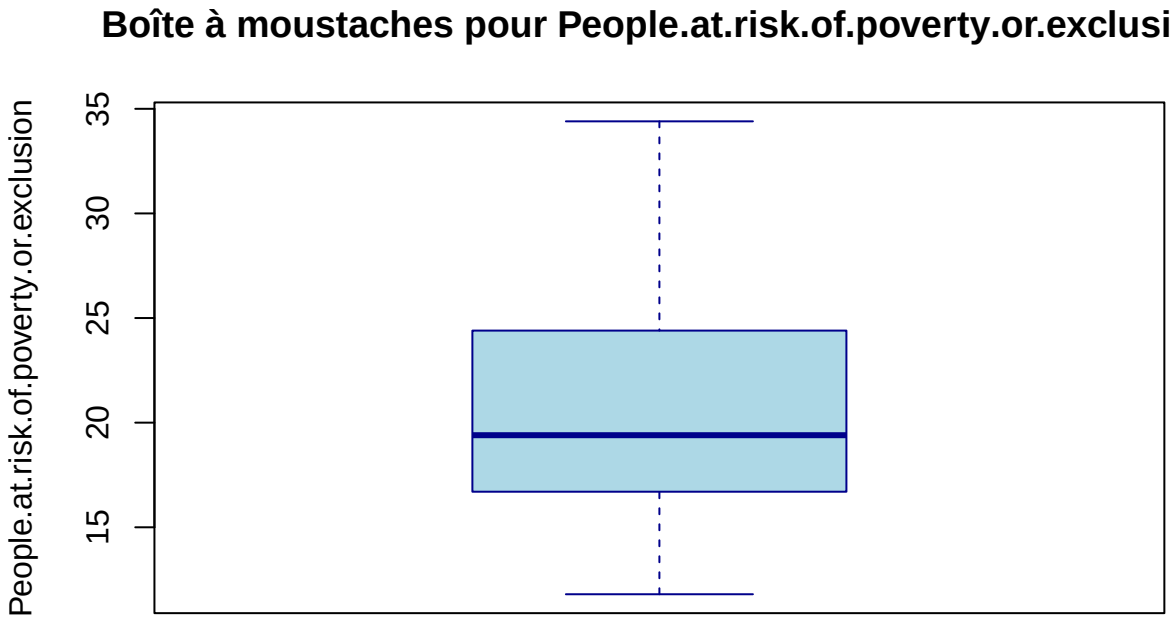

Boîte à moustaches pour First.time.asylum.applicants
First.time.asylum.applicants
0
100000
200000
300000




```r
selected_columns <- colnames(euro_data)[5:6]  # Diagrammes 1 à 2
for (col_name in selected_columns) {
  boxplot(euro_data[[col_name]],
          main = paste("Boîte à moustaches pour", col_name),
          ylab = col_name,
          col = "lightblue",
          border = "darkblue")

  # Interprétation
  cat(paste("\n**Interprétation pour", col_name, ":**\n"))
}
```

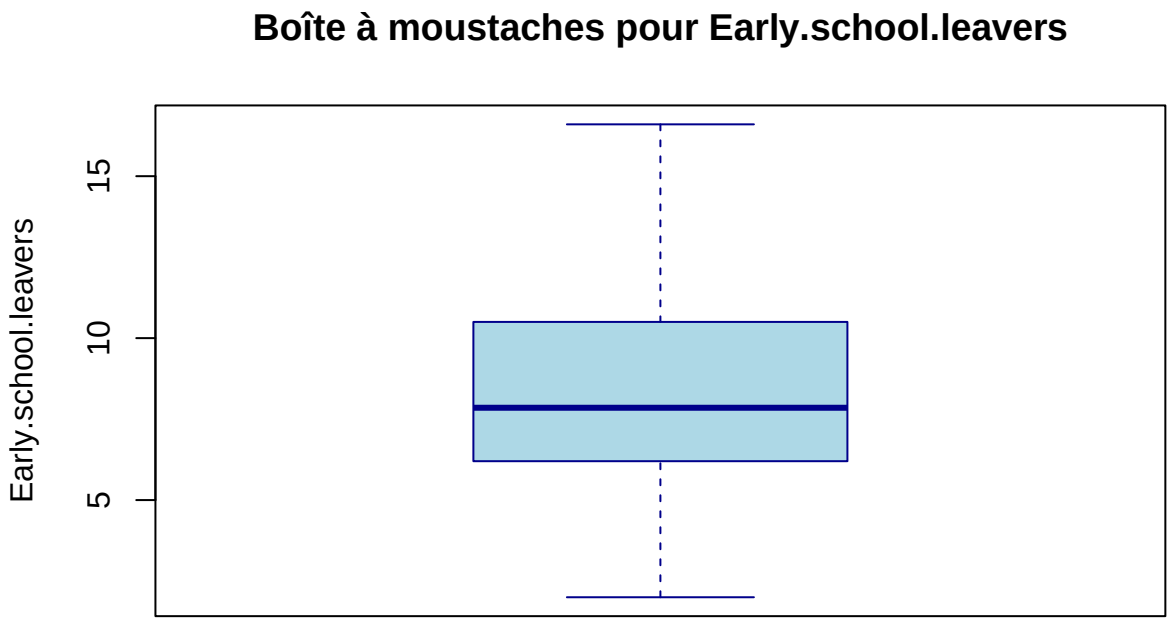



```
##
## **Interprétation pour Minimum.wage :**
```

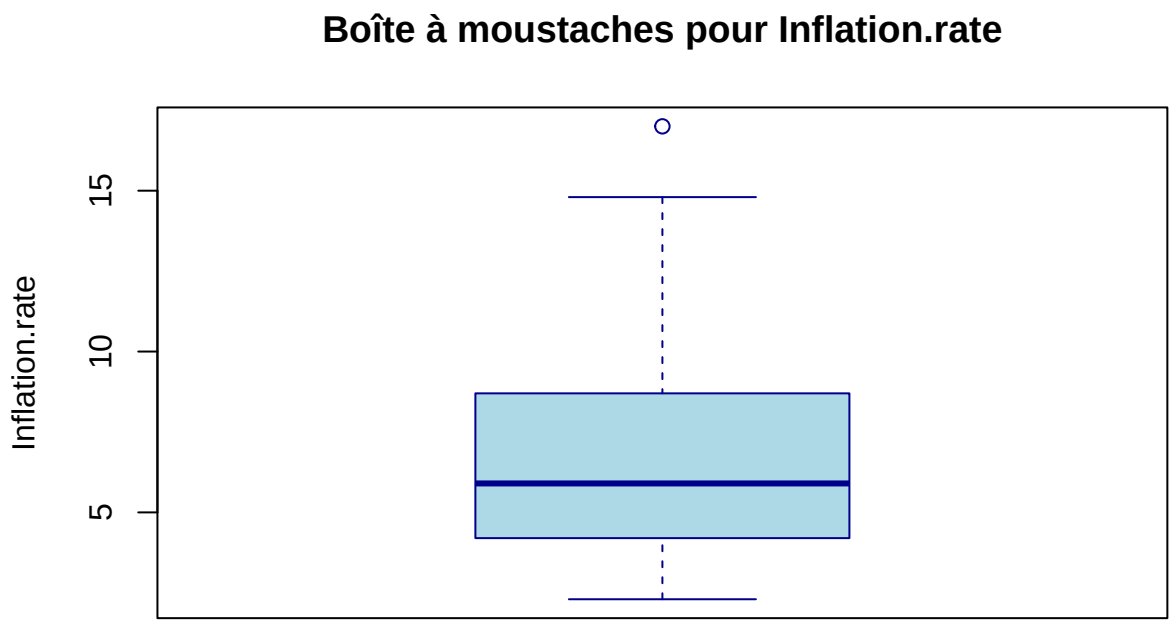



```
##
## **Interprétation pour People.at.risk.of.poverty.or.exclusion :**
```

```r
selected_columns <- colnames(euro_data)[7:8]  # Diagrammes 1 à 2
for (col_name in selected_columns) {
  boxplot(euro_data[[col_name]],
          main = paste("Boîte à moustaches pour", col_name),
          ylab = col_name,
          col = "lightblue",
          border = "darkblue")

  # Interprétation
  cat(paste("\n**Interprétation pour", col_name, ":**\n"))
}
```

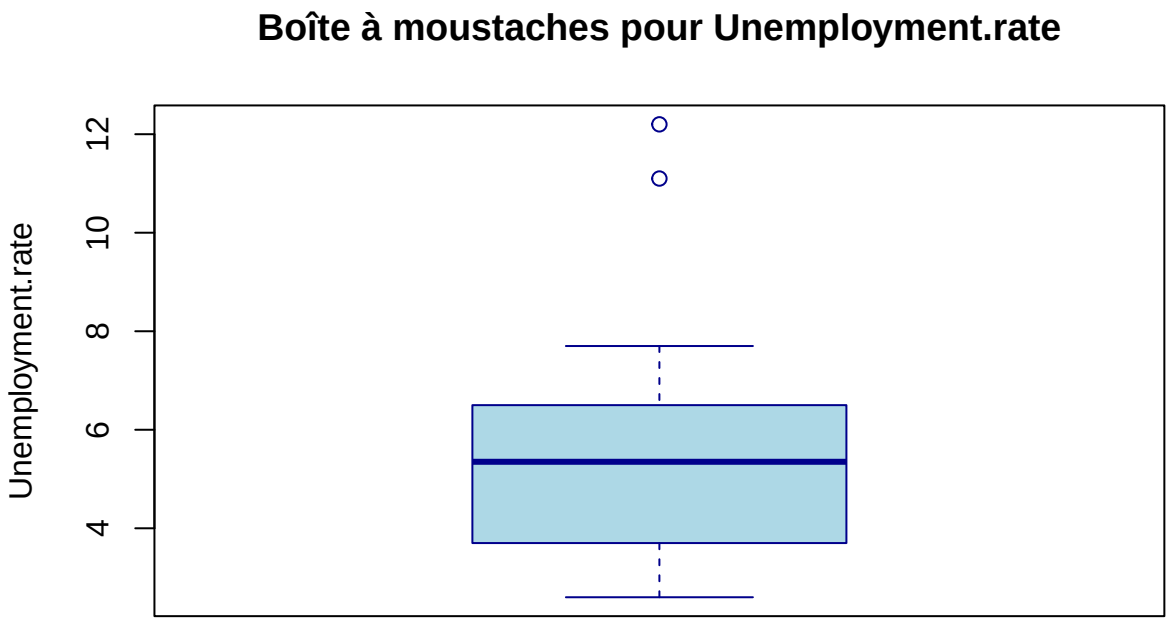



```
##
## **Interprétation pour Early.school.leavers :**
```

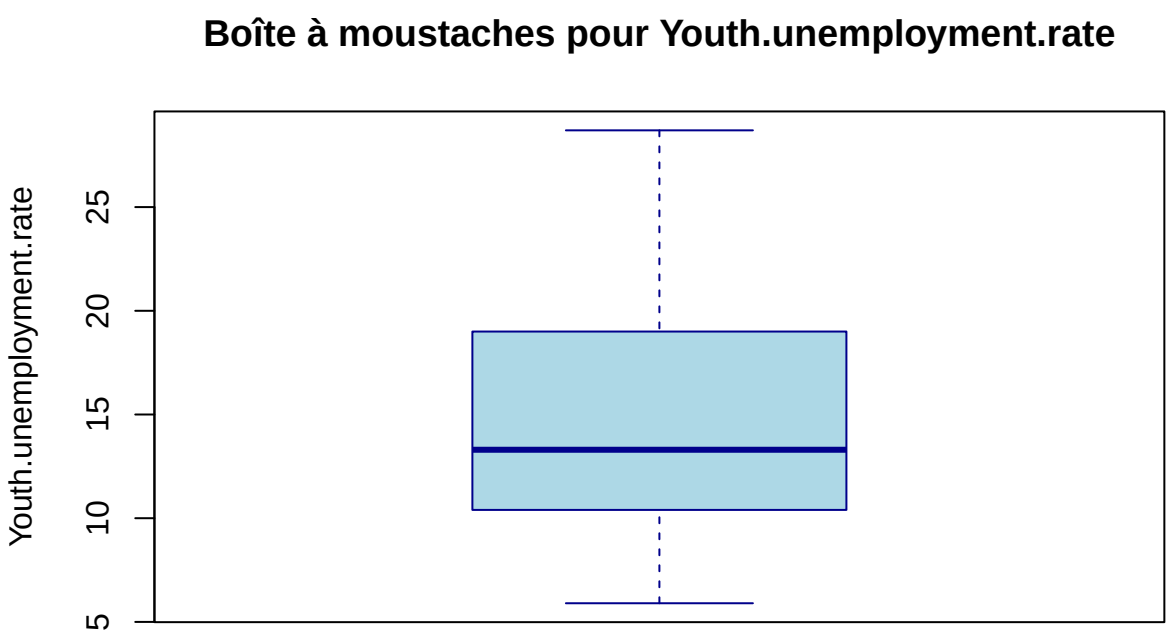



```
##
## **Interprétation pour Inflation.rate :**
```

```r
selected_columns <- colnames(euro_data)[9:10]  # Diagrammes 1 à 2
for (col_name in selected_columns) {
  boxplot(euro_data[[col_name]],
          main = paste("Boîte à moustaches pour", col_name),
          ylab = col_name,
          col = "lightblue",
          border = "darkblue")

  # Interprétation
  cat(paste("\n**Interprétation pour", col_name, ":**\n"))
}
```




```
##
## **Interprétation pour Unemployment.rate :**
```




```
##
## **Interprétation pour Youth.unemployment.rate :**
```

```
# Centrer les données : soustraire la moyenne de chaque colonne

" Question 3 "
```

```
## [1] " Question 3 "
```

```
euro_data_centered <- scale(euro_data, center = TRUE, scale = FALSE)

# Calculer la matrice de variance-covariance
n <- nrow(euro_data)  # Nombre d'observations
V <- (t(euro_data_centered) %*% euro_data_centered) / (n - 1)

# Afficher la matrice
print("Matrice de variance-covariance :")
```

```
## [1] "Matrice de variance-covariance :"
```

```
print(V)
```

```
##                                    Population Youth.population
```

```
## Population                                4.653257e+14    -6.839267e+06
## Youth.population                         -6.839267e+06     3.260931e+00
## First.time.asylum.applicants              1.365717e+12    -1.592480e+04
## Gender.pay.gap                           -6.195038e+06    -2.230724e+00
## Minimum.wage                                       NA               NA
## People.at.risk.of.poverty.or.exclusion             NA               NA
## Early.school.leavers                      1.621818e+07     1.575862e+00
## Inflation.rate                           -6.534739e+06    -2.646345e+00
## Unemployment.rate                         8.017485e+06    -8.740690e-01
## Youth.unemployment.rate                   1.445936e+07    -2.273414e+00
## GDP.per.capita                           -2.857962e+10     2.394673e+04
## Government.gross.debt                     2.994623e+08    -2.145276e+00
## Greenhouse.gas.emissions                 -8.092486e+06     1.863724e+00
## Renewable.energy                         -1.066077e+08     1.010286e+01
## Electricity.prices                        7.486738e+08    -8.085862e-01
## Energy.imports.dependency                 6.282654e+07     4.074517e+00
##                                         First.time.asylum.applicants
## Population                                             1.365717e+12
## Youth.population                                      -1.592480e+04
## First.time.asylum.applicants                           4.950971e+09
## Gender.pay.gap                                         2.341646e+04
## Minimum.wage                                                     NA
## People.at.risk.of.poverty.or.exclusion                           NA
## Early.school.leavers                                   6.578850e+04
## Inflation.rate                                        -5.499202e+04
## Unemployment.rate                                      3.117897e+04
## Youth.unemployment.rate                                1.909011e+04
## GDP.per.capita                                         3.054725e+07
## Government.gross.debt                                  9.319273e+05
## Greenhouse.gas.emissions                              -1.295407e+04
## Renewable.energy                                      -2.707843e+05
## Electricity.prices                                     2.853515e+06
## Energy.imports.dependency                              3.598142e+05
##                                        Gender.pay.gap Minimum.wage
## Population                              -6.195038e+06           NA
## Youth.population                        -2.230724e+00           NA
## First.time.asylum.applicants             2.341646e+04           NA
## Gender.pay.gap                           2.561895e+01           NA
## Minimum.wage                                       NA           NA
## People.at.risk.of.poverty.or.exclusion             NA           NA
## Early.school.leavers                    -1.151034e+00           NA
## Inflation.rate                           5.996276e+00           NA
## Unemployment.rate                       -8.288276e-01           NA
## Youth.unemployment.rate                 -9.091483e+00           NA
## GDP.per.capita                          -2.329693e+04           NA
## Government.gross.debt                   -3.046440e+01           NA
## Greenhouse.gas.emissions                -3.583356e+00           NA
## Renewable.energy                         2.383613e+01           NA
## Electricity.prices                      -5.713678e-01           NA
## Energy.imports.dependency               -3.493792e+01           NA
##                                        People.at.risk.of.poverty.or.exclusion
## Population                                                                NA
## Youth.population                                                          NA
## First.time.asylum.applicants                                              NA
```

