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# AN APPLICATION IN MULTIVARIATE STATISTICAL PROCESS CONTROL FOR POWER SUPPLY CALIBRATION

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## Key Words

Statistical process control; Power calibration; Prediction ellipsoid; Multivariate normal distribution; Multivariate multiple linear regression; Hotelling's  $T^2$ .

## Introduction

Advanced Energy Industries (AE) manufactures direct current (DC), midfrequency, and radio-frequency power supplies for use in thin-film processes. AE's customer base lies primarily in the semiconductor and industrial coating industries, where high power levels are required in sputtering and etching systems to vaporize a material and de-

posit it on its intended target. Typical applications involve power levels ranging from 500 W to 30 kW. In early 1994, AE began production of the Lightning family of DC power supplies for one of its largest customers, Applied Materials. At that time the Lightning family consisted of two models—a 6-kW and a 12-kW version. Prior to production, AE had designed and implemented a computerized automatic test station to exercise units electronically as well as to collect and store test data. The automatic test station uses National Instrument's LabVIEW for Windows® as the controlling software. It was desired to embed real-time statistical process control (SPC) algorithms within the automatic test software to aid manufacturing personnel in identifying and correcting quality problems on-line. The

production rate for Lightning units was expected to be approximately five units per day over two shifts. Subgroup sampling was not deemed appropriate given this relatively low production rate. Two univariate Shewhart-type SPC charts, the individuals chart and the moving-range chart (1), were adapted to the automatic test of several product attributes for the Lightning product family.

Unfortunately, univariate SPC approaches are not readily adaptable to applications involving large sets of similar product attributes (i.e., the multivariate case) (2). The power calibration test automatically determines whether the power error falls within AE's engineering specifications for various input power levels, known as the setpoint power. Power error is defined as the actual power minus the setpoint power. The power calibration test for the Lightning 12k is performed at power levels ranging from 1200 W to 12 kW in 1200-W increments, and, for the Lightning 6k, the test is performed at setpoint power levels ranging from 600 W to 6 kW in 600-W increments. AE's specifications for power error are 1.0% of setpoint power or 0.2% of the full-scale power, whichever is greater. This means that for the Lightning 6k, the power calibration test may be stated mathematically as testing whether  $\text{Abs}(\text{power error W}) \leq \text{Max}\{(0.01 \times \text{setpoint W}), 12 \text{ W}\}$ . For the Lightning 12k, the test determines whether  $\text{Abs}(\text{power error W}) \leq \text{Max}\{(0.01 \times \text{setpoint W}), 24 \text{ W}\}$ . In both cases, the  $\text{Abs}\{\cdot\}$  notation represents the absolute value function.

The data, including the engineering specifications for the first production Lightning 12k unit, are shown in Table 1. A functional power calibration test would simply ensure that the power error for each unit fell within the specification limits at each setpoint power level, as shown in

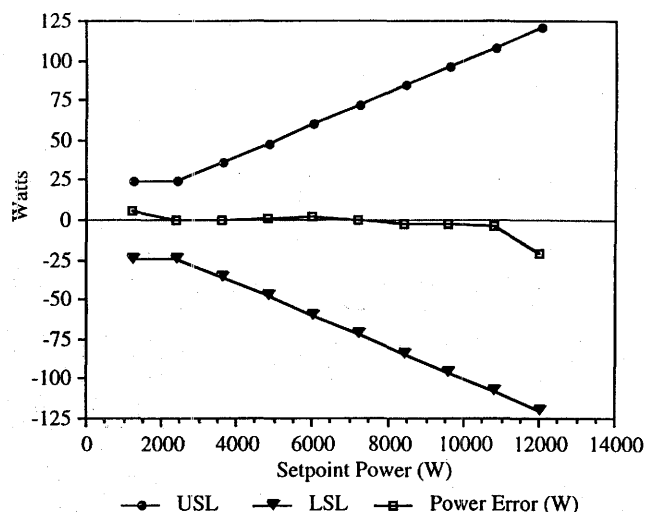


Figure 1. Plot of power error and the specification limits for the calibration data shown in Table 1.

