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9

a) Init $\begin{cases} sum = -14 \\ product = 1 \\ i = 0 \end{cases}$ $\boxed{c = 4}$

Loop: ⊛ $i = 1 \rightarrow product = product . c = 4$

$sum = \cancel{-14 + 5.4}$ $sum + a_1 . product$

$= -14 + 5.4$
$= 6$

⊛ $i = 2 \rightarrow product = product . c = 16$

$sum = sum + a_2 . product$
$= 6 + -7 . 16$
$= 6 - 112$
$= -106$

⊛ $i = 3 \rightarrow product = product . c = 64$

$sum = sum + a_3 . product$
$= -106 + 2 . 64$
$= 22$

$\Rightarrow$ Return 22

b) In the loop:
The loop will repeat n times.
Each time, it operates: $\begin{cases} 2 \text{ multiplications} \\ 1 \text{ addition} \end{cases}$

$\Rightarrow$ number of operations $= (2+1) . n = 3n$

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(10)

a) Init $\begin{cases} result = 2 & c = 4 \\ i = 0 \end{cases}$

<u>Loop</u> : (*) $i = 1$.

$result = result \cdot c + a_2$
$= 2 \cdot 4 + -7$
$= 1$

(*) $i = 2$
$result = result \cdot c + a_1$
$= 1 \cdot 4 + 5$
$= 9$

(*) $i = 3$
$result = result \cdot c + a_0$
$= 9 \cdot 4 - 14$
$= 22$

$\Rightarrow$ Return 22.

b) The loop will repeats ~~10~~ n times
Each loop, it operates $\begin{cases} 1 \text{ addition} \\ 1 \text{ multiplication} \end{cases}$

$\Rightarrow$ number of operations: $(1+1) \cdot n = 2n$

c) number of operations in ex 10: $2 \cdot 98$
number of operations in ex 9: $3 \cdot 98$

$\Rightarrow$ saved operations: $3 \cdot 98 - 2 \cdot 98 = 98$

KOKUYO

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(13)

a) 1st pass:

i = 1 : 5, 6, 3, 4, 8, 2

i = 2 : 5, 3, 6, 4, 8, 2

i = 3 : 5, 3, 4, 6, 8, 2

i = 4 : 5, 3, 4, 6, 8, 2

i = 5 : 5, 3, 4, 6, 2, 8

2nd pass:

i = 1 : 3, 5, 4, 6, 2, 8

i = 2 : 3, 4, 5, 6, 2, 8

i = 3 : 3, 4, 5, 6, 2, 8

i = 4 : 3, 4, 5, 2, 6, 8

3rd pass:

i = 1 : 3, 4, 5, 2, 6, 8

i = 2 : 3, 4, 5, 2, 6, 8

i = 3 : 3, 4, 2, 5, 6, 8

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4th pass:

$i=1: 3,4,2,5,6,8$

$i=2: 3,2,4,5,6,8$

~~i=3~~

5th pass:

$i=1: 2,3,4,5,6,8$

b) To sort for nth element in the list, we need to perform $n-1$ comparisons and the number of comparisons for $n-1$th element.

$$\Rightarrow \begin{cases} B(n) = B(n-1) + n - 1 & n \geq 2 \\ B(1) = 0 \end{cases}$$

c) Base case: $n=1$: $B(1) = \frac{1.(1-1)}{2} = 0$

Assume: $B(k) = \frac{k(k-1)}{2}$

Show:

$$B(k+1) = B(k) + k \quad (rr)$$
$$= \frac{k(k-1)}{2} + k \quad (ih)$$
$$= \frac{k}{2}\left[k-1+2\right]$$
$$= \frac{k(k+1)}{2}$$

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14

a) The worst case happens when the given list is ~~afred~~ already sorted in descending order.

Then, with the 1st pass, number of exchanges is $n-1$. In the last pass, number of exchanges is 1.

$\Rightarrow$ Total number of exchanges is:

$$1+2+\dots+n-1 = \frac{n(n-1)}{2}$$

$\Rightarrow$ Total ~~of~~ number of exchanges and comparisons:

$$\frac{n(n-1)}{2} + \frac{n(n-1)}{2} = n(n-1)$$

b) The best case is when the given list is already sorted in ascending order. Therefore, number of exchanges is 0

$\Rightarrow$ Total number of exchanges and comparisons:

$$\frac{n(n-1)}{2}$$

c) $$\frac{n(n-1)}{2} + \frac{1}{2}\cdot\frac{n(n-1)}{2} = \frac{3n(n-1)}{4}$$

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First, we will prove:

$a \geq F(m+2) \quad \forall m \geq 1$
$b \geq F(m+1) \quad \forall m \geq 1.$

Base case: $m = 1$
$a \geq F(3) = 2$
$b \geq F(2) = 1$
this is true as $a > b \geq 1$.

Assume ~~that~~:
$a \geq F(m+2)$
$b \geq F(m+1)$

Show: m = steps to calculate gcd (a, b)
$\Rightarrow$ $m+1$ = steps to calculate gcd (c, b)
with $c = l.a + b$ $(l \in \mathbb{N})$
We have:
$c = l.a + b$ ~~ln~~
$\geq a + b$
$\geq F(m+2) + F(m+1)$
$\geq F(m+3)$ $\Rightarrow$ q.e.d

There fore, $a \geq F(m+2)$ $\forall m \geq 1$
$\Rightarrow a \geq F(m+2)$ $\forall m \geq 4$

Thứ

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Need to prove: $\left(\frac{3}{2}\right)^{m+1} < F(m+2) \quad \forall m \geq 4$

Base case: $m = 4$

$\Rightarrow \left(\frac{3}{2}\right)^{5} < F(6)$

$\Leftrightarrow 7.59375 < 8$ (true)

Assume: $\left(\frac{3}{2}\right)^{k+1} < F(k+2)$

Show: $\left(\frac{3}{2}\right)^{k+2} < F(k+3)$

We have: $F(k+3) = F(k+2) + F(k+1)$ (rr)

$$> \left(\frac{3}{2}\right)^{k+1} + \left(\frac{3}{2}\right)^{k} \quad \text{(ih)}$$

$$= \left(\frac{3}{2}\right)^{k+2} \cdot \frac{2}{3} + \left(\frac{3}{2}\right)^{k+2} \cdot \frac{4}{9}$$

$$= \left(\frac{3}{2}\right)^{k+2} \cdot \left(\frac{2}{3} + \frac{4}{9}\right)$$

$$= \left(\frac{3}{2}\right)^{k+2} \cdot \frac{10}{9}$$

$$> \left(\frac{3}{2}\right)^{k+2}$$

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40

a)

gcd (89, 55) = gcd (55, 34) m = 1

gcd (55, 34) = gcd (34, 21) m = 2

gcd (34, 21) = gcd (21, 13) m = 3

gcd (21, 13) = gcd (13, 8) m = 4

gcd (13, 8) = gcd (8, 5) m = 5

gcd (8, 5) = gcd (5, 3) m = 6

gcd (5, 3) = gcd (3, 2) m = 7

gcd (3, 2) = gcd (2, 1) m = 8

gcd (2, 1) = gcd (1, 1) m = 9

gcd (1, 1) = 1 m = 10

$\Rightarrow$ 10 divisions

b) $E(89) = 2\log_2 89 \approx 2.(6.48) < 13.$

c) $m < \log_{1.5} 89 - 1 \Leftrightarrow m < 11$

d) $b = 55 \Rightarrow m \le 5.2 = 10.$