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MULTIVARIATE CUMULATIVE SUM CONTROL CHARTS BASED ON PROJECTION PURSUIT

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Abstract: A natural multivariate extension of the two-sided cumulative sum chart is proposed via projection pursuit. A modification is given for improving its performance for the special situation in which the process mean is already shifted at the time the charting begins. Simulation studies show that the new charts have slightly better performance than the competing charts (MC1, MEWMA1 and MEWMA2) in terms of the average delayed run length and standard deviation of the delayed run length, while performing a little worse in terms of the average run length. A distinctive advantage of the proposed charts is that they are more effective than the MC1, MEWMA1, MEWMA2, the combined χ^2-MEWMA1 and the combined χ^2-MEWMA2 charts against the inertia in reacting to mean shifts.

Key words and phrases: Average run length, CUSUM control chart, multivariate control chart, projection pursuit.

1. Introduction

In many quality control problems, several related process characteristics are of interest. Monitoring these characteristics independently can be very misleading (see, for example, Alt (1988); Montgomery (1991, pp.322-324)). Thus, multivariate quality control is necessary and in the past decade, several kinds of multivariate control charts for the process mean have appeared. Most of them are generalizations of the corresponding univariate procedures.

Among the univariate procedures, Shewhart's $\bar{x}$ chart supplemented with run rules has been widely used (see Nelson (1984)). Two very effective alternatives to the $\bar{x}$ chart may be used when detection of small sustained shifts is important (see Montgomery (1991, p.279)): the two-sided cumulative sum (CUSUM) chart and the exponentially weighted moving average (EWMA) chart. As a multivariate counterpart of the $\bar{x}$ chart, the χ^2-chart was first suggested and used by Hotelling (1947) in the testing of bombsights. It is well-known that the χ^2-chart is relatively insensitive to small mean shifts. This disadvantage raises the problem of how to obtain multivariate extensions of run rules, the two-sided CUSUM, and EWMA charts. Woodall and Ncube (1985) suggested a multiple CUSUM chart by using a series of the CUSUM control charts on original characteristics or on principal component axes depending on the type of shift in the mean that is considered to be important to detect. Pignatiello and Runger (1990) showed that the Woodall-Ncube multiple CUSUM chart does not have good average run length (ARL) properties when the process mean shifts in several characteristics simultaneously. To lessen the sensitivity of the multiple univariate CUSUM chart to directions, they recommended using univariate CUSUM charts aimed at several uniformly selected directions. Obviously the more directions, the less the sensitivity. But, at the same time, they found that the resulting control chart is hard to manage when there are three or more characteristics. Hawkins (1993) indicated that under some circumstances separate controls on the regression-adjusted variables by the CUSUM charts can both improve the speed of detection and make the chart signal more easily interpretable. Chan and Li (1995) proposed some run rules for bivariate data patterns.

The above problem also motivates attempts to extend the univariate CUSUM and EWMA statistics to multivariate data. One difficulty encountered with generalizing CUSUM statistics is that there are two cumulative sums for each variable (see Crosier (1986, 1988)) and Pignatiello and Runger (1990) proposed some two-sided methods which require only one cumulative sum, then they generalized these new statistics to higher dimensions. As a result, they obtained two multivariate CUSUM charts: the MCUSUM and MC1 charts. Note that neither of these two charts is a natural multivariate extension of the univariate CUSUM chart. In contrast, the multivariate extension of the EWMA (MEWMA) chart is natural and straightforward, see Lowry, Woodall, Champ and Rigdon (1992). These authors showed that the MCUSUM, MC1 and MEWMA charts preserve the shorter ARLs of the corresponding univariate procedures for detecting small mean shifts while some of these charts can build up an arbitrarily large amount of inertia. For example, if the MEWMA statistic is on one side of the central line when a shift in the other direction occurs, there may be a delayed detection of the shift.