Figure 1. With AE's drive to implement real-time SPC on all products, starting with the Lightning product family, a simple functional test was not sufficient. A method was needed to compare power calibration across the entire operating range to ensure that the performance of each unit undergoing testing was not "statistically different" from that of previous units. Implementation of a univariate SPC approach, such as the individuals and moving-range charts, on the power calibration test for the Lightning automatic test station would require that the power calibration for each unit undergoing testing be compared to the performance of previous units by the 10 setpoint power levels on

Table 1. Power Calibration Test Data for the First Production Lightning 12k Unit

SETPOINT W	ACTUAL W	POWER ERROR	LOWER SPEC. LIMIT	UPPER SPEC. LIMIT
1200	1205.28	5.28	-12.0	12.0
2400	2400.00	0.00	-24.0	24.0
3600	3600.36	0.36	-36.0	36.0
4800	4800.96	0.96	-48.0	48.0
6000	6002.40	2.40	-60.0	60.0
7200	7200.00	0.00	-72.0	72.0
8400	8397.48	-2.52	-84.0	84.0
9600	9598.08	-1.92	-96.0	96.0
10800	10796.76	-3.24	-108.0	108.0
12000	11979.60	-20.40	-120.0	120.0

an individual basis. It would also mean that the test operator would have to interpret more than 10 separate control charts for each unit tested. As this approach is inefficient and, at best, marginally correct, a multivariate technique was needed.

### A Prediction Approach Based on Multivariate Multiple Linear Regression

Because it was desired to compare the calibration of each power supply under test to previous units over the entire operating range, the first step in the development of a multivariate approach for power calibration was to examine the simple linear regression using the setpoint power as the independent variable and the actual power as the dependent variable. Due to the manner in which Lightning units are calibrated prior to the calibration test, it was decided not to fix the intercept at zero in the simple linear regression. Figure 2 shows the regression plot for the data taken from the first production Lightning 12k as given in Table 1. Although the high  $R^2$  value may give the impression that the data for the first production unit was selected to show uncharacteristically high correlation between setpoint and actual power levels, this is not the case. No company, including AE, would remain in the power supply business long if actual power and setpoint power were not highly correlated. Indeed, as we will show later, the strong correlation between actual power and setpoint power is the norm as opposed to the exception, and the fact that actual

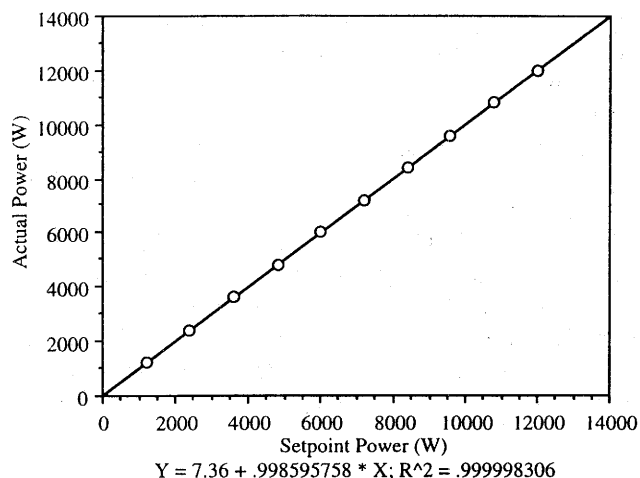


Figure 2. Regression plot for the power calibration data shown in Table 1.