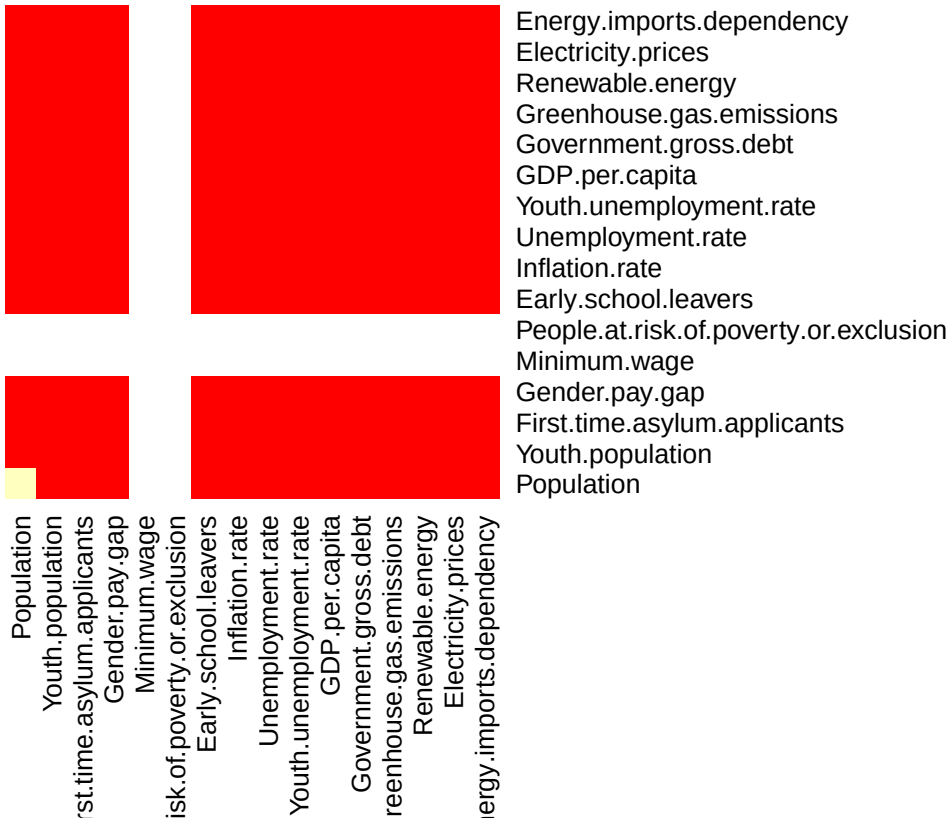
```
## Gender.pay.gap                                                    NA
## Minimum.wage                                                      NA
## People.at.risk.of.poverty.or.exclusion                            NA
## Early.school.leavers                                              NA
## Inflation.rate                                                    NA
## Unemployment.rate                                                 NA
## Youth.unemployment.rate                                           NA
## GDP.per.capita                                                    NA
## Government.gross.debt                                             NA
## Greenhouse.gas.emissions                                          NA
## Renewable.energy                                                  NA
## Electricity.prices                                                NA
## Energy.imports.dependency                                         NA
##                                    Early.school.leavers Inflation.rate
## Population                                1.621818e+07  -6.534739e+06
## Youth.population                          1.575862e+00  -2.646345e+00
## First.time.asylum.applicants              6.578850e+04  -5.499202e+04
## Gender.pay.gap                           -1.151034e+00   5.996276e+00
## Minimum.wage                                        NA             NA
## People.at.risk.of.poverty.or.exclusion              NA             NA
## Early.school.leavers                      1.255172e+01   7.158621e-01
## Inflation.rate                            7.158621e-01   1.211614e+01
## Unemployment.rate                         4.827586e-02  -2.516690e+00
## Youth.unemployment.rate                   6.196552e-01  -3.752138e+00
## GDP.per.capita                           -1.667793e+03  -4.017071e+04
## Government.gross.debt                     6.283103e+00  -2.189462e+01
## Greenhouse.gas.emissions                  1.055172e-01  -4.308966e-01
## Renewable.energy                          1.908069e+01  -9.630897e+00
## Electricity.prices                       -4.490414e+01  -9.986145e+01
## Energy.imports.dependency                -2.727517e+01  -2.880076e+01
##                                    Unemployment.rate
## Population                              8.017485e+06
## Youth.population                       -8.740690e-01
## First.time.asylum.applicants            3.117897e+04
## Gender.pay.gap                         -8.288276e-01
## Minimum.wage                                      NA
## People.at.risk.of.poverty.or.exclusion            NA
## Early.school.leavers                    4.827586e-02
## Inflation.rate                         -2.516690e+00
## Unemployment.rate                       5.049379e+00
## Youth.unemployment.rate                 1.156469e+01
## GDP.per.capita                         -8.682110e+03
## Government.gross.debt                   4.358007e+01
## Greenhouse.gas.emissions               -1.953862e+00
## Renewable.energy                       -8.711034e-01
## Electricity.prices                      1.851538e+01
## Energy.imports.dependency               7.197448e+00
##                                    Youth.unemployment.rate GDP.per.capita
## Population                                    1.445936e+07  -2.857962e+10
## Youth.population                             -2.273414e+00   2.394673e+04
## First.time.asylum.applicants                  1.909011e+04   3.054725e+07
## Gender.pay.gap                               -9.091483e+00  -2.329693e+04
## Minimum.wage                                            NA             NA
## People.at.risk.of.poverty.or.exclusion                  NA             NA
```

```
## Early.school.leavers                              6.196552e-01  -1.667793e+03
## Inflation.rate                                   -3.752138e+00  -4.017071e+04
## Unemployment.rate                                 1.156469e+01  -8.682110e+03
## Youth.unemployment.rate                           3.455114e+01  -2.704445e+04
## GDP.per.capita                                   -2.704445e+04   3.983704e+08
## Government.gross.debt                             1.032097e+02  -1.581849e+05
## Greenhouse.gas.emissions                         -3.952966e+00   2.158401e+04
## Renewable.energy                                 -1.476659e+01   1.228200e+05
## Electricity.prices                               -3.015093e+01   2.644578e+05
## Energy.imports.dependency                         1.513148e+01   5.860834e+04
##                                    Government.gross.debt
## Population                                  2.994623e+08
## Youth.population                           -2.145276e+00
## First.time.asylum.applicants                9.319273e+05
## Gender.pay.gap                             -3.046440e+01
## Minimum.wage                                          NA
## People.at.risk.of.poverty.or.exclusion                NA
## Early.school.leavers                        6.283103e+00
## Inflation.rate                             -2.189462e+01
## Unemployment.rate                           4.358007e+01
## Youth.unemployment.rate                     1.032097e+02
## GDP.per.capita                             -1.581849e+05
## Government.gross.debt                       1.240830e+03
## Greenhouse.gas.emissions                   -1.137271e+01
## Renewable.energy                           -1.327777e+02
## Electricity.prices                          6.659872e+02
## Energy.imports.dependency                   2.703910e+02
##                                    Greenhouse.gas.emissions
## Population                                    -8.092486e+06
## Youth.population                               1.863724e+00
## First.time.asylum.applicants                  -1.295407e+04
## Gender.pay.gap                                -3.583356e+00
## Minimum.wage                                            NA
## People.at.risk.of.poverty.or.exclusion                  NA
## Early.school.leavers                           1.055172e-01
## Inflation.rate                                -4.308966e-01
## Unemployment.rate                             -1.953862e+00
## Youth.unemployment.rate                       -3.952966e+00
## GDP.per.capita                                 2.158401e+04
## Government.gross.debt                         -1.137271e+01
## Greenhouse.gas.emissions                       7.033057e+00
## Renewable.energy                              -2.733149e+00
## Electricity.prices                             1.906025e+01
## Energy.imports.dependency                     -3.686644e+00
##                                    Renewable.energy Electricity.prices
## Population                           -1.066077e+08       7.486738e+08
## Youth.population                      1.010286e+01      -8.085862e-01
## First.time.asylum.applicants         -2.707843e+05       2.853515e+06
## Gender.pay.gap                        2.383613e+01      -5.713678e-01
## Minimum.wage                                    NA                NA
## People.at.risk.of.poverty.or.exclusion          NA                NA
## Early.school.leavers                  1.908069e+01      -4.490414e+01
## Inflation.rate                       -9.630897e+00      -9.986145e+01
## Unemployment.rate                    -8.711034e-01       1.851538e+01
```

```
## Youth.unemployment.rate                   -1.476659e+01      -3.015093e+01
## GDP.per.capita                             1.228200e+05       2.644578e+05
## Government.gross.debt                     -1.327777e+02       6.659872e+02
## Greenhouse.gas.emissions                  -2.733149e+00       1.906025e+01
## Renewable.energy                           3.593812e+02      -4.262927e+02
## Electricity.prices                        -4.262927e+02       6.935113e+03
## Energy.imports.dependency                 -2.903213e+02       7.416931e+02
##                                      Energy.imports.dependency
## Population                                        6.282654e+07
## Youth.population                                  4.074517e+00
## First.time.asylum.applicants                      3.598142e+05
## Gender.pay.gap                                   -3.493792e+01
## Minimum.wage                                                NA
## People.at.risk.of.poverty.or.exclusion                      NA
## Early.school.leavers                             -2.727517e+01
## Inflation.rate                                   -2.880076e+01
## Unemployment.rate                                 7.197448e+00
## Youth.unemployment.rate                           1.513148e+01
## GDP.per.capita                                    5.860834e+04
## Government.gross.debt                             2.703910e+02
## Greenhouse.gas.emissions                         -3.686644e+00
## Renewable.energy                                 -2.903213e+02
## Electricity.prices                                7.416931e+02
## Energy.imports.dependency                         5.925486e+02
```

```r
#  Question 3 suite
V_rounded <- round(V, 2)
# Étape 3 : Créer la heatmap
heatmap(as.matrix(V_rounded),
        main = "Heatmap de la matrice de variance-covariance",
        Colv = NA, Rowv = NA,  # Désactive le clustering
        scale = "none",  # Pas de normalisation supplémentaire
        col = heat.colors(10),  # Palette de couleurs
        margins = c(10, 10))  # Marges pour la lisibilité
```

atmap de la matrice de variance−covariance




```
# Arrondir les valeurs de la matrice à 2 décimales
print("Matrice de variance-covariance (arrondie) :")
```

```
## [1] "Matrice de variance-covariance (arrondie) :"
```

```
round(V, 2)
```

```
##                                        Population Youth.population
## Population                           4.653257e+14      -6839267.06
## Youth.population                    -6.839267e+06             3.26
## First.time.asylum.applicants         1.365717e+12        -15924.80
## Gender.pay.gap                      -6.195038e+06            -2.23
## Minimum.wage                                   NA               NA
## People.at.risk.of.poverty.or.exclusion         NA               NA
## Early.school.leavers                 1.621818e+07             1.58
## Inflation.rate                      -6.534739e+06            -2.65
## Unemployment.rate                    8.017485e+06            -0.87
## Youth.unemployment.rate              1.445936e+07            -2.27
## GDP.per.capita                      -2.857962e+10         23946.73
## Government.gross.debt                2.994623e+08            -2.15
## Greenhouse.gas.emissions            -8.092486e+06             1.86
## Renewable.energy                    -1.066077e+08            10.10
## Electricity.prices                   7.486738e+08            -0.81
## Energy.imports.dependency            6.282654e+07             4.07
##                                    First.time.asylum.applicants
```

```
## Population                                          1.365717e+12
## Youth.population                                   -1.592480e+04
## First.time.asylum.applicants                        4.950971e+09
## Gender.pay.gap                                      2.341646e+04
## Minimum.wage                                                 NA
## People.at.risk.of.poverty.or.exclusion                       NA
## Early.school.leavers                                6.578850e+04
## Inflation.rate                                     -5.499202e+04
## Unemployment.rate                                   3.117897e+04
## Youth.unemployment.rate                             1.909011e+04
## GDP.per.capita                                      3.054725e+07
## Government.gross.debt                               9.319273e+05
## Greenhouse.gas.emissions                           -1.295407e+04
## Renewable.energy                                   -2.707843e+05
## Electricity.prices                                  2.853515e+06
## Energy.imports.dependency                           3.598142e+05
##                                    Gender.pay.gap Minimum.wage
## Population                            -6195038.43           NA
## Youth.population                           -2.23           NA
## First.time.asylum.applicants            23416.46           NA
## Gender.pay.gap                             25.62           NA
## Minimum.wage                                  NA           NA
## People.at.risk.of.poverty.or.exclusion        NA           NA
## Early.school.leavers                       -1.15           NA
## Inflation.rate                              6.00           NA
## Unemployment.rate                          -0.83           NA
## Youth.unemployment.rate                    -9.09           NA
## GDP.per.capita                          -23296.93           NA
## Government.gross.debt                      -30.46           NA
## Greenhouse.gas.emissions                   -3.58           NA
## Renewable.energy                           23.84           NA
## Electricity.prices                         -0.57           NA
## Energy.imports.dependency                  -34.94           NA
##                                    People.at.risk.of.poverty.or.exclusion
## Population                                                             NA
## Youth.population                                                       NA
## First.time.asylum.applicants                                           NA
## Gender.pay.gap                                                         NA
## Minimum.wage                                                           NA
## People.at.risk.of.poverty.or.exclusion                                 NA
## Early.school.leavers                                                   NA
## Inflation.rate                                                         NA
## Unemployment.rate                                                      NA
## Youth.unemployment.rate                                                NA
## GDP.per.capita                                                         NA
## Government.gross.debt                                                  NA
## Greenhouse.gas.emissions                                               NA
## Renewable.energy                                                       NA
## Electricity.prices                                                     NA
## Energy.imports.dependency                                              NA
##                                    Early.school.leavers Inflation.rate
## Population                                 16218178.45    -6534739.21
## Youth.population                                  1.58          -2.65
## First.time.asylum.applicants                  65788.50      -54992.02
```

```
## Gender.pay.gap                                    -1.15           6.00
## Minimum.wage                                        NA             NA
## People.at.risk.of.poverty.or.exclusion              NA             NA
## Early.school.leavers                             12.55           0.72
## Inflation.rate                                    0.72          12.12
## Unemployment.rate                                 0.05          -2.52
## Youth.unemployment.rate                           0.62          -3.75
## GDP.per.capita                                -1667.79      -40170.71
## Government.gross.debt                             6.28         -21.89
## Greenhouse.gas.emissions                          0.11          -0.43
## Renewable.energy                                 19.08          -9.63
## Electricity.prices                              -44.90         -99.86
## Energy.imports.dependency                       -27.28         -28.80
##                                    Unemployment.rate
## Population                               8017484.96
## Youth.population                              -0.87
## First.time.asylum.applicants                31178.97
## Gender.pay.gap                                -0.83
## Minimum.wage                                    NA
## People.at.risk.of.poverty.or.exclusion          NA
## Early.school.leavers                          0.05
## Inflation.rate                               -2.52
## Unemployment.rate                             5.05
## Youth.unemployment.rate                      11.56
## GDP.per.capita                             -8682.11
## Government.gross.debt                         43.58
## Greenhouse.gas.emissions                      -1.95
## Renewable.energy                              -0.87
## Electricity.prices                            18.52
## Energy.imports.dependency                      7.20
##                                    Youth.unemployment.rate GDP.per.capita
## Population                                    14459359.14  -2.857962e+10
## Youth.population                                   -2.27   2.394673e+04
## First.time.asylum.applicants                    19090.11   3.054725e+07
## Gender.pay.gap                                     -9.09  -2.329693e+04
## Minimum.wage                                          NA             NA
## People.at.risk.of.poverty.or.exclusion                NA             NA
## Early.school.leavers                                0.62  -1.667790e+03
## Inflation.rate                                     -3.75  -4.017071e+04
## Unemployment.rate                                  11.56  -8.682110e+03
## Youth.unemployment.rate                            34.55  -2.704445e+04
## GDP.per.capita                                  -27044.45   3.983704e+08
## Government.gross.debt                              103.21  -1.581849e+05
## Greenhouse.gas.emissions                           -3.95   2.158401e+04
## Renewable.energy                                  -14.77   1.228200e+05
## Electricity.prices                                -30.15   2.644578e+05
## Energy.imports.dependency                          15.13   5.860834e+04
##                                    Government.gross.debt
## Population                                 299462337.44
## Youth.population                                 -2.15
## First.time.asylum.applicants                   931927.28
## Gender.pay.gap                                   -30.46
## Minimum.wage                                        NA
## People.at.risk.of.poverty.or.exclusion              NA
```

```
## Early.school.leavers                                    6.28
## Inflation.rate                                        -21.89
## Unemployment.rate                                      43.58
## Youth.unemployment.rate                               103.21
## GDP.per.capita                                    -158184.92
## Government.gross.debt                                1240.83
## Greenhouse.gas.emissions                              -11.37
## Renewable.energy                                      -132.78
## Electricity.prices                                     665.99
## Energy.imports.dependency                              270.39
##                                       Greenhouse.gas.emissions
## Population                                        -8092485.69
## Youth.population                                         1.86
## First.time.asylum.applicants                         -12954.07
## Gender.pay.gap                                          -3.58
## Minimum.wage                                               NA
## People.at.risk.of.poverty.or.exclusion                     NA
## Early.school.leavers                                    0.11
## Inflation.rate                                         -0.43
## Unemployment.rate                                      -1.95
## Youth.unemployment.rate                                -3.95
## GDP.per.capita                                      21584.01
## Government.gross.debt                                 -11.37
## Greenhouse.gas.emissions                                7.03
## Renewable.energy                                       -2.73
## Electricity.prices                                     19.06
## Energy.imports.dependency                              -3.69
##                                       Renewable.energy Electricity.prices
## Population                                -106607660.88       748673833.24
## Youth.population                                 10.10              -0.81
## First.time.asylum.applicants               -270784.28         2853514.54
## Gender.pay.gap                                   23.84              -0.57
## Minimum.wage                                        NA                 NA
## People.at.risk.of.poverty.or.exclusion              NA                 NA
## Early.school.leavers                             19.08             -44.90
## Inflation.rate                                   -9.63             -99.86
## Unemployment.rate                                -0.87              18.52
## Youth.unemployment.rate                         -14.77             -30.15
## GDP.per.capita                                122820.02          264457.76
## Government.gross.debt                           -132.78             665.99
## Greenhouse.gas.emissions                          -2.73              19.06
## Renewable.energy                                359.38            -426.29
## Electricity.prices                             -426.29            6935.11
## Energy.imports.dependency                      -290.32             741.69
##                                       Energy.imports.dependency
## Population                                         62826536.59
## Youth.population                                          4.07
## First.time.asylum.applicants                         359814.25
## Gender.pay.gap                                          -34.94
## Minimum.wage                                               NA
## People.at.risk.of.poverty.or.exclusion                     NA
## Early.school.leavers                                   -27.28
## Inflation.rate                                         -28.80
## Unemployment.rate                                        7.20
```

```
## Youth.unemployment.rate                                  15.13
## GDP.per.capita                                         58608.34
## Government.gross.debt                                    270.39
## Greenhouse.gas.emissions                                  -3.69
## Renewable.energy                                        -290.32
## Electricity.prices                                       741.69
## Energy.imports.dependency                                592.55
```

```r
# Ajouter les noms de lignes et colonnes pour un tableau clair
rownames(V) <- colnames(V) <- colnames(euro_data)

# Afficher sous forme de tableau avec arrondi
print(round(V, 2))
```

```
##                                        Population Youth.population
## Population                           4.653257e+14      -6839267.06
## Youth.population                    -6.839267e+06             3.26
## First.time.asylum.applicants         1.365717e+12        -15924.80
## Gender.pay.gap                      -6.195038e+06            -2.23
## Minimum.wage                                   NA               NA
## People.at.risk.of.poverty.or.exclusion         NA               NA
## Early.school.leavers                 1.621818e+07             1.58
## Inflation.rate                      -6.534739e+06            -2.65
## Unemployment.rate                    8.017485e+06            -0.87
## Youth.unemployment.rate              1.445936e+07            -2.27
## GDP.per.capita                      -2.857962e+10         23946.73
## Government.gross.debt                2.994623e+08            -2.15
## Greenhouse.gas.emissions            -8.092486e+06             1.86
## Renewable.energy                    -1.066077e+08            10.10
## Electricity.prices                   7.486738e+08            -0.81
## Energy.imports.dependency            6.282654e+07             4.07
##                                     First.time.asylum.applicants
## Population                                         1.365717e+12
## Youth.population                                  -1.592480e+04
## First.time.asylum.applicants                       4.950971e+09
## Gender.pay.gap                                     2.341646e+04
## Minimum.wage                                                NA
## People.at.risk.of.poverty.or.exclusion                      NA
## Early.school.leavers                               6.578850e+04
## Inflation.rate                                    -5.499202e+04
## Unemployment.rate                                  3.117897e+04
## Youth.unemployment.rate                            1.909011e+04
## GDP.per.capita                                     3.054725e+07
## Government.gross.debt                              9.319273e+05
## Greenhouse.gas.emissions                          -1.295407e+04
## Renewable.energy                                  -2.707843e+05
## Electricity.prices                                 2.853515e+06
## Energy.imports.dependency                          3.598142e+05
##                                     Gender.pay.gap Minimum.wage
## Population                             -6195038.43           NA
## Youth.population                             -2.23           NA
## First.time.asylum.applicants              23416.46           NA
## Gender.pay.gap                               25.62           NA
## Minimum.wage                                    NA           NA
```

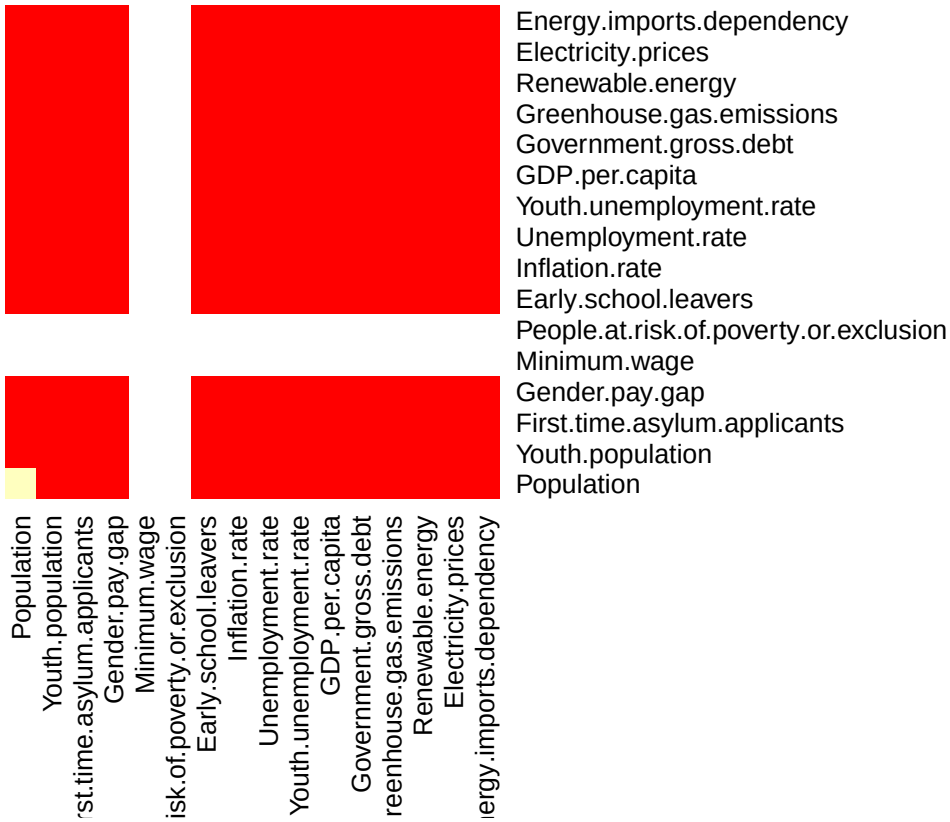
```
## People.at.risk.of.poverty.or.exclusion             NA           NA
## Early.school.leavers                             -1.15           NA
## Inflation.rate                                    6.00           NA
## Unemployment.rate                                -0.83           NA
## Youth.unemployment.rate                          -9.09           NA
## GDP.per.capita                                -23296.93           NA
## Government.gross.debt                            -30.46           NA
## Greenhouse.gas.emissions                          -3.58           NA
## Renewable.energy                                  23.84           NA
## Electricity.prices                                -0.57           NA
## Energy.imports.dependency                        -34.94           NA
##                                   People.at.risk.of.poverty.or.exclusion
## Population                                                            NA
## Youth.population                                                      NA
## First.time.asylum.applicants                                          NA
## Gender.pay.gap                                                        NA
## Minimum.wage                                                          NA
## People.at.risk.of.poverty.or.exclusion                                NA
## Early.school.leavers                                                  NA
## Inflation.rate                                                        NA
## Unemployment.rate                                                     NA
## Youth.unemployment.rate                                               NA
## GDP.per.capita                                                        NA
## Government.gross.debt                                                 NA
## Greenhouse.gas.emissions                                              NA
## Renewable.energy                                                      NA
## Electricity.prices                                                    NA
## Energy.imports.dependency                                             NA
##                                   Early.school.leavers Inflation.rate
## Population                                16218178.45    -6534739.21
## Youth.population                                 1.58          -2.65
## First.time.asylum.applicants                 65788.50      -54992.02
## Gender.pay.gap                                  -1.15           6.00
## Minimum.wage                                       NA             NA
## People.at.risk.of.poverty.or.exclusion             NA             NA
## Early.school.leavers                            12.55           0.72
## Inflation.rate                                   0.72          12.12
## Unemployment.rate                                0.05          -2.52
## Youth.unemployment.rate                          0.62          -3.75
## GDP.per.capita                               -1667.79      -40170.71
## Government.gross.debt                            6.28         -21.89
## Greenhouse.gas.emissions                         0.11          -0.43
## Renewable.energy                                19.08          -9.63
## Electricity.prices                             -44.90         -99.86
## Energy.imports.dependency                      -27.28         -28.80
##                                   Unemployment.rate
## Population                              8017484.96
## Youth.population                             -0.87
## First.time.asylum.applicants             31178.97
## Gender.pay.gap                              -0.83
## Minimum.wage                                   NA
## People.at.risk.of.poverty.or.exclusion         NA
## Early.school.leavers                         0.05
## Inflation.rate                              -2.52
```

```
## Unemployment.rate                                    5.05
## Youth.unemployment.rate                             11.56
## GDP.per.capita                                   -8682.11
## Government.gross.debt                               43.58
## Greenhouse.gas.emissions                            -1.95
## Renewable.energy                                    -0.87
## Electricity.prices                                  18.52
## Energy.imports.dependency                            7.20
##                                       Youth.unemployment.rate GDP.per.capita
## Population                                       14459359.14  -2.857962e+10
## Youth.population                                       -2.27   2.394673e+04
## First.time.asylum.applicants                        19090.11   3.054725e+07
## Gender.pay.gap                                         -9.09  -2.329693e+04
## Minimum.wage                                              NA             NA
## People.at.risk.of.poverty.or.exclusion                    NA             NA
## Early.school.leavers                                    0.62  -1.667790e+03
## Inflation.rate                                         -3.75  -4.017071e+04
## Unemployment.rate                                      11.56  -8.682110e+03
## Youth.unemployment.rate                                34.55  -2.704445e+04
## GDP.per.capita                                     -27044.45   3.983704e+08
## Government.gross.debt                                 103.21  -1.581849e+05
## Greenhouse.gas.emissions                               -3.95   2.158401e+04
## Renewable.energy                                      -14.77   1.228200e+05
## Electricity.prices                                    -30.15   2.644578e+05
## Energy.imports.dependency                              15.13   5.860834e+04
##                                       Government.gross.debt
## Population                                    299462337.44
## Youth.population                                    -2.15
## First.time.asylum.applicants                       931927.28
## Gender.pay.gap                                        -30.46
## Minimum.wage                                              NA
## People.at.risk.of.poverty.or.exclusion                    NA
## Early.school.leavers                                    6.28
## Inflation.rate                                        -21.89
## Unemployment.rate                                      43.58
## Youth.unemployment.rate                               103.21
## GDP.per.capita                                    -158184.92
## Government.gross.debt                                1240.83
## Greenhouse.gas.emissions                              -11.37
## Renewable.energy                                      -132.78
## Electricity.prices                                     665.99
## Energy.imports.dependency                              270.39
##                                       Greenhouse.gas.emissions
## Population                                        -8092485.69
## Youth.population                                        1.86
## First.time.asylum.applicants                       -12954.07
## Gender.pay.gap                                        -3.58
## Minimum.wage                                              NA
## People.at.risk.of.poverty.or.exclusion                    NA
## Early.school.leavers                                    0.11
## Inflation.rate                                         -0.43
## Unemployment.rate                                      -1.95
## Youth.unemployment.rate                                -3.95
## GDP.per.capita                                      21584.01
```

```
## Government.gross.debt                                -11.37
## Greenhouse.gas.emissions                               7.03
## Renewable.energy                                      -2.73
## Electricity.prices                                    19.06
## Energy.imports.dependency                             -3.69
##                                     Renewable.energy Electricity.prices
## Population                          -106607660.88       748673833.24
## Youth.population                            10.10              -0.81
## First.time.asylum.applicants          -270784.28         2853514.54
## Gender.pay.gap                              23.84              -0.57
## Minimum.wage                                   NA                 NA
## People.at.risk.of.poverty.or.exclusion         NA                 NA
## Early.school.leavers                        19.08             -44.90
## Inflation.rate                              -9.63             -99.86
## Unemployment.rate                           -0.87              18.52
## Youth.unemployment.rate                    -14.77             -30.15
## GDP.per.capita                          122820.02          264457.76
## Government.gross.debt                     -132.78             665.99
## Greenhouse.gas.emissions                    -2.73              19.06
## Renewable.energy                           359.38            -426.29
## Electricity.prices                         -426.29            6935.11
## Energy.imports.dependency                  -290.32             741.69
##                                     Energy.imports.dependency
## Population                                       62826536.59
## Youth.population                                        4.07
## First.time.asylum.applicants                       359814.25
## Gender.pay.gap                                        -34.94
## Minimum.wage                                             NA
## People.at.risk.of.poverty.or.exclusion                   NA
## Early.school.leavers                                  -27.28
## Inflation.rate                                        -28.80
## Unemployment.rate                                       7.20
## Youth.unemployment.rate                                15.13
## GDP.per.capita                                      58608.34
## Government.gross.debt                                 270.39
## Greenhouse.gas.emissions                               -3.69
## Renewable.energy                                     -290.32
## Electricity.prices                                    741.69
## Energy.imports.dependency                             592.55
```

```r
#  Question 3 suite
V_rounded <- round(V, 2)

# Étape 3 : Créer la heatmap
heatmap(as.matrix(V_rounded),
        main = "Heatmap de la matrice de variance-covariance",
        Colv = NA, Rowv = NA,  # Désactive le clustering
        scale = "none",  # Pas de normalisation supplémentaire
        col = heat.colors(10),  # Palette de couleurs
        margins = c(10, 10))  # Marges pour la lisibilité
```

