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MATHEMATICS

Area under curve $\frac{1}{x}$ is infinite, volume of revolution $\frac{1}{x}$ is π?

Asked 1 year, 7 months ago Active 29 days ago Viewed 128 times

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Stumbled across this weird phenomenon using the equation $y = \frac{1}{x}$.

Surface Area: When you calculate the surface area under the curve from 1 to ∞

$$\int_1^\infty \frac{1}{x}dx = \lim_{a\to\infty}\int_1^a \frac{1}{x}dx = \lim_{a\to\infty}[\ln|x|]_1^a = \lim_{a\to\infty}(\ln|a| - ln\,|1|) = \infty$$

Volume of revolution : When you calculate the volume of the revolution from 1 to ∞

$$\pi\int_1^\infty \left(\frac{1}{x}\right)^2 dx = \pi\lim_{a\to\infty}\int_1^a \frac{1}{x^2}dx = \pi\lim_{a\to\infty}\left[-\frac{1}{x}\right]_1^a = \pi * (1-0) = \pi$$

How can it be that an object with an infinite surface area under his curve has a finite volume when you rotate it around the axis?

I get the math behind it and I'm assuming there is nothing wrong with the math. But it seems very contra-intuitive because if you rotate an infinite surface area just a little fraction it should have an infinite volume, that's what my intuition tells me?. So can someone explain to me why this isn't like that, that an infinite surface area rotated around the axis can have a finite volume?

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edited Mar 20 '19 at 10:07
YuiTo Cheng
4,452 9 19 51

asked Mar 20 '19 at 10:06
Daan Seuntjens
177 1 10

This is Gabriel's horn. Here is the Wikipedia page for it, for anyone curious: en.wikipedia.org/wiki/Gabriel%27s_Horn. See the paradox sections especially. – Minus One-Twelfth Mar 20 '19 at 10:12

Well, your intuition is wrong. And that is probably the most standard example that you can have rotate an infinite area and get a solid of finite volume. (And @MinusOne-Twelfth have already given you the name of this construction) – Henrik supports the community Mar 20 '19 at 10:14

By the way, you have calculated the area under the curve, whereas the surface area of revolution is given by

$$\int_1^\infty 2\pi \cdot \frac{1}{x}\sqrt{1+\left(-\frac{1}{x^2}\right)^2}\,dx.$$

See e.g. tutorial.math.lamar.edu/Classes/CalcII/SurfaceArea.aspx. – Minus One-Twelfth Mar 20 '19 at 10:18

How can a line have infinite length but finite area? – Gunnar Sveinsson Mar 20 '19 at 16:30

For what it's worth, $\int \frac{\sqrt{1+x^4}}{x^3}dx = \frac{1}{2}\ln\left(\sqrt{1+x^4}+x^2\right) - \frac{\sqrt{1+x^4}}{2x^2} + C$. – J.G. Oct 11 at 9:51

2 Answers

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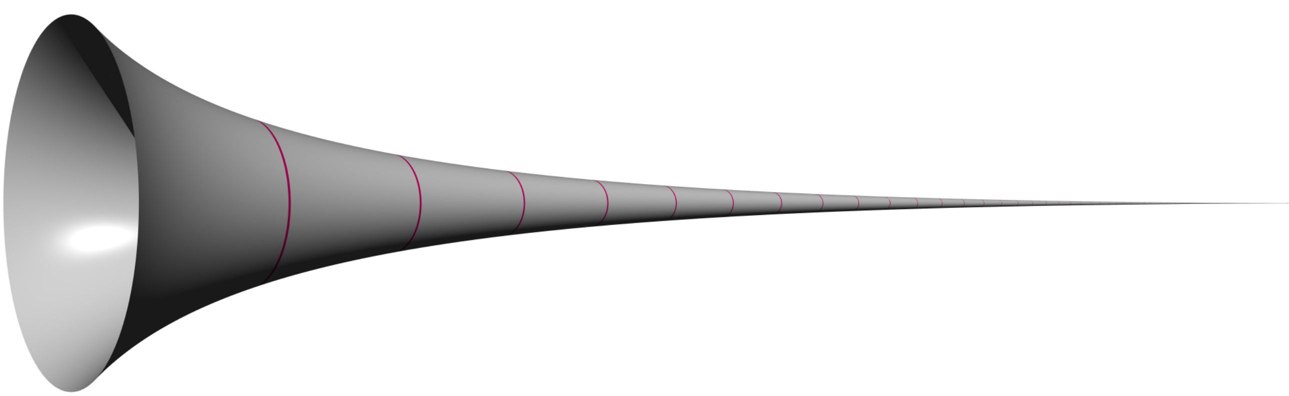
As @Minus One-Twelfth pointed out in the comments: this phenomenon is called Gabriel's horn.

Gabriel's horn is a geometric figure which has infinite surface area but finite volume.



How you can interpret the phenomenon:

You can treat the horn as a stack of disk on top of each other with radii that are diminishing. Every disk has a radius $r = \frac{1}{x}$ and an area of πr^2 or $\frac{\pi}{x^2}$.

- The sum of all the radii creates a series that is the same as the surface area under the curve

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- The sum of all the area's of all the disks creates a series that is the same as the volume of the revolution.

The series $\frac{1}{x}$ diverges but $\frac{1}{x^2}$ converges. So the area under the curve is infinite and the volume of the revolution is finite.

This creates a paradox: you could fill the inside of the horn with a fixed volume of paint, but couldn't paint the inside surface of the horn. This paradox can be explained by using a 'mathematically correct paint', meaning that the paint can be spread out infinitely thin. Therefore, a finite volume of paint can paint an infinite surface.

edited Oct 11 at 9:30

answered Mar 20 '19 at 11:59

Daan Seuntjens

177 1 10

0

A simple way to visualize this is in terms of Pappus's 2^{nd} Centroid Theorem.

Pappus's 2^{nd} Centroid Theorem says the volume of a planar area of revolution is the product of the area A and the length of the path traced by its centroid R, i.e., $2\pi R$. The bottom line is that the volume is given simply by $V = 2\pi RA$. The centroid of a volume is given by

$$\mathbf{R} = \frac{\int_A \mathbf{r} dA}{\int_A dA} = \frac{1}{A}\int_A \mathbf{r} dA$$

Now you can see that the product RA essentially eliminates any problems with the area and you are left with a *proper* intergral.

answered Mar 20 '19 at 16:14

Cye Waldman

5,143 2 8 26

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