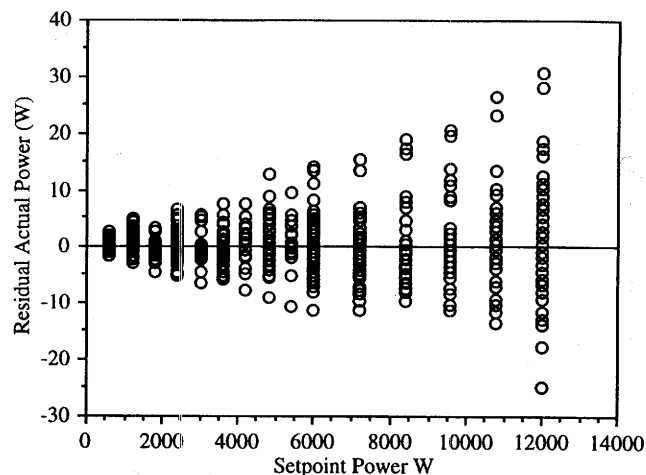


Figure 3. Residuals versus fitted plot for the pretransform Phase I simple linear regression.

and setpoint power are highly correlated provides the basis for the method which will be developed in this article.

Sullivan and Woodall (3) advocate the use of two phases for multivariate control of process parameters. In Phase I, estimates of the in-control process parameters are obtained from a historical sample. In Phase II, the estimates from Phase I are used to detect out-of-control situations for the process parameters. For development of this case, the first 23 production Lightning 6k and 66 Lightning 12k units served as the historical sample for Phase I. The actual power was regressed on the setpoint power for the 89 units comprising the Phase I sample. Based on the residuals versus fitted plot shown in Figure 3, the residuals displayed strong evidence of nonhomogeneity of variance (i.e., that the standard deviation of the residuals increases with setpoint power). The results for several well-known tests for unequal variances (4,5) are shown in Table 2. These tests provide solid evidence that the data do not support a conclusion for homogeneity of variance for the residuals at different setpoint power levels.

Table 2. Tests for Unequal Variance for the Residuals for the Untransformed Linear Regression

TEST	F-RATIO	DF NUM	DF DEN	PROB > F
O'Brien [0.5]	8.8597	14	875	0.0000
Brown-Forsythe	14.3645	14	875	0.0000
Levene	15.7389	14	875	0.0000
Bartlett	34.2161	14	—	0.0000

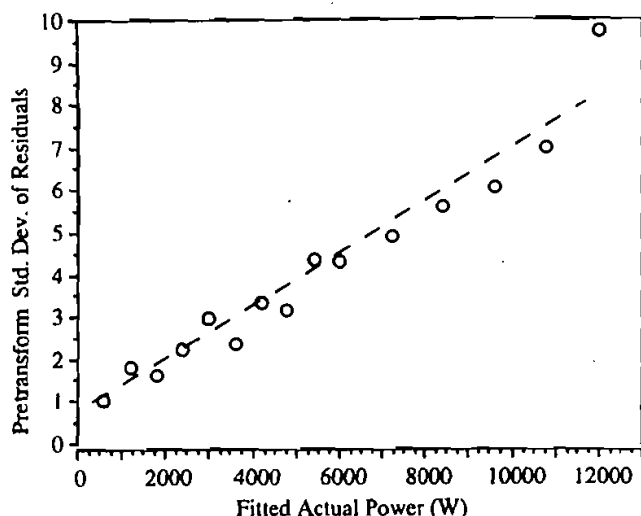


Figure 4. Scatterplot of the standard deviation of actual power residuals versus fitted actual power.

For the setpoint power calibration test, nonhomogeneity of variance is expected. This is the reason that AE's engineering specification limits for the power calibration test are a fixed percentage of setpoint power. As shown in Figure 1, the specification limits are funnel-shaped in order to compensate for increased variability in the power error as the setpoint power increases. Unfortunately, this implies that simple linear regression is inappropriate for analysis of the power calibration data, unless a variance stabilizing transformation (4) or weighted least squares (6) is used. Stabilizing the variance using a simple nonlinear transformation was the more expedient of the two approaches, as standard analysis techniques and software could be used on the transformed data.

