Bayesian Econometrics
Semester 2, 2014

ASSIGNMENT 1 (due September 12th 4:00pm)

Part I: Computer Exercises

Note: *For some of the computer exercises you have to submit the program files and hard copies of the figures you have created to answer the questions. Please make sure that your programs are clearly written and readable and easy to follow for a third party and the graphs clearly labelled.*

Report all results in your answers.

[9] Question 1 (Monte Carlo Integration and Inverse Transform Method): Suppose that the posterior distribution for a parameter θ is given by a normal distribution with mean 0.5 and variance 2 truncated below at zero, $N_{(0,+\infty)}(0.5,2)$

a) Write a short computer program that generates S=1000 draws from this truncated Normal distribution and compute the mean and variance based on the generated draws from the distribution.
(i) Write a short pseudo code with the key steps/components of the program.
(ii) Implement the program in Matlab.

b) Extend your program to plot the estimated distribution of θ (histogram of the draws or kernel-smoothed graph).

c) *Based on your draws*
(i) Compute and report the probability that θ lies between 0 and 1,
(ii) Compute and report the 0.05 and 0.95 quantiles of the distribution.
What is the length of the interval (distance between values of θ at the 5% and 95% quantiles) ?

d) Can you find a 90% interval with a smaller length? Extend your program to find the highest posterior 90% credibility interval for θ as the 90% (interval with the shortest length). Report the interval and the interval length.

Submit the m-file for the final program and call it yourname_assign1q1.m

[4] Question 2 (Method of Composition): Suppose you want to generate draws from a student-t density $t(\nu, \mu, \sigma^2)$ and assume there is no function available to generate draws from a student-t density directly.

Use the result that if

$$h(\lambda_i) = G\left(\frac{\upsilon}{2}, \frac{\upsilon}{2}\right) \quad and \quad f(u_i|\mu, \sigma^2, \lambda_i) = N(\mu, \lambda_i^{-1}\sigma^2)$$

then

$$f(u_i|\sigma^2) = \int f(u_i|\sigma^2, \lambda_i)\ h(\lambda_i)\ d\lambda_i = t(\upsilon, \mu, \sigma^2)$$

and the available matlab functions to generate draws from the normal and Gamma distributions.

(a) Write a short matlab program that generates draws from a student-t density with 5 degrees of freedom, mean of 1 and variance of 2. The program should also compute the mean and the variance of the draws.
(i) Write a pseudo code for the program.
(ii) Implement the program in mablab code.

(b) Report the mean and the variance of the draws.

Submit the m-file for the final program and call it yourname_assign1q2.m

[6] Question 3 (Linear Regression Model)

Use the estimation programs and the data from the house-price example in ch3.zip posted for Lab2. Refer to the handout for chapter 3 from Koop's book with description and notation of simple linear model with conjugate priors.

a) Plot the predictive distribution for a new observation y*, with x* = [1 5000 2 2 1], under the informative prior.
Then, compute the interval in which you would expect the predicted outcome to lie within 90% probability under the informative prior.

Change the prior mean to be zero for all elements in the beta vector.

b) Compute and report the 90% HPD intervals for all elements in β and for σ^2.

c) Consider a model that assumes that only the number of bathrooms and bedrooms affect the house price but not the lot size and number of storeys (call it model 1).
Compare the original model with no restrictions (call it model 2).
Compute and report the Bayes Factor for model 1 over model 2. Briefly interpret.

Just provide the modified or additional code that you created to answer the questions.

Part II: **Analytical Exercises**

[8] Questions 4: Suppose you have n observations, 1=1,2,…,n on independent coin tosses where $y_i = 1$ refers to the coin coming up heads and $y_i = 0$ the coin coming up tails. The probability that the coin comes up with heads is $P(y_i = 1) = \pi$ and the probability that the coin comes up with tails is $P(y_i = 0) = 1 - \pi$.

a) Write down the likelihood, the prior and then the expression for the posterior distribution, assuming a uniform prior distribution on π.
Working with the exact form of Bayes theorem (not the proportional version), derive the posterior distribution for π.

b) What kind/type of prior distribution is the uniform prior? Explain your answer referring to the concepts of conjugate, (im)proper an uninformative priors.
Are there any advantages/disadvantages for using such a prior (in general)?

c) Suppose you are told that a previous experiment with the coin was done with s tosses and s_1 came up heads. How can you incorporate this prior information in your Bayesian analysis from (b)?
Change what you need to change and derive the posterior distribution again. Use the notion of bayesian updating (assume uniform prior on π in the outset).

[6] Question 5: Consider the simple model

$$y_i = \mu_i + \varepsilon_i \ , \quad \varepsilon_i \sim N(0,1)$$

where $\mu_i = \mu$.

Suppose we have $i = 1,2, \ldots, n$ observations.

(a) Write down the likelihood for vector y in matrix form. Define the mean vector as $\mu \times i_n$ where i_n is the identity vector.

Use the proportional version of Bayes Theorem and write down the expression for the posterior density of μ assuming that $p(\mu) = N(b_0, B_0)$, where both hyperparameters are scalars.

b) Derive the mean and variance of the normal posterior density of μ using the completion of squares method. Show your work.

[3] **Question 6**: Consider the Accept Reject Method discussed in lecture 7 and the case when $f(y) = k * r(y)$ and k is unknown, i.e. we don't know the normalizing constant of $f(y)$.

Show that by setting the acceptance probability at $r(y)/cg(y)$ the method will give you draws from the target density $f(y)$.