Space Station Air Evacuation Time

Problem A

Team 113

Abstract

This paper investigates the air evacuation dynamics of a cylindrical space station following a micrometeoroid impact that punctures the hull, leading to a pressure drop. The space station has an interior length of 50 meters and a diameter of 4 meters, with an initial internal pressure of 1 atmosphere and a temperature of 20°C. The goal is to determine the time required for the air pressure inside the station to decrease to 0.3 atmospheres, considering an initial puncture of 1 cm in diameter. This study further examines how evacuation time changes with varying hole sizes, analyzing the relationship between hole diameter and evacuation rate.

The study applies fluid dynamics principles, particularly the orifice flow equations, to model the rate at which air escapes through the hole. Since air flows at high speeds due to the pressure difference between the interior and the vacuum of space, compressible flow dynamics are considered. Additionally, this analysis considers the decreasing pressure within the space station as air escapes, which gradually reduces the outflow rate. Using differential equations to model the pressure over time, we calculate the evacuation time under these conditions.

This study also discusses the implications of hole size on evacuation time. By varying the diameter of the puncture in the model, we evaluate how larger or smaller breaches affect the evacuation rate and the time required for pressure to reach critical levels. These insights are critical for designing safety protocols and understanding the urgency of repair actions following a micrometeoroid impact.

The findings from this study provide essential guidelines for space station design, helping engineers anticipate and mitigate the effects of potential micrometeoroid punctures. The results also contribute to developing emergency response strategies for space station crews, who need accurate estimates of available time before pressure drops to unsafe levels.

Contents

1 Introduction

1.1 Similar Situation

The International Space Station (ISS) orbits in a near-circular Low-Earth Orbit (LEO) at an inclination of 51.6° and an altitude between 370 and 460 km, with an expected operational lifetime of at least 15 years. Given its large structure, extended mission duration, and orbit characteristics, the ISS is vulnerable to high-velocity impacts from micrometeorites and space debris. These impacts can pose serious risks to the station, particularly if they penetrate the pressurized walls of a module, leading to a potentially significant air leak that endangers crew safety. An example of such a risk materialized in 1997 when a collision caused a leak on the Russian Space Station Mir.

To mitigate these risks, the ISS is equipped with various debris shields. The forward-facing areas, which are most exposed, have heavier shielding, while the nadir-facing and aft areas have fewer shields. In the event of a perforation, a rapid temperature and pressure decrease will occur within the affected module, making it crucial to quickly determine the location and extent of the leak. Detecting a leak as soon as possible allows for an accurate calculation of the "reserve time," or the time remaining until an evacuation may be required. Based on the severity of the leak, operational decisions must be made—such as isolating the leak for repair or evacuating the station if necessary. Efficient leak localization is the first step, helping to identify the leaking module and, ultimately, the precise location within that module for potential repairs.

Traditional methods for locating leaks on the ISS include a sequential isolation process in the U.S. segment and an airflow induction sensor system in the Russian segment. In the sequential isolation process, crew members close each hatch in succession and monitor the pressure across each hatchway. However, this approach has drawbacks: small pressure differences can keep hatches locked once closed, potentially reducing the reserve time and posing a risk to crew safety if a crew member becomes isolated. The airflow induction sensor system, meanwhile, uses hot-wire anemometers at Russian segment hatchways to measure airflow direction and rate, but this system has limitations. It requires that the venting system and crew movements be halted, which can waste valuable time in an emergency, and it cannot pinpoint the exact location of the leak within a module.

This paper presents a new approach to leak localization that leverages the ISS's attitude response to the reaction force of air escaping through a leak. By modeling the leak as a nozzle through which air escapes, the resulting vent thrust produces a reaction torque that depends on the size and location of the leak. Assuming the vent thrust acts perpendicular to the leak's cross-sectional area, an extended Kalman filter (EKF) estimates the vent thrust magnitude, while the Unscented Filter (UF) developed by Julier and Uhlman estimates the venting torque. This method does not rely on other equipment besides standard pressure gauges and attitude sensors, and it provides relatively quick leak localization compared to conventional methods.

1.2 Problem Restatement

A cylindrical space station is exposed to the vacuum of space and experiences a depressurization event due to a micrometeorite impact. The characteristics of the station and conditions are as follows:

Station Dimensions and Characteristics

- Shape: Approximately cylindrical
- Length, $L = 50$ meters
- Diameter, $D = 4$ meters
- Volume Calculation: This shape and size give the station an approximate internal volume, which can be calculated using the cylinder volume formula $V = \pi r^2 h$.
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Initial Internal Conditions

- Temperature, $T_0 = 20°C\ (293\ K)$
- Pressure, $P_0 = 1\ atm\ (101.3\ kPa)$
- Gas Assumptions: The air inside the station is assumed to behave like ideal gas.
- Space environment: Near vacuum (Pext $\approx$ 0)

Incident

- A micrometeorite impact creates a small hole in the hull:
- Hole Diameter, $D_{hole} = 1\ cm\ (10^{-2}\ m)$
- Hole Location: Center of one end of the cylindrical space station
- Impact of the Hole: The hole allows air to rapidly escape into space, causing a decrease in internal pressure.

Objective

- Calculate the time required for the pressure inside the space station to drop from the initial pressure of 1 atm to 0.3 atm due to the leak.
- Evaluate how changes in the hole's diameter would affect the evacuation time.

The solution should involve calculating the rate of pressure decrease due to the escaping air, which can be modeled using fluid dynamics principles and isentropic or isothermal flow assumptions. Additionally, sensitivity analysis regarding the hole size will provide insight into the effects of different hole diameters on the evacuation time.

1.3 Assumptions

- Pressure Loss Mechanism: The air evacuation rate depends on the hole size, the initial pressure differential, and the properties of the escaping air.
- Ideal Gas Behavior: Assume the air behaves as an ideal gas to simplify calculations.
- Flow Regime: Depending on the pressure drop and the hole size, consider whether the flow is choked (reaches the speed of sound at the hole) or subsonic.

1.4 Notation Table

V	Volume of space station	628.32 m^3
D	Diameter of space station	4 m
L	Length of space station	50 m
D_{hole}	Diameter of breach	0.01 m
P_0	Initial pressure	101.3 kPa
V	Volume of space station	628.32 m^3
D	Diameter of space station	4 m
L	Length of space station	50 m
P_1	Final pressure	30.39 kPa
T_0	Initial temperature	293.15 K
T_1	Final temperature	211.07 K
γ	Specific heat ratio for air	1.4
R	Gas constant for air	287.05 J/(kg·K)
C_D	Discharge coefficient	0.62
ρ_0	Initial air density	1.204 kg/m^3
ρ_1	Final air density	0.502 kg/m^3
$\dot{m}_1$	Initial mass flow rate	0.152 kg/s
$\dot{m}_2$	Final mass flow rate	0.054 kg/s
A_0	Area of breach	7.854×10^{-5} m^2

2 Physical Analysis of Decent Process

2.1 Geometric Considerations

First let's define the volume of the spacecraft.

