7 $(\exists x)[P(x) \to Q(x)] \to [(\forall x)P(x) \to (\exists x)Q(x)]$

$\to$ $(\exists x)[P(x) \to Q(x)] \wedge (\forall x)P(x) \to (\exists x)Q(x)$

1.	$(\exists x)[P(x) \to Q(x)]$	hyp
2.	$P(a) \to Q(a)$	1, ei
3.	$(\forall x)P(x)$	hyp
4.	$P(a)$	3, ui
5.	$Q(a)$	2,4 mp
6.	$(\exists x)Q(x)$	5 eg

9

a) Domain: whole numbers

$P(x,y)$: $x+y<0$

$Q(x,y)$: $x+y>0$

An example to prove that given wff is invalid:

$x=1$

$(\exists y)P(x,y)$: choose $y=-2$

$(\exists y)Q(x,y)$: choose $y=1$

However, $(\forall x)(\exists y)[P(x,y) \wedge Q(x,y)]$ can not produce any y that makes both $P(x,y)$ and $Q(x,y)$ ~~to~~ true.

b) $(\exists y)P(x,y)$: this "y" in this expression only applies to $P(x,y)$

$(\exists y)Q(x,y)$: this "y" in this expression only applies to $Q(x,y)$

However, from step 1 to step 2, the proof assumes that both "y"s in the 2 expressions are the same.

(10)

a) Domain: whole numbers

$Q(x,y)$: $y > x$.

b) In step 3, y is a free variable, which means that step 4 cannot use ug to deduce.

(13) $(\exists x)(\exists y)P(x,y) \to (\exists y)(\exists x)P(x,y)$

1. $(\exists x)(\exists y)P(x,y)$ hyp
2. $(\exists y)P(a,y)$ 1, ei
3. $P(a,b)$ 2, ei
4. $(\exists x)P(x,b)$ 3, eg
5. $(\exists y)(\exists x)P(x,y)$ 4, eg

(15) $(\forall x)P(x) \wedge (\exists x)[P(x)]' \to (\exists x)Q(x)$

1. $(\forall x)P(x)$ hyp
2. $(\exists x)[P(x)]'$ hyp
3. $P(a)$ 1, ui
4. $[P(a)]'$ 2, ei
5. $Q(a)$ 3, 4 inc
6. $(\exists x)Q(x)$ 5, eg

(18) $(\exists x)[R(x) \vee S(x)] \to (\exists x)R(x) \vee (\exists x)S(x)$.

1. $(\exists x)[R(x) \vee S(x)]$ hyp
2. $R(a) \vee S(a)$ 1, ei
3. $\cancel{(\exists x)[R(x)}[(\exists x)R(x)]'$ temp. hyp
4. $(\forall x)[R(x)]'$ 3, neg
5. $[R(a)]'$ 4, ui
6. $S(a)$ 2,5 ds
7. $(\exists x)S(x)$ 6, eg

~~8. $(\exists x)R(x) \vee (\exists x)S(x)$ ∃~~

8. $[(\exists x)R(x)]' \to (\exists x)S(x)$ temp. hyp discharged

9. $[[(\exists x) R(x)]']' \vee (\exists x) S(x)$ 8, imp

10. $(\exists x) R(x) \vee (\exists x) S(x)$ 9, dn

22 $[(\forall x) P(x) \to (\forall x) Q(x)] \to (\forall x)[P(x) \to Q(x)]$.

1. $(\forall x) P(x) \to (\forall x) Q(x)$
2. $[(\forall x) P(x)]' \vee (\forall x) Q(x)$

Domain: whole numbers

$P(x): x = 0$
$Q(x): x > 0$

Since $P(x): x \neq 0$ / a $(\forall x) P(x)$ is false
$\Rightarrow [(\forall x) P(x) \to (\forall x) Q(x)]$ is always true.

Choose $x = 0$, then $(\forall x)[P(x) \to Q(x)]$ is false.

23 $(\exists x)(\forall y) Q(x,y) \to (\forall y)(\exists x) Q(x,y)$

1. $(\exists x)(\forall y) Q(x,y)$ hyp
2. $(\forall y) Q(a,y)$ 1, ei
3. $Q(a,y)$ 2, ui
4. $(\exists x) Q(x,y)$ 3, eg
5. $(\forall y)(\exists x) Q(x,y)$ 4, ug

(26) $(\forall y)[Q(x,y) \to P(x)] \to [(\exists y) Q(x,y) \to P(x)]$

$\to (\forall y)[Q(x,y) \to P(x)] \wedge (\exists y) Q(x,y) \to P(x)$

1. $(\forall y)[Q(x,y) \to P(x)]$ hyp
2. $(\exists y) Q(x,y)$ hyp
3. $Q(x,a)$ 2, ei
4. $Q(x,a) \to P(x)$ 1, ui
5. $P(x)$ 3,4 mp

(28) $(\forall x)(P(x) \vee Q(x)) \wedge (\exists x) Q(x) \to (\exists x) P(x)$.