The first step in determining the appropriate transformation was to find the relationship of the standard deviation of the actual power residuals to the fitted actual power. Figure 4 shows that a linear relationship between the two quantities is plausible. A linear relationship between the standard deviation of the dependent variable and the independent variable implies that a natural log transformation on the dependent variable is appropriate (4). However, regressing the natural log of the actual power onto the untransformed setpoint power will artificially introduce

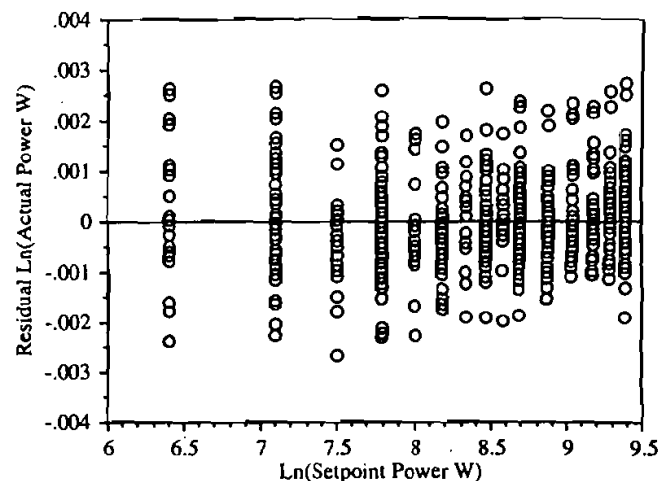


Figure 5. Residuals versus fitted plot for the posttransform Phase I simple linear regression.

curvature into the relationship between the actual power and the setpoint power. Therefore, the natural log transformation was applied to both variables in order to maintain the ideal linear relationship with slope equal to 1 and intercept equal to 0. As evidenced in Figure 5, the application of the natural log transformation to both variables appears to induce a more constant standard deviation for the actual power residuals. Subsequent to the log transform, the tests for unequal variances among the residuals, as shown in Table 3, still force us to conclude that nonhomogeneity of variance is present. However, the test statistics in Table 3 have been reduced such that the conclusion for nonhomogeneity of variance is significantly weakened when compared to the test statistics for the untransformed simple linear regression. Because no variance-stabilizing transform guarantees passage of the tests for unequal variances subsequent to the transform, we will proceed on our subjective interpretation of Figure 5 and on the simplicity of this particular transform.

Performed individually for each of the first 89 units tested, simple linear regression yields 89 estimates of two statistics—namely the slope and the intercept.\* It would be desirable to find that the intercept is identically 0 and the slope identically 1 for all units tested. However, with the absence of outliers and no diagnostic problems with the

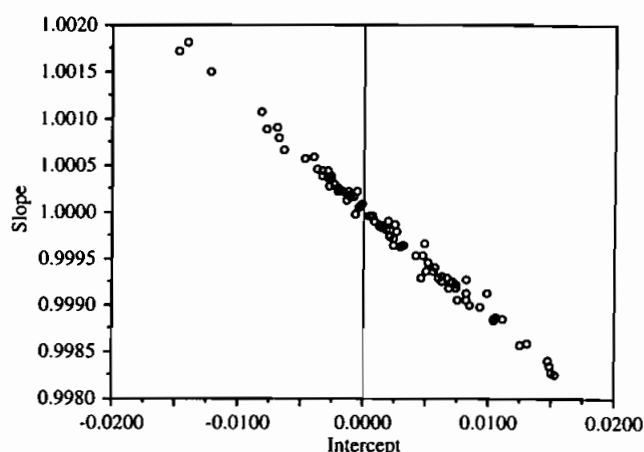
\*The informed reader may question the use of the slope and intercept from the simple linear regression as the statistics of interest when the 10-dimensional vector of power errors was available. The slope and intercept were chosen because the power error data tended to be correlated, thus making the variance-covariance matrix singular. The correlation in the power error data was due partially to the physics of the design and partially to overfitting the regression model during the actual calibration of the units prior to test. The solution to this problem is the subject of another discussion, however.