In this article, we develop a natural multivariate extension of the CUSUM chart, namely PPCUSUM, via projection pursuit. These charts are restricted to multivariate normal observations. We prove that the PPCUSUM chart can be viewed as the limit of the Pignatiello-Runger multiple CUSUM chart (see Pignatiello and Runger (1990, pp.183-184)) as the number of the uniformly selected directions is increasing to infinity. We point out that the PPCUSUM chart can perform better than the Pignatiello-Runger multiple CUSUM chart and that it is relatively easy to implement, Section 3. The PPCUSUM chart has two advantages over the competing charts (MC1 and MEWMA). First, it is more effective in coping with the inertia problems, Section 6, and with the shifts which occur when the process mean has already been in-control for some time, Section 5. Second, the shift-time (the time in which the shift occurs) is often relatively easy to detect by plotting some univariate CUSUM chart, Section 2.

The article is organized as follows. The basis for the PPCUSUM chart is given in Section 2. A method of improving its speed of the PPCUSUM chart in detecting a mean shift, which occurs at the time the charting begins, is also proposed in Section 2. The basic properties of the new charts, including the empirical approximate formulae of the in-control ARLs, are developed in Section 3. The procedure for operating the PPCUSUM chart is given in Section 4. An extensive comparison of the new charts with the other existing multivariate control charts is presented in Sections 5 and 6. In Section 5 some illustrative examples are presented. Conclusions are presented in Section 7.

2. An Alternative Multivariate CUSUM Chart

Suppose that the $p-$quality characteristics of a process, denoted by X, are normally distributed with mean μ_0 and covariance matrix Σ_0 when the process is in-control. The aim of a multivariate control chart is to detect any shift in the process mean from μ_0 by taking successive independent samples of size n from the process. The performance of a multivariate control chart is often measured by the average run length (ARL) and the standard deviation of the run length (SRL). For convenience we consider, in what follows, the standardized version $\bar{y}_i = \sqrt{n}\Sigma_0^{-1/2}(\bar{x}_i - \mu_0)$, where $\bar{x}_i$ is the sample mean of the ith sample of size n. So the $\bar{y}_i$ are independent and normally distributed with mean 0 and covariance matix $I_{p\times p}$ as long as the process is in-control. Let μ be the mean of X when the process is out-of-control. Then the mean of the out-of-control $\bar{y}_i$ will be $\sqrt{n}\Sigma_0^{-1/2}(\mu - \mu_0)$. For simplicity, we assume that the covariance matrix of y_i remains $I_{p\times p}$ when the process is out-of-control. In the following let $||\cdot||$ denote the Euclidean norm of a vector. Let k be the reference value of the CUSUM chart with respect to the standardized version $\bar{y}_i$. For a direction a with $||a|| = 1$, define the CUSUM statistics: $C_0^a = 0$, $C_i^a = \max\{0, C_{i-1}^a + a^T\bar{y}_i - k\}$, $1 \le i < +\infty$.

Healy (1987) pointed out that if the process mean shifts along a direction b_0, the CUSUM chart (with any control limit, say h_o) for the projections $\{b_0^T\Sigma_0^{-1}\bar{x}_i$, $i = 1, 2, \ldots\}$ (or $\{a_0^T\bar{y}_i,\ i = 1, 2, \ldots\}$ with $a_0 = \Sigma_0^{-1/2}b_0$), will give the optimal ARL performance. The definitions of such optimality can be found, for example, in Pollak (1985) and Moustakides (1986). Note that by definition, the above CUSUM chart is equivalent to the CUSUM chart (with the control limit $h_1 = h_0/(b_0^T\Sigma_0^{-1/2}b_0)^{1/2}$) for the projections $\{a_1^T\bar{y}_i,\ i = 1, 2, \ldots\}$ with $a_1 = \Sigma_0^{-1/2}b_1$ and $b_1 = b_0/(b_0^T\Sigma_0^{-1}b_0)^{1/2}$. This implies there exists a direction a_1 with $||a_1|| = 1$ such that the CUSUM chart for the projections of $\{\bar{y}_i,\ i = 1, 2, \ldots\}$ along the direction a_1 also has the optimal ARL performance. For simplicity of notation, we concentrate on the case $||a_0|| = 1$ in the following. For the case $||a_0|| \neq 1$, the argument is the same if we replace a_0 by a_1. Note that a control chart can be viewed as repeated significance tests (see Alt (1988)). Hence, Healy's result implies that when the first i samples are obtained, one should use the statistic $C_i^{a_0}$ to test whether there is a shift along the direction a_0. In practice a_0 is unknown and, therefore, such test cannot be applied directly.

