

1- Generate All Binary Strings of Length N

$n=3$
 $\rightarrow 0 \rightarrow 010$
 $\rightarrow 1 \rightarrow 011$
 $\rightarrow 2 \rightarrow 101$

$n=3$
 $2^3 = 8$ $2^3 - 1 = 7$

$n=4$
 $2^4 - 1 = 15$
 $2^5 - 1 = 31$
 $0 \rightarrow 0000$
 $1 \rightarrow 0001$
 $2 \rightarrow 0010$
 $3 \rightarrow 0011$
 $4 \rightarrow 0100$
 $5 \rightarrow 0101$
 $6 \rightarrow 0110$
 $7 \rightarrow 0111$

① Base condition \rightarrow

(0, 1)

$n=3$
 2
 1
 $0 \rightarrow$ return output

$0 \rightarrow 0$
 $1 \rightarrow 1$
 $gB(n-1, "0")$
 $gB(n-1, "1")$

$8 \rightarrow 1000$ write

$gB(0, "1011")$
 $gB(0, "1101")$
 $gB(0, "1010")$
 $gB(0, "1100")$

$gB(0, "1011")$
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 $gB(0, "1100")$

```

void generateBinary(int n, string out){
    if(n==0){
        cout<<out<<endl;
        return;
    }
    generateBinary(n-1, out+"0");
    generateBinary(n-1, out+"1");
}
    
```

```

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```

```

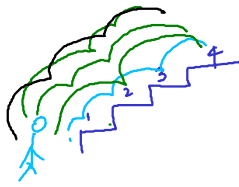
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}
    
```

output

000
 001
 010
 001
 100
 101
 110
 111

not

- 1 step
- 2 step.



✓ 1 3 2 ✓

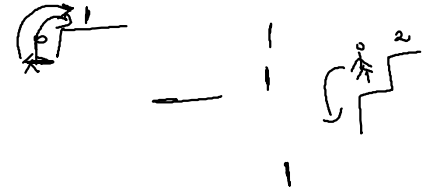
① Base case

($n=0$, $n=-1$) return 1

$$\begin{array}{ccc} cw(n-1) & + & cw(n-2) \\ \downarrow & & \downarrow \\ \text{1st step} & & \text{2nd step} \end{array}$$

$$\begin{array}{l} 1+1+1+1 \\ 2+2 \\ 1+1+2 \\ 2+1+1 \\ 1+2+1 \end{array}$$

5 combinations

$$n=4 \quad n=0$$


```

int countWays(int n){
    if(n==0 || n==1)
        return 1;
    return countWays(n-1)+countWays(n-2);
}

int countWays(int n){
    if(n==0 || n==1)
        return 1;
    return countWays(n-1)+countWays(n-2);
}

```

Handwritten annotations: The first code block has a green '4' above the opening brace and a green '3' above the closing brace. The second code block has a green '3' above the opening brace and a green '3' above the closing brace. A green arrow points from the '3' above the first closing brace to the '3' above the second opening brace. Below the first code block, there is a green '3' and 'cw(2)'. Below the second code block, there is a green '3' and 'cw(1)'.

```
int countWays(int n){
    if(n==0 || n==1)
        return 1;
    return countWays(n-1)+countWays(n-2);
}

int countWays(int n){
    if(n==0 || n==1)
        return 1;
    return countWays(n-1)+countWays(n-2);
}
```

5

$$\begin{aligned} & f(0) \\ & f(1) \\ & f(2) \\ & f(3) \\ & f(4) \\ & \dots \end{aligned}$$

Call Stack

3- Print All Balanced Parentheses Combinations

() → BP

{ } α BP

$\begin{matrix} & (& (& (&) &) &) \\ & | & | & | & | & | & | \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{matrix}$
 (3-3)

Balanced P

open > 0 ⇒ (
 close > open ⇒)
 (())

n=3

()()()
 (())()
 (()())
 ()(())
 ((())

5 combinations

3 3

3 > 0 ✓
 2 > 0 ✓
 1 > 0 ✓
 0 > 0 ✗
 3 > 0
 2 > 0
 1 > 0
 0 > 0 ✗

(Logical

((
 (((
 ((()
 ((()
 (((()))

Base

return //

bp(3,3,"")