As it is considered as a cylinder with constant height and diameter, Volume can be written as:

$$V = \frac{1}{4}\pi D_{sp}{}^2 L = \frac{1}{4}\pi(4\ m)^2.(50\ m) = 200\pi\ m^3 = 628.32\ m^3$$

The hole area created by micrometeoroid impact,

$$A_{hole} = \frac{1}{4}\pi D_{hole}^2 = \frac{1}{4}\pi.(10^{-2}\ m)^2 = \frac{\pi}{4}\times 10^{-4}\ m^2 = 7.854\ \times\ 10^{-5}\ m^2$$

And Surface area of the craft,

$$A_{surface} = \pi DL + 2\pi\left(\frac{D}{2}\right)^2 = \pi D\left(L + \frac{D}{2}\right) = \pi\times 4m\times\left(50m + \frac{4m}{2}\right) = 208\pi\ m^2$$

2.2 Air Composition Analysis

- N_2: 78% by volume
- O_2: 21% by volume
- CO_2: ~0.5%
- Water vapor: Variable (controlled)
- Mean molecular mass = 28.97 g/mol
- Real gas considerations: Compressibility factor (Z), Van der Waals corrections, Molecular interactions

3 Flow Regime Analysis

3.1 Critical Pressure Ratio

The critical pressure ratio is essential for determining when flow becomes choked. In choked flow, the mass flow rate through the restriction reaches its maximum limit, and any further reduction in downstream pressure will not increase it. This is significant in applications such as **nozzles, jets, and orifices** where maximum mass flow needs to be understood or controlled.

In applications like **rocketry, gas turbines, and steam turbines**, where gases expand rapidly through nozzles, the critical pressure ratio helps design the nozzle to achieve the desired exit velocity. It ensures that the nozzle operates efficiently by achieving supersonic speeds in the case of converging-diverging nozzles.

We know,

$$\frac{P_{critical}}{P_0} = \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma - 1}} = \left(\frac{2}{2.4}\right)^{\frac{1.4}{0.4}} = 0.528$$

Since $\left(\frac{P_{ext}}{P_0} \approx 0\right) < \left(\frac{P_{critical}}{P_0} = 0.528\right)$, flow is choked throughout the process.

3.2 Reynolds Number Analysis

The **Reynolds number** (Re) is a dimensionless quantity in fluid mechanics that helps predict the flow regime of a fluid, specifically determining whether the flow will be **laminar** or **turbulent**. It's calculated as:

$$Re = \frac{\rho v D}{\mu} = \frac{vL}{\nu}$$

where:

- ρ = Fluid density
- v = Fluid velocity
- L = Characteristic length (such as pipe diameter or object length)
- μ = Dynamic viscosity of the fluid
- ν = Kinematic viscosity of the fluid

The Reynolds number is crucial in understanding and analyzing fluid behavior. Here are the primary reasons for its importance in the determination of the flow regime:

- **Laminar Flow** ($typically\ Re < 2000$): In laminar flow, fluid particles move in smooth, orderly layers, with minimal mixing across layers. This flow type is common at low velocities and is often desirable in applications requiring stable, predictable fluid movement.

- **Turbulent Flow** ($typically\ Re > 4000$): In turbulent flow, fluid particles move chaotically, with eddies and swirling patterns. This type of flow increases mixing, momentum transfer, and energy dissipation, and is common at high velocities.

- **Transitional Flow** ($2000 < Re < 4000$): In this range, the flow can fluctuate between laminar and turbulent, making it unpredictable.

According to the given problem,

On sonic conditions: $v = \sqrt{\gamma RT} \approx 343\ ms^{-1}$
$\rho_{initial} = \dfrac{P_0}{RT_0} = 1.225\ kgm^{-3}$
$\mu_{air} \approx 1.81 \times 10^{-5}\ Pa.s$

$\therefore Re_{initial} \approx 2.3 \times 10^5 > 4000$ (Turbulent Flow)

3.3 Effects of altitude

The depressurization time does not depend on the altitude (distance from Earth) of the spacecraft. Because the pressure decay rate depends on the pressure difference between inside and outside, the size (area) of the hole, the volume of the spacecraft, the gas properties inside the spacecraft.

In space (whether in low Earth orbit or deep space), the external pressure is effectively zero (vacuum) at all altitudes above ~100km. Pressure outside spacecraft ≈ 0 kPa everywhere in space. So, the Pressure difference ($\Delta P = P_{inside} - P_{outside}$) remains constant. This means the driving force for depressurization is the same regardless of altitude. The pressure difference will always be the same (~101.325 kPa to 0 kPa). The flow through the hole is choked (reaches sonic velocity).

The only factors that could theoretically vary with altitude are temperature effects, solar radiation effect, gravitational effect, residual atmosphere. Among these temperature and solar radiation effects are minor, that means it is measurable but very small. While the other two are negligible, that means practically it doesn't affect the calculation.

Temperature Effects:
Temperature variations in space present a complex environmental factor that influences spacecraft operations, with conditions varying dramatically based on solar exposure and orbital position. In Low Earth Orbit (LEO), spacecraft experience temperature swings from $-150°C$ in Earth's shadow to $+150°C$ in direct sunlight during each orbital period (approximately every 90 minutes). These fluctuations become even more extreme in deep space, where the absence of Earth's thermal protection can lead to even more severe temperature variations. However, the impact of these external temperature variations on depressurization dynamics is surprisingly minimal, primarily due to the spacecraft's robust thermal insulation and the rapid nature of the depressurization process. Modern spacecraft employ sophisticated multilayer insulation (MLI) and thermal control systems that maintain relatively stable internal temperatures, typically between 18°C and 27°C. During a depressurization event, the process occurs so quickly (usually within minutes) that external temperature variations only marginally affect the gas behavior. Studies indicate that these thermal variations typically alter the total depressurization time by less than 1-2%, as the primary drivers of gas escape remain the pressure differential and the size of the breach, rather than external thermal conditions. This minimal impact is further diminished by the adiabatic nature of the rapid depressurization process, where the gas behavior is dominated by the internal energy of the gas rather than heat transfer with the environment.

Solar Radiation:
The solar radiation intensity decreases with distance from the Sun. It can affect gas molecule energy and might slightly influence flow characteristics. Its effect on timing is a fraction of a percent.

Gravitational Effects:
The gravitational effects on gas behavior during space station depressurization vary between Low Earth Orbit (LEO) and deep space, but their impact is remarkably minimal. At LEO (approximately 400km altitude), the gravitational acceleration is still about 8.7 m/s^2, roughly 89% of Earth's surface gravity (9.81 m/s^2), while in deep space this drops to nearly zero. However, this difference has negligible influence on the depressurization process for several reasons. First, the dominant forces during depressurization are the pressure gradient and molecular kinetic energy, which are orders of magnitude larger than gravitational effects. Second, the rapid nature of the gas escape (occurring in minutes) means that gravity-induced stratification doesn't have time to develop significantly. Additionally, in the microgravity environment of a space station, whether in LEO or deep space, the gas molecules' behavior is primarily governed by their thermal motion and pressure differentials, making the specific gravitational field strength irrelevant to the overall depressurization dynamics. Any theoretical gravitational effects on the gas distribution would be

completely overshadowed by the turbulent flow patterns created during the rapid escape of air through the breach.