atmap de la matrice de variance−covariance




```
# Question 4 améliorée
# Transformer la matrice de corrélation en format long
# Calculer la matrice de corrélation
correlation_matrix <- cor(euro_data, use = "complete.obs")

# Afficher un aperçu de la matrice
print("Matrice de corrélation (arrondie) :")
```

```
## [1] "Matrice de corrélation (arrondie) :"
```

```
round(correlation_matrix, 2)
```

```
##                                        Population Youth.population
## Population                                   1.00            -0.11
## Youth.population                            -0.11             1.00
## First.time.asylum.applicants                 0.90            -0.07
## Gender.pay.gap                              -0.06            -0.25
## Minimum.wage                                 0.25             0.63
## People.at.risk.of.poverty.or.exclusion       0.13            -0.43
## Early.school.leavers                         0.32             0.03
## Inflation.rate                              -0.11            -0.52
## Unemployment.rate                            0.14            -0.15
## Youth.unemployment.rate                      0.08            -0.15
## GDP.per.capita                               0.01             0.71
## Government.gross.debt                        0.41            -0.06
```

```
## Greenhouse.gas.emissions                    -0.13             0.30
## Renewable.energy                            -0.22            -0.18
## Electricity.prices                           0.40             0.23
## Energy.imports.dependency                    0.04             0.49
##                                     First.time.asylum.applicants
## Population                                                  0.90
## Youth.population                                           -0.07
## First.time.asylum.applicants                                1.00
## Gender.pay.gap                                              0.07
## Minimum.wage                                                0.34
## People.at.risk.of.poverty.or.exclusion                      0.16
## Early.school.leavers                                        0.36
## Inflation.rate                                             -0.25
## Unemployment.rate                                           0.19
## Youth.unemployment.rate                                     0.03
## GDP.per.capita                                              0.10
## Government.gross.debt                                       0.38
## Greenhouse.gas.emissions                                   -0.06
## Renewable.energy                                           -0.16
## Electricity.prices                                          0.48
## Energy.imports.dependency                                   0.16
##                                     Gender.pay.gap Minimum.wage
## Population                                  -0.06         0.25
## Youth.population                            -0.25         0.63
## First.time.asylum.applicants                 0.07         0.34
## Gender.pay.gap                               1.00        -0.30
## Minimum.wage                                -0.30         1.00
## People.at.risk.of.poverty.or.exclusion      -0.14        -0.35
## Early.school.leavers                        -0.01        -0.10
## Inflation.rate                               0.43        -0.63
## Unemployment.rate                           -0.05        -0.09
## Youth.unemployment.rate                     -0.29        -0.19
## GDP.per.capita                              -0.37         0.90
## Government.gross.debt                       -0.12         0.03
## Greenhouse.gas.emissions                    -0.24         0.51
## Renewable.energy                             0.43        -0.13
## Electricity.prices                          -0.04         0.43
## Energy.imports.dependency                   -0.40         0.37
##                                     People.at.risk.of.poverty.or.exclusion
## Population                                                           0.13
## Youth.population                                                    -0.43
## First.time.asylum.applicants                                         0.16
## Gender.pay.gap                                                      -0.14
## Minimum.wage                                                        -0.35
## People.at.risk.of.poverty.or.exclusion                               1.00
## Early.school.leavers                                                 0.43
## Inflation.rate                                                      -0.04
## Unemployment.rate                                                    0.43
## Youth.unemployment.rate                                              0.45
## GDP.per.capita                                                      -0.32
## Government.gross.debt                                                0.04
## Greenhouse.gas.emissions                                            -0.28
## Renewable.energy                                                     0.09
## Electricity.prices                                                  -0.18
```

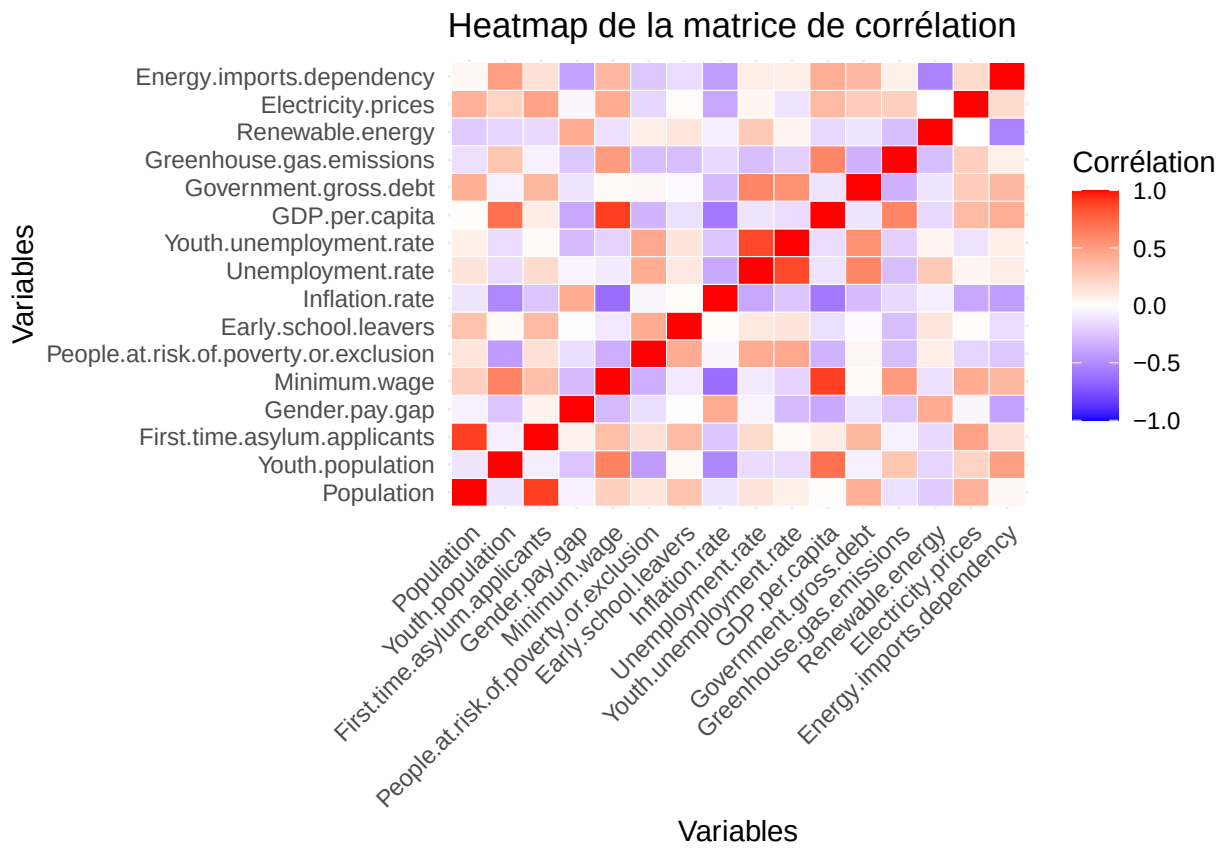
```
## Energy.imports.dependency                                    -0.24
##                                     Early.school.leavers Inflation.rate
## Population                                          0.32          -0.11
## Youth.population                                    0.03          -0.52
## First.time.asylum.applicants                        0.36          -0.25
## Gender.pay.gap                                     -0.01           0.43
## Minimum.wage                                       -0.10          -0.63
## People.at.risk.of.poverty.or.exclusion              0.43          -0.04
## Early.school.leavers                                1.00           0.01
## Inflation.rate                                      0.01           1.00
## Unemployment.rate                                   0.12          -0.38
## Youth.unemployment.rate                             0.14          -0.25
## GDP.per.capita                                     -0.13          -0.58
## Government.gross.debt                              -0.02          -0.29
## Greenhouse.gas.emissions                           -0.28          -0.16
## Renewable.energy                                    0.13          -0.07
## Electricity.prices                                  0.02          -0.38
## Energy.imports.dependency                          -0.15          -0.42
##                                     Unemployment.rate
## Population                                       0.14
## Youth.population                                -0.15
## First.time.asylum.applicants                     0.19
## Gender.pay.gap                                  -0.05
## Minimum.wage                                    -0.09
## People.at.risk.of.poverty.or.exclusion           0.43
## Early.school.leavers                             0.12
## Inflation.rate                                  -0.38
## Unemployment.rate                                1.00
## Youth.unemployment.rate                          0.87
## GDP.per.capita                                  -0.12
## Government.gross.debt                            0.62
## Greenhouse.gas.emissions                        -0.28
## Renewable.energy                                 0.28
## Electricity.prices                               0.05
## Energy.imports.dependency                        0.09
##                                     Youth.unemployment.rate GDP.per.capita
## Population                                             0.08           0.01
## Youth.population                                      -0.15           0.71
## First.time.asylum.applicants                           0.03           0.10
## Gender.pay.gap                                        -0.29          -0.37
## Minimum.wage                                          -0.19           0.90
## People.at.risk.of.poverty.or.exclusion                 0.45          -0.32
## Early.school.leavers                                   0.14          -0.13
## Inflation.rate                                        -0.25          -0.58
## Unemployment.rate                                      0.87          -0.12
## Youth.unemployment.rate                                1.00          -0.15
## GDP.per.capita                                        -0.15           1.00
## Government.gross.debt                                  0.57          -0.12
## Greenhouse.gas.emissions                              -0.20           0.61
## Renewable.energy                                       0.06          -0.17
## Electricity.prices                                    -0.12           0.36
## Energy.imports.dependency                              0.09           0.42
##                                     Government.gross.debt
## Population                                           0.41
```

```
## Youth.population                                    -0.06
## First.time.asylum.applicants                         0.38
## Gender.pay.gap                                      -0.12
## Minimum.wage                                         0.03
## People.at.risk.of.poverty.or.exclusion               0.04
## Early.school.leavers                                -0.02
## Inflation.rate                                      -0.29
## Unemployment.rate                                    0.62
## Youth.unemployment.rate                              0.57
## GDP.per.capita                                      -0.12
## Government.gross.debt                                1.00
## Greenhouse.gas.emissions                            -0.34
## Renewable.energy                                    -0.11
## Electricity.prices                                   0.27
## Energy.imports.dependency                            0.37
##                                      Greenhouse.gas.emissions
## Population                                            -0.13
## Youth.population                                       0.30
## First.time.asylum.applicants                          -0.06
## Gender.pay.gap                                        -0.24
## Minimum.wage                                           0.51
## People.at.risk.of.poverty.or.exclusion                -0.28
## Early.school.leavers                                  -0.28
## Inflation.rate                                        -0.16
## Unemployment.rate                                     -0.28
## Youth.unemployment.rate                               -0.20
## GDP.per.capita                                         0.61
## Government.gross.debt                                 -0.34
## Greenhouse.gas.emissions                               1.00
## Renewable.energy                                      -0.28
## Electricity.prices                                     0.25
## Energy.imports.dependency                              0.08
##                                      Renewable.energy Electricity.prices
## Population                                      -0.22                0.40
## Youth.population                                -0.18                0.23
## First.time.asylum.applicants                    -0.16                0.48
## Gender.pay.gap                                   0.43               -0.04
## Minimum.wage                                    -0.13                0.43
## People.at.risk.of.poverty.or.exclusion           0.09               -0.18
## Early.school.leavers                             0.13                0.02
## Inflation.rate                                  -0.07               -0.38
## Unemployment.rate                                0.28                0.05
## Youth.unemployment.rate                          0.06               -0.12
## GDP.per.capita                                  -0.17                0.36
## Government.gross.debt                           -0.11                0.27
## Greenhouse.gas.emissions                        -0.28                0.25
## Renewable.energy                                 1.00                0.00
## Electricity.prices                               0.00                1.00
## Energy.imports.dependency                       -0.54                0.19
##                                      Energy.imports.dependency
## Population                                            0.04
## Youth.population                                      0.49
## First.time.asylum.applicants                          0.16
## Gender.pay.gap                                       -0.40
```

```
## Minimum.wage                                          0.37
## People.at.risk.of.poverty.or.exclusion               -0.24
## Early.school.leavers                                 -0.15
## Inflation.rate                                       -0.42
## Unemployment.rate                                     0.09
## Youth.unemployment.rate                               0.09
## GDP.per.capita                                        0.42
## Government.gross.debt                                 0.37
## Greenhouse.gas.emissions                              0.08
## Renewable.energy                                     -0.54
## Electricity.prices                                    0.19
## Energy.imports.dependency                             1.00
```

```r
# Transformer la matrice de corrélation en format long
correlation_long <- reshape2::melt(round(correlation_matrix, 2))
```

```r
# Heatmap de la matrice de corrélation
ggplot(correlation_long, aes(Var1, Var2, fill = value)) +
  geom_tile(color = "white") +
  scale_fill_gradient2(low = "blue", high = "red", mid = "white",
                       midpoint = 0, limit = c(-1, 1),
                       name = "Corrélation") +
  theme_minimal() +
  labs(title = "Heatmap de la matrice de corrélation",
       x = "Variables",
       y = "Variables") +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
```




```
head(euro_data)
```

```
##   Population Youth.population First.time.asylum.applicants Gender.pay.gap
## 1    9104772             16.9                         56135           18.4
## 2   11742796             17.8                         29260            5.0
## 3    6447710             13.2                         22390           13.0
## 4    3850894             15.9                          1635           12.5
## 5     920701             19.8                         11660           10.2
## 6   10827529             15.1                          1130           17.9
##   Minimum.wage People.at.risk.of.poverty.or.exclusion Early.school.leavers
## 1         1766                                  17.5                  8.6
## 2         1994                                  18.7                  6.2
## 3          477                                  32.2                  9.3
## 4          840                                  19.9                  2.0
## 5         1000                                  16.7                 10.5
## 6          764                                  11.8                  6.4
##   Inflation.rate Unemployment.rate Youth.unemployment.rate GDP.per.capita
## 1            7.7               5.1                    10.4          37460
## 2            2.3               5.5                    16.1          37300
## 3            8.6               4.3                    12.1           7850
## 4            8.4               6.1                    19.0          14750
## 5            3.9               6.1                    16.9          27720
## 6           14.8               2.6                     8.3          18480
##   Government.gross.debt Greenhouse.gas.emissions Renewable.energy
## 1                  77.8                      8.3             33.8
```

```
## 2                 105.2                      9.3             13.8
## 3                  23.1                      9.1             19.1
## 4                  63.0                      6.8             27.9
## 5                  77.3                     10.5             19.4
## 6                  44.0                     11.1             18.2
##   Electricity.prices Energy.imports.dependency
## 1              288.5                     74.5
## 2              377.2                     74.0
## 3              119.4                     37.1
## 4              154.3                     60.3
## 5              351.9                     92.0
## 6              303.9                     41.8
```

```r
# Question 5
# Remplacer les NA par la moyenne
euro_data_replace_na <- apply(euro_data, 2, function(x) ifelse(is.na(x), mean(x, na.rm = TRUE), x))

# Calculer les composantes principales
res <- prcomp(euro_data_replace_na, scale = TRUE, center = TRUE)

# Vérification de l'orthogonalité et des normes
orthogonality_check <- t(res$rotation) %*% res$rotation
cat("Produit scalaire des vecteurs propres (orthogonalité) :\n")
```

```
## Produit scalaire des vecteurs propres (orthogonalité) :
```

```r
print(round(orthogonality_check, 2))
```

```
##      PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10 PC11 PC12 PC13 PC14 PC15 PC16
## PC1    1   0   0   0   0   0   0   0   0    0    0    0    0    0    0    0
## PC2    0   1   0   0   0   0   0   0   0    0    0    0    0    0    0    0
## PC3    0   0   1   0   0   0   0   0   0    0    0    0    0    0    0    0
## PC4    0   0   0   1   0   0   0   0   0    0    0    0    0    0    0    0
## PC5    0   0   0   0   1   0   0   0   0    0    0    0    0    0    0    0
## PC6    0   0   0   0   0   1   0   0   0    0    0    0    0    0    0    0
## PC7    0   0   0   0   0   0   1   0   0    0    0    0    0    0    0    0
## PC8    0   0   0   0   0   0   0   1   0    0    0    0    0    0    0    0
## PC9    0   0   0   0   0   0   0   0   1    0    0    0    0    0    0    0
## PC10   0   0   0   0   0   0   0   0   0    1    0    0    0    0    0    0
## PC11   0   0   0   0   0   0   0   0   0    0    1    0    0    0    0    0
## PC12   0   0   0   0   0   0   0   0   0    0    0    1    0    0    0    0
## PC13   0   0   0   0   0   0   0   0   0    0    0    0    1    0    0    0
## PC14   0   0   0   0   0   0   0   0   0    0    0    0    0    1    0    0
## PC15   0   0   0   0   0   0   0   0   0    0    0    0    0    0    1    0
## PC16   0   0   0   0   0   0   0   0   0    0    0    0    0    0    0    1
```

```r
norms <- apply(res$rotation, 2, function(col) sqrt(sum(col^2)))
cat("Normes des vecteurs propres (1 attendu) :\n")
```

```
## Normes des vecteurs propres (1 attendu) :
```

```r
print(round(norms, 2))
```

```
##  PC1  PC2  PC3  PC4  PC5  PC6  PC7  PC8  PC9 PC10 PC11 PC12 PC13 PC14 PC15 PC16
##    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1    1
```

