Homework 1

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Problem 1

```
set.seed(42)

roll_dice <- function(n){
    x <- runif(n)
    x[x >= 5/6          ] = 6
    x[x >= 4/6 & x < 5/6] = 5
    x[x >= 3/6 & x < 4/6] = 4
    x[x >= 2/6 & x < 3/6] = 3
    x[x >= 1/6 & x < 2/6] = 2
    x[           x < 1/6] = 1
    x
}
```

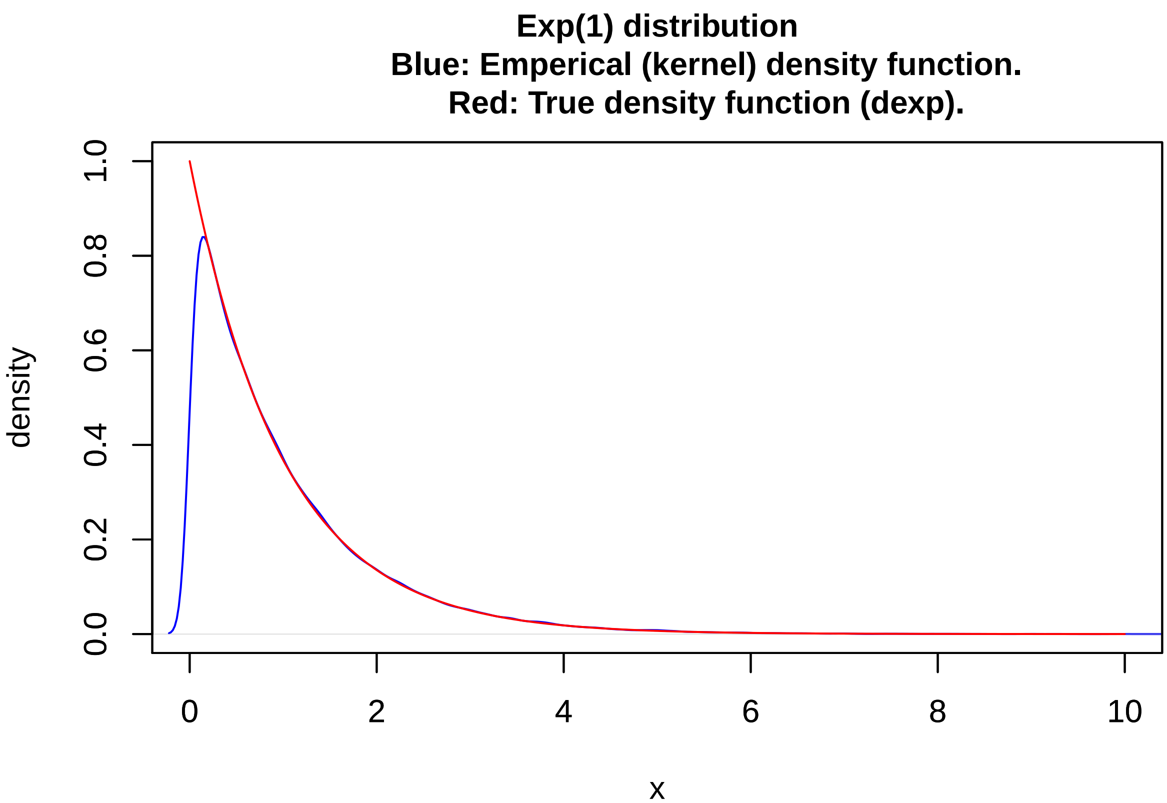
```
R <- 100000

rolls <- roll_dice(R)
roll_freq <- table(rolls) / R
barplot(roll_freq, main="Dice rolls. Observed relative frequencies")
```

Problem 2

As shown in lecture in Section 1.3 and specifically in Ex1.1, exponential distribution can be realized as $-\frac{log(u)}{\lambda}$, where u comes from $U[0, 1]$.

```
exponential <- function (n, rate=1) {
    u <- runif(n)
    x <- -log(u) / rate
    x
}
```

```
gen_exp <- exponential(R, rate=1)
exp_seq <- seq(0, 10, 0.01)
plot(density(gen_exp),
     xlim=c(0,10), ylim=c(0,1),
     col="blue", xlab="x", ylab="density", cex.main=1,
     main="Exp(1) distribution
           Blue: Emperical (kernel) density function.
           Red: True density function (dexp).")
par(new=TRUE)
plot(exp_seq, dexp(exp_seq), type="l",
     xlim=c(0,10), ylim=c(0,1),
     col="red", xlab="", ylab="", main="")
```



Problem 3

1) According to property ii), $N(t + s) - N(t)$ is independent of $N(t)$.
2) From property iii) we know, that $N(t) \sim Poi(\lambda t)$, which means, that $N(t)$ depends on t.
3) Let's assume, that $N(t + s) - N(t)$ depends on t. Then both $N(t + s) - N(t)$ and $N(t)$ depend on