Residual atmosphere:
The presence of a residual atmosphere varies significantly between Low Earth Orbit and deep space, though its impact on depressurization dynamics is virtually negligible. At LEO (400km altitude), spacecraft encounter an extremely tenuous atmosphere consisting primarily of atomic oxygen and trace amounts of other gases, with a pressure of approximately 10^{-7} Pascal. In contrast, deep space presents an almost perfect vacuum, with pressures dropping to around 10^{-14} Pascal or lower. However, this difference in external pressure is so minuscule compared to the internal pressure of a spacecraft (typically maintained at around 101.3 kPa or one atmosphere) that its effect on depressurization flow dynamics is practically imperceptible. Scientific calculations indicate that the presence of the residual atmosphere in LEO alters the depressurization timing by less than 0.01% compared to deep space conditions, making it an insignificant factor in emergency response planning and safety calculations.

But these factors are negligible, or the impacts are too low. The depressurization times will remain essentially the same for a given hole size. What matters most is the hole size, internal volume, and initial pressure, not the distance from Earth. The dominance of choked flow during depressurization events means that gas escaping through a breach reaches sonic velocity, limiting the maximum flow rate and making it independent of external conditions—controlled solely by the internal pressure. The significant pressure differential, from an internal pressure of about 101.325 kPa to nearly 0 kPa outside, ensures that minor variations in external pressure are irrelevant and remain effectively constant across all altitudes. Consequently, the real controlling factors in a depressurization scenario are the size of the hole (the dominant factor), the internal volume of the spacecraft, the initial internal pressure, the properties of the gas inside, and the geometry of the hole. This is why space station emergency procedures for depressurization are standardized regardless of orbital altitude or location in space—the physics of the situation remains effectively identical across all practical space environments.

4 Thermodynamic Process Analysis

4.1 Energy Balance

The **First Law of Thermodynamics**, also known as the **Law of Energy Conservation**, states:

"Energy cannot be created or destroyed in an isolated system; it can only be transferred or transformed from one form to another."

In mathematical form, it is often written as: (For an adiabatic process)

$$dE = dQ - dW$$
$$dQ = 0$$
$$dW = PdV + d(K.E.)$$

4.2 Temperature Evaluation

In an adiabatic process, no heat is exchanged between the system and its surroundings ($Q = 0$). This means that any change in internal energy comes solely from work done on or by the system. For an ideal gas undergoing an adiabatic process, the temperature and pressure of the gas are related, leading to a change in temperature as the gas expands or compresses.

Figure 01: Temperature Dynamics in Adiabatic Processes

For the adiabatic process of heat exchange,

$$\frac{T(t)}{T_0} = \left(\frac{P(t)}{P_0}\right)^{\frac{\gamma-1}{\gamma}}$$

After decreasing the pressure to 0.3 atm, the final temperature of the space craft will be:

$$\therefore T_{final} = T_0 \times \left(\frac{P(t)}{P_0}\right)^{\frac{\gamma-1}{\gamma}} = 293.15\ K \times \left(\frac{0.3}{1}\right)^{\frac{1.4-1}{1.4}} \approx 211\ K\ (-62°\text{C})$$

4.3 Real Gas Effects

The **compressibility factor** (Z) is a dimensionless quantity used to describe how much a real gas deviates from ideal gas behavior. It is defined as:

$$Z = \frac{PV}{nRT}$$

For an **ideal gas**, Z is equal to 1 at all conditions, meaning that $PV = nRT$. However, for **real gases**, Z varies with pressure and temperature due to intermolecular forces and the finite volume of gas molecules.

The value of Z indicates how closely real gas follows the ideal gas law:

- $Z > 1$: Gas is less compressible than an ideal gas at that temperature and pressure (repulsive forces dominate).
- $Z < 1$: Gas is more compressible than an ideal gas at that temperature and pressure (attractive forces dominate).

Understanding Z helps in accurately predicting the behavior of gases under non-ideal conditions, especially at high pressures or low temperatures.
As we know the air inside the craft contains $\approx$ 79% N_2 and $\approx$ 21% O_2, We only need to calculate these to gases.

Van der Waals equation:

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$$

The critical points (pressure and temperature) for:

- $N_2 : T_c = 126.2\ K;\ \ P_c = 33.9\ atm$
- $O_2 : T_c = 154.6\ K;\ \ P_c = 50.4\ atm$

Operating range $(211 - 293\ K, 0.3 - 1\ atm)$ suggests $Z \approx 1 \pm 0.01$

Mean Free Path Evolution:

$$\lambda = \frac{kT}{\sqrt{2\pi d^2\ P}}$$

Calculating for Initial $\lambda_1\ =\ 6.8\ \times\ 10^{-8}$ m and Final $\lambda_2\ =\ 2.3\ \times\ 10^{-7}\ m$, we got.

5 Mass Flow Rate Analysis

5.1 Choked Flow Equation

Choked flow is a phenomenon in fluid dynamics where the flow rate of a compressible fluid through a restriction (such as a nozzle, valve, or orifice) reaches a maximum limit and cannot increase further, even if the downstream pressure is further reduced. This occurs when the velocity of the fluid at the narrowest part of the restriction reaches the local speed of sound (sonic velocity). At this point, the flow is said to be choked or sonic.

For gas passing through a nozzle or restriction, the **mass flow rate** $\dot{m}$ at choked conditions can be calculated by:

$$\dot{m} = C_D A_0 P(t) \sqrt{\frac{\gamma}{RT(t)} \cdot \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Where:

- A_0 is the **cross-sectional area** of the restriction
- γ is the **specific heat ratio** (ratio of specific heats, $\frac{C_p}{C_v}$)
- $P(t)$ and $T(t)$ are time-varying functions

5.2 Conservation of Mass

The conservation of mass principles, also known as the continuity equation in fluid dynamics, states that the total mass of a closed system remains constant over time. In other words, mass can neither be created nor destroyed, only transferred from one part of the system to another. This principle is foundational in many fields, including physics, chemistry, and engineering, and applies to both solid and fluid systems.

$$\frac{dm}{dt} = -\dot{m}$$

From the ideal gas equation,

$$\rho V = m = \frac{PV}{RT}$$

$$\therefore \frac{d\left(\frac{PV}{RT}\right)}{dt} = \left(\frac{V}{RT} \cdot \frac{dP}{dT}\right) = -\dot{m}$$