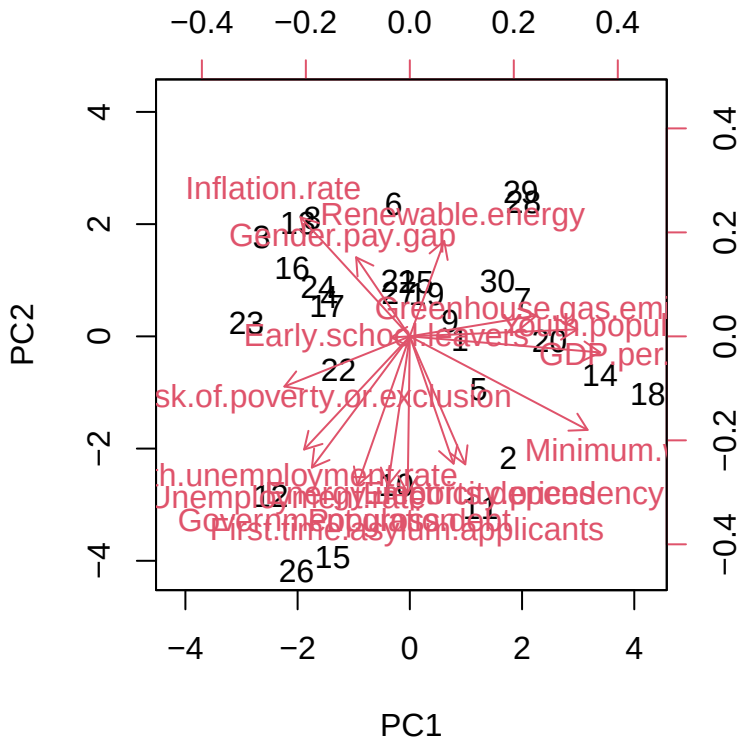
```r
# Afficher les premières coordonnées dans la nouvelle base
cat("Premières coordonnées des observations :\n")
```

```
## Premières coordonnées des observations :
```

```r
head(res$x)
```

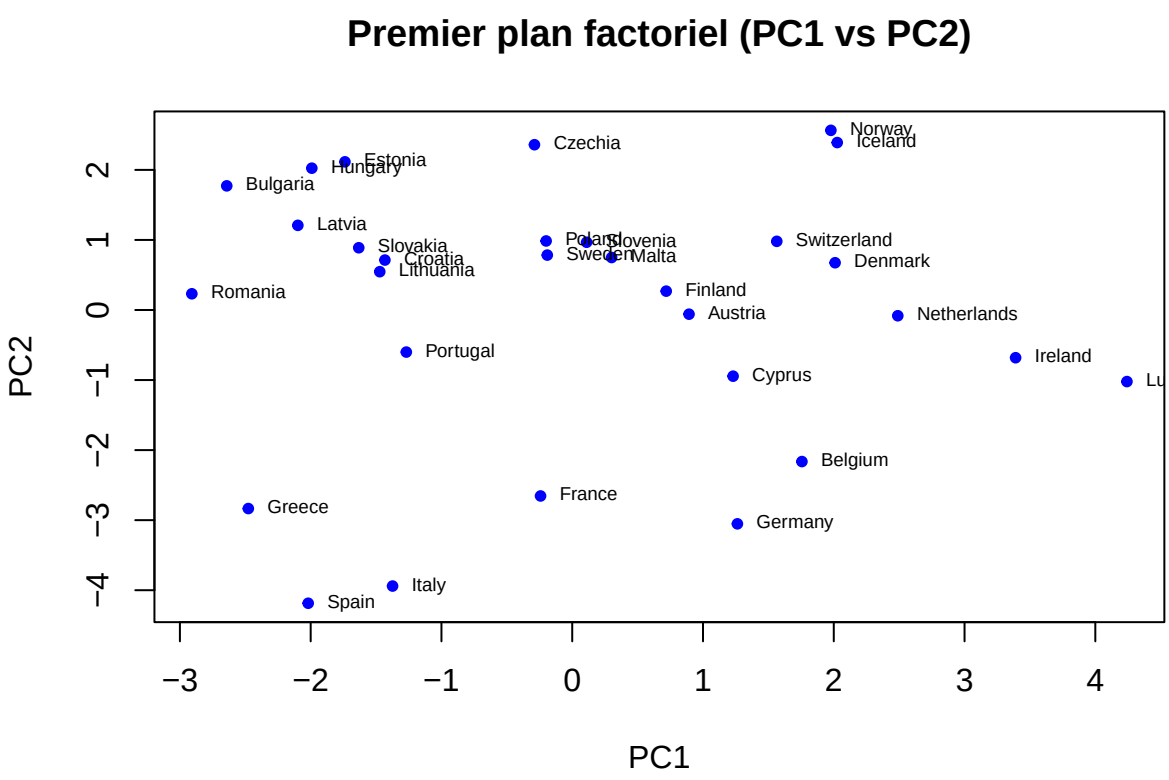
```
##             PC1         PC2         PC3         PC4        PC5        PC6
## [1,]  0.8930784 -0.05981786 -0.97098080 -0.32722747  0.9238143 -0.4927803
## [2,]  1.7565034 -2.16393542 -0.07834658  1.31816585 -0.2325251 -0.1199267
## [3,] -2.6417622  1.77214093 -0.25429763 -0.23821400 -1.6690826  1.9269231
## [4,] -1.4323209  0.71295161 -0.13114739  1.72626176  0.5961859 -0.0925010
## [5,]  1.2302770 -0.94486199  0.27801253  1.08399558 -0.3331520 -1.0172092
## [6,] -0.2887514  2.35926409 -2.92285398  0.04385271 -0.1945308 -0.9634055
##             PC7          PC8        PC9       PC10       PC11       PC12
## [1,]  0.1009824  0.651241168  0.6247011 -0.2092549  0.2453571  0.5611628
## [2,] -0.3036975  0.514094776 -1.1343872 -0.1550909 -0.6500718  0.4742277
## [3,] -0.1860566  0.006445367  0.4886804 -1.1327358 -0.1528996 -0.3188954
## [4,] -0.1856331 -0.928931619  0.1125664 -0.3373321 -0.2422261 -0.2552262
## [5,]  0.4715290  1.995736250 -0.3374791  0.6185508 -0.1764931 -0.9572414
## [6,] -1.3480963  0.346571689 -0.3554556  0.7335514  0.3869734 -0.1430733
##            PC13        PC14        PC15        PC16
## [1,]  0.4778485  0.29771859 -0.17539637 -0.14631681
## [2,]  0.3592356 -0.10884273 -0.25695322  0.09789929
## [3,]  0.1384509 -0.30770441  0.12484341  0.06258464
## [4,] -0.5752980  0.25874542 -0.35628808  0.18537500
## [5,] -0.4161035 -0.02523226  0.02481071  0.02486392
## [6,]  0.1021369 -0.16528975  0.15868235  0.08522987
```

```r
# Biplot pour visualiser les résultats
biplot(res, scale = 0)
```




```r
# Question 6
# Étape 1 : Calculer les composantes principales (si non déjà fait)
euro_data_replace_na <- apply(euro_data, 2, function(x) ifelse(is.na(x), mean(x, na.rm = TRUE), x))
res <- prcomp(euro_data_replace_na, scale = TRUE, center = TRUE)

# Étape 2 : Récupérer les coordonnées dans la nouvelle base
```

```r
coord <- res$x

# Étape 3 : Ajouter les noms des pays comme noms de lignes
tmp <- read.csv("data/euro.csv", header = TRUE, sep = ";")
rownames(coord) <- tmp[, 1]

# Étape 4 : Créer le graphique du premier plan factoriel
plot(coord[, 1], coord[, 2],
     xlab = "PC1", ylab = "PC2",
     main = "Premier plan factoriel (PC1 vs PC2)",
     pch = 20, col = "blue")

# Étape 5 : Ajouter les noms des pays sur le graphique
text(coord[, 1], coord[, 2], labels = rownames(coord), pos = 4, cex = 0.6)
```




```
# Question 7
# Extraire les variances (valeurs propres)
variances <- res$sdev^2

# Calculer le pourcentage de variance expliquée
pourcentages <- variances / sum(variances) * 100

# Calculer la variance expliquée cumulée
cumul <- cumsum(pourcentages)

# Afficher les informations pour contrôle
cat("Valeurs propres :\n")
```

```
## Valeurs propres :
```

```
print(round(variances, 2))
```

```
##  [1] 3.64 3.52 2.03 1.86 1.28 0.87 0.75 0.56 0.45 0.35 0.21 0.17 0.16 0.09 0.04
## [16] 0.03
```

```
cat("\nPourcentages de variances expliquées :\n")
```

```
##
## Pourcentages de variances expliquées :
```

```r
print(round(pourcentages, 2))
```

```
##  [1] 22.72 22.02 12.72 11.60  8.02  5.43  4.70  3.50  2.80  2.16  1.28  1.05
## [13]  0.99  0.56  0.26  0.19
```

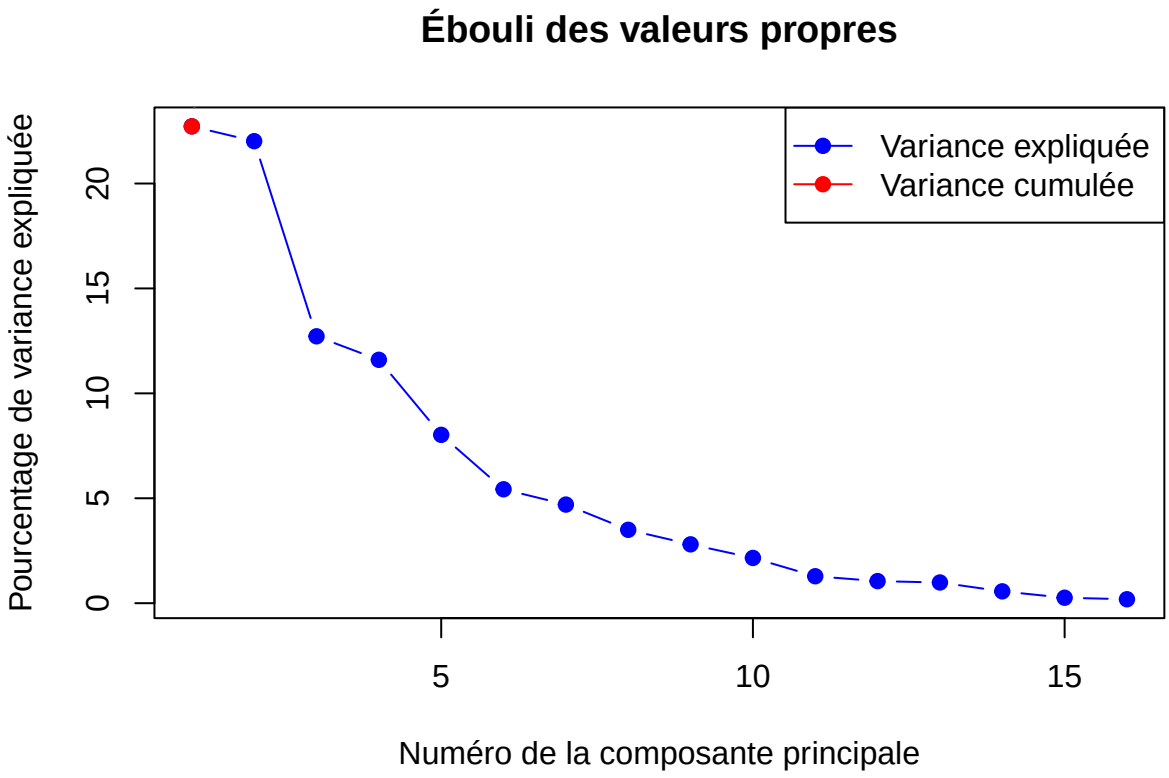
```r
cat("\nVariance expliquée cumulée :\n")
```

```
##
## Variance expliquée cumulée :
```

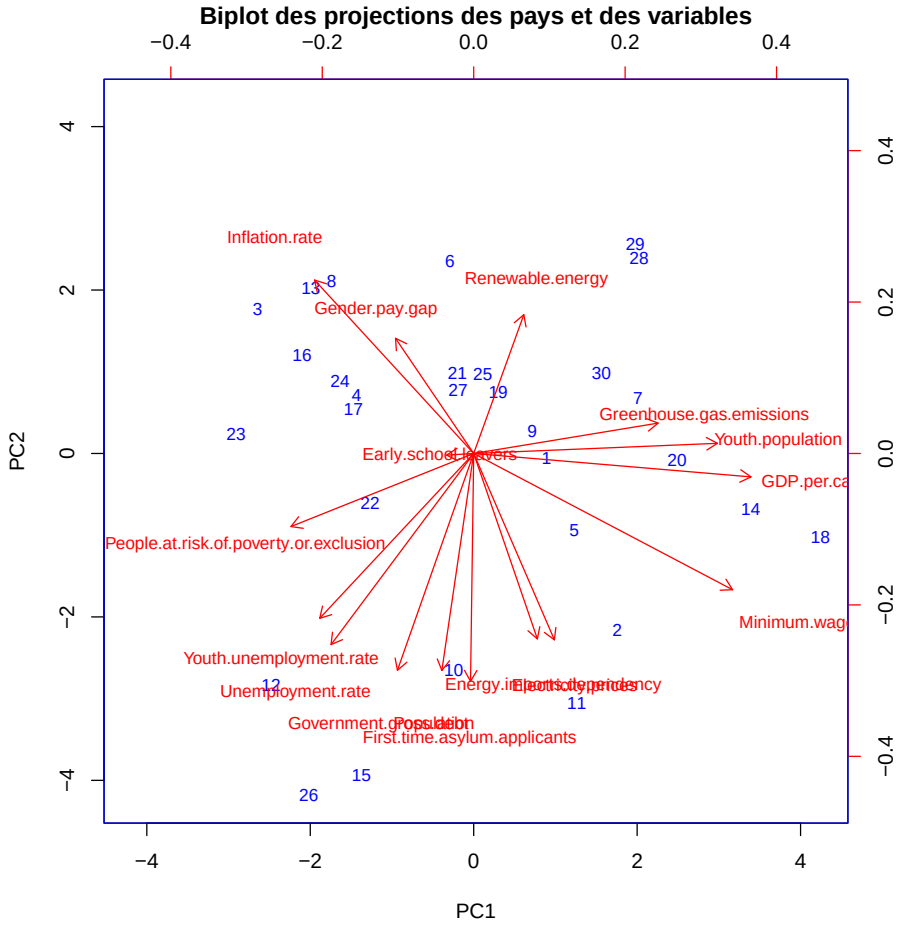
```r
print(round(cumul, 2))
```

```
##  [1]  22.72  44.74  57.46  69.06  77.08  82.51  87.21  90.70  93.51  95.66
## [11]  96.95  98.00  98.99  99.55  99.81 100.00
```

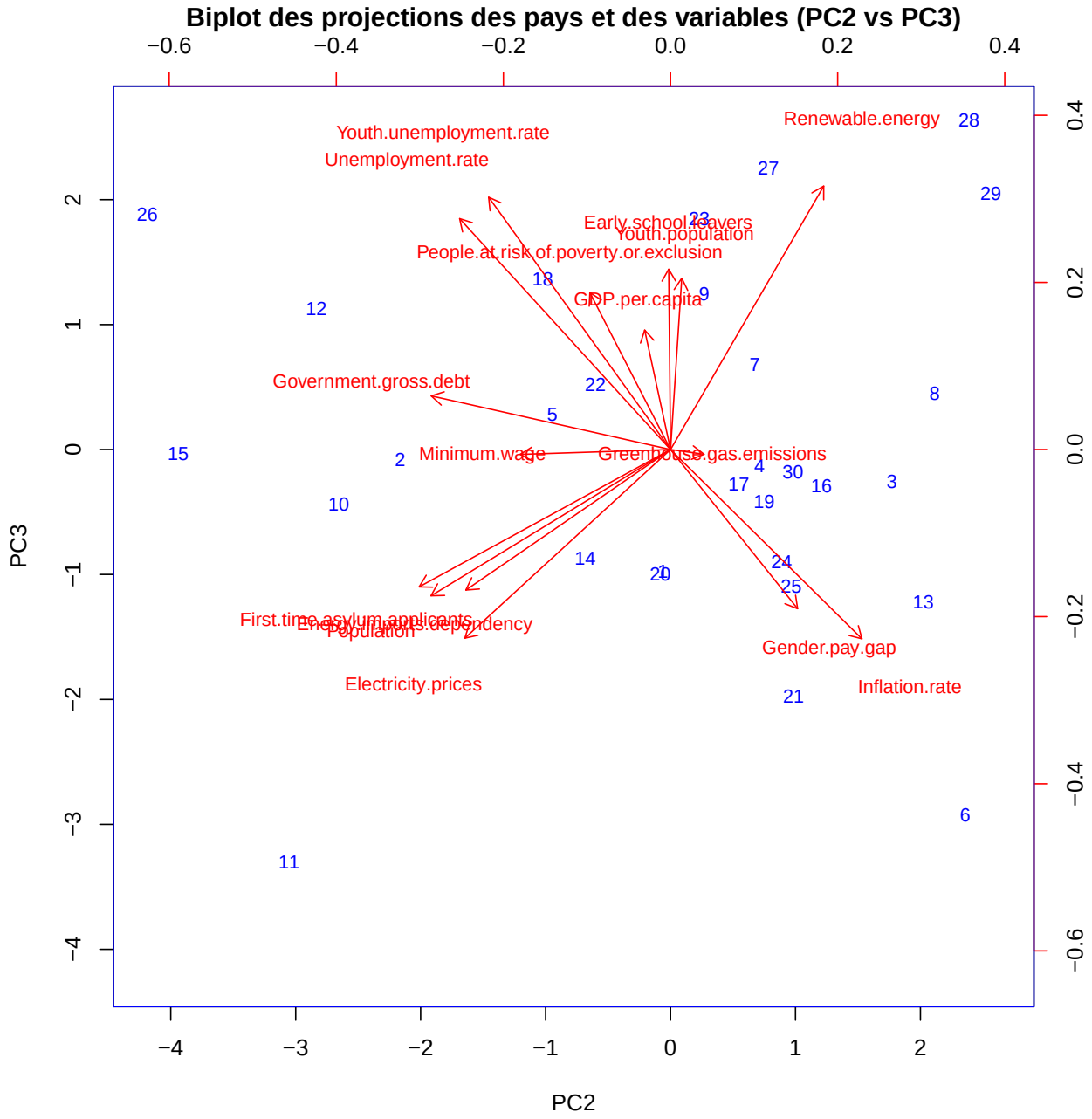
```r
# Question 7
# Créer un graphique combiné
plot(pourcentages, type = "b", pch = 19, col = "blue",
     xlab = "Numéro de la composante principale",
     ylab = "Pourcentage de variance expliquée",
     main = "Ébouli des valeurs propres")
lines(cumul, type = "b", pch = 19, col = "red")  # Ajouter la variance cumulée

# Ajouter une légende
legend("topright", legend = c("Variance expliquée", "Variance cumulée"),
       col = c("blue", "red"), pch = 19, lty = 1)
```




```r
# Question 8
# Créer le biplot avec des couleurs et des tailles ajustées
biplot(res, scale = 0,
       main = "Biplot des projections des pays et des variables",
       cex = 0.8,  # Taille des points et flèches
       col = c("blue", "red"))  # Points des pays en bleu, flèches des variables en rouge
```