```

void balancedParentheses(int open, int close, string out){
    if(open==0 && close==0){
        cout<<out<<endl;
        return;
    }
    if(open>0) balancedParentheses(open-1,close,out+"(");
    if(close>open) balancedParentheses(open,close-1,out+")");
}
  
```

```

void balancedParentheses(int open, int close, string out){
    if(open==0 && close==0){
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        return;
    }
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```

5 combinations

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    }
    if(open>0) balancedParentheses(open-1, close, out+"(");
    if(close>open) balancedParentheses(open, close-1, out+")");
}

```

Output
((()))

→ 4

4- Convert a Decimal Number to Binary Using Recursion

✓ Lecture

2, 3, 4, ... 0 & 1

10 ⇒ 1010

4 → (100)

10 | 884
80 ↓
84
80
4

num = 10

num ⇒ num/10

num ⇒ num%10

n == 0 return

2 | 10
2 | 5 - 0
2 | 2 - 1
2 | 1 - 0
0 - 1
1010
=

1) BC

2) CTF ✓ Tail recursion.

3) P

dB(10)

```

void decimalToBinary(int n){
    if(n==0) return;
    decimalToBinary(n/2);
    cout<<n%2;
}

```

```

void decimalToBinary(int n){
    if(n==0) return;
    decimalToBinary(n/2);
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}

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```

void decimalToBinary(int n){
    if(n==0) return;
    decimalToBinary(n/2);
    cout<<n%2;
}

```

1/2 1/2 | 1
0
1

1010 //

↑

Hlw: BTD

100 ⇒ 4

Lecture
Number
BTD, DTB

5- Count Number of Ways to Express N as Sum of 1, 3, and 4

$$n = 5$$

$$1) 1 + 1 + 1 + 1 + 1$$

$$2) 1 + 1 + 3$$

$$3) 1 + 4$$

$$4) 3 + 1 + 1$$

$$5) 1 + 4$$

$$6) 1 + 3 + 1$$

6 combinations.

2nd

$$n = 0 \quad (1)$$

$$n < 0 \quad (0)$$

$$cw(n-1) + cw(n-3) + cw(n-4)$$

6 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(4) 4+1+1 cw(2) cw(1)

main -> 6

4 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(3) 2+1+1 cw(1) + cw(0)

2 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(2) 1+1+0

1 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(1) 1+0+0 cw(-1) cw(-2)

1 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(0) 1 cw(-1) cw(-2) cw(-3)

0 {
 int countSumWays(int n){
 if(n==0) return 1;
 if(n<0) return 0;
 return countSumWays(n-1)+countSumWays(n-3)+countSumWays(n-4);
 }
 cw(-1) cw(-2) cw(-3)

6- Count Number of Digits in a Number

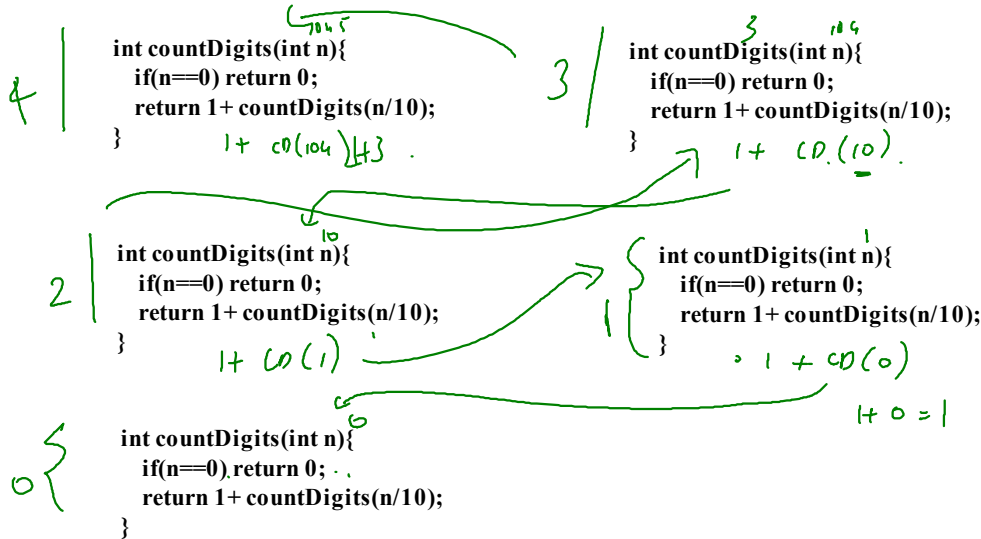
If num \Rightarrow 1045
 op \Rightarrow 4

num = 10

o/p \Rightarrow 2

cd(1045)

$1 + (n/10)$



4 //

7-Sum of Digits of a Number

n = 0

(cs)

n = 1234

$1 + 2 + 3 + 4 \Rightarrow 10$

n = 123④

$\%10$

$n \Rightarrow n/10$

recursion

n = 123

rem = $n \% 10 \Rightarrow 3$

sum = 0 + ③

$n = n/10$ (12)

n = 12

rem = 2

sum = 5

n = 1

n = 1

rem = 1

sum = 6

n = 0 \rightarrow return 0

