(2)

a) p: 8 is even → true
q: 6 is odd → false

$\Rightarrow$ $p \vee q$ = true $\vee$ false = true

b) p: 8 is even → true
q: 6 is odd → false

$\Rightarrow$ $p \wedge q$ = true $\wedge$ false = false

c) p: 8 is odd → false
q: 6 is odd → false

$\Rightarrow$ $p \vee q$ = false $\vee$ false = false

d) p: 8 is odd → false
q: 6 is odd → false

$\Rightarrow$ $p \wedge q$ = false $\wedge$ false = false

e) p: 8 is odd → false
q: 6 is odd → false

$\Rightarrow$ $p \to q$ = false → false = true

f) p: 8 is even → true
q: 6 is odd → false

$\Rightarrow p \to q =$ true → false = false

g) p: 8 is odd → false
q: 6 is even → true

$\Rightarrow p \to q =$ false → true = true

h) p : 8 is odd → false
q : 6 is even → true

$\Rightarrow p \wedge q =$ false $\wedge$ true = ~~true~~ false

r: $8 < 6$ → false

$\Rightarrow (p \wedge q) \to r$ ~~= true → false = false~~

= false → false
= true

(9)

a)

1. correct
2. incorrect
3. correct.

b) q: Cucumbers are seedy
p: Cucumbers are green

Given statement: $p' \cancel{\vee} q$ $p \wedge q$

Then, negation forms: $(p \wedge q)' \Leftrightarrow p' \vee q'$

$\Rightarrow$ Correct negations: 2

c) p: $2 < 7$
q: 3 is odd

Given statement: $p \wedge q$

Then, negation forms: $(p \wedge q)' \Leftrightarrow p' \vee q'$

$\Rightarrow$ Correct negations: 4

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d) ~~p//.~~ p: the food is good
q: the service is excellent.

⇒ Given statement: $p' \wedge q'$

⇒ Negation forms: $(p' \wedge q')' \longleftrightarrow p \vee q$

⇒ Sentence: Either The food is good or the service is ~~g~~ excellent.

e) r: the price is high
p: the food is good
q: the service is excellent

⇒ Given statement: $r \rightarrow (p \wedge q)$

⇒ Negation form: $r \wedge (p \wedge q)' \Longleftrightarrow r \wedge (p' \vee q')$

⇒ Sentence:

The ~~high/is~~ price is high, and either the food is not good or the service is not excellent.

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c) $B \to (A \wedge C)$

d) $\cancel{(B' \vee C') \to A}$ $\quad A \to (B' \vee C')$

e) $\cancel{A' \wedge C'}$

$A \wedge [C' \to (B' \vee C)]$

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b) Violets aren't blue, or, if roses are red, then sugar is sweet.

c) Sugar is sweet and roses are not red, if and only if violets are blue.

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a) H: the horse is fresh
K: the knight will win

$\Rightarrow \quad H \to K$

b) K: the knight will win
H: the horse is fresh
A: the amor is strong.

$\Rightarrow \quad K \leftrightarrow (H \land A)$

c) H: A fresh horse
K: the knight to win

$\Rightarrow \quad K \rightarrow H$

d) K: the knight will win
A: the armor is strong

$\Rightarrow \quad K \leftrightarrow A$

e) K: the knight to win
H: the horse is fresh
A: the amor is strong

$\Rightarrow \quad (A \lor H) \rightarrow K$

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a)

A	B	$A \to B$	A'	$A' \vee B$	$(A \to B) \leftrightarrow (A' \vee B)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

→ tautology

$(A \wedge B) \vee C \to A \wedge (B \vee C)$

b)

A	B	C	$A \wedge B$	$(A \wedge B) \vee C$	$B \vee C$	$A \wedge (B \vee C)$	
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	F	F	F	T
F	T	T	F	T	T	F	F
F	T	F	F	F	T	F	T
F	F	T	F	T	T	F	F
F	F	F	F	F	F	F	T

c)

A	B	A'	B'	$A' \vee B'$	$(A' \vee B')'$	$A \wedge (A' \vee B')$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	T	F	F

d)

A	B	$A \wedge B$	A'	$A \wedge B \to A'$
T	T	T	F	F
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

e)

A	B	C	$A \to B$	$A \vee C$	$B \vee C$	$(A \vee C) \to (B \vee C)$	$(A \to B) \to [(A \vee C) \to (B \vee C)]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	F	F	T	T

$\Rightarrow$ tautology

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a) Using contradiction to prove.

$$\Rightarrow \begin{cases} A : \text{false} \\ (A \wedge B) \wedge B' : \text{true} \end{cases}$$