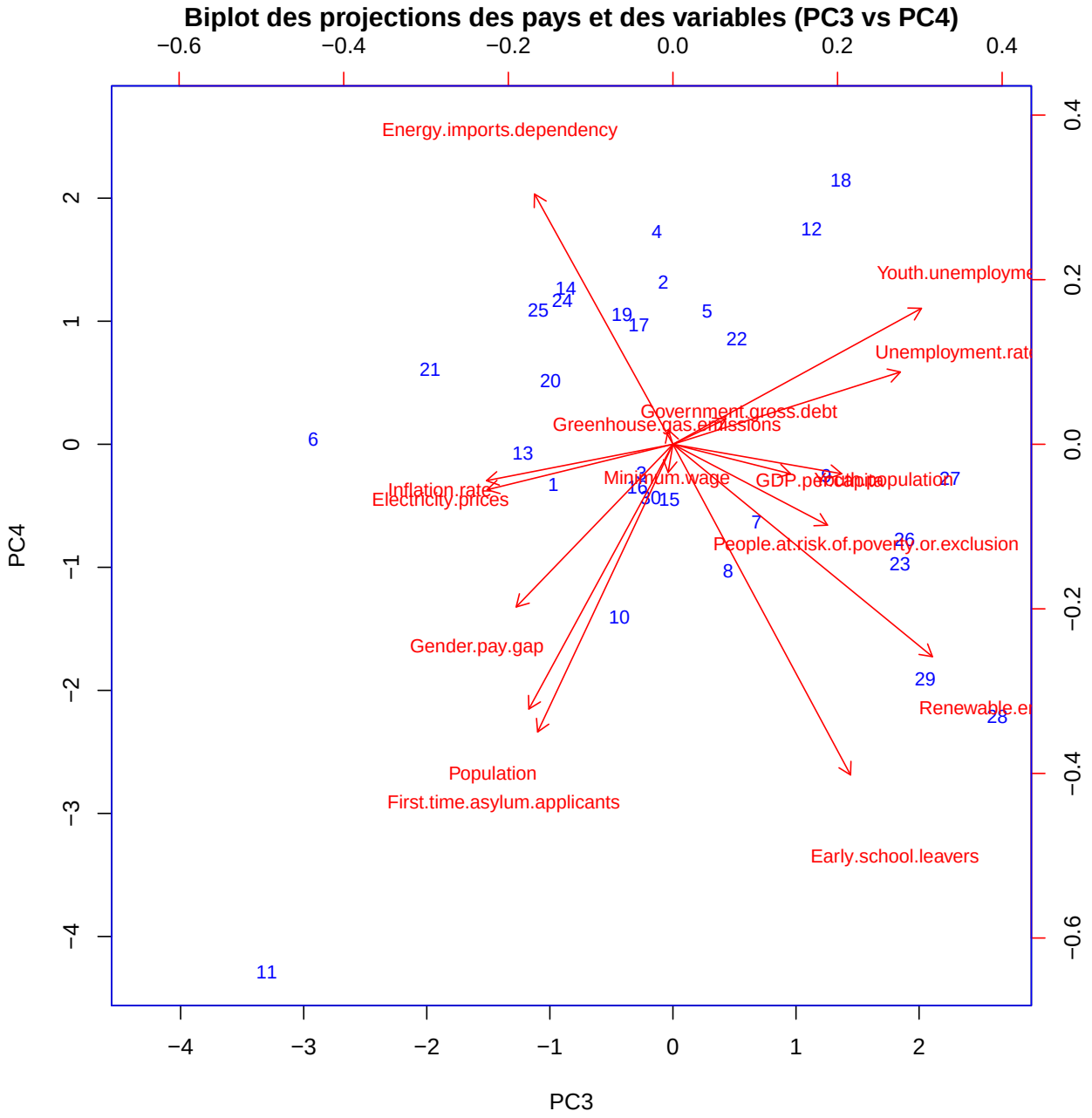



```r
# Question 9 Observez d'autres plans factoriels, et commentez.
# Plan PC2 vs PC3
biplot(res, choices = c(2, 3), scale = 0,
       main = "Biplot des projections des pays et des variables (PC2 vs PC3)",
       cex = 0.8,  # Taille des points et flèches
       col = c("blue", "red"))  # Points des pays en bleu, flèches des variables en rouge
```




```r
# Plan PC3 vs PC4
biplot(res, choices = c(3, 4), scale = 0,
       main = "Biplot des projections des pays et des variables (PC3 vs PC4)",
       cex = 0.8,  # Taille des points et flèches
```

```
       col = c("blue", "red"))  # Points des pays en bleu, flèches des variables en rouge
```



```r
# Question 10. Déterminez quelles sont les variables les mieux représentées par le premier plan factori
# Extraire les charges des vecteurs propres (coefficients de res$rotation)
loadings <- res$rotation

# Calculer le cos² pour le plan PC1-PC2
cos2_PC1_PC2 <- rowSums(loadings[, 1:2]^2)  # Somme des carrés des charges sur PC1 et PC2

# Afficher les cos²
cat("Cos² des variables sur le plan PC1-PC2 :\n")
```

```
## Cos² des variables sur le plan PC1-PC2 :
```

```r
print(round(cos2_PC1_PC2, 2))
```

```
##                              Population                      Youth.population
##                                    0.13                                  0.16
##           First.time.asylum.applicants                        Gender.pay.gap
##                                    0.14                                  0.05
##                           Minimum.wage People.at.risk.of.poverty.or.exclusion
##                                    0.23                                  0.11
##                   Early.school.leavers                       Inflation.rate
##                                    0.00                                  0.15
##                      Unemployment.rate               Youth.unemployment.rate
##                                    0.16                                  0.14
##                         GDP.per.capita                 Government.gross.debt
##                                    0.21                                  0.14
##               Greenhouse.gas.emissions                       Renewable.energy
##                                    0.10                                  0.06
##                     Electricity.prices             Energy.imports.dependency
##                                    0.11                                  0.10
```

```r
# Identifier les variables les mieux représentées
best_variables <- names(sort(cos2_PC1_PC2, decreasing = TRUE))
cat("\nVariables les mieux représentées par PC1-PC2 (par ordre de qualité) :\n")
```

```
##
## Variables les mieux représentées par PC1-PC2 (par ordre de qualité) :
```

```r
print(best_variables)
```

```
##  [1] "Minimum.wage"
##  [2] "GDP.per.capita"
##  [3] "Youth.population"
##  [4] "Unemployment.rate"
##  [5] "Inflation.rate"
##  [6] "Government.gross.debt"
##  [7] "First.time.asylum.applicants"
##  [8] "Youth.unemployment.rate"
##  [9] "Population"
## [10] "Electricity.prices"
## [11] "People.at.risk.of.poverty.or.exclusion"
```

```
## [12] "Energy.imports.dependency"
## [13] "Greenhouse.gas.emissions"
## [14] "Renewable.energy"
## [15] "Gender.pay.gap"
## [16] "Early.school.leavers"
```

```r
# Question 11
# Étape 1 : Extraire les scores des individus
scores <- res$x  # Les coordonnées des individus dans le nouvel espace

# Étape 2 : Extraire les variances des composantes principales
variances <- res$sdev^2  # Valeurs propres

# Étape 3 : Calculer les contributions des individus sur chaque composante
# Contribution = (scores^2) / (variance de la composante)
contributions <- sweep(scores^2, 2, variances, "/")

# Étape 4 : Résumé des contributions
cat("Contributions des individus sur chaque composante principale :\n")
```

```
## Contributions des individus sur chaque composante principale :
```

```r
print(round(contributions, 2))  # Contributions arrondies
```

```
##         PC1  PC2  PC3  PC4  PC5  PC6   PC7  PC8  PC9 PC10 PC11 PC12 PC13 PC14
##  [1,] 0.22 0.00 0.46 0.06 0.66 0.28  0.01 0.76 0.87 0.13 0.29 1.87 1.45 0.98
##  [2,] 0.85 1.33 0.00 0.94 0.04 0.02  0.12 0.47 2.87 0.07 2.06 1.34 0.82 0.13
##  [3,] 1.92 0.89 0.03 0.03 2.17 4.27  0.05 0.00 0.53 3.72 0.11 0.60 0.12 1.05
##  [4,] 0.56 0.14 0.01 1.61 0.28 0.01  0.05 1.54 0.03 0.33 0.29 0.39 2.10 0.74
##  [5,] 0.42 0.25 0.04 0.63 0.09 1.19  0.30 7.12 0.25 1.11 0.15 5.45 1.10 0.01
##  [6,] 0.02 1.58 4.20 0.00 0.03 1.07  2.42 0.21 0.28 1.56 0.73 0.12 0.07 0.30
##  [7,] 1.11 0.13 0.23 0.21 0.66 0.22  0.26 0.70 0.28 1.96 1.71 0.03 0.01 0.72
##  [8,] 0.83 1.27 0.10 0.57 0.04 0.93  4.24 1.01 1.45 0.49 1.85 0.45 0.01 0.90
##  [9,] 0.14 0.02 0.76 0.04 1.05 0.38  0.33 0.04 0.32 0.00 1.20 0.02 5.32 0.07
## [10,] 0.02 2.00 0.09 1.06 0.29 0.54  0.00 3.02 0.24 0.01 3.33 1.48 2.41 0.43
## [11,] 0.44 2.65 5.35 9.89 0.06 0.43  0.01 0.01 0.33 0.12 0.06 0.95 0.03 0.47
## [12,] 1.69 2.28 0.62 1.65 1.23 0.62  1.67 0.51 1.02 4.76 0.18 0.16 0.71 0.71
## [13,] 1.09 1.16 0.73 0.00 0.60 3.79  1.43 0.07 2.56 0.79 1.29 3.53 0.00 0.01
## [14,] 3.16 0.13 0.37 0.87 0.08 1.16  0.85 0.67 0.04 0.17 0.27 4.07 1.54 0.23
## [15,] 0.52 4.41 0.00 0.11 0.45 0.15  0.01 0.06 3.52 0.00 3.39 0.12 0.01 0.63
## [16,] 1.21 0.41 0.04 0.07 0.78 1.52  0.09 1.56 0.83 0.00 0.01 1.46 0.00 1.69
## [17,] 0.60 0.08 0.04 0.51 0.00 1.08  0.10 0.15 0.00 0.02 0.23 0.01 0.02 7.92
## [18,] 4.95 0.30 0.92 2.48 3.90 1.30  0.75 1.28 2.35 0.30 1.92 0.15 0.35 0.13
## [19,] 0.02 0.16 0.08 0.60 0.35 0.03 11.17 0.11 0.11 0.57 0.29 0.45 0.11 0.18
## [20,] 1.70 0.00 0.49 0.14 0.00 0.00  0.81 0.23 1.43 0.36 3.32 0.01 0.01 0.02
## [21,] 0.01 0.28 1.91 0.20 0.99 0.10  1.09 2.97 2.49 0.00 0.31 1.05 1.60 0.06
## [22,] 0.44 0.10 0.13 0.39 0.49 0.26  0.25 0.03 0.50 0.19 0.13 0.10 0.64 0.76
## [23,] 2.33 0.02 1.67 0.51 5.34 1.24  1.12 0.07 0.85 1.12 0.21 2.12 0.00 0.42
## [24,] 0.73 0.22 0.40 0.74 0.26 0.57  0.01 0.03 1.18 1.74 0.27 0.52 0.01 0.47
## [25,] 0.00 0.27 0.59 0.64 0.00 0.11  0.01 1.65 2.51 0.76 0.07 0.25 7.63 0.03
## [26,] 1.12 4.97 1.74 0.32 0.01 0.01  0.01 0.04 1.04 1.37 0.12 2.03 0.94 0.65
## [27,] 0.01 0.17 2.49 0.04 2.13 0.21  0.01 3.00 0.49 3.21 0.01 0.00 0.66 1.73
## [28,] 1.13 1.62 3.41 2.64 1.77 4.17  0.50 0.66 0.34 3.40 0.01 0.01 0.81 2.98
```

```
## [29,] 1.08 1.87 2.07 1.96 0.30 0.23  0.00 1.02 0.00 0.15 1.78 0.03 0.00 4.37
## [30,] 0.67 0.27 0.02 0.10 4.97 3.10  1.32 0.01 0.30 0.60 3.43 0.23 0.53 0.24
##       PC15 PC16
##  [1,] 0.74 0.71
##  [2,] 1.59 0.32
##  [3,] 0.37 0.13
##  [4,] 3.05 1.14
##  [5,] 0.01 0.02
##  [6,] 0.61 0.24
##  [7,] 1.16 0.32
##  [8,] 0.25 0.18
##  [9,] 0.22 2.69
## [10,] 1.72 0.18
## [11,] 3.44 0.80
## [12,] 0.01 1.17
## [13,] 0.10 0.09
## [14,] 0.30 1.16
## [15,] 0.24 0.81
## [16,] 0.68 0.07
## [17,] 1.73 0.26
## [18,] 0.18 0.25
## [19,] 0.16 0.49
## [20,] 0.10 1.09
## [21,] 0.81 3.21
## [22,] 1.95 4.43
## [23,] 1.82 0.17
## [24,] 2.12 0.14
## [25,] 0.12 4.48
## [26,] 3.52 0.52
## [27,] 1.56 1.31
## [28,] 0.06 0.02
## [29,] 0.36 0.89
## [30,] 0.02 1.67
```

```
# Étape 5 : Calcul des contributions totales par individu (somme sur toutes les composantes)
contributions_totales <- rowSums(contributions)
cat("\nContributions totales par individu :\n")
```

```
##
## Contributions totales par individu :
```

```
print(round(contributions_totales, 2))
```

```
##  [1]  9.50 12.96 16.01 12.26 18.13 13.44  9.72 14.57 12.61 16.81 25.03 18.96
## [13] 17.24 15.08 14.42 10.42 12.74 21.50 14.88  9.73 17.08 10.81 18.99  9.42
## [25] 19.12 18.42 17.04 23.55 16.11 17.47
```

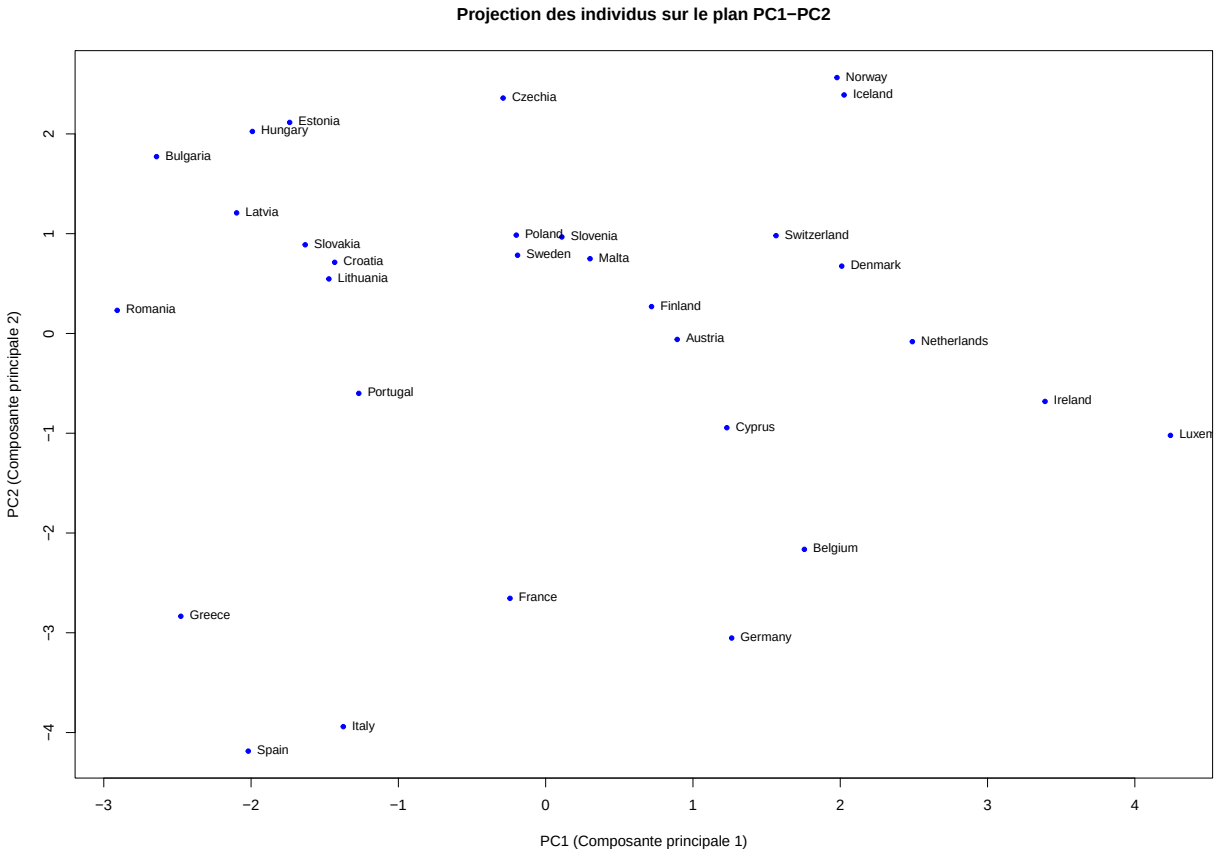
```
# Étape 6 : Identifier les individus avec des contributions faibles
seuil <- 1 / nrow(scores)  # Seuil théorique de contribution moyenne
individus_faibles <- names(contributions_totales[contributions_totales < seuil])
cat("\nIndividus ayant une contribution faible (en dessous du seuil) :\n")
```

```
##
## Individus ayant une contribution faible (en dessous du seuil) :
```

```
print(individus_faibles)
```

```
## NULL
```

```
# question 12
# Étape 1 : Extraire les scores
coord <- res$x  # Scores des individus

# Étape 2 : Ajouter les noms des pays
tmp <- read.csv("data/euro.csv", header = TRUE, sep = ";")  # Charger les noms
rownames(coord) <- tmp[, 1]

# Étape 3 : Créer le graphique
plot(coord[, 1], coord[, 2],
     xlab = "PC1 (Composante principale 1)",
     ylab = "PC2 (Composante principale 2)",
     main = "Projection des individus sur le plan PC1-PC2",
     pch = 20, col = "blue")

# Ajouter les noms des individus
text(coord[, 1], coord[, 2], labels = rownames(coord), pos = 4, cex = 0.8)
```



```r
# TP2 ****************
```

```r
# Question 1
#  Après avoir déterminé les deux matrices de dissimilarités en utilisant respectivement une métrique E
# et une métrique réduite, effectuez une classification ascendante hiérarchique des données fondée pour
# matrice sur le saut minimum.
# Matrice de dissimilarité Euclidienne (non normalisée)
dissimilarity_euclidean <- as.matrix(dist(euro_data, method = "euclidean"))

# Normaliser les données
euro_data_normalized <- scale(euro_data)

# Matrice de dissimilarité Euclidienne (normalisée)
dissimilarity_reduced <- as.matrix(dist(euro_data_normalized, method = "euclidean"))

# Afficher les matrices
cat("Matrice de dissimilarité Euclidienne (non normalisée) :\n")
```

```
## Matrice de dissimilarité Euclidienne (non normalisée) :
```

```r
print(round(dissimilarity_euclidean, 2))
```

```
##             1          2          3          4          5          6          7
## 1         0.0  2638160.9  2657441.6  5254209.8  8184197.7  1723739.7  3172609.6
## 2   2638160.9        0.0  5295172.6  7891982.7 10822113.6   915893.4  5810224.2
## 3   2657441.6  5295172.6        0.0  2596908.1  5527055.2  4379883.5   517377.9
## 4   5254209.8  7891982.7  2596908.1        0.0  2930238.9  6976636.0  2082102.7
## 5   8184197.7 10822113.6  5527055.2  2930238.9        0.0  9906837.9  5012023.0
## 6   1723739.7   915893.4  4379883.5  6976636.0  9906837.9        0.0  4894993.5
## 7   3172609.6  5810224.2   517377.9  2082102.7  5012023.0  4894993.5        0.0
## 8   7739095.7 10376966.3  5081864.8  2485011.2   445423.9  9461646.0  4566922.4
## 9   3541179.2  6178875.8   884403.3  1713222.9  4643283.9  5263592.7   369016.5
## 10 59068272.1 56430300.0 61725394.4 64322245.8 67252408.7 57345630.8 62240489.5
## 11 75254567.8 72616667.8 77911743.7 80508619.6 83438748.0 73532049.3 78426873.1
## 12  1309339.5  1329246.9  3966447.0  6563330.6  9493397.5   417425.3  4481796.4
## 13   498677.6  2143374.4  3152120.1  5748850.2  8679061.1  1227792.4  3667289.2
## 14  3833770.1  6471512.3  1178083.5  1421690.0  4350916.7  5556402.2   661626.8
## 15 49892485.3 47254514.4 52549606.4 55146459.4 58076621.7 48169847.0 53064707.3
## 16  7222010.5  9859856.2  4564752.4  1967886.6   962468.6  8944522.6  4049836.8
## 17  6247781.6  8885592.0  3590504.5   993615.7  1936652.9  7970250.9  3075606.3
## 18  8444257.2 11082114.6  5787427.3  3190822.5   265931.4 10166927.0  5271935.1
## 19  8562908.8 11200787.2  5905729.2  3308864.2   378816.7 10285481.2  5390666.0
## 20  8706540.1  6068506.0 11363651.2 13960476.9 16890619.4  6983909.5 11878694.2
## 21 27649015.7 25010959.5 30306030.3 32902842.6 35833037.6 25926208.1 30821105.6
## 22  1412975.2  1226591.2  4068976.5  6665728.9  9595927.6   310914.0  4584084.3
## 23  9949920.8  7311827.9 12606844.4 15203656.9 18133855.5  8227027.8 13121964.2
## 24  3676462.9  6314104.5  1019192.6  1577899.5  4508119.1  5398737.4   505152.5
## 25  6987988.3  9625861.3  4330788.3  1733946.6  1196292.5  8710559.9  3815806.0
## 26 38980730.5 36342803.8 41637883.5 44234753.4 47164894.8 37258173.3 42153012.4
## 27  1464078.3  1261488.4  4207644.3  6889503.2  9915736.2   317260.5  4739409.5
## 28  9319046.8 12139076.8  6478490.5  3702359.4   569987.2 11160612.2  5927755.2
## 29  3734910.3  6459056.0   992533.1  1692844.2  4718327.5  5513902.8   458665.5
```

```
## 30   301607.7  3023539.1  2446003.6  5127617.3  8153693.3  2078822.6  2977400.6
##             8          9         10         11         12         13         14
## 1   7739095.7  3541179.23 59068272 75254568  1309339.5   498677.6  3833770.1
## 2  10376966.3  6178875.82 56430300 72616668  1329246.9  2143374.4  6471512.3
## 3   5081864.8   884403.26 61725394 77911744  3966447.0  3152120.1  1178083.5
## 4   2485011.2  1713222.94 64322246 80508620  6563330.6  5748850.2  1421690.0
## 5    445423.9  4643283.92 67252409 83438748  9493397.5  8679061.1  4350916.7
## 6   9461646.0  5263592.70 57345631 73532049   417425.3  1227792.4  5556402.2
## 7   4566922.4   369016.54 62240490 78426873  4481796.4  3667289.2   661626.8
## 8         0.0  4198142.43 66807245 82993600  9048259.5  8233861.0  3905930.1
## 9   4198142.4        0.00 62609165 78795544  4850339.3  4035840.0   294757.5
## 10 66807244.6 62609165.06        0 16186913 57759062.7 58573415.9 62901731.7
## 11 82993600.2 78795543.54 16186913        0 73945362.1 74759828.2 79088088.5
## 12  9048259.5  4850339.35 57759063 73945362        0.0   816305.6  5143049.7
## 13  8233861.0  4035839.96 58573416 74759828   816305.6        0.0  4328749.0
## 14  3905930.1   294757.49 62901732 79088088  5143049.7  4328749.0        0.0
## 15 57631457.5 53433380.51  9175789 25362422 48583274.2 49397631.5 53725951.5
## 16   517133.4  3681040.00 66290127 82476490  8531161.6  7716736.2  3388911.8
## 17  1491399.1  2706784.64 65315861 81502231  7556922.1  6742465.0  2414819.3
## 18   708356.7  4903380.29 67512337 83698686  9753540.9  8939201.5  4610612.9
## 19   823917.6  5021931.53 67631081 83817438  9872100.7  9057701.2  4729577.0
## 20 16445468.8 12247370.11 50361800 66548190  7397378.3  8211691.5 12539950.7
## 21 35387852.2 31189774.11 31419547 47606198 26339802.2 27153993.1 31482393.0
## 22  9150738.2  4952681.53 57656534 73842947   116587.4   916896.3  5245494.4
## 23 17688665.7 13490605.64 49118621 65305082  8640704.0  9454810.0 13783290.5
## 24  4062909.8   136788.50 62744354 78930740  4985522.6  4170952.5   167299.3
## 25   751126.5  3447031.16 66056150 82242504  8297165.5  7482779.5  3154818.4
## 26 46719740.1 42521678.83 20087624 36273877 37671519.1 38485952.9 42814244.4
## 27  9455987.4  5120180.96 59542299 76259541   124863.7   952600.0  5422417.0
## 28  1046036.9  5533605.49 72465604 89769540 10718665.1  9848070.9  5220936.5
## 29  4258721.0    85416.58 64739923 81457137  5087099.3  4245993.3   224880.3
## 30  7694009.0  3358237.98 61304387 78021579  1651970.6   812140.9  3660253.9
##          15         16         17         18         19       20       21
## 1  49892485  7222010.5  6247781.6  8444257.2  8562908.8  8706540 27649016
## 2  47254514  9859856.2  8885592.0 11082114.6 11200787.2  6068506 25010959
## 3  52549606  4564752.4  3590504.5  5787427.3  5905729.2 11363651 30306030
## 4  55146459  1967886.6   993615.7  3190822.5  3308864.2 13960477 32902843
## 5  58076622   962468.6  1936652.9   265931.4   378816.7 16890619 35833038
## 6  48169847  8944522.6  7970250.9 10166927.0 10285481.2  6983909 25926208
## 7  53064707  4049836.8  3075606.3  5271935.1  5390666.0 11878694 30821106
## 8  57631458   517133.4  1491399.1   708356.7   823917.6 16445469 35387852
## 9  53433381  3681040.0  2706784.6  4903380.3  5021931.5 12247370 31189774
## 10  9175789 66290127.4 65315860.8 67512336.5 67631081.0 50361800 31419547
## 11 25362422 82476490.1 81502231.0 83698685.7 83817438.5 66548190 47606198
## 12 48583274  8531161.6  7556922.1  9753540.9  9872100.7  7397378 26339802
## 13 49397632  7716736.2  6742465.0  8939201.5  9057701.2  8211692 27153993
## 14 53725952  3388911.8  2414819.3  4610612.9  4729577.0 12539951 31482393
## 15        0 57114340.6 56140074.3 58336558.1 58455294.8 41186016 22243808
## 16 57114341        0.0   974273.0  1224209.5  1341023.6 15928356 34870729
## 17 56140074   974273.0        0.0  2197538.9  2315257.5 14954089 33896458
## 18 58336558  1224209.5  2197538.9        0.0   131665.5 17150563 36092993
## 19 58455295  1341023.6  2315257.5   131665.5        0.0 17269291 36211688
## 20 41186016 15928356.0 14954089.2 17150563.2 17269290.8        0 18942493
## 21 22243808 34870728.6 33896457.8 36092992.5 36211687.6 18942493        0
```

```
## 22 48480750  8633615.5  7659343.8  9856017.3  9974572.6  7294799 26237116
## 23 39942840 17171542.2 16197272.4 18393885.7 18512506.5  1244054 17699189
## 24 53568569  3545785.7  2571513.5  4768452.2  4886751.3 12382589 31324945
## 25 56880363   234200.3   740373.1  1457455.8  1574941.4 15694366 34636765
## 26 10911881 46202627.6 45228366.0 47424850.3 47543579.1 30274323 11332659
## 27 50065591  8921916.1  7915695.8 10184216.3 10306809.1  7528867 27092497
## 28 62656277  1598777.4  2640185.5   295313.7   165765.1 18626576 38876876
## 29 55263219  3724730.3  2718655.6  4986532.4  5109387.3 12726502 32290151
## 30 51827680  7159963.7  6153770.5  8422071.9  8544791.2  9290960 28854655
##             22       23          24         25       26          27         28
## 1   1412975.22  9949921  3676462.90  6987988.3 38980731  1464078.30  9319046.8
## 2   1226591.23  7311828  6314104.51  9625861.3 36342804  1261488.36 12139076.8
## 3   4068976.47 12606844  1019192.57  4330788.3 41637884  4207644.26  6478490.5
## 4   6665728.92 15203657  1577899.47  1733946.6 44234753  6889503.25  3702359.4
## 5   9595927.61 18133856  4508119.13  1196292.5 47164895  9915736.15   569987.2
## 6    310914.02  8227028  5398737.42  8710559.9 37258173   317260.51 11160612.2
## 7   4584084.33 13121964   505152.48  3815806.0 42153012  4739409.52  5927755.2
## 8   9150738.20 17688666  4062909.79   751126.5 46719740  9455987.39  1046036.9
## 9   4952681.53 13490606   136788.50  3447031.2 42521679  5120180.96  5533605.5
## 10 57656533.80 49118621 62744354.29 66056150.0 20087624 59542298.67 72465604.4
## 11 73842947.40 65305082 78930739.77 82242504.0 36273877 76259541.21 89769540.2
## 12   116587.45  8640704  4985522.60  8297165.5 37671519   124863.68 10718665.1
## 13   916896.26  9454810  4170952.55  7482779.5 38485953   952599.98  9848070.9
## 14  5245494.44 13783290   167299.32  3154818.4 42814244  5422416.95  5220936.5
## 15 48480749.68 39942840 53568568.57 56880363.2 10911881 50065591.20 62656277.5
## 16  8633615.51 17171542  3545785.73   234200.3 46202628  8921916.12  1598777.4
## 17  7659343.84 16197272  2571513.54   740373.1 45228366  7915695.76  2640185.5
## 18  9856017.34 18393886  4768452.21  1457455.8 47424850 10184216.28   295313.7
## 19  9974572.55 18512507  4886751.32  1574941.4 47543579 10306809.06   165765.1
## 20  7294799.49  1244054 12382588.81 15694365.8 30274323  7528867.06 18626576.4
## 21 26237115.97 17699189 31324944.91 34636764.8 11332659 27092496.59 38876876.5
## 22        0.00  8537935  5087830.52  8399650.6 37569072    27013.79 10828235.2
## 23  8537935.36        0 13625760.74 16937580.4 29031207  8812907.74 19955666.8
## 24  5087830.52 13625761        0.00  3311831.9 42656870  5259871.68  5389159.8
## 25  8399650.60 16937580  3311831.85        0.0 45968645  8680248.27  1848726.3
## 26 37569072.06 29031207 42656870.30 45968644.6        0 38796051.75 50991159.3
## 27    27013.79  8812908  5259871.68  8680248.3 38796052        0.00 10833487.5
## 28 10828235.15 19955667  5389159.79  1848726.3 50991159 10833487.50        0.0
## 29  5192801.16 14010600    84618.29  3482978.5 43993668  5197695.75  5453536.8
## 30  1757788.42 10575120  3498101.70  6918252.8 40558111  1762330.42  9009575.6
##             29         30
## 1   3734910.33   301607.7
## 2   6459056.01  3023539.1
## 3    992533.11  2446003.6
## 4   1692844.17  5127617.3
## 5   4718327.55  8153693.3
## 6   5513902.84  2078822.6
## 7    458665.48  2977400.6
## 8   4258721.03  7694009.0
## 9     85416.58  3358238.0
## 10 64739922.94 61304386.8
## 11 81457137.37 78021579.4
## 12  5087099.33  1651970.6
## 13  4245993.26   812140.9
```

```
## 14   224880.27  3660253.9
## 15 55263218.86 51827680.0
## 16  3724730.34  7159963.7
## 17  2718655.56  6153770.5
## 18  4986532.40  8422071.9
## 19  5109387.27  8544791.2
## 20 12726501.58  9290960.2
## 21 32290151.02 28854655.5
## 22  5192801.16  1757788.4
## 23 14010599.65 10575119.5
## 24    84618.29  3498101.7
## 25  3482978.53  6918252.8
## 26 43993667.87 40558110.9
## 27  5197695.75  1762330.4
## 28  5453536.80  9009575.6
## 29        0.00  3435576.1
## 30  3435576.09        0.0
```

```
cat("\nMatrice de dissimilarité Euclidienne (normalisée) :\n")
```