The projection pursuit method is a powerful tool for constructing an estimate of a_0. It often views a_0 as the most interesting direction in terms of some unknown index (objective function) and is implemented by the following steps (see, for example, Huber (1985)). The first step is to select a sample index (projection pursuit index) which is approximately proportional to the unknown index, and quantitatively measures how interesting a direction is. The second step is to obtain the most interesting direction in terms of the index just selected, and draw a further inference from the projected data in that direction.

In the following paragraph, we first use the projection pursuit method to derive an estimate, say $\hat{a}_0$, of a_0 when the first i samples are obtained, and approximate $C_i^{a_0}$ by $C_i^{\hat{a}_0}$. Then, we obtain a control chart by plotting $C_i^{\hat{a}_0}$ against the sample number i. This chart, called PPCUSUM, is a projection pursuit approximation of Healy's optimal CUSUM chart when the underlying direction of shift is unknown.

Note that $\Sigma_0^{-1/2}(\mu - \mu_0) = a_0||\Sigma_0^{-1/2}(\mu - \mu_0)||$ by the definition of a_0. The method of estimating a_0 is based on the fact that, as a function of a, the deviation between $a^T\Sigma_0^{-1/2}\mu$ and $a^T\Sigma_0^{-1/2}\mu_0$ attains the maximum at a_0. That is, a_0 is the most interesting direction if we take such a deviation as our index. However that deviation function depends on the unknown parameter μ and, thus, should be replaced by some sample index. We choose C_i^a as a sample index because, by standard arguments, we can show that C_i^a is approximately proportional to $|a^T\mu - a^T\mu_0|$ almost surely when $|a^T\mu - a^T\mu_0| > k$, and i is large. Then we estimate a_0 by the direction which gives the maximum value of C_i^a. The CUSUM $C_i^{a_0}$ can be estimated by $C_i^{\hat{a}_0} = \max_{||a||=1} C_i^a$. Thus the PPCUSUM chart is equivalent to a control chart which indicates an out-of-control signal as soon as $\max_{||a||=1} C_i^a > h$, where h is the upper control limit.

Note that the estimate $\hat{a}_0$ is automatically adjusted with respect to successive samples. So the above idea of using Healy's result to contruct a multivariate control chart is different from those in the literature where the CUSUM charts are often used along several predetermined directions simultaneously (see, for example, Hawkins (1993)).

Note that it may be possible to use the normal model based likelihood method to address the multivariate quality control problem under consideration. The main motivation of using the projection pursuit method is that we show

(i) our procedure is essentially an extension of both the Pignatiello-Runger and Healy charts; (ii) the procedure can be easily extended to the other settings (for instance, monitoring multivariate covariance and using some robust or nonparametric quality indices) because the projection pursuit does not require a normal model (see Liu and Singh (1993) and Chan and Zhang (1996) for more details).

For each i, let $C_{ij} = ||\bar{y}_j + \cdots + \bar{y}_i|| - (i-j+1)k = \sqrt{n}\{(\bar{x}_j - \mu_0 + \cdots + \bar{x}_i - \mu_0)^T \Sigma_0^{-1}(\bar{x}_j - \mu_0 + \cdots + \bar{x}_i - \mu_0)\}^{1/2} - (i-j+1)k, 1 \leq j \leq i$, $C_i = \max\{0, C_{ii}, \ldots, C_{i1}\}$. In the following we show that $\max_{||a||=1} C_i^a = C_i$, $i = 1, 2, \ldots$. This makes the calculation of $\max_{||a||=1} C_i^a$ simple.