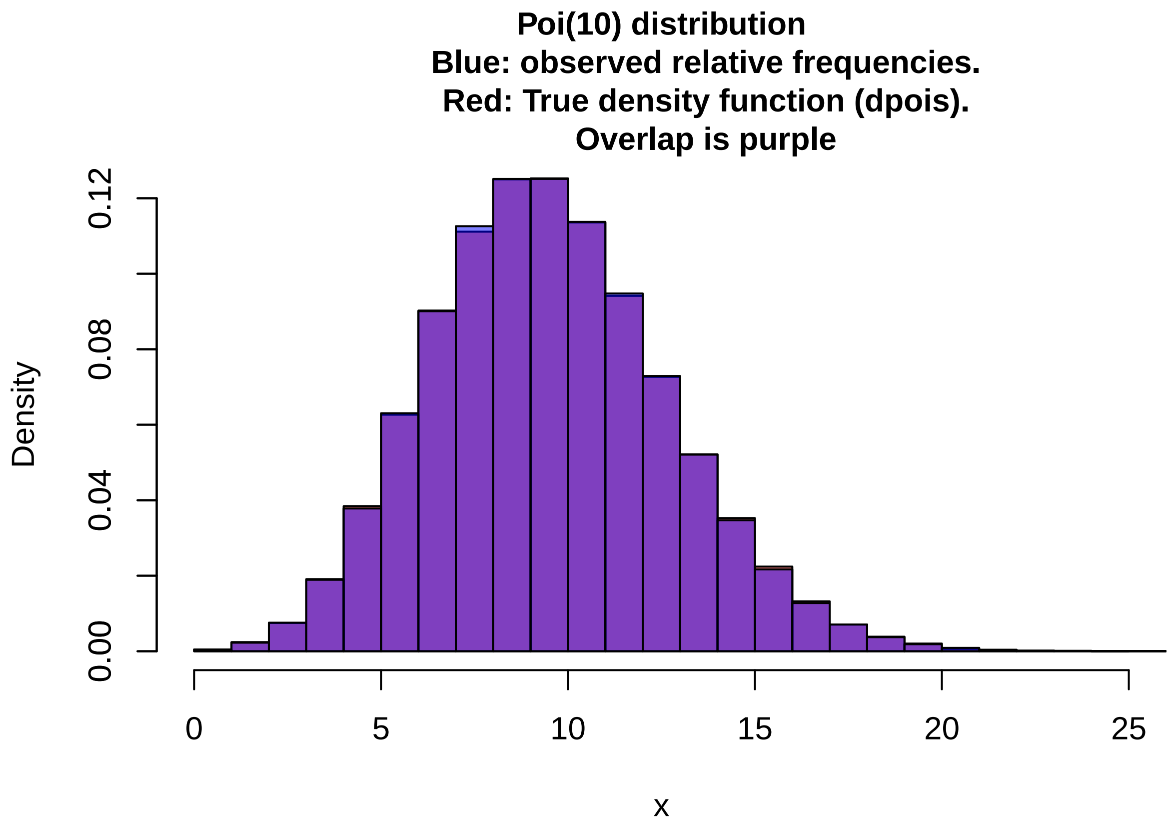
t, which makes them correlated. In this case, assumption of independence is violated. Therefore, $N(t+s) - N(t)$ is independent of t.

4) If $N(t+s) - N(t)$ is independent of starting point t, it can only depend on the interval length s. Therefore, $N(t+s) - N(t) = N(s)$.
5) Property iii) states, that $N(t) \sim Poi(\lambda t)$, therefore $N(t+s) - N(t) = N(s) \sim Poi(\lambda s)$.

```
poisson_single <- function(rate=1){
    counter <- 0
    sum <- 0
    repeat{
        sum <- sum + exponential(1, rate)
        if(sum > 1){
            break
        }
        counter <- counter + 1
    }
    counter
}

poisson <- function(n, rate=1){
    vec <- numeric(n)
    for (i in 1:n){
        vec[i] <- poisson_single(rate)
    }
    vec
}
```

```
R <- 100000
rate <- 10

blue <- rgb(0, 0, 1, alpha=0.5)
red  <- rgb(1, 0, 0, alpha=0.5)
```

```
gen_poi <- poisson(R, rate)
plot_seq <- seq(1, 25)
hist(gen_poi, freq=FALSE, breaks=25,
    xlim=c(0,25),
    col=red, xlab='x', cex.main=1,
    main="Poi(10) distribution
          Blue: observed relative frequencies.
          Red: True density function (dpois).
          Overlap is purple")
barplot(dpois(plot_seq, rate), space=0, add=TRUE,
    xlim=c(0,25),
    col=blue)
```


Poi(10) distribution
Blue: observed relative frequencies.
Red: True density function (dpois).
Overlap is purple
Density
0.00
0.04
0.08
0.12
0
5
10
15
20
25
x