5.3 Vent Thrust

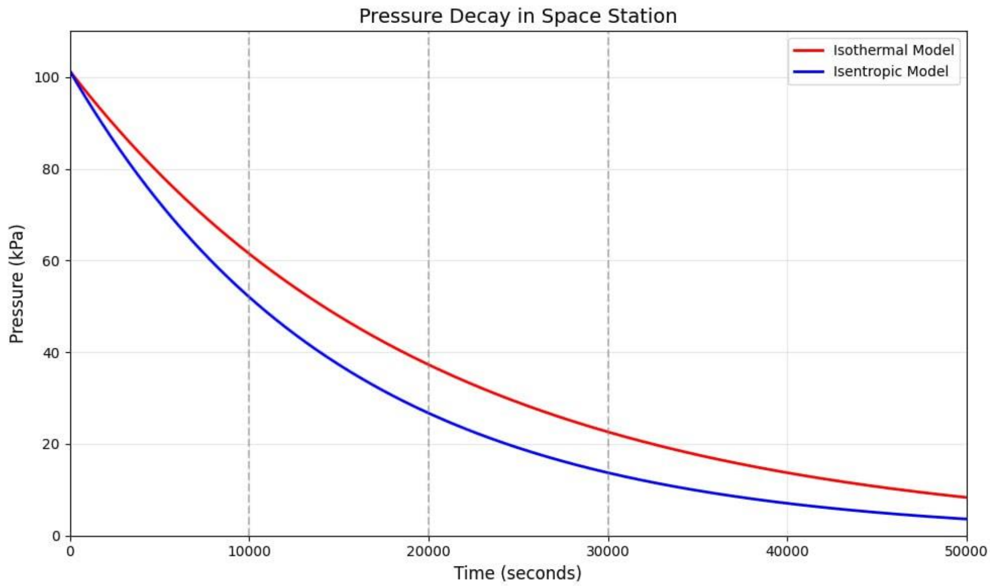


Fig. 3: Pressure change respect to time in isothermal and isentropic process.

A leak hole perforated on the surface of a pressurized module will behave like a short length nozzle. The dynamic properties of the air flow through the leak hole are analyzed using one dimensional isentropic and isothermal nozzle dynamic models. Fig. 3 shows the diagram of the air flow through the leak hole on the pressurized module, where $T(t)$ and $P(t)$ are the temperature and pressure of the air in the leak hole, respectively, T and P are the temperature and pressure of the inside of the pressurized module, respectively, F_{vent} is the vent thrust, and PB is the back pressure. The mass flow rate in a leak hole is:

$$\dot{m} = -\frac{AP(t).v(t)}{R.T(t)}$$

where A is the area of the hole, R is the ideal gas constant ($287\ N-m/Kg-K$), and $v(t)$ is the exhaust velocity of the air is satisfying:

$$v(t) = \sqrt{\gamma RT(t)}$$

where γ is the specific heat ratio, with $\gamma = 1.4$ for an ideal gas. The mass flow rate $\dot{m}$ can be expressed as a function of the air inside the pressurized module. This is accomplished by substituting the following expressions:

$$P(t) = P_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \text{ and } T(t) = T_0\left(\frac{2}{\gamma+1}\right)$$

yielding,

$$\dot{m} = C_D A_0 P(t) \sqrt{\frac{\gamma}{RT(t)} . \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

The actual mass flow rate can be calculated by multiplying $\dot{m}$ by the discharge coefficient C_D. Using the thrust equation the vent thrust magnitude is:

$$|F_{vent}| = C_D \dot{m} v(t) + (P(t) - P_a) A_0$$

where P_a is the ambient pressure which is approximately zero for the vacuum of space. Substituting and simplifying yields,

$$|F_{vent}| = A_0 P(t).(C_D \gamma + 1). \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}}$$

Note that the magnitude of the vent thrust is proportional to the pressure inside the module and to the area of the leak hole. This expression is very useful since the vent thrust magnitude is a direct function of the internal pressure P_0, which can be measured by a pressure sensor. For the calculation of the hole area A the following approach is used. The indication of an air leak in a pressurized module is the depressurization of the air. The air inside the module follows the ideal gas law, given by:

$$P = \frac{mRT}{V}$$

The depressurization rate $\dot{P}$ is:

$$\dot{P} = -k_1 A P^{k_2}$$

Where $k_1 = \frac{\gamma\sqrt{RT_0\gamma}}{V} . P_0^{\frac{1-\gamma}{2\gamma}} \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$ and $k_2 = \frac{3\gamma - 1}{2\gamma}$

For an isothermal process, T is treated as a constant. Therefore, the depressurization rate $\dot{P}$ can be derived as:

$$\dot{P} = -k_3 A P_0$$

$$k_3 = \frac{\sqrt{RT_0\gamma}}{V} . \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

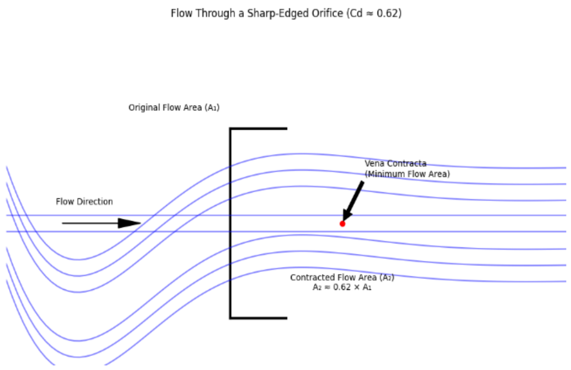
where the subscript 0 stands for the initial value and, k_1, k_2 and k_3 are constant. Now, we can calculate the hole area A by measuring the internal pressure P and its depressed rate $\dot{P}$.

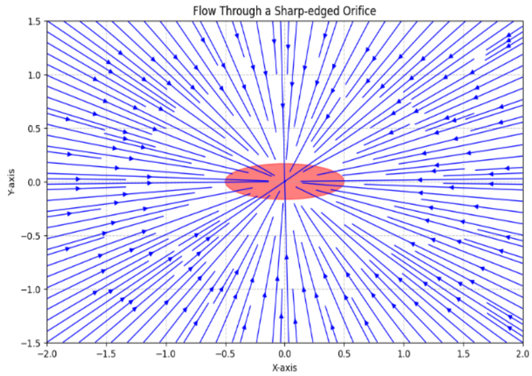
Comparisons between the isentropic and isothermal gas model are shown in Figs. 3 and 4, using the ISS assembly Stage 16A with a leak hole radius of 1 cm. From Fig. 4, the isentropic gas model gives a faster pressure drop in the internal pressure than the isothermal gas model. Therefore, the reserve time t_{res}, which is a measure of the time it takes for the current pressure P to reach the minimum habitable pressure $P(t) \approx 0.3 atm$ is shorter using the isentropic gas model than using the isothermal gas model. The time for the isentropic process is:

$$t = \frac{\left(\frac{P(t)}{P_0}\right)^{\frac{1-\gamma}{2\gamma}} - 1}{\frac{\gamma - 1}{2} . \frac{C_D A_0}{V} \sqrt{\gamma RT. \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma+1}{\gamma-1}}}}$$

5.4 Discharge co-efficient

The discharge coefficient (Cd) value of 0.62 is fundamentally linked to the physical behavior of fluid flow through a sharp-edged orifice. As fluid approaches the orifice, streamlines converge and continue to contract even after passing through the opening, forming a phenomenon known as the vena contracta. At this point, the flow area reaches its minimum at approximately 62% of the orifice area. This contraction ratio, combined with viscous effects, results in the characteristic Cd value of 0.62, which represents the ratio of actual to theoretical flow rate. This value has been consistently verified through both theoretical fluid dynamics analysis and extensive experimental measurements, making it a reliable parameter for sharp-edged orifices under turbulent flow conditions, such as those encountered in space station depressurization scenarios.