For each i, $C_i^a = \max\{0, a^T\bar{y}_i - k, a^T(\bar{y}_{i-1} + \bar{y}_i) - 2k, \ldots, a^T(\bar{y}_1 + \cdots + \bar{y}_i) - ik\}$ by induction. Then, it is obvious that for each fixed i and $0 \leq j \leq i-1$, $\max_{||a||=1} C_i^a \geq \max_{||a||=1}\{a^T(\bar{y}_{i-j} + \cdots + \bar{y}_i) - (j+1)k\}$, $= ||\bar{y}_{i-j} + \cdots + \bar{y}_i|| - (j+1)k$ which implies $\max_{||a||=1} C_i^a \geq C_i$. On the other hand, for each a, $||a|| = 1$, $C_i^a \leq \max\{0, ||\bar{y}_i|| - k, \ldots, ||\bar{y}_1 + \cdots + \bar{y}_i|| - ik\} = C_i$, which gives the inequality $\max_{||a||=1} C_i^a \leq C_i$.

By definition, all one needs to do is to recursively add the y-vectors together in calculating $\{C_i\}$. It seems that the MEWMA is easier to compute than the PPCUSUM (see Runger and Prabhu (1996)).

When an out-of-control signal appears in the PPCUSUM chart, interpretation is needed. Let $S_0 = 0$, $S_i = \sum_{j=1}^{i} \bar{y}_j$ for $i = 1, 2, \ldots$. Then $C_i = \max\{0, ||S_i - S_{i-1}|| - k, ||S_i - S_{i-2}|| - 2k, \ldots, ||S_i - S_0|| - ik\}$ for $i = 1, 2, \ldots$. This implies that when an out-of-control signal appears in the PPCUSUM at the i_0th time period, there will exist i_1 such that $||S_{i_0} - S_{i_1}|| - (i_0 - i_1)k = \max_{||a||=1} C_{i_0}^a > h$, $i_0 > i_1$. We use $\Sigma_0^{1/2}(S_{i_0} - S_{i_1})$ to estimate the direction of shift in the mean of X, where $\hat{a}_0 = (S_{i_0} - S_{i_1})/||S_{i_0} - S_{i_1}||$ gives the maximum value of $C_{i_0}^a$. We make a CUSUM control chart for the projections of the transformed samples $\{\bar{y}_i, \ i = 1, 2, \ldots\}$ in that estimated direction. Let $S_{ui}^{\hat{a}_0}, 1 \leq i < \infty$ and $S_{li}^{\hat{a}_0}, 1 \leq i < \infty$ denote the associated two sequences of the CUSUM values (see, for example, Hawkins and Olwell (1998, p.27) for the definitions). Now the problem of estimating the shift-time can be solved by existing methods for the univariate CUSUM chart (see Hawkins and Olwell (1998, pp.20-21 and pp.26-27); Montgomery (1991)). Since the shift is upward in the direction $\hat{a}_0$, here we adopt the method in Hawkins and Olwell (1998): estimate the shift-time by $i_3 + 1$, where $i_3 = \min\{i \leq i_0 : S_{ui}^{\hat{a}_0} = 0\}$. It is helpful to choose those of relatively larger absolute values from p components of $\Sigma_0^{1/2}(S_{i_0} - S_{i_1})/||S_{i_0} - S_{i_1}||$ to diagnose the causes of the signal. We can also use the charts suggested by Pignatiello and Runger (1990) and Hawkins (1993) to interpret the signal.

In practice there is a special situation in which the process mean is already shifted at the time the charting begins. In this situation Lucas and Crosier (1982) proposed the Fast Initial Response (FIR) CUSUM chart for a univariate normal mean by giving the CUSUM a "head start". It is difficult to extend their method to the multivariate setting directly. We propose the following tentative solution.