```

int sumDigits(int n){
    if(n==0) return 0;
    return (n%10)+sumDigits(n/10);
}

```

Handwritten notes for sumDigits: 123, 3, 2+sum(1), 1, 1+sum(0), 0.

```

int sumDigits(int n){
    if(n==0) return 0;
    return (n%10)+sumDigits(n/10);
}

```

Handwritten notes for sumDigits: 123, 3, 2+sum(1), 1, 1+sum(0), 0.

8-Product of Digits of a Number -- Homework

$n = 123$
 $1 \times 2 \times 3 = 6$

9- Reverse a Number Using Recursion

(C++)

$n = 1234$
 $op \Rightarrow 4321$

$rem \Rightarrow n \% 10 \Rightarrow 4$
 $rev \Rightarrow rev \times 10 + rem$
 $\Rightarrow 0 + 4 \Rightarrow 4$
 $n = n / 10 (123)$

$n = 12$
 $rem = 2$
 $rev = 43 \times 10 + 2$
 $\Rightarrow 430 + 2$
 $\Rightarrow 432$
 $n = 1$

$n = 1$
 $rem = 1$
 $rev = 432 \times 10 + 1$
 $\Rightarrow 4321$
 $n = 0$
 B.C.

$n = 123$
 $rem = 3$
 $rev \Rightarrow 4 \times 10 + 3$
 $\Rightarrow 40 + 3 = 43$
 $n = 12$

```

int reverseNumber(int n, int reverse=0){
    if(n==0) return reverse;
    return reverseNumber(n/10, reverse*10+(n%10));
}

```

Handwritten notes: 1234, 12, 43, 1, 432, 0, 4321.

```

int reverseNumber(int n, int reverse=0){
    if(n==0) return reverse;
    return reverseNumber(n/10, reverse*10+(n%10));
}

```

Handwritten notes: 123, 4, 12, 43, 0, 4321.

```

int reverseNumber(int n, int reverse=0){
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```

Handwritten notes: 12, 43, 1, 432, 0, 4321.

```

int reverseNumber(int n, int reverse=0){
    if(n==0) return reverse;
    return reverseNumber(n/10, reverse*10+(n%10));
}

```

Handwritten notes: 0, 4321, 4321.

```

int reverseNumber(int n, int reverse=0){
    if(n==0) return reverse;
    return reverseNumber(n/10, reverse*10+(n%10));
}

```

Handwritten notes: 1, 432, 0, 4321.

10-Count Zeros in a Number

$$n = 102040$$

$$o/p: 3 //$$

Day 200 → H/W.

$$int count = (n \% 10 == 0) ? 1 : 0$$

$$count + cZ(n/10)$$

11- Count Number of Times a Digit Occurs in a Number → H/W

12-Check if a Number is a Power of 2 Using Recursion

$$\begin{array}{l} n=0 \rightarrow \text{false} \\ n=1 \rightarrow \text{true} \\ n=4 \rightarrow \text{true} \\ n=3 \rightarrow \text{false} \end{array}$$

$$\begin{array}{l} 2^0 \rightarrow 1 \\ 2^1 \rightarrow 2 \\ 2^2 \rightarrow 4 \\ 2^3 \rightarrow 8 \\ \vdots \end{array}$$

$$\begin{array}{l} n=0 \rightarrow \text{false} \\ n=1 \rightarrow \text{true} \end{array}$$