**Table 3.** Tests for Unequal Variance for the Residuals for the Posttransform Linear Regression

TEST	F-RATIO	DF NUM	DF DEN	PROB > F
O'Brien [0.5]	4.9872	14	875	0.0000
Brown-Forsythe	5.4032	14	875	0.0000
Levene	5.6283	14	875	0.0000
Bartlett	4.6628	14	—	0.0000

model, a slope-intercept pair close to the ideal point [1, 0] would indicate good correlation between the setpoint power and the actual power. For the 89 individual simple linear regressions comprising the Phase I sample, the slope-intercept pairs from the individual simple linear regressions are shown in Figure 6.

The problem now becomes how to determine which Phase II slope-intercept pairs are "statistically different" from the Phase I pairs. Because slope-intercept pairs can be treated as observations from a bivariate normal distribution, the  $100(1 - \alpha)\%$  prediction region for the Phase I slope-intercept pairs can easily be calculated. Each Phase II slope-intercept pair can then be compared to the Phase I  $100(1 - \alpha)\%$  prediction region to determine whether or not the slope-intercept pair for the current unit under test is consistent with the distribution of Phase I pairs. If the slope-intercept pair falls within the 99% prediction ellipse, then the unit is considered to be like Phase I units in terms of power calibration. If not, then the unit is recalibrated and retested. We chose to implement the prediction method outlined above, coupled with tests for outliers in the residuals, a test for the mean squared error of the residuals, and

**Figure 6.** Scattergram of slope-intercept pairs for the posttransform Phase I data.

pass/fail runs testing for identification of on-line calibration problems. We chose not to implement a true multivariate quality control chart because of the practical considerations involved in having operators interpret out-of-control conditions for the multivariate case graphically. Using a prediction-based approach rather than a multivariate control chart, the results of the multivariate testing can be condensed to a simple go-no go criterion which is easily integrated with the real-time nature of the automated power calibration test. A discussion of some of the relevant problems with on-line interpretation of out-of-control conditions can be found in Refs. 7-9.

To accomplish this method, the problem is formulated in terms of multivariate multiple linear regression. Suppose that a model is postulated which relates  $m$  responses,  $y_1, y_2, \dots, y_m$  to a set of independent variables,  $z_1, z_2, \dots, z_r$ , where each response is assumed to be linearly related to the independent variables (10-12). This relationship is stated as

$$y_1 = \beta_{01} + \beta_{11}z_1 + \dots + \beta_{r1}z_r + \varepsilon_1,$$

$$y_2 = \beta_{02} + \beta_{12}z_1 + \dots + \beta_{r2}z_r + \varepsilon_2,$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$y_m = \beta_{0m} + \beta_{1m}z_1 + \dots + \beta_{rm}z_r + \varepsilon_m.$$

In vector-matrix notation and assuming that there are  $n$  replications for each set of  $m$  responses, the statement of the model becomes

$$\begin{matrix} \mathbf{Y} & = & \mathbf{Z} & \boldsymbol{\beta} & + & \boldsymbol{\varepsilon} \\ (n \times m) & & [n \times (r + 1)] & [(r + 1) \times m] & & (n \times m) \end{matrix}$$

where  $E(\varepsilon_i) = 0$ ,  $\text{Cov}(\varepsilon_i, \varepsilon_k) = \sigma_{(ik)}\mathbf{I}$  for  $i, k = 1, 2, \dots, m$ , where the  $m$  observations on the  $j$ th trial have covariance matrix  $\Sigma$ , and where  $\boldsymbol{\beta}$  and  $\sigma_{ik}$  are unknown;  $\mathbf{I}$  represents the  $m \times m$  identity matrix. The implication is that observations from different trials are assumed to be uncorrelated.