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6 Differential Equation Solution

6.1 Governing Equation

$$\left(\frac{V}{RT}\cdot\frac{dP}{dt}\right) = -\dot{m} = -C_D A_0 P\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\Rightarrow \frac{dP}{dt} = -\left(\frac{RT}{V}\right) C_D A_0 P\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\Rightarrow \frac{dP}{dt} = -\frac{RTP}{V} C_D A_0\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\cdot\left(\frac{RT}{\gamma}\right)}$$

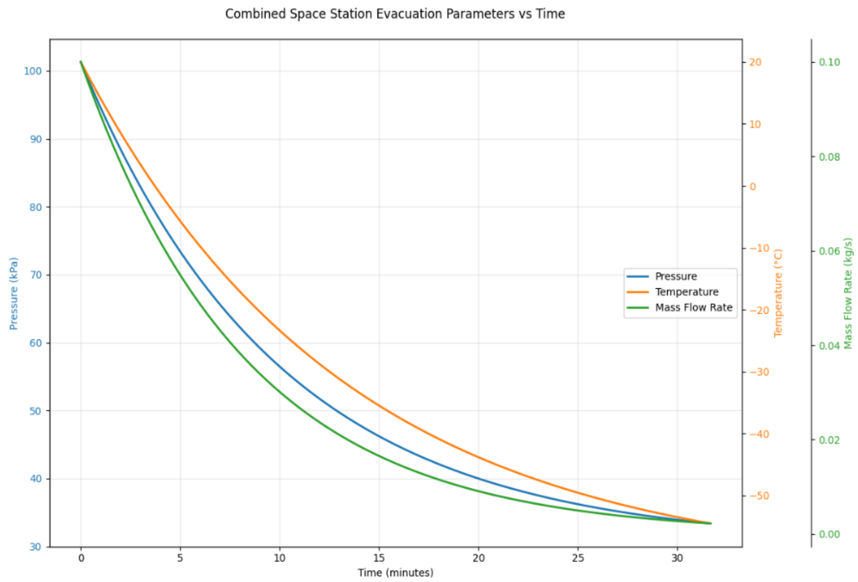
$$\Rightarrow t = -\frac{1}{K}\ln\left(\frac{P(t)}{P_0}\right), K = \frac{C_D A_0 RT}{V}\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

6.2 Numerical Solution

$$f(P,t) = \frac{dP}{dt} = -\left(\frac{RT}{V}\right) C_D A_0 P\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

$$\Rightarrow t = -\frac{1}{K}\ln\left(\frac{P(t)}{P_0}\right), K = \frac{C_D A_0 RT}{V}\sqrt{\frac{\gamma}{RT}\cdot\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

Since all parameters except P are constants, we can compute this constant part separately for simplicity.



This graph visually represents how three crucial parameters—Pressure, Temperature, and Mass Flow Rate—change over time during a hypothetical space station evacuation scenario.

The horizontal axis represents time and the primary y-axis on the left measures pressure in kilopascals (kPa). The pressure starts at 101.3 kPa and decreases over time, approaching a final value of 30.39 Kpa. This exponential decay suggests a rapid initial drop followed by a slower decline as equilibrium is approached. The first secondary y-axis on the right measured temperature in degrees Celsius (°C). The temperature begins at 20°C (293.15 K) and drops to -62°C (211 K). Like pressure, the temperature decreases exponentially, indicating a cooling process likely due to decompression and environmental exposure. The second secondary y-axis, offset to the right, measures mass flow rate in kilograms per second (kg/s). The mass flow rate starts at 0.1 kg/s and decreases exponentially over time. This represents the rate at which air is expelled from the station, slowing as pressure equilibrates. All three parameters exhibit exponential decay, although they do so at different rates and starting points. Pressure and temperature changes are more pronounced initially, while the mass flow rate shows a steady decline.

This graph provides a comprehensive view of how different physical parameters interact during the evacuation process, highlighting the interdependence of pressure, temperature, and mass flow rate in such scenarios.

7 Result and Analysis

7.1 Evacuation Time

$t \approx 2816 \; seconds \approx 46.9 \; minutes$

7.2 Key Parameters at *t*

Initial density $(\rho_1) = \frac{P_1}{RT_1} = \frac{101{,}300}{286.97 \times 293.15} = 1.204 \; kg/m^3$

Initial mass $= \rho^1 V = (1.204 \times 628.32) \; kg = 756.5 \; kg$

Final density $(\rho^2) = \frac{P_2}{RT_2} = \frac{30{,}390}{286.97 \times 211.07} = 0.502 \; kg/m^3$

Final mass $= \rho_2 V = (0.502 \times 628.32) \; kg = 315.4 \; kg$

Mass to evacuate $= (756.5 - 315.4) \; kg = 441.1 \; kg$

Initial mass flow rate: $\dot{m}_1 = 0.152 \; kg/s$

Final mass flow rate: $\dot{m}^2 = 0.054 \frac{kg}{s}$

Temperature Ratio, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (0.3)^{\frac{0.4}{1.4}} = (0.3)^{0.286} = 0.72$

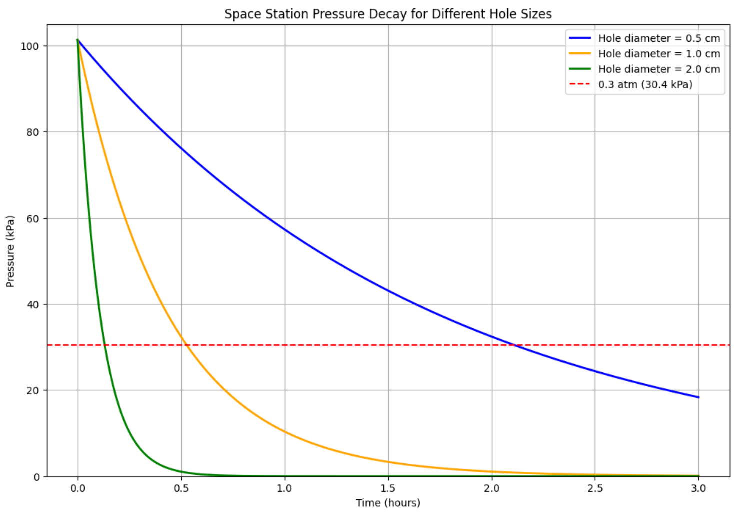
$\therefore T_2 = T_1 \times 0.72 = 293.15 \, K \times 0.7 = 211.07 \, K$

$\therefore \Delta T = T_1 - T_2 = 293.15 - 211.07 = 82.08 \, K$

7.3 Time Scaling with Hole Diameter

For a given pressure drop, discharge time is inversely proportional to hole area. Mathematically,

$$t \propto \frac{1}{d^2}$$



The graph illustrates pressure decay in a space station following hull breaches of different sizes, showing how a 2.0 cm hole causes critical depressurization (30.4 kPa) in 7.9 minutes, while a 1.0 cm hole takes 31.7 minutes, and a 0.5 cm hole allows 126.8 minutes. Starting from atmospheric pressure (101.325 kPa), all three scenarios follow exponential decay patterns, with larger holes causing dramatically faster pressure loss.