Domain : whole numbers.
$P(x)$: $x < 1$
$Q(x)$: $x \geq 1$.

Choose $x = 1$, then the given wff is invalid.

(31)
a) 1. $(\forall x)(M(x) \to P(x))$ hyp
2. $(\forall x)(S(x) \to M(x))$ hyp
3. $M(x) \to P(x)$ 1, ui
4. $S(x) \to M(x)$ 2, ui

5. $S(x) \rightarrow P(x)$ 3,4 hs

6. $(\forall x)[S(x) \rightarrow P(x)]$ 5, ug

b) 1. $(\forall x)(M(x) \rightarrow P(x))$ hyp

2. $(\exists x)(S(x) \wedge M(x))$ hyp

3. $M(a) \rightarrow P(a)$ 1, ui

4. $S(a) \wedge M(a)$ 2, ei

5. $S(a)$ 4, simp

6. $M(a)$ 4, simp

7. $P(a)$ 3,6 mp

8. $S(a) \wedge P(a)$ 5, 7 con

9. $(\exists x)(S(x) \wedge P(x))$ 8, eg

b) 1. $(\forall x)[M(x) \rightarrow (P(x))']$ hyp

2. $(\exists x)[S(x) \wedge M(x)]$ hyp

3. $M(a) \rightarrow (P(a))'$ 1, ui

4. $S(a) \wedge M(a)$ 2, ei

5. $S(a)$ 4, simp

~~6. $M(a)$ simp~~

~~7. $(P(a))'$ 3,6 mp~~

b) 1. $(\forall x)[M(x) \to (P(x))']$ hyp

2. $(\forall x)[S(x) \to M(x)]$ hyp

3. $M(x) \to (P(x))'$ 1, ui

4. $S(x) \to M(x)$ 2, ui

5. $\cancel{M(x)}\ S(x) \to (P(x))'$ 3,4 hs

6. $(\forall x)(S(x) \to (P(x))')$ 5, ug

d) 1. $(\forall x)[M(x) \to (P(x))']$ hyp

2. $(\exists x)[\cancel{M}\ S(x) \wedge M(x)]$ hyp

3. $M(a) \to (P(a))'$ 1, ui

4. $S(a) \wedge M(a)$ 2, ei

5. $S(a)$ 4, simp

6. $M(a)$ 4, simp

7. $(P(a))'$ 4,6 mp

8. $S(a) \wedge (P(a))'$ 5,7 con

9. $(\exists x)[S(x) \wedge (P(x))']$ 8, eg.

34

1. $(\exists x)[A(x) \wedge (N(x))']$ hyp
2. $(\forall x)[G(x) \rightarrow N(x)]$ hyp
3. $(\forall x)[G(x) \vee C(x)]$ hyp
4. $A(a) \wedge (N(a))'$ 1, ei
5. $G(a) \rightarrow N(a)$ 2, ui
6. $G(a) \vee C(a)$ 3, ui
7. $(N(a))'$ 4, simp
8. $(N(a))' \rightarrow (G(a))'$ 5, cont
9. $(G(a))'$ 7,8 mp
10. $A(a)$ 4, simp
11. $C(a)$ 6,9 ds
12. $A(a) \wedge C(a)$ 10,11 con
13. $(\exists x)[A(x) \wedge C(x)]$ 12, eg

42

1. $(\forall x)[F(x) \to (\exists y)[C(y) \wedge O(x,y)]]$ hyp

2. $(\forall x)(\forall y)[\cancel{D(x)}\ O(x,y) \wedge C(y) \to (D(x))']$ hyp

3. $F(x) \to (\exists y)[C(y) \wedge O(x,y)]$ 1, ui

4. $F(x) \to C(a) \wedge O(x,a)$ 3, ei

5. $(\forall y)[O(x,y) \wedge C(y) \to (D(x))']$ 2, ui

6. $O(x,y) \wedge C(y) \to (D(x))'$ 5, ui

7. $D(x)$ temp. hyp

~~8. $O(x,y) \wedge C(y)$ 6, 7~~

8. $(D(x))''$ 7, dn

9. $(O(x,y) \wedge C(y))'$ 6, 8, mt

10. $(\forall y)(O(x,y) \wedge C(y))'$ 9, ug

11. $[(\exists y)[O(x,y) \wedge C(y)]]'$ 10, neg

12. $(F(x))'$ 3, 11 mt

13. $D(x) \to (F(x))'$ temp. hyp discharged

14. $\cancel{\forall(x)}\ (\forall x)[D(x) \to (F(x))']$ 13, ug