$$(n \% 2 \neq 0) \rightarrow \text{false} \quad n = \frac{2}{2}$$

$$\frac{n=16}{2}$$

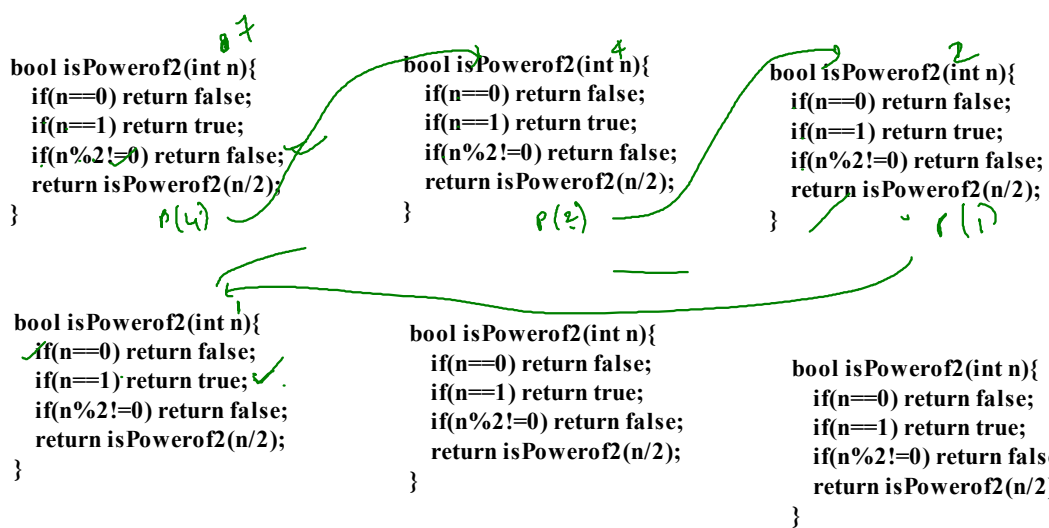
$$\frac{n=8}{2}$$

$$\frac{n=4}{2}$$

$$\frac{n=1}{2} \rightarrow \text{true}$$

$$n=17 \Rightarrow 1 \quad (n \% 2 \neq 0) \checkmark$$

mechanical



true

8

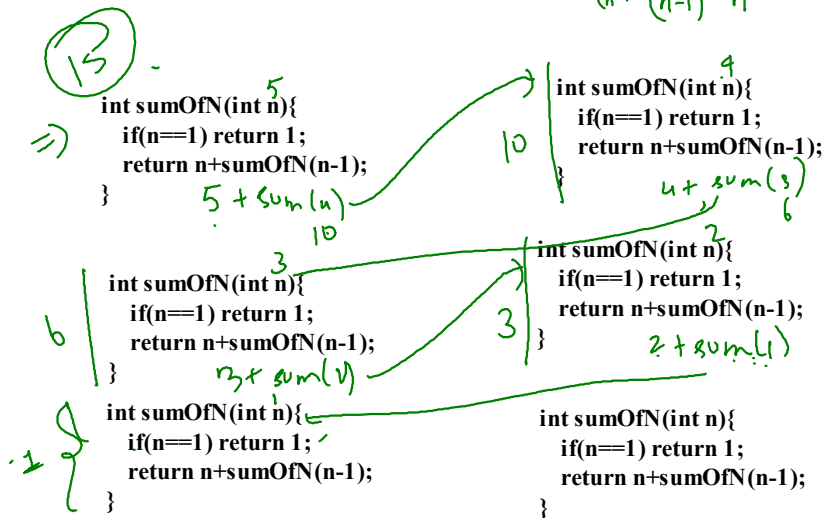
3

14- Print sum from 1 to n.

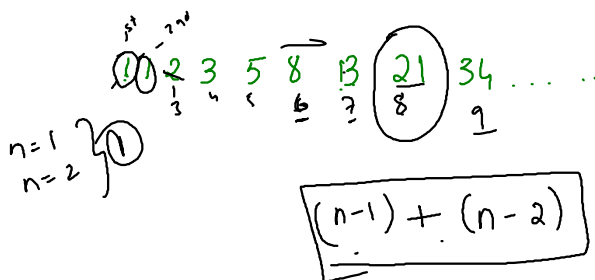
$$n=5 \\ 1+2+3+4+5 \Rightarrow 15 //$$

← (n-1) (n-1) n

$$n + (n-1) \\ 4 + (n-1)$$



14- Calculate nth Fibonacci number



n=8

21

6 → 8