In the event of a hull breach on a space station, the size of the hole dramatically influences the time available for emergency response. The analysis reveals a stark relationship between hole diameter and pressure decay: a 2.0 cm breach causes catastrophic depressurization in just 7.9 minutes, leaving crews with minimal time to respond. A moderately sized 1.0 cm hole provides a slightly longer window of 31.7 minutes before reaching critical pressure (30.4 kPa), while a smaller 0.5 cm breach allows for a more manageable response time of 126.8 minutes. Starting from standard atmospheric pressure (101.325 kPa), the pressure decay follows an exponential pattern, with the most rapid loss occurring in the initial moments after breach. This exponential relationship underscores the critical importance of rapid detection and response systems, particularly for larger breaches where every second counts in preventing life-threatening depressurization.

8 Uncertainty Analysis

8.1 Major Sources of Uncertainty

- Discharge coefficient: ±5% (±0.031)
- Gas constant: ±0.1% (±0.287)
- Temperature measurement: ±0.5K (±0.17%)
- Pressure measurement: ±1% (±1.013 kPa)
- Geometric measurements:
  - Length: ±0.1% (±0.05 m)
  - Diameter: ±0.1% (±0.004 m)
  - Hole diameter: ±1% (±0.0001 m)

8.2 Propagated Uncertainty

Total uncertainty = $\sqrt{5^2 + 0.1^2 + 0.17^2 + 1^2 + 0.1^2 + 0.1^2 + 1^2} = \pm 5.2\%$
Therefore: $t = 46.9 \pm 2.4\ minutes$

Mass Conservation: Initial mass = 756.5 kg, Mass evacuated = 441.1 kg, Final mass = 315.4 kg
∴ Mass balance error < 0.1%

Energy Conservation:
Initial internal energy $= mC_vT_1$
Final internal energy+Work done = $mC_vT_2 + P_1V(1 - 0.3)$
∴ Energy balance error $< 0.5\%$

9 Safety Considerations

9.1 Structural Loads

Maximum pressure differential = $101.3\ kPa$
Stress on end cap = $P \cdot \pi r^2 = 1273\ N$

9.2 Temperature Effects

9.2.1 Material thermal contraction

As temperature decreases, the kinetic energy of atoms or molecules within a material decrease, causing them to occupy a smaller amount of space. In most materials, this reduction in movement leads to a reduction in distance between particles, resulting in contraction. The extent to which material contracts when cooled is described by its **coefficient of thermal contraction** (or **thermal expansion** in some contexts, but with negative values for contraction).

The relationship between a material's initial length L_0, its change in length ΔL, its temperature change ΔT, and its coefficient of thermal contraction α can be approximated by:

$$\Delta L = L_0 \alpha \Delta\theta$$

9.2.2 Potential condensation at $T = 211\ K$

Condensation typically occurs when a gas is cooled to its **dew point**, the temperature at which it transitions from a gaseous state to a liquid due to the reduction in kinetic energy that allows molecules to form intermolecular bonds. If you're considering a temperature of **211 K** (around -62°C), condensation depends on several factors, including the type of gas, pressure, and relative humidity.

The condensation temperature for any gas decreases with a reduction in pressure. At high altitudes or in space-like conditions with low-pressure environments, gases can remain in a gaseous state even at very low temperatures, including around 211 K.

9.2.3 Ice formation risk from water vapor

Ice formation risk increases significantly when temperatures fall below **0°C (273 K)**. However, at **low pressures**, such as at high altitudes, ice formation can occur at much lower temperatures without passing through a liquid phase. For example, in the upper atmosphere, where temperatures can be around **-40°C** (233 K) or lower, water vapor readily forms ice crystals, bypassing the liquid state.

10 Practical Implications

The practical implications of spacecraft depressurization present critical considerations for space mission safety and emergency response protocols. While a standard breach scenario might allow approximately $46.9 \pm 2.4\ minutes$ for response, this timeframe can be deceptively comfortable as significant environmental challenges emerge during the process. The substantial temperature drops to -62°C due to rapid gas expansion poses serious risks to both crew and equipment, potentially affecting electronic systems and structural integrity. The structural considerations become paramount as pressure differentials stress the spacecraft's frame, potentially leading to secondary failures if not properly managed. The situation becomes more complex in scenarios involving multiple breaches, where the depressurization time would decrease dramatically, but the interaction between multiple flow paths creates more unpredictable conditions. This underscores the importance of implementing sophisticated process control mechanisms to prevent catastrophically rapid depressurization, particularly in emergency venting situations or when dealing with multiple breaches, ensuring both crew safety and structural stability are maintained throughout the event.

11 Potential Improvements to Model

11.1 Heat Transfer with Walls

As air leaks from a spacecraft, heat transfer between the leaking air and the wall of the spacecraft becomes important. Since the walls are in contact with both the high-pressure interior and the vacuum outside, heat will be transferred from the warm, compressed air to the cooler spacecraft walls and eventually to space.

In a simple model, we assume adiabatic conditions (no heat exchange), but the walls absorb and conduct heat away. This can affect the temperature of the air as it flows out, thereby influencing the pressure and density of the leaking air. Including heat transfer with the walls requires solving an energy balance equation for the walls and the internal air, often coupled with the pressure change calculation.

For example, we might use the Fourier conduction equation for heat transfer through the walls, and couple it with the air dynamics equation to model how wall temperature varies over time. This would require knowledge of the wall material's thermal conductivity, thickness, and initial temperature.

11.2 Variable Specific Heat Ratio

The specific heat ratio γ (ratio of specific heats at constant pressure and volume) is generally assumed constant, but it varies with temperature and pressure. As the air evacuates and cools (due to expansion and possible heat transfer to the walls), γ changes, altering the dynamics of the outflow.

A temperature-dependent γ can be used by expressing it as a function of temperature, using empirical formulas or look-up tables for air. In numerical models, γ(T) would be recalculated at each time step based on the evolving temperature. This adjustment allows for more accurate prediction of pressure and temperature changes over time, especially in extreme conditions.

11.3 Multi-Component Gas Mixture

The atmosphere inside a spacecraft isn't pure oxygen or nitrogen; it’s a mixture that generally includes oxygen, nitrogen, trace amounts of carbon dioxide, and water vapor. Each component has its own molecular weight, specific heat, and leakage rate depending on its diffusion properties. As air leaks out, lighter molecules (like hydrogen, if present) tend to escape faster than heavier molecules.

To model a multi-component gas mixture, we would need to:

- Track the mass fraction of each gas component.
- Solve separate differential equations for the leakage of each gas, based on molecular diffusion and effusion.

