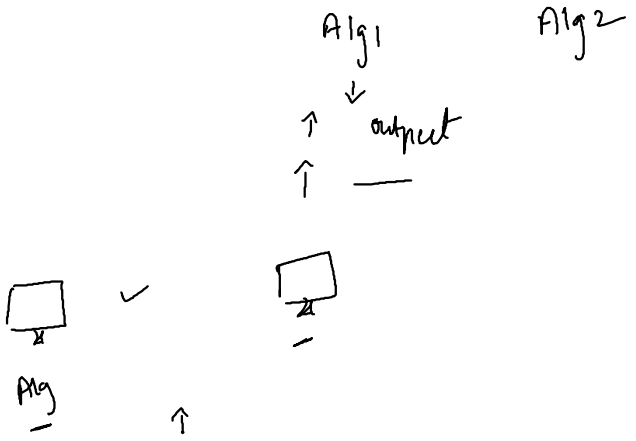
Asymptotic Notations

TC => O(n)
Ω(n)
Θ(n)

1) Big O = Worst case
2) Big Ω = Best case ✓
3) Big Theta(Θ) = Tight estimate (worst + best)

Alg1 Alg2

↓ output



why AN?

5 Alg1 → 1 sec ✓
5 Alg2 → 10 sec

1 1 crore — ?

Alg

tool

1) Upper Bound

maximum

retire 50

45 50 51

Pizza → 30 min

O

2) Lower Bound

Overestimate

Pizza => 30 min

to

(=> 2 hrs)

Ω

3) Tight Bound

Exact steps

Bus 10 min

Θ

n=5

0,1,2,3,4 Code Example

```
for(int i=0 ; i<n ; i++) {
    cout<<i;
}
```

O(n)

Ω(n²)

Θ(n)

TB => 20 min ✓

UB => 30 min ✓

LB => 2 ghante ✓

1) **Big O Notation (WorstCase) - Upper Bound (O)**

Maximum time

-ve +ve

TC = O(n)

My 1 ghante,

Big O

Maths → Big O

Industry → order of

1) MD → Big O

Algi → traditional tareeke se chai banana

$1 \to t_1$ $O(n)$

$2 \to 2t_1$

$3 \to 3t_1$

⋮

$n \to nt_1$

$T(n) = nt_1 + b$

Big O → mighe worst case ka andaza de do

maximum

$= n$

$\Rightarrow O(n)$ → ic 2min ✗

1min ✗

Big O

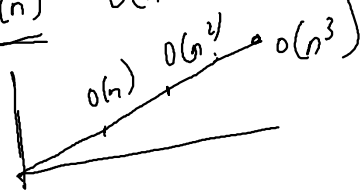
Note:

$O(n) \subset O(n^2) \subset O(n^3)$ ✓

✓ Upper Bound

upper Limit

$O(n)$ $O(n^2)$ $O(n^3)$



✓ $O(n) \subset O(n^2) \subset O(n^3)$

✓ $\boxed{O(n)}$ $O(n^2)$ ✗

minin

$O(n) \to O(n^2)$ ✓

Example

1) $f(n) = 3n + 5$

$\Rightarrow n$

$\Rightarrow O(n)$

Big O → maximum time → worst case → upper bound

guarance → code isse zyaada slow nahi chalega.

2) ID → Big O → order of → worst case

3 log

1st Ananya → n (fastest)

2nd Bhavna → n^2 (thoda slow)

3rd Suresh → n^3 (bahut slow)

1 Race

3 Race

Ananya → Price

✓ $\boxed{O(n)}$ $O(n^2)$ $O(n^3)$

$O(1)$ ✓

$O(1)$, $O(n)$

$T(n) = 2n + 5$

$\Rightarrow n$

$\Rightarrow O(n)$

$O(n)$ ✓

$O(n^2)$ Medium

$O(n^3)$ Bahut slow

Big O ✓

Code kitna slow ho sakta hai worse-case me, uska upper limit batata hai

→ Code isse zyaada slow nahi chalega

→ Shayad fast chale → but not slower.