Since $(A \wedge B) \wedge B'$: true

$$\Rightarrow \begin{cases} A \wedge B : \text{true} \\ B' : \text{true} \end{cases} \iff \begin{cases} A \wedge B : \text{true} \\ B : \text{false} \quad (1) \end{cases}$$

~~c) Using contradiction to prove~~

~~$\Rightarrow \left\{ A \wedge B \right.$~~

a) $[B' \wedge (A \to B)] \to A'$

Using contradiction to prove

$$\Rightarrow \begin{cases} A' : \text{False} \\ B' \wedge (A \to B) : \text{true} \quad (1) \end{cases}$$

$$\Leftrightarrow \begin{cases} A : \text{true} \\ B' : \text{true} \quad (\text{from } (1)) \\ A \to B : \text{true} \quad (\text{from } (1)) \end{cases}$$

$$\Leftrightarrow \begin{cases} A : \text{true} & (2) \\ B : \text{false} & (3) \\ A \to B : \text{true} & (4) \end{cases}$$

$\xrightarrow{(2),(3)}$ $A \to B$: false , which conflicts with (4)

$\Rightarrow$ q.e.d

c) $(A \vee B) \wedge A' \to B$

Using contradiction to prove

$$\Rightarrow \begin{cases} B : \text{False} \\ (A \vee B) \wedge A' : \text{true} \quad (1) \end{cases}$$

$$\Leftrightarrow \begin{cases} B : \text{false} \\ A' : \text{true} \quad (\text{from } (1)) \\ A \vee B : \text{true} \quad (\text{from } (1)) \end{cases}$$

$(\Leftarrow)$ $\begin{cases} B: \text{false} \\ A: \text{false} \\ A \vee B: \text{true} \quad (2) \end{cases}$

Since A: false and B: false

$\Rightarrow$ $A \vee B$: false, which conflicts with (2)

$\Rightarrow$ q.e.d

(47)

a) Using truth table

A	B	$A \vee B$	A'	B'	$A' \wedge B'$	$(A' \wedge B')'$	$(A \vee B) \leftrightarrow (A' \wedge B')'$
T	T	T	F	F	F	T	T
T	F	T	F	T	F	T	T
F	T	T	T	F	F	T	T
F	F	F	T	T	T	F	T

b) Using truth table

A	B	$A \to B$	B'	$A \wedge B'$	$(A \wedge B')'$	$(A \to B) \leftrightarrow (A \wedge B')'$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(48)

(*) $A \wedge B$ is equivalent to $(A' \vee B')'$

A	B	$A \wedge B$	A'	B'	$A' \vee B'$	$(A' \vee B')'$	$(A \wedge B) \leftrightarrow (A' \vee B')'$
T	T	T	F	F	F	T	T
T	F	F	F	T	T	F	T
F	T	F	T	F	T	F	T
F	F	F	T	T	T	F	T

(*) $A \rightarrow B$ is equivalent to $A' \vee B$

A	B	$A \rightarrow B$	A'	$A' \vee B$	$(A \rightarrow B) \leftrightarrow (A' \vee B)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

(49)

(*) $A \vee B$ is equivalent to $A' \rightarrow B$

A	B	$A \vee B$	A'	$A' \rightarrow B$	$(A \vee B) \leftrightarrow (A' \rightarrow B)$
T	T	T	F	T	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	F	T

(*) $A \wedge B$ is equivalent to $(A \to B')'$

A	B	$A \wedge B$	B'	$A \to B'$	$(A \to B')'$	$(A \wedge B) \leftrightarrow (A \to B')'$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	F	F	T	F	T
F	F	F	T	T	F	T

(57)

We have: (A, B, C and D mean manchines A, B, C and D is infected)

a. $D \to C$

b. $C \to A$

c. $D' \to (B' \wedge C)$

d. $A \to (B \vee C')$

Examine 2 cases:

(*) If D is true, then C is true.
Since C is true, A is true.
Since A is true, then $B \vee C'$ is true. But C is true, then B is true

$\Rightarrow$ A, B, C and D are true

$\Rightarrow$ In this case, 4 machines are infected.

(*) If D is false, then D' is true.
Since D' is true, then $B' \wedge C$ is true.
Since $B' \wedge C$ is true, then B' is true and C is true.
Since C is true, then A is true
Since A is true, then $B \vee C'$ is true
Since C is true and $B \vee C'$ is true, then B is true

Since both B and B' are true, then this case cannot happen.

$\Rightarrow$ Conclusion: All 4 machines are infected.