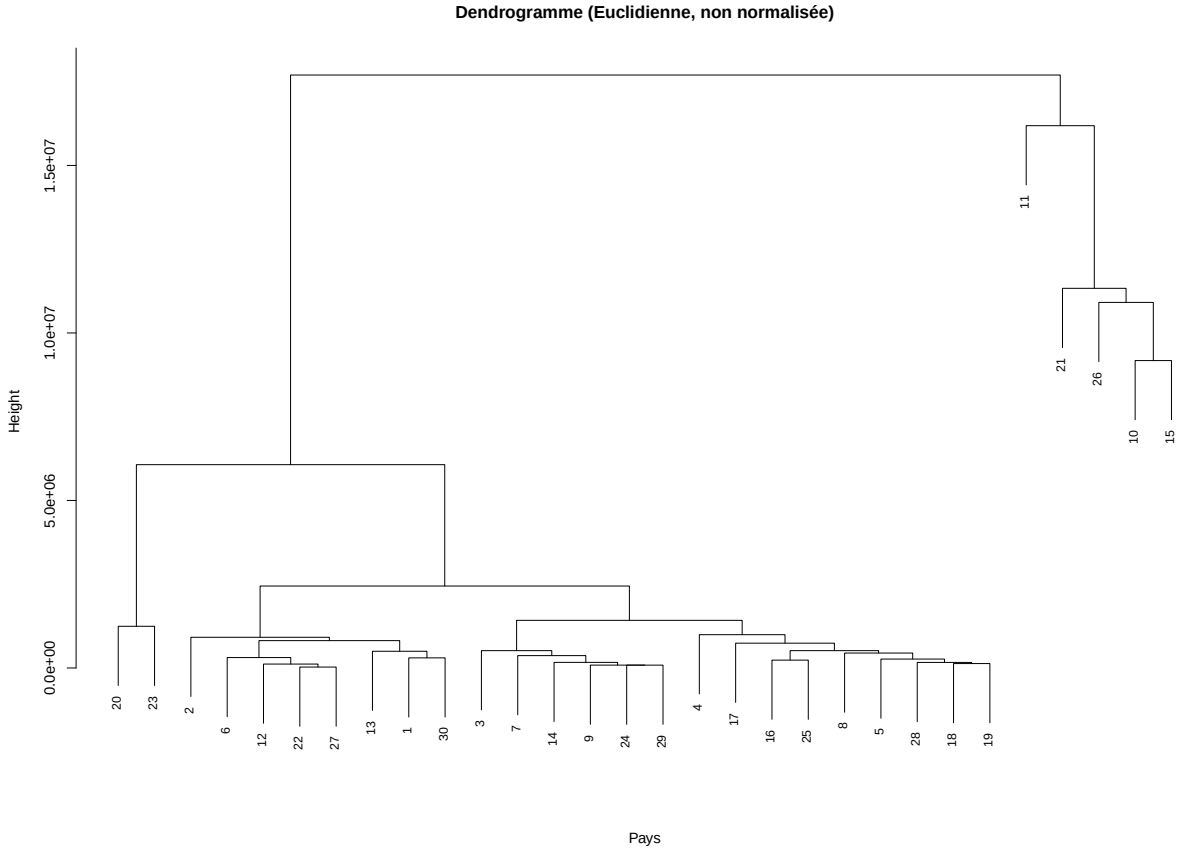
```
##
## Matrice de dissimilarité Euclidienne (normalisée) :
```

```
print(round(dissimilarity_reduced, 2))
```

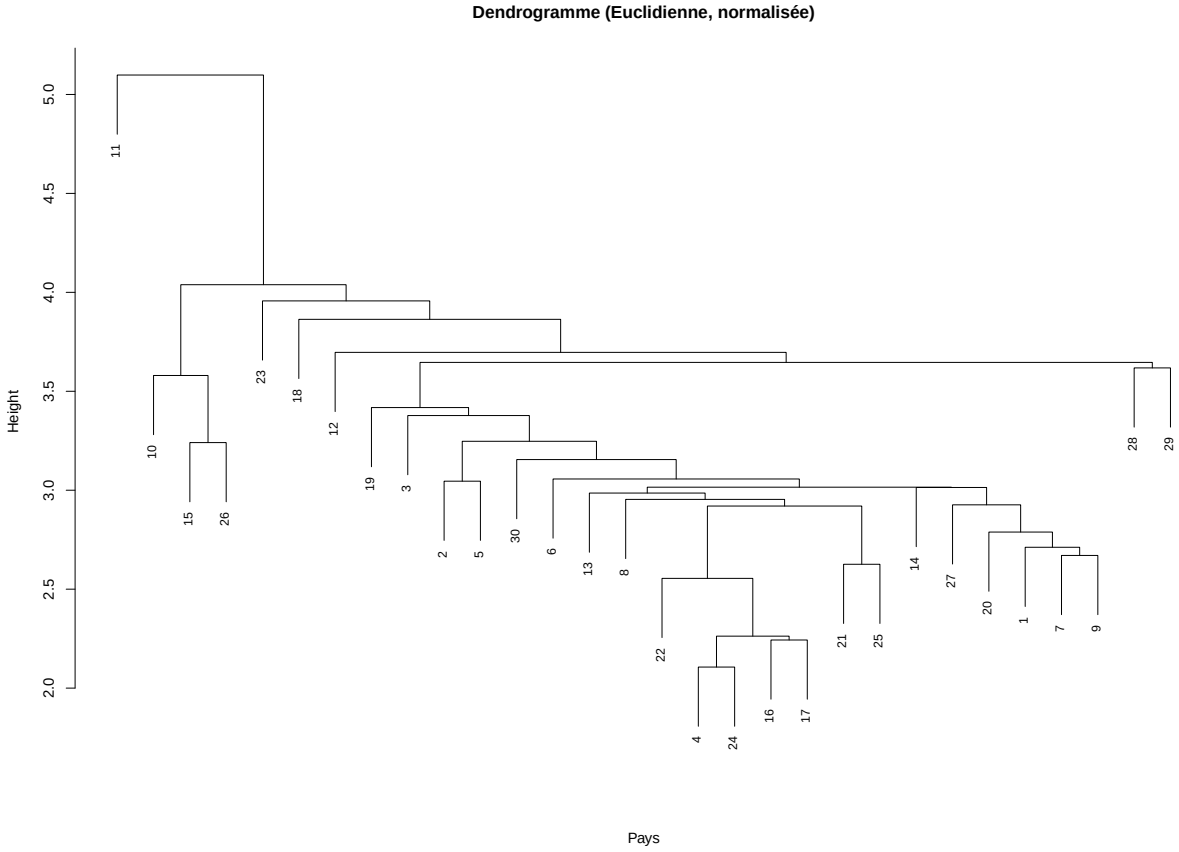
```
##       1    2    3    4    5    6    7    8    9   10   11   12   13   14   15
## 1  0.00 3.80 5.55 3.97 3.61 4.15 3.04 4.70 2.71 4.04 5.87 5.60 4.48 3.95 5.40
## 2  3.80 0.00 6.87 4.91 3.05 6.20 4.09 6.63 4.12 4.43 6.90 5.34 6.83 3.25 4.39
## 3  5.55 6.87 0.00 4.26 6.58 5.39 6.13 3.53 5.61 6.52 8.25 6.66 4.71 7.13 6.77
## 4  3.97 4.91 4.26 0.00 4.79 4.65 4.87 4.29 3.81 4.94 8.36 4.59 4.18 5.66 5.74
## 5  3.61 3.05 6.58 4.79 0.00 5.64 3.65 5.96 4.01 5.21 7.36 5.63 5.87 4.10 5.24
## 6  4.15 6.20 5.39 4.65 5.64 0.00 5.36 4.59 5.30 6.46 7.50 7.65 4.00 5.79 7.31
## 7  3.04 4.09 6.13 4.87 3.65 5.36 0.00 4.84 2.67 5.01 6.95 6.94 5.88 3.89 6.41
## 8  4.70 6.63 3.53 4.29 5.96 4.59 4.84 0.00 4.14 6.11 8.09 6.43 4.87 6.66 7.09
## 9  2.71 4.12 5.61 3.81 4.01 5.30 2.67 4.14 0.00 4.28 7.33 5.32 5.22 4.86 5.73
## 10 4.04 4.43 6.52 4.94 5.21 6.46 5.01 6.11 4.28 0.00 5.10 5.01 5.98 5.82 3.58
## 11 5.87 6.90 8.25 8.36 7.36 7.50 6.95 8.09 7.33 5.10 0.00 8.79 8.22 7.12 6.18
## 12 5.60 5.34 6.66 4.59 5.63 7.65 6.94 6.43 5.32 5.01 8.79 0.00 6.90 7.16 4.51
## 13 4.48 6.83 4.71 4.18 5.87 4.00 5.88 4.87 5.22 5.98 8.22 6.90 0.00 7.13 6.86
## 14 3.95 3.25 7.13 5.66 4.10 5.79 3.89 6.66 4.86 5.82 7.12 7.16 7.13 0.00 6.54
## 15 5.40 4.39 6.77 5.74 5.24 7.31 6.41 7.09 5.73 3.58 6.18 4.51 6.86 6.54 0.00
## 16 3.91 5.73 3.58 3.47 5.39 4.51 4.54 2.95 4.15 5.47 7.62 5.60 4.51 6.25 5.94
## 17 3.49 4.76 3.38 2.26 4.42 4.55 4.40 3.82 3.85 5.04 7.63 4.96 4.08 5.32 5.32
## 18 6.32 4.96 8.16 6.98 5.60 8.09 5.79 8.15 6.06 7.37 9.10 8.26 8.53 3.86 7.64
## 19 3.92 4.87 5.17 4.07 4.00 5.59 4.24 6.04 4.63 5.61 7.80 6.71 4.31 5.34 6.34
## 20 2.79 3.57 6.17 4.55 3.80 5.20 3.15 5.98 3.70 4.62 6.42 6.81 5.62 3.01 6.32
## 21 4.26 4.89 4.65 3.46 5.12 3.06 5.07 4.96 4.83 5.16 7.14 6.74 4.46 5.11 5.86
## 22 3.28 3.90 4.84 2.55 3.73 5.30 4.22 4.64 3.02 4.07 7.53 3.70 4.55 5.57 4.18
## 23 6.37 6.72 3.96 5.49 6.36 7.12 6.26 5.05 5.90 6.23 8.59 6.76 5.37 7.83 5.67
## 24 3.47 5.31 4.47 2.11 4.54 3.71 4.81 4.00 3.91 5.04 7.90 5.09 2.99 5.85 5.81
## 25 3.37 4.17 4.78 2.92 4.59 3.75 4.19 5.03 3.65 5.16 7.51 6.13 4.42 4.90 5.82
## 26 6.17 5.76 7.12 6.23 5.89 8.60 6.76 6.98 5.66 4.18 7.29 4.31 7.44 7.68 3.24
## 27 4.68 5.57 5.89 3.95 5.55 6.49 3.66 4.65 2.93 5.19 8.63 6.26 5.96 6.14 6.52
```

```
## 28 6.24 7.31 7.09 7.46 6.48 7.17 5.44 6.42 5.21 7.79 9.45 9.25 7.11 7.27 8.84
## 29 4.96 6.67 6.61 6.09 6.24 6.39 3.65 5.37 3.74 6.63 8.53 8.53 6.30 6.18 8.27
## 30 3.63 5.44 6.33 5.08 5.48 6.00 3.15 5.72 4.06 5.89 7.31 7.40 6.66 4.76 7.31
##      16   17   18   19   20   21   22   23   24   25   26   27   28   29   30
## 1  3.91 3.49 6.32 3.92 2.79 4.26 3.28 6.37 3.47 3.37 6.17 4.68 6.24 4.96 3.63
## 2  5.73 4.76 4.96 4.87 3.57 4.89 3.90 6.72 5.31 4.17 5.76 5.57 7.31 6.67 5.44
## 3  3.58 3.38 8.16 5.17 6.17 4.65 4.84 3.96 4.47 4.78 7.12 5.89 7.09 6.61 6.33
## 4  3.47 2.26 6.98 4.07 4.55 3.46 2.55 5.49 2.11 2.92 6.23 3.95 7.46 6.09 5.08
## 5  5.39 4.42 5.60 4.00 3.80 5.12 3.73 6.36 4.54 4.59 5.89 5.55 6.48 6.24 5.48
## 6  4.51 4.55 8.09 5.59 5.20 3.06 5.30 7.12 3.71 3.75 8.60 6.49 7.17 6.39 6.00
## 7  4.54 4.40 5.79 4.24 3.15 5.07 4.22 6.26 4.81 4.19 6.76 3.66 5.44 3.65 3.15
## 8  2.95 3.82 8.15 6.04 5.98 4.96 4.64 5.05 4.00 5.03 6.98 4.65 6.42 5.37 5.72
## 9  4.15 3.85 6.06 4.63 3.70 4.83 3.02 5.90 3.91 3.65 5.66 2.93 5.21 3.74 4.06
## 10 5.47 5.04 7.37 5.61 4.62 5.16 4.07 6.23 5.04 5.16 4.18 5.19 7.79 6.63 5.89
## 11 7.62 7.63 9.10 7.80 6.42 7.14 7.53 8.59 7.90 7.51 7.29 8.63 9.45 8.53 7.31
## 12 5.60 4.96 8.26 6.71 6.81 6.74 3.70 6.76 5.09 6.13 4.31 6.26 9.25 8.53 7.40
## 13 4.51 4.08 8.53 4.31 5.62 4.46 4.55 5.37 2.99 4.42 7.44 5.96 7.11 6.30 6.66
## 14 6.25 5.32 3.86 5.34 3.01 5.11 5.57 7.83 5.85 4.90 7.68 6.14 7.27 6.18 4.76
## 15 5.94 5.32 7.64 6.34 6.32 5.86 4.18 5.67 5.81 5.82 3.24 6.52 8.84 8.27 7.31
## 16 0.00 2.24 8.37 4.97 5.58 4.56 3.34 4.75 3.38 4.05 6.44 4.25 7.22 5.92 4.51
## 17 2.24 0.00 6.90 3.66 4.48 3.76 2.59 4.56 2.55 3.18 5.81 4.23 7.28 6.12 4.60
## 18 8.37 6.90 0.00 6.52 4.82 6.73 6.97 8.30 7.45 6.46 8.06 6.83 7.37 6.70 6.86
## 19 4.97 3.66 6.52 0.00 3.42 4.71 3.85 5.61 4.00 3.69 6.98 5.47 7.00 5.88 4.88
## 20 5.58 4.48 4.82 3.42 0.00 4.26 4.53 7.07 4.66 3.66 7.08 5.25 6.42 5.15 3.98
## 21 4.56 3.76 6.73 4.71 4.26 0.00 4.37 6.02 3.67 2.63 7.40 5.64 7.07 6.26 5.81
## 22 3.34 2.59 6.97 3.85 4.53 4.37 0.00 4.90 2.73 3.27 4.73 3.82 7.09 5.86 4.75
## 23 4.75 4.56 8.30 5.61 7.07 6.02 4.90 0.00 5.53 6.04 5.68 5.60 6.66 6.96 7.54
## 24 3.38 2.55 7.45 4.00 4.66 3.67 2.73 5.53 0.00 3.28 6.25 4.52 7.59 6.17 5.25
## 25 4.05 3.18 6.46 3.69 3.66 2.63 3.27 6.04 3.28 0.00 7.02 4.79 6.74 5.53 4.71
## 26 6.44 5.81 8.06 6.98 7.08 7.40 4.73 5.68 6.25 7.02 0.00 6.18 9.12 8.42 7.90
## 27 4.25 4.23 6.83 5.47 5.25 5.64 3.82 5.60 4.52 4.79 6.18 0.00 6.36 4.17 4.34
## 28 7.22 7.28 7.37 7.00 6.42 7.07 7.09 6.66 7.59 6.74 9.12 6.36 0.00 3.62 7.28
## 29 5.92 6.12 6.70 5.88 5.15 6.26 5.86 6.96 6.17 5.53 8.42 4.17 3.62 0.00 4.63
## 30 4.51 4.60 6.86 4.88 3.98 5.81 4.75 7.54 5.25 4.71 7.90 4.34 7.28 4.63 0.00
```