Example: Sharmaji ka beta (Rohan)

Bus

Big O ⇒ 30 min

✓10 min ✓

✓30 min ✓

$f(n) = n^2 + n + 3$

$= n^2 + n$

$\Rightarrow n^2$

Max $\Rightarrow O(n^2)$ ✓

$n = 2$ ✓

$n^2 = 4$

$n = 2$

✓ **f(n) is O(g(n)) if there exist constants c and n_0 such that:**

$f(n) \leq c \times g(n)$ for all $n \geq n_0$

$f(n) \Rightarrow$ zyada se zyaada

↙ ↘ input

time

$g(n) \rightarrow$ Big O

$f(n) \leq c\,g(n)$

Example

$f(n) = n^2 + n + 3$

$= O(n^2)$

$g(n) = n^2$

$n = 1$, $c = 5$

$5 \leq 1$ ✗

$5 \leq 5$ ✓

$c = 5$ ∀ $n \geq n_0$

$f(n) = 1^2 + 1 + 3 \Rightarrow 5$

$g(n)\ n^2 = 1^2 = 1$

$= 5(1) \Rightarrow 5$

27 $f(n) = n^3 + 1$

$O(n^3)$ ✓

✓

$g(n) = n^3$

$f(n) \leq c\,g(n)$

$n \geq n_0$

$c = 1$, $c = 2$

$1^3 + 1 = 2$

$1^3 = 1$ ✗

$1^3 + 1 = 2$

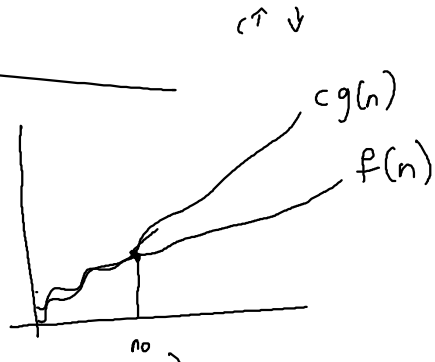
$2^3 \Rightarrow 8$

$2 \leq 8$ ✓

$\boxed{O(n^3)} \rightarrow O(n^4) \Rightarrow O(n^5)$

$\rightarrow O(n^3)$

Big



max ✓ $O(n!)$

$O(2^n)$

$O(n^3)$

$O(n^2)$

$O(n \log n)$

$O(n)$

$O(\log N)$

least $O(1)$

$n + \log N + n \log n + n^2$

$\Rightarrow O(n^2)$

$\Rightarrow$ Different Big O

1) Constant time $O(1)$

2) Linear time $O(n)$

3) Quadratic $O(n^2)$

4) Cubic $O(n^3)$

5) logarithmic $O(\log n)$

2) Big Omega Notation Ω (Best case / lower bound)

↳ Sabse kam time mai kitna jaldi kaam hosakta hai

lucky ≡ favorite

key = 3

$\Rightarrow$ $\boxed{3}$ 3 -1

$\Omega g(n)$

f(n) is Ω(g(n)) if there exist constants c and n_0 such that:
$c \times g(n) <= f(n)$ for all $n \geq n_0$

1) $f(n) = 5n^2 + 4n + 2$

$= \Omega(n^2)$

$5n^2 \rightarrow$ quadratic

$4n \rightarrow$ Linear time

$+2$ constant

$f(n) \geq n^2 \quad \forall\ n \geq$ some value

$n^2 \leq f(n) \quad \forall\ n \geq$ some n_0

$n = 1000$

$5(1000)^2 + 4(1000) + 2$

$\Rightarrow \boxed{5000000} + 4000 + 2$

$\Rightarrow 5004002$

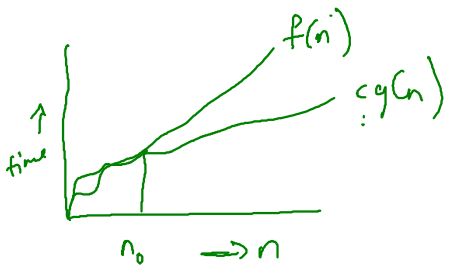
$f(n) = 5n^2 + 4n + 1$

$= \Omega(n^2)$

$g(n) = n^2$

$c = 1$ =)

$5n^2 \geq n^2$, $\boxed{c = 1}$



$f(n) \in \Omega(n^2)$

3) Big Theta Θ → Tight bound

↳ Exact steps.

25-30

$f(n) = n^2 + n + 3$

Big O = n^2 ✓

Big Ω = n^2 ✓

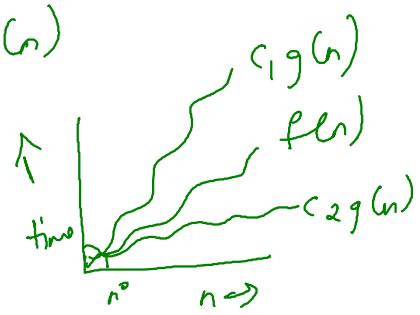
Big theta Θ = n^2

$f(n)$ is $\Theta g(n)$ if it is both $Og(n)$ & $\Omega g(n)$

$f(n) \leq c_1 g(n)$

$c_2 g(n) \leq f(n)$

$c_2 g(n) \leq f(n) \leq c_1 g(n)$



Big O → worst case (zyada se zyada time)

Big Ω → Best case (kam se kam time)

Big Θ → Tight bound (fix time range)