Note that when the process mean is in-control, the statistic C_i plotted in the PPCUSUM chart will be within the decision interval $[0, h]$ and away from the upper control limit h for $1 \leq i \leq m$ and a large m. Our simulation experience shows that in this setting $(1-\lambda)^{i-t_i+1}h$ is very small, where $0 < \lambda < 1$ is a suitably chosen constant and $t_i = \min\{j : 1 \leq j < i, C_{ij} = C_i\}$. If we add $(1-\lambda)^{i-t_i+1}h$ to each C_i, $1 \leq i \leq m$, the resulting sequence should be still within the decision interval $[0, h]$. However, when a large mean shift occurs, there is a higher chance that C_{ii} takes a large value. Thus $\{C_i\}$ will soon cross the upper control limit. Our simulation experience indicates that $(1-\lambda)^{i-t_i+1}h$ is not very small in this setting. Consequently, adding $(1-\lambda)^{i-t_i+1}h$ to C_i will cause C_i to cross the upper control limit more quickly. We are thus led to modifying the PPCUSUM chart by adding $(1-\lambda)^{i-t_i+1}h$ to the statistic C_i. The new statistic is denoted by CM_i and the modified chart is referred to as FPCUSUM. It can be shown that the FPCUSUM chart exhibits a fast initial response feature (the numerical results similar to Lucas and Crosier (1982) are omitted but available from the authors). We use Example 5.1 (Case 1) in Section 4 to show this feature. First, we use the PPCUSUM chart with $k = 0.5, h = 5$. The corresponding PPCUSUM values are $(C_1, C_2, C_3, C_4, C_5) = (0.629919, 2.166418, 4.460767, 5.7131002, 2.9141109)$. The PPCUSUM chart gives an out-control-control signal at the $4-$th time period. Now we apply the FPCUSUM chart with $\lambda = 0.55$ and with the same k and h. The corresponding FPCUSUM values become $(CM_1, CM_2, CM_3, CM_4, CM_5) = (3.379919, 4.916418, 7.210767, 9.7006002, 5.6641109)$. The FPCUSUM chart gives an out-of-control signal at the third time period. Thus the FPCUSUM chart results in a faster detection at the expense of a smaller in-control ARL (see Figure 1).

3. Basic Properties

The method of Pignatiello and Runger (1990, p.185) can be applied directly to prove the first property.

Property 1. Both the PPCUSUM and FPCUSUM charts are directionally invariant. That is, the ARL and SRL of the PPCUSUM and FPCUSUM charts depend on the shift μ only through the distance $d = d(\mu, \mu_0) = \{(\mu-\mu_0)^T \Sigma_0^{-1}(\mu-\mu_0)\}^{1/2}$.

This property makes the calculation and comparison of the ARL and SRL easier because we need only consider shifts of the form $\mu - \mu_0 = (\delta, \ldots, \delta)$ and $\Sigma_0 = I_{p\times p}$.

Property 2. When $p = 1$ the PPCUSUM chart reduces to the two-sided CUSUM chart, the PPCUSUM chart is a natural multivariate extension of the univariate CUSUM chart.

Pignetiello and Runger (1990) suggested a multiple CUSUM chart: use univariate CUSUM charts along several uniformly selected directions, say a_j, $1 \leq j \leq l$, simultaneously. In the language of repeated significance tests, this chart is equivalent to using $C_i^{a_j}$, $1 \leq j \leq l$, to test the hypothesis $\mu = \mu_0$ simultaneously when the first i samples are obtained. If we use the same control limit, say h, for all these charts, then an out-of-control signal appears in one of these charts if and only if $\max_{1\leq j\leq l} C_i^{a_j} > h$. Note that if l tends to infinity, then $\max_{1\leq j\leq l} C_i^{a_j}$ converges to $\max_{||a||=1} C_i^{a}$, the statistic used in the PPCUSUM. In this sense we have the next property.

Property 3. The PPCUSUM chart is the limit of the Pignatiello-Runger multiple CUSUM chart when the number of uniformly selected directions tends to infinity.

Note that in higher dimension a very large l is required, and in this case the computation of $\max_{1\leq j\leq l} C_i^{a_j}$ is harder than that of C_i.

Table 3.1. The in-control ARL and SRL of the PPCUSUM charts with $k = 0.5$, for $p = 2, 3, 4$ and various control limits h.