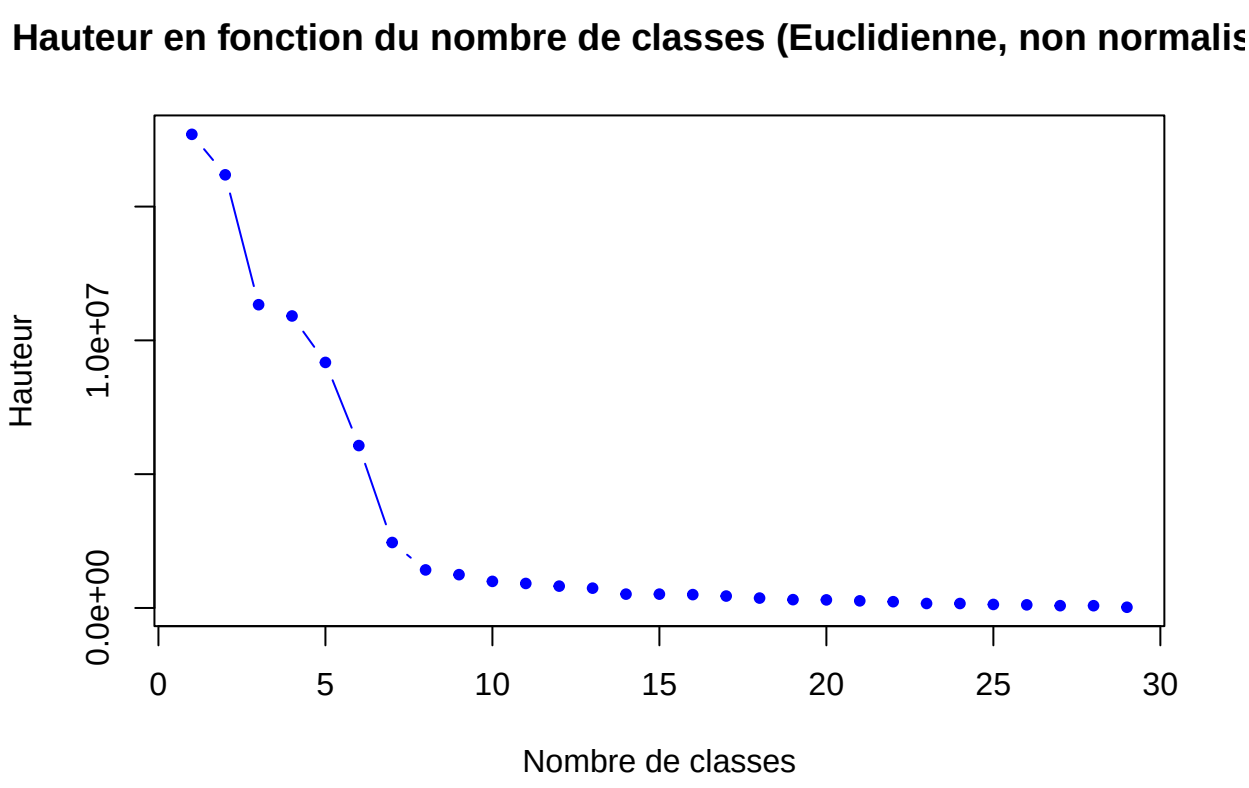
```r
# Question 1 suite
# CAH avec la matrice non normalisée
cah_single_euclidean <- hclust(dist(euro_data, method = "euclidean"), method = "single")

# CAH avec la matrice normalisée
cah_single_reduced <- hclust(dist(euro_data_normalized, method = "euclidean"), method = "single")

# Dendrogramme pour les données non normalisées
plot(cah_single_euclidean,
     main = "Dendrogramme (Euclidienne, non normalisée)",
     xlab = "Pays", sub = "", cex = 0.8)
```



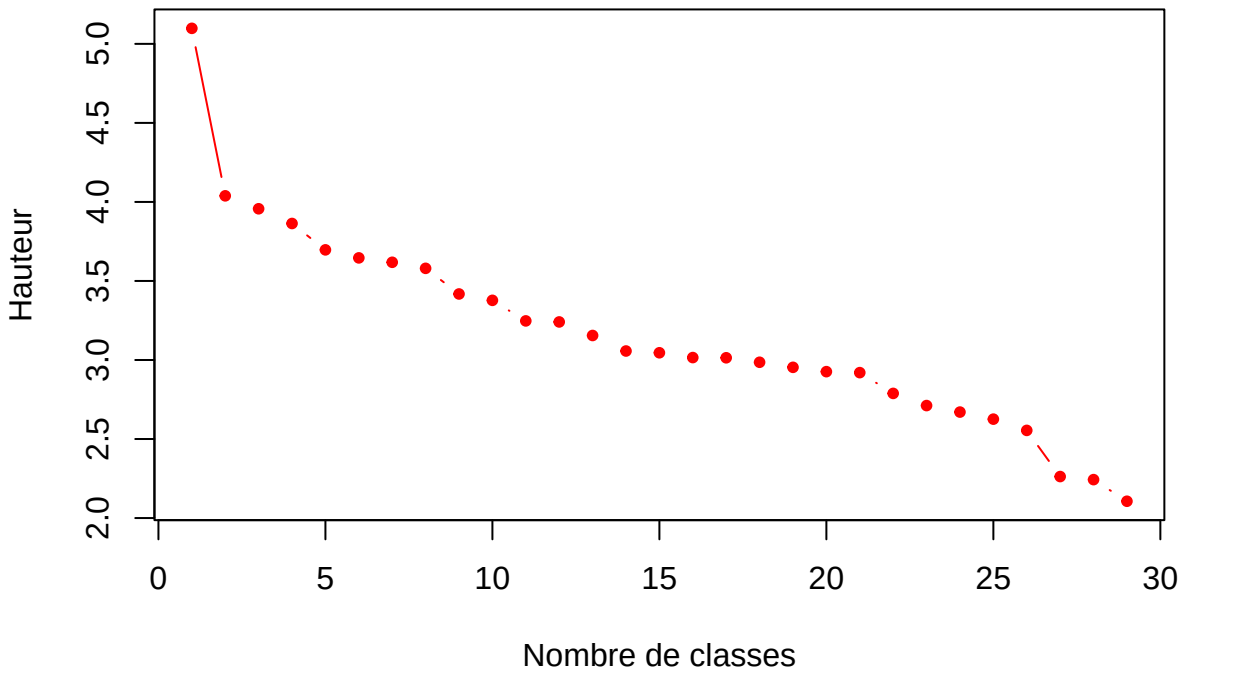

```
# Dendrogramme pour les données normalisées
plot(cah_single_reduced,
     main = "Dendrogramme (Euclidienne, normalisée)",
     xlab = "Pays", sub = "", cex = 0.8)
```




```r
# Question 2
# Récupérer les hauteurs des fusions
heights_euclidean <- sort(cah_single_euclidean$height, decreasing = TRUE)
heights_reduced <- sort(cah_single_reduced$height, decreasing = TRUE)

# Courbe des hauteurs pour la matrice non normalisée
plot(1:length(heights_euclidean), heights_euclidean,
     type = "b", col = "blue", pch = 20,
     xlab = "Nombre de classes",
     ylab = "Hauteur",
     main = "Hauteur en fonction du nombre de classes (Euclidienne, non normalisée)")
```




```r
# Courbe des hauteurs pour la matrice normalisée
plot(1:length(heights_reduced), heights_reduced,
     type = "b", col = "red", pch = 20,
     xlab = "Nombre de classes",
     ylab = "Hauteur",
     main = "Hauteur en fonction du nombre de classes (Euclidienne, normalisée)")
```




```r
# Question 3
# Fixer le nombre de classes (par exemple 4, basé sur la question 2)
k <- 4  # Ajuster selon la coupure choisie
classes <- cutree(cah_single_reduced, k = k)

# Ajouter les classes aux données
euro_data_with_classes <- data.frame(euro_data_normalized, Classe = classes)
# Fonction pour calculer le centre de gravité
calculate_center <- function(data, classes) {
  aggregate(. ~ Classe, data = data, FUN = mean)
}

# Centres de gravité
centers <- calculate_center(euro_data_with_classes, classes)
cat("Centres de gravité :\n")
```

```
## Centres de gravité :
```

```r
print(centers)
```

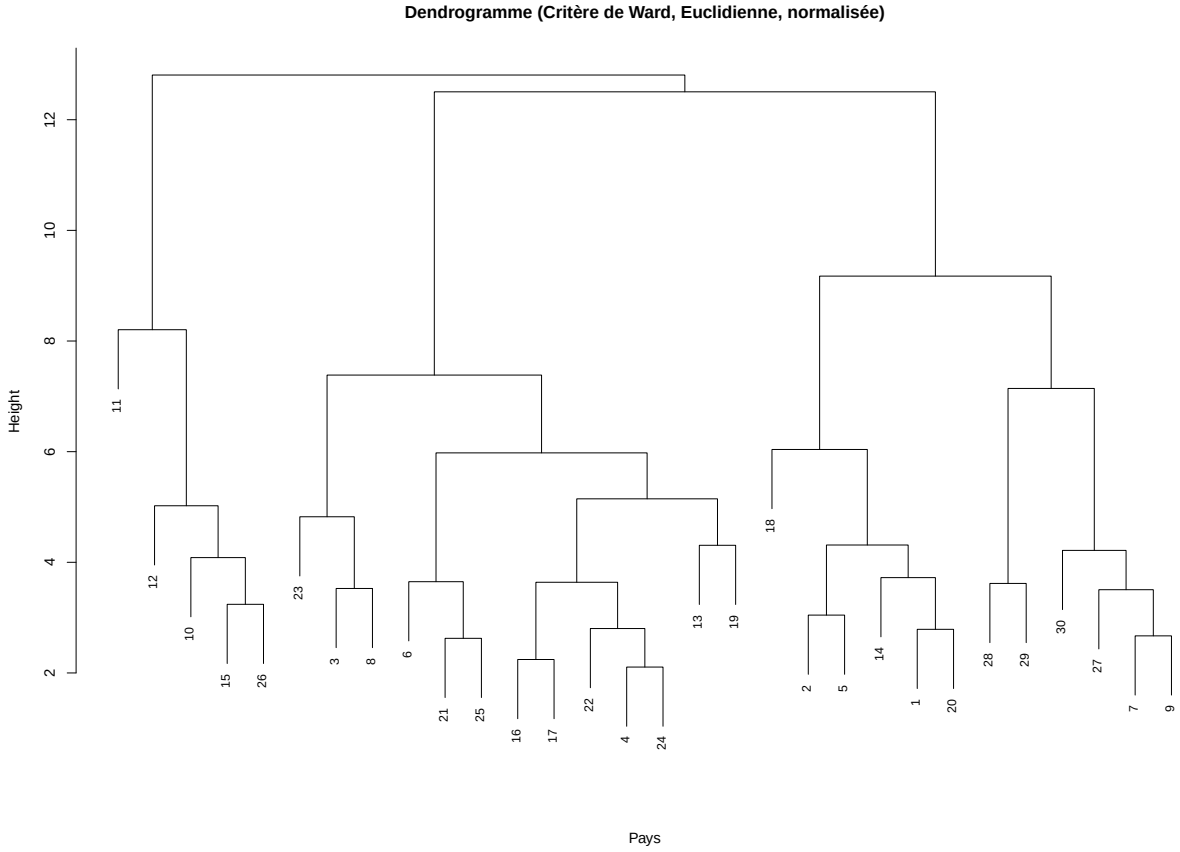
```
##   Classe Population Youth.population First.time.asylum.applicants
## 1      1 -0.3638724      -0.09519565                    -0.3345654
## 2      2  1.9919270      -0.31934053                     1.5517476
## 3      3  3.1944601      -0.39317649                     4.1619449
## 4      4  0.1671058      -0.50393043                    -0.3739529
```

```
##   Gender.pay.gap Minimum.wage People.at.risk.of.poverty.or.exclusion
## 1     0.08401396  -0.05205426                              -0.1556766
## 2    -0.65922266   0.25663429                               0.6338058
## 3     1.06621527   1.36679084                               0.1302341
## 4    -1.54169854  -1.04355427                               2.7061971
##   Early.school.leavers Inflation.rate Unemployment.rate Youth.unemployment.rate
## 1           -0.3481198      0.1261332       -0.10087154             -0.09664735
## 2            0.5927446     -0.5401019        1.56054209              1.35022754
## 3            1.2137151     -0.2528137       -1.09475291             -1.53623117
## 4            2.2863006      0.8101529        0.01780086              1.16876059
##   GDP.per.capita Government.gross.debt Greenhouse.gas.emissions
## 1     -0.1577956           -0.09869767                0.1213823
## 2     -0.1497554            1.54594803               -0.6561108
## 3      0.2074731           -0.01353189                0.3871305
## 4     -1.0971873           -0.43368304               -0.9326325
##   Renewable.energy Electricity.prices Energy.imports.dependency
## 1       -0.2961284         -0.0170000                0.1781534
## 2       -0.5234557          0.5904368                0.4177909
## 3       -0.5076307          2.1122596                0.4232683
## 4       -0.3335557         -0.7456611               -1.0638536
```

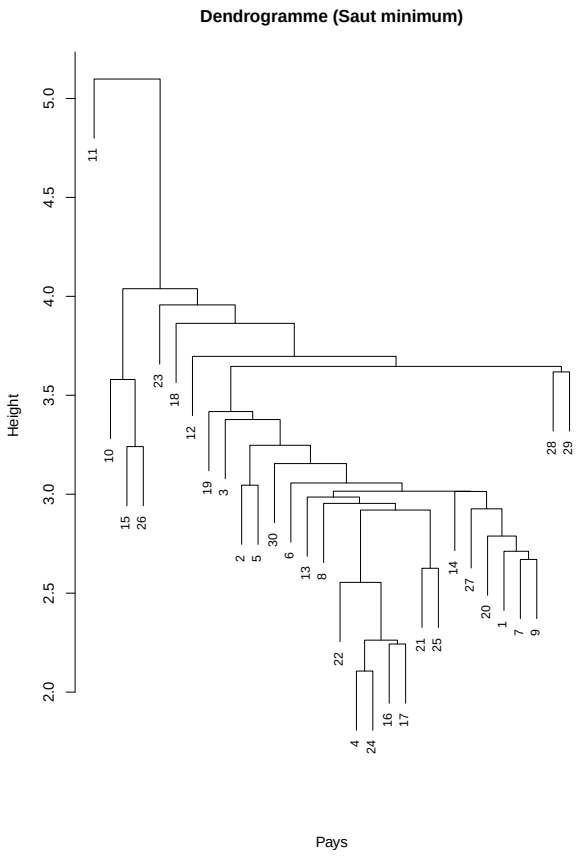
```r
# Calculer l'inertie intra-classe
# Question 3 suite
calculate_inertia <- function(data, centers, classes) {
  inertia <- 0
  for (class in unique(classes)) {
    class_data <- data[classes == class, ]
    class_center <- centers[class, -ncol(centers)]  # Exclure la colonne des classes
    inertia <- inertia + sum(rowSums((class_data - class_center)^2))
  }
  return(inertia)
}

# Inertie totale
inertia <- calculate_inertia(euro_data_normalized, centers, classes)
cat("\nInertie intra-classe totale : ", round(inertia, 2), "\n")
```

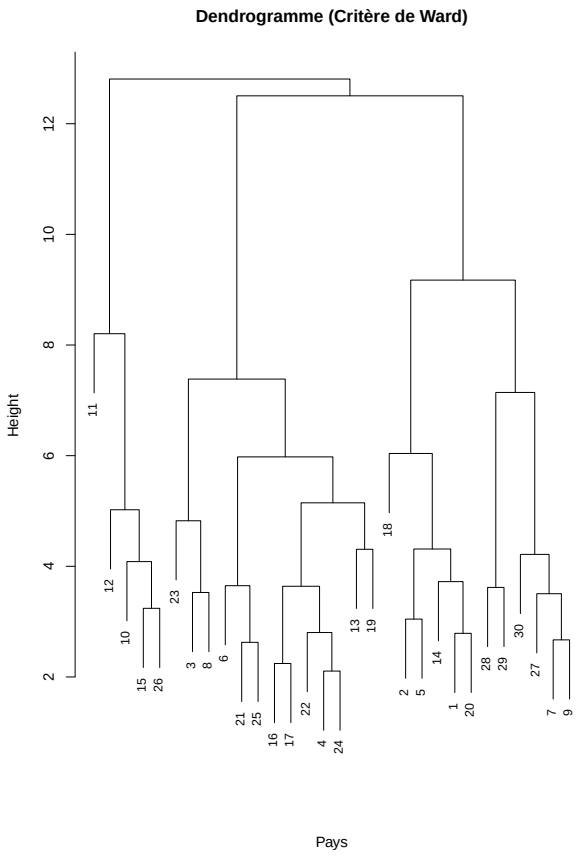
```
##
## Inertie intra-classe totale :  132.85
```

```r
# Question 4
# Classification ascendante hiérarchique avec le critère de Ward
cah_ward <- hclust(dist(euro_data_normalized, method = "euclidean"), method = "ward.D2")

# Représenter le dendrogramme
plot(cah_ward,
     main = "Dendrogramme (Critère de Ward, Euclidienne, normalisée)",
     xlab = "Pays", sub = "", cex = 0.8)
```




```r
par(mfrow = c(1, 2))  # Afficher les deux dendrogrammes côte à côte

# Dendrogramme avec le saut minimum
plot(cah_single_reduced,
     main = "Dendrogramme (Saut minimum)",
     xlab = "Pays", sub = "", cex = 0.8)

# Dendrogramme avec Ward
plot(cah_ward,
     main = "Dendrogramme (Critère de Ward)",
     xlab = "Pays", sub = "", cex = 0.8)
```






```r
par(mfrow = c(1, 1))  # Réinitialiser l'affichage
```

```r
# Qustion 4  Effectuez une autre classification en utilisant le critère de Ward. Commentez les différen
# Découpage en classes avec les deux méthodes
classes_single <- cutree(cah_single_reduced, k = 4)
classes_ward <- cutree(cah_ward, k = 4)

# Ajouter les classes aux données
euro_data_with_classes <- data.frame(euro_data_normalized, Classe_Single = classes_single, Classe_Ward =

# Afficher un aperçu
head(euro_data_with_classes)
```

```
##   Population Youth.population First.time.asylum.applicants Gender.pay.gap
## 1 -0.2941425        0.1052162                    0.28349361     1.20451373
## 2 -0.1718499        0.6036090                   -0.09845354    -1.44291393
## 3 -0.4173177       -1.9437317                   -0.19608989     0.13763990
## 4 -0.5377000       -0.4485535                   -0.49105968     0.03885528
## 5 -0.6735369        1.7111484                   -0.34858451    -0.41555394
## 6 -0.2142795       -0.8915692                   -0.49823673     1.10572911
##   Minimum.wage People.at.risk.of.poverty.or.exclusion Early.school.leavers
## 1    0.8677402                              -0.5670191           0.02822593
## 2    1.2628220                              -0.3346014          -0.64919646
## 3   -1.3658578                               2.2800979           0.22580746
```

```
## 4   -0.7368461                              -0.1021837          -1.83468565
## 5   -0.4595957                              -0.7219642           0.56451866
## 6   -0.8685400                              -1.6710032          -0.59274459
##   Inflation.rate Unemployment.rate Youth.unemployment.rate GDP.per.capita
## 1      0.2355764       -0.20470989              -0.7706675      0.2660926
## 2     -1.3157803       -0.02670129               0.1990466      0.2580762
## 3      0.4941358       -0.56072710              -0.4814545     -1.2174325
## 4      0.4366782        0.24031161               0.6924098     -0.8717275
## 5     -0.8561190        0.24031161               0.3351468     -0.2219025
## 6      2.2753231       -1.31726367              -1.1279305     -0.6848465
##   Government.gross.debt Greenhouse.gas.emissions Renewable.energy
## 1            0.38958609              0.01005534        0.1781191
## 2            1.16743348              0.38713049       -0.8768806
## 3           -1.16326982              0.31171546       -0.5973057
## 4           -0.03056505             -0.55555739       -0.1331058
## 5            0.37539180              0.83962068       -0.5814807
## 6           -0.56994828              1.06586577       -0.6447807
##   Electricity.prices Energy.imports.dependency Classe_Single Classe_Ward
## 1          0.5788290                0.66564454             1           1
## 2          1.6439448                0.64510418             1           1
## 3         -1.4517357               -0.87077425             1           2
## 4         -1.0326540                0.08229837             1           2
## 5          1.3401407                1.38455708             1           1
## 6          0.7637533               -0.67769488             1           2
```

```
# Fixer le nombre de classes
# question 5
# Vérifier les NA ou NaN dans les données normalisées
cat("Y a-t-il des NA/NaN dans les données ?\n")
```

```
## Y a-t-il des NA/NaN dans les données ?
```

```
print(any(is.na(euro_data_normalized)))
```

```
## [1] TRUE
```

```
# Si des NA sont présents, afficher leur localisation
if (any(is.na(euro_data_normalized))) {
  cat("Position des NA/NaN :\n")
  print(which(is.na(euro_data_normalized), arr.ind = TRUE))
  euro_data_normalized <- apply(euro_data_normalized, 2, function(x) {
  ifelse(is.na(x), mean(x, na.rm = TRUE), x)
})
print(any(is.na(euro_data_normalized)))
}
```

```
## Position des NA/NaN :
##      row col
## [1,]  27   5
## [2,]  28   5
## [3,]  29   5
## [4,]  30   5
## [5,]  28   6
## [1] FALSE
```

```r
euro_data_normalized <- apply(euro_data_normalized, 2, function(x) {
  ifelse(is.na(x), mean(x, na.rm = TRUE), x)
})
print(any(is.na(euro_data_normalized)))
```

```
## [1] FALSE
```

```r
# Suite question 5
# Fixer le nombre de classes
k <- 4

# Appliquer k-means
set.seed(42)  # Fixer la graine pour rendre les résultats reproductibles
kmeans_result <- kmeans(euro_data_normalized, centers = k, nstart = 10)

# Afficher les résultats
cat("Classes affectées par k-means :\n")
```

```
## Classes affectées par k-means :
```

```r
print(kmeans_result$cluster)
```

```
##  [1] 3 2 4 4 2 4 3 4 3 1 1 1 4 2 1 4 4 2 4 2 4 4 4 4 4 4 1 3 3 3 3
```

```r
cat("\nCentres des classes :\n")
```

```
##
## Centres des classes :
```

```r
print(round(kmeans_result$centers, 2))
```

```
##   Population Youth.population First.time.asylum.applicants Gender.pay.gap
## 1       1.79            -0.44                         1.83          -0.08
## 2      -0.38             1.10                        -0.24          -0.98
## 3      -0.41             0.79                        -0.30           0.41
## 4      -0.32            -0.68                        -0.45           0.19
##   Minimum.wage People.at.risk.of.poverty.or.exclusion Early.school.leavers
## 1         0.30                                  0.63                0.33
## 2         1.20                                 -0.44               -0.50
## 3         0.34                                 -0.50                0.42
## 4        -0.76                                  0.19               -0.16
##   Inflation.rate Unemployment.rate Youth.unemployment.rate GDP.per.capita
## 1          -0.53             1.21                    0.90          -0.18
## 2          -0.92            -0.28                   -0.13           1.04
## 3          -0.44            -0.17                   -0.40           0.88
## 4           0.79            -0.27                   -0.08          -0.81
##   Government.gross.debt Greenhouse.gas.emissions Renewable.energy
## 1                  1.48                   -0.34           -0.50
## 2                 -0.12                    1.16           -0.81
## 3                 -0.34                   -0.03            1.47
```

```
## 4                -0.34                    -0.29           -0.29
##   Electricity.prices Energy.imports.dependency
## 1               0.79                     0.51
## 2               0.74                     1.03
## 3              -0.26                    -0.72
## 4              -0.45                    -0.20
```

