1 MEWMA chart for single time series vectors

time series $X_1, X_2, \ldots, X_T$, $X_t \in \mathbb{R}^p$.

In-control process: $\mathbb{E}(X_t) = \mu_0$

Out-of-control process: $\mathbb{E}(X_t) = \mu_0 + \delta$ with shift $\delta \neq 0$

- MEWMA process

$$\tilde{X}_t = \lambda X_t + (1-\lambda)\tilde{X}_{t-1} \text{ for } t = 1, 2 \ldots$$

given λ... smoothing parameter, δ

or

$$\tilde{X}_t = \Lambda X_t + (I - \Lambda)\tilde{X}_{t-1}$$

with diagonal matrix Λ
out-of-control signal if

$$T^2 = \left(\tilde{X}_t - \mu_0\right)^T \Sigma^{-1} \left(\tilde{X}_t - \mu_0\right) > h,$$

h threshold value

References:

Cynthia A. Lowry, William H. Woodall, Charles W. Champ and Steven E. Rigdon (1992) A Multivariate Exponentially Weighted Moving Average Control Chart. *Technometrics*, Vol. 34, 46-53

MEWMA chart

Lee, M. H. and Khoo, M. (2006). Optimal statistical design of a multivariate EWMA chart based on ARL and MRL, *Communications in Statistics: Simulation and Computation*, Vol. 35, 831-847.

MEWMA chart,

H. G. Kramer & L.V. Schmid (1997) Ewma charts for multivariate time series, Sequential Analysis: Design Methods and Applications, 16:2, 131-154, DOI: 10.1080/07474949708836378

MEWMA chart with a matrix as smoothing parameter

Stoumbos, Zachary G.; Sullivan, Joe H. . (2002). Robustness to Non-Normality of the Multivariate EWMA Control Chart. Journal of Quality Technology, 34(3), 260–276.

2 MEWMA chart for averages of time series vectors

- MEWMA process

$$\tilde{X}_t = \lambda \bar{X}_t + (1-\lambda)\tilde{X}_{t-1} \text{ for } t = 1, 2 \ldots$$

3 ARL average run length

function $ARL(\delta, \lambda, h)$ computes ARL

$ARL_0 = ARL(0, \lambda, h)$

We want to calculate

$$\arg \min_{\lambda, h: ARL_0(\lambda, h) = 370.4} ARL(\delta, \lambda, h)$$

4 New Ideas

time series (phase II) $X_1, X_2, \dots, X_T$, $X_t \in \mathbb{R}^p$.

In-control process: $\mathbb{E}(X_t) = \mu_0$ known

$\text{cov}(X_t) = \Sigma$ unknown

Estimated covariance matrix $\hat{\Sigma}$: data from phase I

- MEWMA process

$$\tilde{X}_t = \lambda \bar{X}_t + (1 - \lambda) \tilde{X}_{t-1} \text{ for } t = 1, 2 \dots$$

- out-of-control signal if

$$T^2 = \left(\tilde{X}_t - \mu_0\right)^T \hat{\Sigma}^{-1} \left(\tilde{X}_t - \mu_0\right) > h$$

h threshold value

- out-of-control process with shift $d \in \mathbb{R}^p$ where $\mathbb{E}(X_t) = \mu_0 + d$

$$\delta = d^T \hat{\Sigma}^{-1} d$$

$ARL_0 = ARL(0, \lambda, h)$

The aim is to simulate the process and to calculate

$$\arg \min_{\lambda, h: ARL_0(\lambda, h) = 370.4} ARL(\delta, \lambda, h)$$

- MEWMA process with different λ's

$$\tilde{X}_t = \Lambda \bar{X}_t + (I - \Lambda) \tilde{X}_{t-1} \text{ for } t = 1, 2 \dots$$

where $\Lambda = \text{diag}(\lambda_1, \dots, \lambda_p)$ a diagonal matrix

Reason for different λ's: different magnitude of the components of d

Literature:

M. A. Mahmoud & P. E. Maravelakis (2010) The Performance of the MEWMA Control Chart when Parameters are Estimated, Communications in Statistics - Simulation and Computation, 39:9, 1803-1817

influence of estimated parameters, ARL depending on λ, p and number of items in phase I, h is chosen to yield $ARL_0 = 200$ in control

Lee, M. H. and Khoo, M. (2006). Optimal statistical design of a multivariate EWMA chart based on ARL and MRL, *Communications in Statistics: Simulation and Computation*, Vol. 35, 831-847.

similar idea but no estimated parameters