$p=2$			$p=3$			$p=4$		
h	ARL	SRL	h	ARL	SRL	h	ARL	SRL
4.75	107	98.6	5.75	108	103	7.00	143	133
5.00	133	124	6.00	132	125	7.50	212	195
5.50	206	196	6.25	163	153	7.75	260	246
5.75	261	249	6.50	195	187	8.00	305	285
6.00	326	311	6.75	245	232	8.25	378	340
6.25	406	359	7.00	300	275	8.50	446	392
6.50	485	421	7.25	365	337	8.75	524	437
7.00	689	504	7.50	450	391	9.00	628	486

For the convenience of using the PPCUSUM chart, some simulated in-control ARL and SRL values of various control limits are presented in Table 3.1. These values are the sample mean and sample standard deviation of the simulated run lengths. In these simulations, the number of replications is 6000. The relative standard error of the estimated ARL can be estimated by $SRL/(ARL \cdot u^{1/2})$, where u is the number of the replications. Thus the estimated relative standard errors of these simulations are smaller than 0.03. It follows from the approximation formula of Siegmund (1985) for the in-control ARL of the univariate

CUSUM chart with control limit h that $\log(ARL)$ can be approximated by a linear function of h when ARL is large. This motivates using some linear functions to approximate the in-control ARLs of the PPCUSUM chart. To this end, we apply the least squares fitting (see, for example, the S-PLUS Guide to Statistical and Mathematical Analysis, version 3.3) to the simulated ARLs for various h in Table 3.1. This results in a useful approximate formula.

For the in-control ARL of the PPCUSUM chart, we have $\log(ARL) = c_0 + c_1 h$ where

$$c_o = \begin{cases} 0.6899, \text{ when } & p = 2, k = 0.5; \\ -0.0120, \text{ when } & p = 3, k = 0.5; \\ -0.1714, \text{ when } & p = 4, k = 0.5; \end{cases}$$

$$c_1 = \begin{cases} 0.8438, \text{ when } & p = 2, k = 0.5; \\ 0.8159, \text{ when } & p = 3, k = 0.5; \\ 0.7368, \text{ when } & p = 4, k = 0.5. \end{cases}$$

Let R_P^2 be the residual error of the above fitting. Then

$$R_P^2 = \begin{cases} 0.001390, \text{ when } & p = 2, k = 0.5; \\ 0.000076, \text{ when } & p = 3, k = 0.5; \\ 0.000396, \text{ when } & p = 4, k = 0.5. \end{cases}$$

Similar tables and formulae for the FPCUSUM chart are available from the authors.

4. Procedure for Operating the New Charts

We begin with discussing the choice of the parameter λ of the FPCUSUM chart. The simulation results in Ngai and Zhang (1998) suggest that in general if we want to detect large mean shifts quickly, then a small λ is preferable, and that if both small and large mean shifts are considered, a relatively good choice of λ is as follows: $\lambda = 0.55$ when $p = 2$, and $\lambda = 0.6$ when $p = 3, 4, 5$.

Let μ_1 be the out-of-control value of interest. Then, like the univariate case, the parameter k of the PPCUSUM and FPCUSUM charts is usually chosen so that $k/\sqrt{n}$ is about one half of the Mahalanobis distance $d(\mu_1, \mu_0) = ((\mu_1 - \mu_0)^T \Sigma_0^{-1}(\mu_1 - \mu_0))^{1/2}$.

In general the procedure for operating the PPCUSUM and FPCUSUM charts consist of three steps. Take the PPCUSUM as an example.

Step 1. Estimate μ_0 and Σ_0 from past data. Calculate $\Sigma_0^{-1/2}$.

Step 2. Specify the in-control ARL and SRL, the out-of-control shift of interest and the parameter k. Use the empirical formulae mentioned above to choose the parameter h.

Step 3. Transform the ith sample mean $\bar{x}_i$ by $\bar{y}_i = \sqrt{n}\Sigma_0^{-1/2}(\bar{x}_i - \mu_0)$; calculate C_i and t_i for each i. Then, plot C_i against i and check whether the plotted point falls outside the decision interval $[0, h]$. If the plotted point falls outside the decision interval, then an out-of-control signal is given. Use the proposal in Section 2 to estimate the direction of the shift and the shift-time.

