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$$= 2 \cdot 2^{m} \cdot (2^{m} \cdot 2 - 1) + (2^{m+1})^{2}$$

$$= 2 \cdot 2^{2m+1} - 2 \cdot 2^{m} + 2^{2m+2}$$

$$= 2^{m+1}\left(2^{m+1} + 2^{m+1} - 1\right)$$

$$= 2^{m+1}\left(2 \cdot 2^{m+1} - 1\right).$$

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Show:

$$S(k+1) = S(k) + 2k+1 \quad (rr)$$
$$= k^2 + 2k+1 \quad (ih)$$
$$= (k+1)^2$$

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$S(1) = 1 \qquad S(2) = \cancel{2}\ 3$

$S(n) = S(n-1) + S(n-2) \quad n \geq 3$

Explain: A string of length n which has no two consecutive zero's can be generated in 2 ways:

+ Append 1 at the end of a string ~~lenght~~ length n-1 which has no two consecutive zeros
 $\Rightarrow$ there are $S(n-1)$ strings

+ Append 10 at the end of a string length n-2 which has no two consecutive zeros
 $\Rightarrow$ there are $S(n-2)$ strings

$\Rightarrow S(n) = S(n-1) + S(n-2)$

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Base case: ~~S(1)~~ $n=1$: $S(1) = 1$ $(2.1 - 1) = 1$ (true)
Assume: $S(k) = k.(2k-1)$ with $k = 2^m$
Show:
$$S(2^{m+1}) = 2.S(2^m) + (2^{m+1})^2 \quad (rr)$$

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$\Leftrightarrow \log_{\frac{6}{5}} 4 + 1 \le n$

$\Leftrightarrow 8.6 \le n$

$\rightarrow \boxed{n = 9.}$

Conclusion

(23)

$S(1) = 1$
$S(n) = S(n-1) + n \quad n \geq 2$

Base case: $n = 1$: ~~S(n)~~ $S(1) = \frac{1.(1+1)}{2} = 1$

Assume: $S(k) = \frac{k(k+1)}{2}$

Show:

$S(k+1) = S(k) + k+1$ (rr)
$= \frac{k(k+1)}{2} + k+1$
$= \cancel{k\left(\frac{k+1}{2} + 1\right)}\ (k+1)\left[\frac{k}{2} + 1\right]$
$= \frac{(k+1)(k+2)}{2}$

(24) $S(1) = 1$
$S(n) = S(n-1) + 2n - 1$

Base case: $n = 1$: $S(1) = 1^2 = 1$
Assume: $S(k) = k^2$

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b)
Base case: $n=1$: $S(1) = 10^{1+2} = 1000$ (true)
Assume: $S(k) = 10^{k+2}$
Show:
$S(k+1) = 10 \cdot S(k)$ (recurrence relation)
$= 10 \cdot 10^{k+2}$ (inductive hyp)
$= 10^{k+3}$

c) $S(21) = 10^{21+2} = 10^{23}$

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a) $S(1) = 3$
$S(n) = 5 \cdot S(n-1) - 6^{n-2} \quad n \geq 2$

b)
Base case: $n=1$: $S(1) = 4 \cdot 5^{1-1} - 6^{1-1} = 3$ (true)
Assume: $S(k) = 4 \cdot 5^{k-1} - 6^{k-1}$
Show:
$S(k+1) = 5 \cdot S(k) - 6^{k-1}$ (rr)
$= 5 \cdot (4 \cdot 5^{k-1} - 6^{k-1}) - 6^{k-1}$ (ih)
$= 4 \cdot 5^{k} - 5 \cdot 6^{k-1} - 6^{k-1}$
$= 4 \cdot 5^{k} - 6^{k}$

c) $S(n) \leq 0$
$\Leftrightarrow 4 \cdot 5^{n-1} - 6^{n-1} \leq 0$
$\Leftrightarrow 4 \leq \left(\frac{6}{5}\right)^{n-1}$
$\Leftrightarrow \log_{\left(\frac{6}{5}\right)} 4 \leq n-1$

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(1)

Base case: $n = 1$: $S(n) = 5.1 = 5$ (true)
Assume: $S(K) = 5.K$
Show: $S(K+1) = S(K) + 5$ (recurrence relation)
$= 5.K + 5$ (inductive hypothesis)
$= 5(K+1)$

(6)

Base case: $n = 1$: $S(1) = 1^2 = 1$ (true)
Assume: $S(K) = K^2$
Show: $S(K+1) = \cancel{S(K-1)}\ S(K) + (2K+1)$
$= K^2 + 2K + 1$
$= (K+1)^2$

(12)

Base case: $n = 1$: $S(1) = 3^{1-1}.(1+1)! = 2$ (true)
Assume: $S(K) = 3^{K-1}.(K+1)!$
Show:
$S(K+1) = 3(K+2).S(K)$ (recurrence relation)
$= 3.(K+2).3^{K-1}.(K+1)!$ (inductive hyp)
$= 3^K.(K+2)!$

(15)

a) $S(1) = 1000$
$S(n) = 10.S(n-1)$ $n \geq 2$

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