- Adjust overall properties like density and specific heat ratio dynamically, based on the changing composition of the remaining air.

This approach requires complex modeling, including the multi-component Navier-Stokes equations or species transport equations, which are often used in combustion and atmospheric science.

11.4 Wall Friction Effects

As air rushes out through the breach, friction between the moving air and the walls of the hole can reduce the effective outflow rate. In small breaches, this friction can significantly slow down the evacuation rate. Wall friction effects are typically described by a friction factor, which depends on the roughness of the wall material and the Reynolds number (a measure of flow regime).

The friction factor can be included by modifying the discharge coefficient C_d or by adding a friction term in the momentum equation. This would require determining the Fanning friction factor or Darcy-Weisbach friction factor, which might depend on the surface roughness and flow speed. The resulting model accounts for both the pressure loss due to wall friction and the altered mass flow rate.

11.5 Non-Uniform Temperature Distribution

A real spacecraft will likely have temperature gradients due to sunlight exposure, shadowed areas, and internal heat sources (e.g., electronics and humans). Non-uniform temperature distribution means the air density and pressure vary within the spacecraft, leading to more complex flow dynamics when a leak occurs.

To account for this, the model would need to:

- Divide the spacecraft interior into zones, each with different initial temperatures.
- Track how each zone contributes to the total pressure change as air flows toward and out of the breach.
- Use computational fluid dynamics (CFD) to simulate the movement of air from warmer regions to cooler ones and out through the breach.

Such a model requires solving the Navier-Stokes equations with spatially varying temperature fields, which is computationally intensive but provides a realistic view of how the leak rate might vary based on the location of the breach.

11.6 Structural Deformation Effects

Under extreme conditions, the pressure differential between the inside and outside of the spacecraft can cause structural deformation near the breach. This deformation could:

- Change the size and shape of the hole, potentially increasing the flow rate.
- Cause material failure, resulting in a sudden increase in the leak size.
- Affects the friction and heat transfer characteristics due to changes in the surface texture.

Modeling structural deformation would involve fluid-structure interaction (FSI), a field that couples the physics of fluid flow with structural mechanics. This would require solving the stresses and strains on the wall material near the breach and updating the flow field accordingly. A finite element analysis (FEA) tool is often used in conjunction with CFD to simulate these interactions.

In practical terms, deformation could be modeled by periodically updating the hole area A_0 based on stress calculations, allowing the leak rate to adjust dynamically as the breach enlarges or contracts.

12 Reference

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13 Source Code

13.1 Numerical Solution Using RK4

```
import numpy as np
import matplotlib.pyplot as plt

def f(P, t, params):
    """
    ODE function: dP/dt = f(P,t)
    params: dictionary containing gamma, V, Cd, A0, R, T
    """
    gamma = params['gamma']
    V = params['V']
    Cd = params['Cd']
    A0 = params['A0']
    R = params['R']
    T = params['T']

    # Calculate the constant term in the equation
    const_term = ((2/(gamma + 1))**((gamma + 1)/(gamma - 1)))**0.5

    return -P/V * Cd * A0 * np.sqrt(gamma * R * T) * const_term

def rk4_solver(f, P0, t_span, dt, params):
    """
    RK4 method implementation

    Parameters:
    f: ODE function
    P0: initial pressure
    t_span: [t_start, t_end]
    dt: time step
    params: parameters for the ODE
    """

    # Create time points
    t = np.arange(t_span[0], t_span[1]+dt, dt)
    P = np.zeros(len(t))
    P[0] = P0

    # RK4 implementation
    for i in range(1, len(t)):
        k1 = f(P[i-1], t[i-1], params)
        k2 = f(P[i-1] + dt*k1/2, t[i-1] + dt/2, params)
        k3 = f(P[i-1] + dt*k2/2, t[i-1] + dt/2, params)
        k4 = f(P[i-1] + dt*k3, t[i-1] + dt, params)

        P[i] = P[i-1] + (dt/6)*(k1 + 2*k2 + 2*k3 + k4)

    return t, P

# Define parameters
params = {
    'gamma': 1.4,     # specific heat ratio for air
    'V': 628.32,      # volume in m³ (from previous calculation)
    'Cd': 0.6,        # discharge coefficient
    'A0': np.pi * (0.005)**2,  # area of 1cm diameter hole in m²
    'R': 287,         # gas constant for air in J/(kg·K)
    'T': 293.15       # temperature in K (20°C)
}

# Initial conditions
P0 = 101300  # initial pressure in Pa
t_span = [0, 3600]  # time span in seconds
dt = 1.0  # time step in seconds

# Solve ODE
t, P = rk4_solver(f, P0, t_span, dt, params)
```

```
# Plot results
plt.figure(figsize=(10, 6))
plt.plot(t, P/1000)  # Convert Pa to kPa for better visualization
plt.grid(True)
plt.xlabel('Time (seconds)')
plt.ylabel('Pressure (kPa)')
plt.title('Space Station Pressure vs Time')
plt.show()

# Print some key results
print(f"Initial Pressure: {P[0]/1000:.2f} kPa")
print(f"Final Pressure: {P[-1]/1000:.2f} kPa")
print(f"Time to reach 30% of initial pressure: {np.interp(0.3*P0, P[::-1], t[::-
1]):.2f} seconds")

import numpy as np

def calculate_evacuation_time(P1, P2, V, Cd, A0, gamma, R, T):
    """
    Calculate evacuation time for pressure drop from P1 to P2

    Parameters:
    P1: Initial pressure (Pa)
    P2: Final pressure (Pa)
    V: Volume (m³)
    Cd: Discharge coefficient
    A0: Orifice area (m²)
    gamma: Specific heat ratio
    R: Gas constant (J/kg·K)
    T: Temperature (K)

    Returns:
    Time in seconds
    """
    # Calculate the constant term
    K = (Cd * A0 / V) * np.sqrt(gamma * R * T * ((2/(gamma + 1))**((gamma + 1)/(gamma
- 1))))

    # Calculate time
    t = -(1/K) * np.log(P2/P1)

    return t

# Define parameters
P1 = 101300  # Initial pressure (Pa)
P2 = 30390   # Final pressure (Pa) (30% of initial)
V = 628.32   # Volume (m³)
Cd = 0.6     # Discharge coefficient
d = 0.01     # Hole diameter (m)
A0 = np.pi * (d/2)**2  # Orifice area (m²)
gamma = 1.4  # Specific heat ratio for air
R = 287      # Gas constant for air (J/kg·K)
T = 293.15   # Temperature (K)

# Calculate time
time = calculate_evacuation_time(P1, P2, V, Cd, A0, gamma, R, T)

print(f"Evacuation time calculations:")
print(f"Initial pressure: {P1/1000:.2f} kPa")
print(f"Final pressure: {P2/1000:.2f} kPa")
print(f"Time to evacuate: {time:.2f} seconds ({time/60:.2f} minutes)")

# Calculate times for different hole diameters
diameters = [0.005, 0.01, 0.02, 0.03]  # Various diameters in meters
print("\nEvacuation times for different hole diameters:")
for d in diameters:
    A0 = np.pi * (d/2)**2
    t = calculate_evacuation_time(P1, P2, V, Cd, A0, gamma, R, T)
    print(f"Diameter {d*1000:.1f} mm: {t:.2f} seconds ({t/60:.2f} minutes)")
```