For the case of the Lightning power calibration test, the first step is to calculate the mean vector for all slope-intercept pairs. This could be accomplished by simply calculating the arithmetic average of each set of statistics, or, to be consistent with the notation of multivariate multiple linear regression, the mean vector is calculated using  $\hat{\boldsymbol{\beta}} = (\mathbf{Z}^T\mathbf{Z})^{-1}\mathbf{Z}^T\mathbf{Y}$ , where  $r = 0$ ,  $m = 2$ ,  $n$  units have been tested, and the vector  $\mathbf{Z}$  has dimension  $n \times 1$  and is composed of 1's. The matrix  $\mathbf{Y}$  has dimension  $n \times 2$  and is composed of the vector of slopes in the first column and

the vector of intercepts in the second column. Performing the calculation for the first 89 production units, the mean vector is found to be [0.9997730, 0.0023176]. This result implies that the expected slope-intercept combination is very close to the ideal slope-intercept pair [1, 0]. If the  $100(1 - \alpha)\%$  confidence ellipse for the mean of the slope-intercept pairs includes [1, 0], then one might conclude that the process is centered. The calculation of the  $100(1 - \alpha)\%$  confidence ellipse (10-12) is based on Hotelling's  $T^2$ , where Hotelling's  $T^2$  is given by

$$T^2 = \left( \frac{\hat{\beta}^T \mathbf{z}_0 - \beta^T \mathbf{z}_0}{\sqrt{\mathbf{z}_0^T (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{z}_0}} \right)^T \left[ \frac{n \hat{\Sigma}}{n - r - 1} \right]^{-1} \left( \frac{\hat{\beta}^T \mathbf{z}_0 - \beta^T \mathbf{z}_0}{\sqrt{\mathbf{z}_0^T (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{z}_0}} \right);$$

where  $\mathbf{z}_0 = 1$  represents the desired value of the independent variables for prediction and the estimate for  $\Sigma$  for the Phase 1 sample is

$$\hat{\Sigma} = \frac{(\mathbf{Y} - \mathbf{Z}\hat{\beta})(\mathbf{Y} - \mathbf{Z}\hat{\beta})^T}{n} = \begin{bmatrix} 3.928 \times 10^{-5} & -4.5123 \times 10^{-6} \\ -4.5123 \times 10^{-6} & 5.231 \times 10^{-7} \end{bmatrix}.$$

The  $100(1 - \alpha)\%$  confidence ellipsoid (7-9) for  $\beta^T \mathbf{z}_0$ , based on Hotelling's  $T^2$ , is given by the following inequality:

$$\begin{aligned} & \left( \beta^T \mathbf{z}_0 - \hat{\beta}^T \mathbf{z}_0 \right)^T \left[ \frac{n \hat{\Sigma}}{n - r - 1} \right]^{-1} \left( \beta^T \mathbf{z}_0 - \hat{\beta}^T \mathbf{z}_0 \right) \\ & \leq \left( \mathbf{z}_0^T (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{z}_0 \right) \left( \frac{m(n - r - 1)}{n - r - m} \right) F_{(\alpha, m, n - r - m)}. \end{aligned}$$

Setting  $\alpha = 0.01$  in the above yields the 99% simultaneous confidence ellipsoid for the mean of the slope-intercept pairs.

The  $100(1 - \alpha)\%$  simultaneous prediction interval for the individual slope-intercept pairs is also based on Hotelling's  $T^2$  and is given by

$$\begin{aligned} & \left( \mathbf{Y}_0 - \hat{\beta}^T \mathbf{z}_0 \right)^T \left[ \frac{n \hat{\Sigma}}{n - r - 1} \right]^{-1} \left( \mathbf{Y}_0 - \hat{\beta}^T \mathbf{z}_0 \right) \\ & \leq \left( 1 + \mathbf{z}_0^T (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{z}_0 \right) \left( \frac{m(n - r - 1)}{n - r - m} \right) F_{(\alpha, m, n - r - m)}. \end{aligned}$$