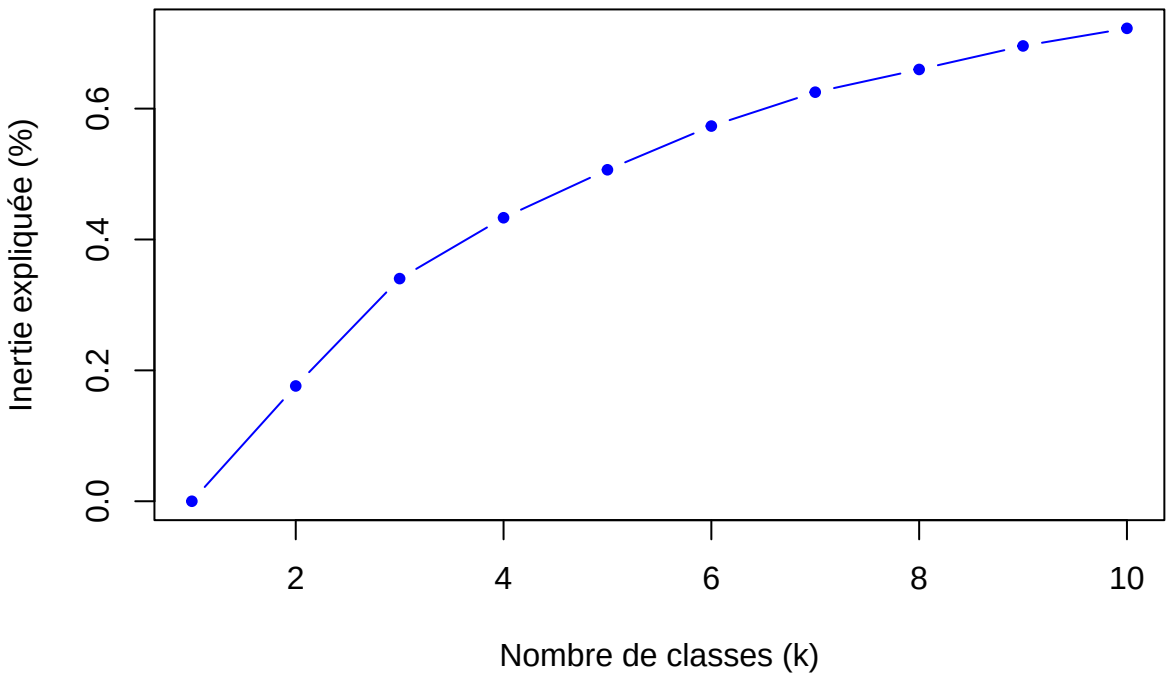
```r
cat("\nInertie intra-classe totale :\n")
```

```
##
## Inertie intra-classe totale :
```

```r
print(round(kmeans_result$tot.withinss, 2))
```

```
## [1] 260.15
```

```r
# Initialiser les variables
max_k <- 10  # Tester jusqu'à 10 classes
inertie_totale <- sum(scale(euro_data_normalized, center = TRUE, scale = FALSE)^2)
inertie_intra <- numeric(max_k)
inertie_expliquee <- numeric(max_k)

# Calculer l'inertie intra-classe pour chaque k
for (k in 1:max_k) {
  set.seed(42)  # Graine pour reproductibilité
  kmeans_result <- kmeans(euro_data_normalized, centers = k, nstart = 10)
  inertie_intra[k] <- kmeans_result$tot.withinss
  inertie_expliquee[k] <- 1 - (inertie_intra[k] / inertie_totale)
}

# Tracer la courbe
plot(1:max_k, inertie_expliquee, type = "b", pch = 20, col = "blue",
     xlab = "Nombre de classes (k)", ylab = "Inertie expliquée (%)",
     main = "Inertie expliquée en fonction du nombre de classes")
```



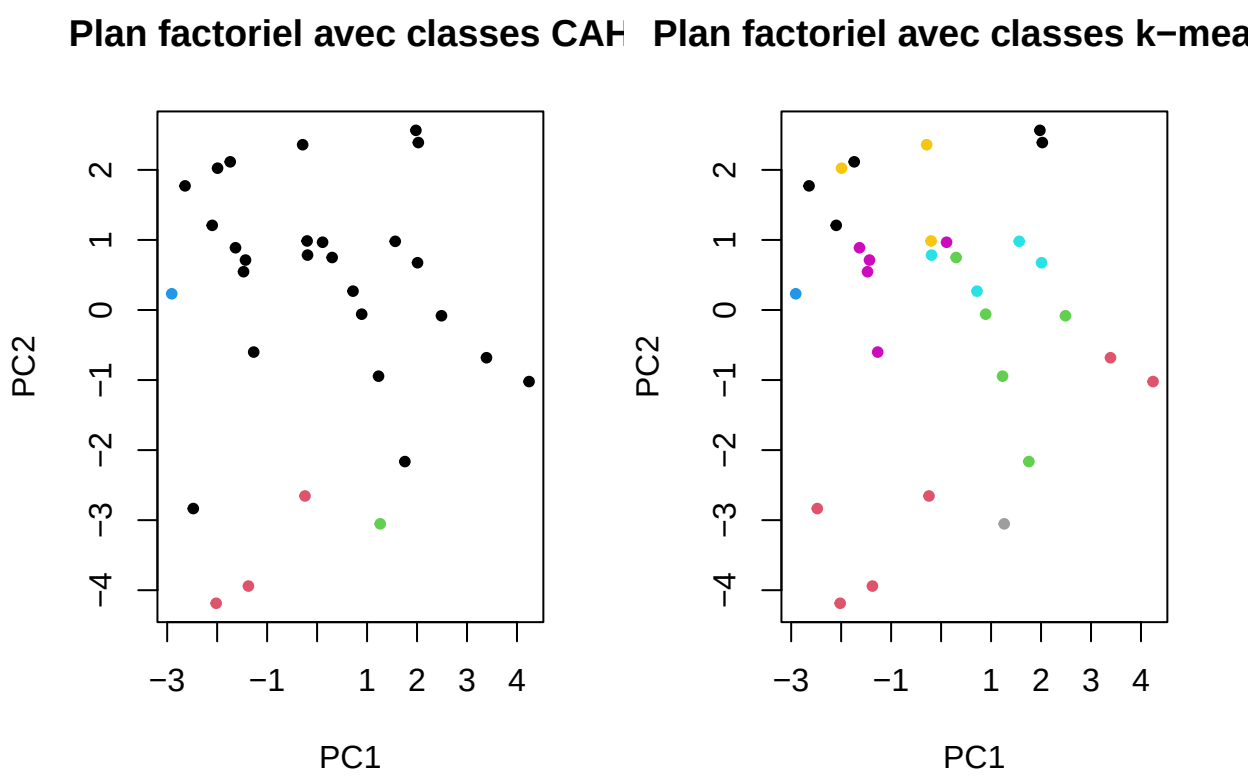

```r
# Question 7
# Étape 1 : Effectuer l'ACP
# Effectuer l'ACP sur les données normalisées
res_acp <- prcomp(euro_data_normalized, center = TRUE, scale. = TRUE)

# Résumé de l'ACP pour comprendre les variances expliquées
cat("Résumé de l'ACP :\n")
```

```
## Résumé de l'ACP :
```

```r
print(summary(res_acp))
```

```
## Importance of components:
##                           PC1    PC2    PC3    PC4     PC5     PC6     PC7
## Standard deviation     1.9068 1.8769 1.4265 1.3623 1.13291 0.93197 0.86710
## Proportion of Variance 0.2272 0.2202 0.1272 0.1160 0.08022 0.05428 0.04699
## Cumulative Proportion  0.2272 0.4474 0.5746 0.6906 0.77079 0.82507 0.87207
##                           PC8     PC9    PC10    PC11    PC12    PC13    PC14
## Standard deviation     0.74801 0.66980 0.58743 0.45336 0.41016 0.39737 0.30054
## Proportion of Variance 0.03497 0.02804 0.02157 0.01285 0.01051 0.00987 0.00565
## Cumulative Proportion  0.90704 0.93508 0.95664 0.96949 0.98000 0.98987 0.99552
##                           PC15    PC16
## Standard deviation     0.2040 0.17357
## Proportion of Variance 0.0026 0.00188
## Cumulative Proportion  0.9981 1.00000
```

```r
# Étape 2 : Récupérer les résultats de classification
# Fixer le nombre de classes optimal
k <- 4  # Ajuster selon les résultats précédents

# Classes obtenues par CAH
classes_cah <- cutree(cah_single_reduced, k = k)

# Classes obtenues par k-means
classes_kmeans <- kmeans_result$cluster

# Étape 3 : Représenter les classes dans le plan factoriel
par(mfrow = c(1, 2))  # Afficher les deux graphiques côte à côte

# Représentation des classes CAH
plot(res_acp$x[, 1], res_acp$x[, 2],
     col = classes_cah,
     pch = 20,
     xlab = "PC1", ylab = "PC2",
     main = "Plan factoriel avec classes CAH")

# Représentation des classes k-means
plot(res_acp$x[, 1], res_acp$x[, 2],
     col = classes_kmeans,
     pch = 20,
     xlab = "PC1", ylab = "PC2",
     main = "Plan factoriel avec classes k-means")
```




```r
par(mfrow = c(1, 1))  # Réinitialiser l'affichage
```

```r
# Étape 1 : Effectuer l'ACP
# Effectuer l'ACP sur les données normalisées
res_acp <- prcomp(euro_data_normalized, center = TRUE, scale. = TRUE)

# Résumé de l'ACP pour comprendre les variances expliquées
cat("Résumé de l'ACP :\n")
```

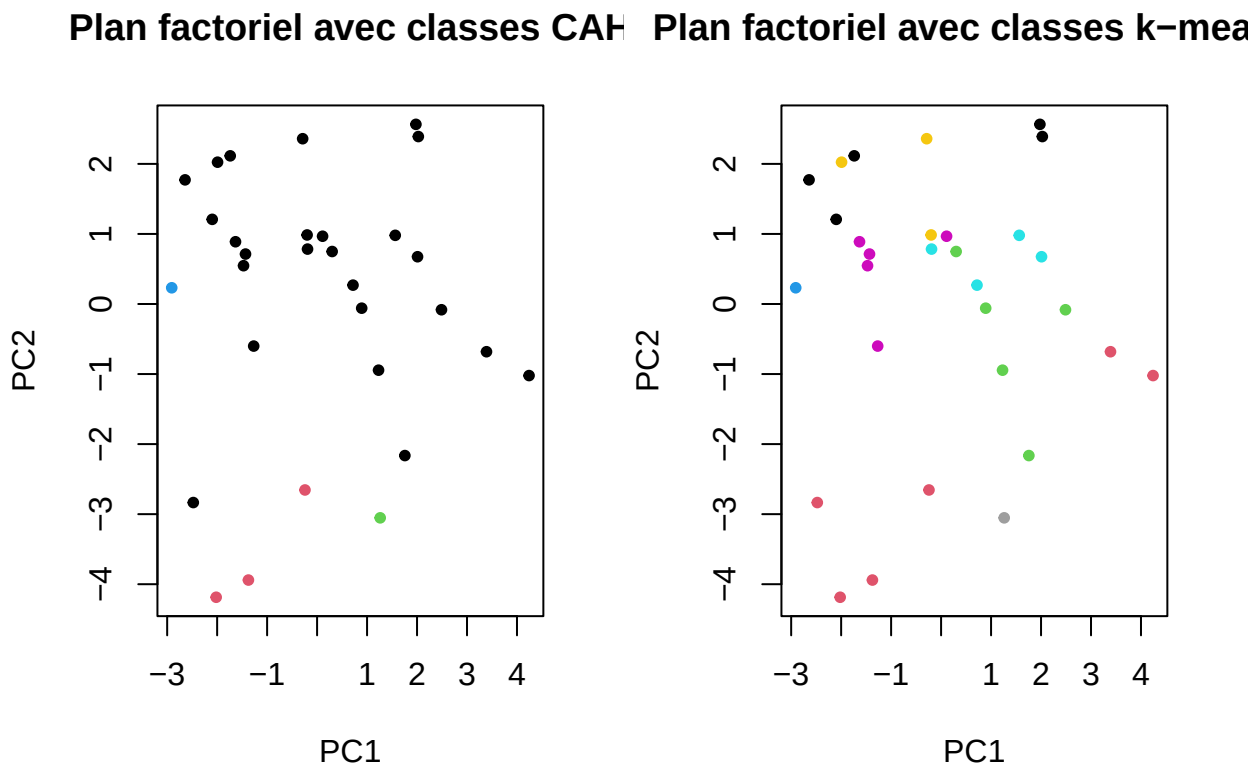
```
## Résumé de l'ACP :
```

```r
print(summary(res_acp))
```

```
## Importance of components:
##                           PC1    PC2    PC3    PC4     PC5     PC6     PC7
## Standard deviation     1.9068 1.8769 1.4265 1.3623 1.13291 0.93197 0.86710
## Proportion of Variance 0.2272 0.2202 0.1272 0.1160 0.08022 0.05428 0.04699
## Cumulative Proportion  0.2272 0.4474 0.5746 0.6906 0.77079 0.82507 0.87207
##                           PC8     PC9    PC10    PC11    PC12    PC13    PC14
## Standard deviation     0.74801 0.66980 0.58743 0.45336 0.41016 0.39737 0.30054
## Proportion of Variance 0.03497 0.02804 0.02157 0.01285 0.01051 0.00987 0.00565
## Cumulative Proportion  0.90704 0.93508 0.95664 0.96949 0.98000 0.98987 0.99552
##                           PC15    PC16
## Standard deviation     0.2040 0.17357
```

```
## Proportion of Variance 0.0026 0.00188
## Cumulative Proportion  0.9981 1.00000
```

```r
# Étape 2 : Récupérer les classes des deux méthodes
# Fixer le nombre optimal de classes (par exemple, k = 4)
k <- 4

# Classes obtenues par CAH
classes_cah <- cutree(cah_single_reduced, k = k)

# Classes obtenues par k-means
classes_kmeans <- kmeans_result$cluster

# Étape 3 : Ajouter les classes au tableau des coordonnées de l'ACP
coord_acp <- data.frame(res_acp$x[, 1:2], Classe_CAH = as.factor(classes_cah), Classe_Kmeans = as.facto

# Étape 4 : Représenter les classes dans le plan factoriel
par(mfrow = c(1, 2))  # Afficher les deux graphiques côte à côte

# Représentation des classes CAH
plot(coord_acp$PC1, coord_acp$PC2,
     col = coord_acp$Classe_CAH,
     pch = 20,
     xlab = "PC1", ylab = "PC2",
     main = "Plan factoriel avec classes CAH")

# Représentation des classes k-means
plot(coord_acp$PC1, coord_acp$PC2,
     col = coord_acp$Classe_Kmeans,
     pch = 20,
     xlab = "PC1", ylab = "PC2",
     main = "Plan factoriel avec classes k-means")
```




```r
par(mfrow = c(1, 1))  # Réinitialiser l'affichage
```

```r
# Étape 1 : Définir une zone restreinte dans le plan factoriel
zone_restante <- coord_acp[coord_acp$PC1 > -1 & coord_acp$PC1 < 1 & coord_acp$PC2 > -1 & coord_acp$PC2 <

# Afficher les pays dans la zone restreinte
cat("Pays dans la zone sélectionnée :\n")
```

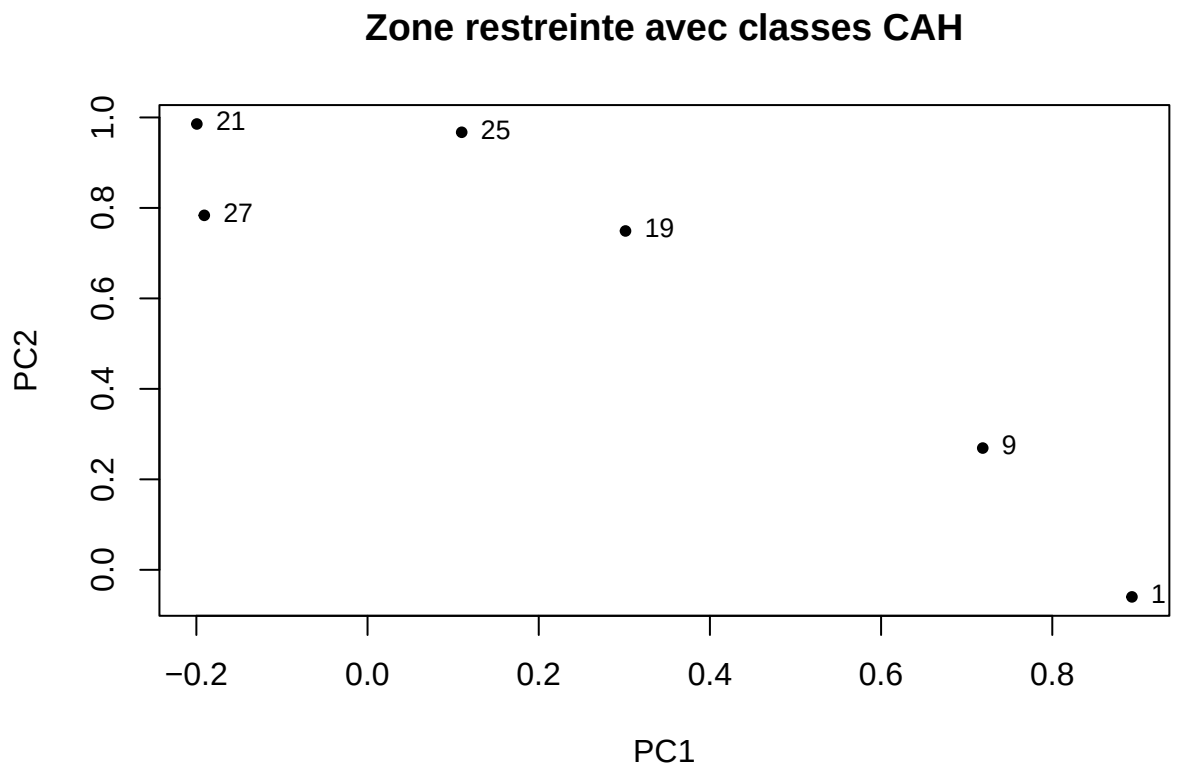
```
## Pays dans la zone sélectionnée :
```

```r
print(zone_restante)
```

```
##           PC1         PC2 Classe_CAH Classe_Kmeans
## 1   0.8930784 -0.05981786          1             3
## 9   0.7187603  0.26913474          1             5
## 19  0.3013992  0.74908977          1             3
## 21 -0.1994403  0.98555691          1             7
## 25  0.1100162  0.96722107          1             6
## 27 -0.1907443  0.78363267          1             5
```

```r
# Étape 2 : Représenter graphiquement la zone
plot(zone_restante$PC1, zone_restante$PC2,
     col = zone_restante$Classe_CAH,
```

```
     pch = 20,
     xlab = "PC1", ylab = "PC2",
     main = "Zone restreinte avec classes CAH")

# Ajouter les noms des pays
text(zone_restante$PC1, zone_restante$PC2, labels = rownames(zone_restante), pos = 4, cex = 0.8)
```




```
tinytex::is_tinytex()
```

```
## [1] TRUE
```

Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.