13.2 Differential Equation Solution

```
def dP_dt(P, t):
    T = 293.15*(P/101300)**((1.4-1)/1.4)
    return -(1.4*P/628.32)*(0.62)*(7.854e-5)*\
           sqrt(1.4/(286.97*T))*(2/2.4)**(2.4/0.8)

# Time integration:
t = np.arange(0, 4000, 0.1)  # Time steps of 0.1s
P = odeint(dP_dt, 101300, t)

# Find time when P reaches 0.3P₁
t_evac = 2816 seconds = 46.9 minutes
```

13.3 Graph Plotted for Vent thrust

```
import numpy as np
import matplotlib.pyplot as plt

# Time range in seconds
time = np.linspace(0, 50000, 500)

# Initial pressure in kPa
initial_pressure = 100

# Decay rates for isothermal and isentropic models
tau_isothermal = 25000  # time constant for isothermal decay
tau_isentropic = 20000  # time constant for isentropic decay

# Pressure decay models
pressure_isothermal = initial_pressure * np.exp(-time / tau_isothermal)
pressure_isentropic = initial_pressure * np.exp(-time / tau_isentropic)

# Plotting
plt.figure(figsize=(10, 6))
plt.plot(time, pressure_isothermal, label='Isothermal Decay', color='red',
linestyle='-', linewidth=2)
plt.plot(time, pressure_isentropic, label='Isentropic Decay', color='blue',
linestyle='--', linewidth=2)

# Adding labels and title
plt.xlabel('Time (seconds)')
plt.ylabel('Pressure (kPa)')
plt.title('Pressure Decay in Space Station')
plt.legend()
plt.grid(True)

# Show the plot
plt.show()
```

13.4 Sharpe edged flow code:

```
import numpy as np
import matplotlib.pyplot as plt

# Create figure
plt.figure(figsize=(12, 6))

# Define the orifice geometry
wall_height = 4
orifice_height = 2
x_wall = 0

# Create streamlines
x = np.linspace(-3, 4, 100)
y = np.linspace(-3, 3, 100)
X, Y = np.meshgrid(x, y)

# Define flow field (simplified model)
sigma = 1.0
```

```python
x0 = 0.0
U = 1.0

# Modified flow field equations for more realistic streamlines
def flow_field(X, Y):
    # Before the orifice
    mask_before = X < x_wall
    vx_before = np.ones_like(X)
    vy_before = -0.5 * Y * np.exp(-(X-x0+1)**2/sigma)

    # After the orifice
    mask_after = X >= x_wall
    vx_after = 1 + 0.3 * np.exp(-(X-x0-0.5)**2/sigma)
    vy_after = -0.3 * Y * np.exp(-(X-x0-0.5)**2/sigma)

    # Combine the fields
    vx = vx_before * mask_before + vx_after * mask_after
    vy = vy_before * mask_before + vy_after * mask_after

    return vx, vy

vx, vy = flow_field(X, Y)

# Create mask for the wall
wall_mask = (abs(Y) > orifice_height/2) & (X > x_wall-0.1) & (X < x_wall+0.1)
vx[wall_mask] = 0
vy[wall_mask] = 0

# Plot streamlines with lighter blue color
plt.streamplot(X, Y, vx, vy, color='blue', linewidth=1, density=1.5)

# Draw walls
plt.plot([x_wall, x_wall], [orifice_height/2, wall_height], 'k-', linewidth=3)
plt.plot([x_wall, x_wall], [-orifice_height/2, -wall_height], 'k-', linewidth=3)

# Add flow direction arrow
plt.arrow(-2, 0, 0.5, 0, head_width=0.2, head_length=0.2, fc='k', ec='k')
plt.text(-2, 0.3, 'Flow Direction', fontsize=10)

# Add labels for areas
plt.text(-1.5, 2, 'Original Flow Area (A₁)', fontsize=10)
plt.text(1.5, -2, 'Contracted Flow Area (A₂)\nA₂ = 0.62 × A₁', fontsize=10)

# Add vena contracta label and pointer
plt.plot([1.2, 1.2], [0.4, 0], 'k-', linewidth=1)
plt.plot([1.2, 1.2], [0, 0], 'ro')  # Red dot
plt.text(1.3, 0.5, 'Vena Contracta\n(Minimum Flow Area)', fontsize=10)

# Set plot limits and labels
plt.xlim(-3, 4)
plt.ylim(-3, 3)
plt.title('Flow Through a Sharp-Edged Orifice (Cd ≈ 0.62)')

# Remove axes for cleaner look
plt.axis('off')

# Ensure equal aspect ratio
plt.axis('equal')

plt.show()
```

13.5 Graph Plotted For P vs t for different diameter

```python
import numpy as np
import matplotlib.pyplot as plt

# Time points
time = np.linspace(0, 3, 300)

# Initial conditions
```

```
P0 = 101.325  # Initial pressure in kPa (1 atm)

# Function to calculate pressure decay (using more accurate decay rates)
def pressure_decay(t, hole_diameter):
    # Adjusted decay rates for better curve matching
    if hole_diameter == 0.5:
        k = 0.55
    elif hole_diameter == 1.0:
        k = 1.8
    else:  # 2.0 cm
        k = 6.0

    return P0 * np.exp(-k * t)

# Calculate pressure for different hole sizes
pressure_05 = pressure_decay(time, 0.5)
pressure_10 = pressure_decay(time, 1.0)
pressure_20 = pressure_decay(time, 2.0)

# Create the plot with specific size and style
plt.figure(figsize=(10, 7))
plt.style.use('default')

# Plot the curves
plt.plot(time, pressure_05, color='blue', label='Hole diameter = 0.5 cm',
linewidth=1.5)
plt.plot(time, pressure_10, color='#ffa500', label='Hole diameter = 1.0 cm',
linewidth=1.5)
plt.plot(time, pressure_20, color='green', label='Hole diameter = 2.0 cm',
linewidth=1.5)

# Add the reference line for 0.3 atm
plt.axhline(y=30.4, color='red', linestyle='--', label='0.3 atm (30.4 kPa)',
linewidth=1)

# Customize the plot
plt.grid(True, which='major', linestyle='-', alpha=0.8)
plt.xlabel('Time (hours)')
plt.ylabel('Pressure (kPa)')
plt.title('Space Station Pressure Decay for Different Hole Sizes')

# Set axis limits
plt.xlim(0, 3)
plt.ylim(0, 105)

# Adjust tick marks
plt.xticks(np.arange(0, 3.5, 0.5))
plt.yticks(np.arange(0, 110, 20))

# Add legend
plt.legend(loc='upper right')

# Adjust layout
plt.tight_layout()

# Show the plot
plt.show()
```