Figure 7 shows the 99% prediction ellipsoid for slope-intercept pairs in the Phase I sample. The 99% confidence ellipsoid for the mean of the slope-intercept pairs is obscured by the observations in Figure 7. However, it does not include the ideal point [1, 0]. This implies that the power calibration process, as measured by this multivariate technique, was not centered and that some work is required to center the process. Figure 7 provides a reasonable "sanity" check for the method, as it implies that only 3 out of a total of 89 Lightning units would have failed this component of the production testing if the Phase I prediction ellipse had been implemented during the initial production runs. Having gathered test data and calculated the 99% prediction ellipse for the Phase I sample, the method was integrated with the automatic power calibration test software and applied to Phase I units. The algorithm was implemented such that the test software paused for the

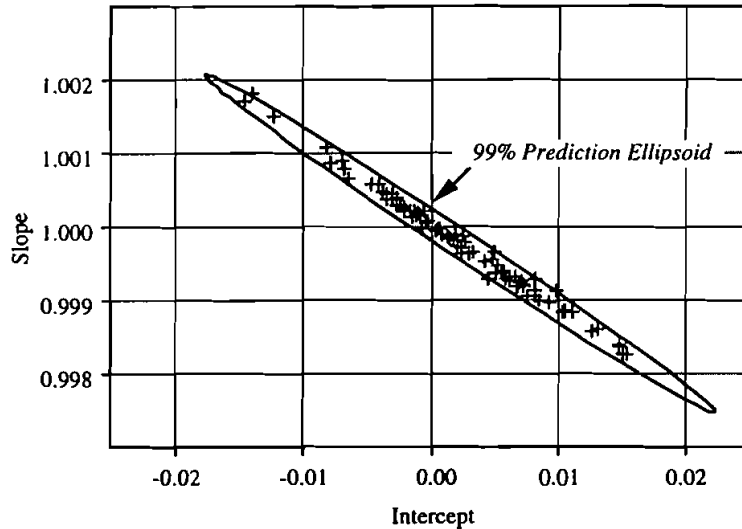


Figure 7. Initial Phase I 99% prediction ellipsoid for slope-intercept pairs.

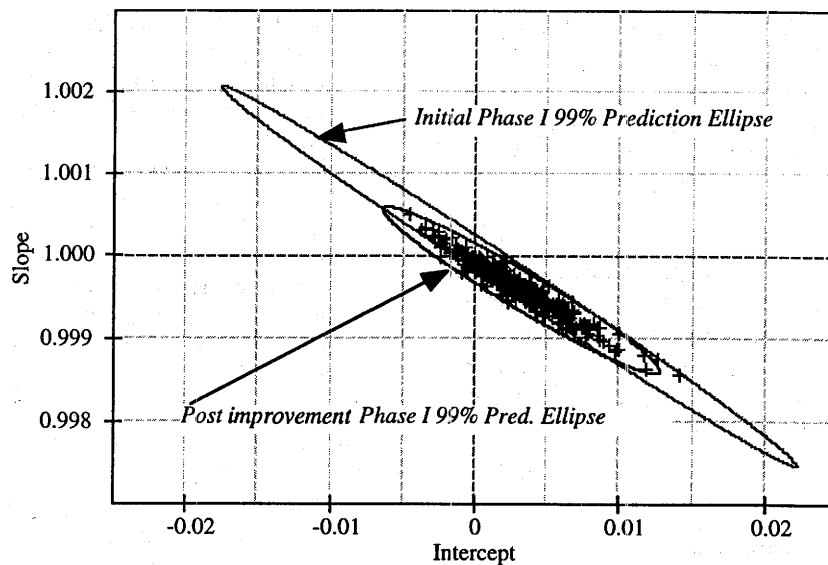


Figure 8. Phase I 99% prediction ellipsoids before and after power calibration improvement efforts with data for the first 300 units subsequent to the process change.

operator to take action only if the unit failed the multivariate prediction test, a test for outliers, a test for the magnitude of the mean-squared error (MSE) for the residuals, or runs tests. If all went well and the unit passed these tests, the additional test time consumed by these statistical tests was very small.

### Conclusion

The method was used successfully for several months, and several quality problems were identified on-line by test operators. The 99% prediction ellipse was updated periodically but did not change significantly from the initial Phase I prediction ellipse. Then in the fall of 1994, AE made changes to its power calibration procedures to try to center the calibration process and improve overall power calibration performance. Figure 8 shows the results of this improvement effort by comparing the initial 99% prediction ellipse and postimprovement Phase I 99% prediction ellipse. The area within the 99% prediction ellipse was decreased significantly, implying that the process variability in the slope-intercept pairs had been decreased. The post-improvement prediction ellipse has performed well since the power calibration process was changed. One particular advantage of this method is that it can be readily adapted to the power calibration tests for AE's other product lines.

A top-notch design, a highly motivated test and production staff, and real-time SPC methods integrated with the

automatic test software have yielded notable quality results during the first months of production of Lightning power supplies. Figure 9 shows the process capability of the power calibration test in terms of the capability indices  $C_p$  and  $C_{pk}$  by setpoint power level for the first 632 Lightning units produced. Figure 9 implies that the Lightning power calibration performance appears to be a capable and centered process, as evidenced by the small difference between  $C_p$  and  $C_{pk}$ .

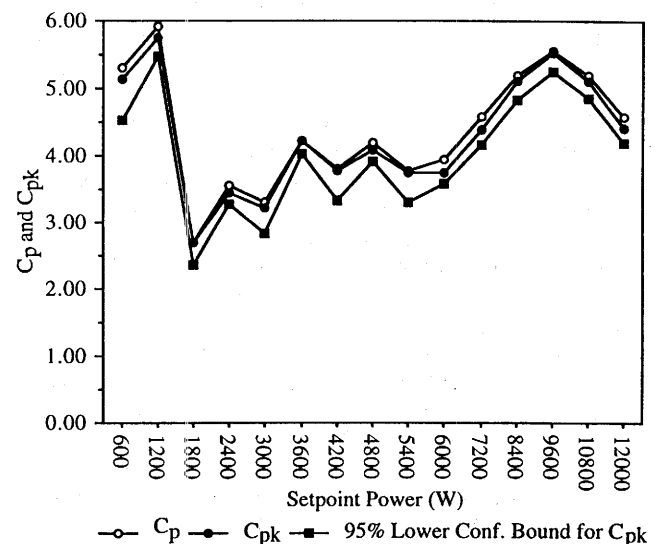


Figure 9. Process capability by setpoint power for the first 632 units.



The  $100(1 - \alpha)\%$  lower confidence interval for  $C_{pk}$  is also shown in Figure 8. It is calculated using

$$C_{pkL} = \hat{C}_{pk} \left[ 1 - z_{\alpha} \left[ \left( \frac{n-1}{n-3} \right) - \left( \frac{n-1}{2} \right) \left\{ \frac{\Gamma[\frac{1}{2}(n-2)]}{\Gamma[\frac{1}{2}(n-1)]} \right\}^2 \right]^{1/2} \right],$$

where  $\hat{C}_{pk}$  is the estimator for  $C_{pk}$ ,  $C_{pkL}$  is the  $100(1 - \alpha)\%$  lower confidence estimate,  $z_{\alpha}$  is the upper  $\alpha$  quantile of the standard normal distribution,  $\alpha = 0.05$ ,  $n$  is the sample size, and  $\Gamma(\cdot)$  is the gamma function (13). For the operating range of interest to AE's customer base, the lower confidence bound for  $C_{pk}$  is well above 2.0. This implies that for this particular set of tests, AE's corporate objective of six-sigma quality has been achieved and exceeded.

### Acknowledgments

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