5. Performance of the New Charts

In this section, the ARL, SRL, ADRL and SDRL performances of the PP-CUSUM, MEWMA1, MEWMA2 and MC1 charts are compared in Tables 5.1 and 5.2 for $p = 2, 5$ via Monte Carlo simulation. The χ^2-chart, Crosier's MCUSUM chart and the multiple CUSUM chart have not been involved in this study because they have been compared with the MEWMA2 chart in Lowry, Woodall, Champ and Rigdon (1992), and it has been shown that the MEWMA2 chart performs slightly better. See Appendix 1 for the definitions of MC1, MEWMA1 and MEWMA2 charts.

In Lowry, Woodall, Champ and Rigdon (1992), the ARL performances of the MC1, MEWMA1 and MEWMA2 charts are compared only under initial out-of-control conditions. This is in contrast to how quality control charts are applied in practice: a shift often occurs after the process mean has been in-control for some time. We call it a delayed shift. This issue has been investigated by Siegmund (1985), Pollak (1985), Moustakides (1986) and Ritov (1990), among others. Here we address the issue according to the scheme used by Siegmund (1985). That is, we make a comparison of the PPCUSUM, MC1, MEWMA1 and MEWMA2 charts in terms of the average delayed run length (ADRL) and the standard deviation of the delayed run lengths (SDRL) under the following model: for a fixed integer v the process mean is in-control before the first $v - 1$ samples, and out-of-control after the vth sample (including the vth sample). ADRL and SDRL are equal to ARL and SRL when $v = 1$. Mathematically, the ADRL and SDRL under the above model are just the conditional expectation and conditional standard deviation, $E((RL-v+1)|RL \geq v)$ and $(Var((RL-v+1)|RL \geq v))^{1/2}$. After simulation, we choose v so that these quantities are near their limits, the steady-state ADRL and SDRL: $v = 15$ is a reasonable choice. In this study, we adopt Yashchin's approach to assume that there is no false alarm before the shift (see, for example, Yashchin (1985); Reynolds, Amin and Arnold (1990)). Similar to Taylor (1968), if the control chart signals during the first $v - 1$ samples, then this sequence of samples is discarded and the count of samples is reset to zero.

Each simulated value of the delayed run length is obtained by subtracting $v-1$ from a simulated run length.

Table 5.1. ARL and SRL comparison.

$p=2$							
d	0.0	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h=5, k=0.5$							
ARL	133	27.4	9.33	5.41	3.82	3.00	2.51
SRL	124	21.3	4.71	2.10	1.22	0.84	0.64
MC1 with $h=4.33, k=0.5$							
ARL	131	26.2	8.57	4.80	3.40	2.70	2.27
SRL	126	21.3	4.83	2.01	1.18	0.82	0.60
MEWMA1 with $h=\sqrt{7.88}, r=0.1$							
ARL	132	20.8	6.96	3.67	2.40	1.75	1.41
SRL	135	17.5	4.61	2.21	1.30	0.86	0.60
MEWMA1 with $h=3.077, r=0.4$							
ARL	132	39.2	10.6	4.76	2.86	2.03	1.58
SRL	128	38.4	8.73	3.21	1.62	1.02	0.71
MEWMA2 with $h=2.763, r=0.1$							
ARL	132	23.7	9.18	5.64	4.11	3.29	2.74
SRL	123	16.3	4.18	1.94	1.18	0.83	0.65
$p=5$							
d	0.0	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h=8, k=0.5$							
ARL	163	37.5	12.6	7.44	5.36	4.22	3.51
SRL	154	28.1	5.71	2.41	1.44	0.98	0.74
MC1 with $h=6.55, k=0.5$							
ARL	163	35.4	10.5	6.15	4.42	3.51	2.93
SRL	159	30.2	5.53	2.25	1.30	0.90	0.68
MEWMA1 with $h=\sqrt{14.3}, r=0.1$							
ARL	161	31.4	9.62	4.95	3.13	2.26	1.75
SRL	164	26.5	6.21	2.79	1.63	1.09	0.78
MEWMA1 with $h=\sqrt{15.97}, r=0.4$							
ARL	161	65.3	17.6	6.99	3.88	2.62	1.97
SRL	156	63.4	15.4	4.98	2.27	1.33	0.90
MEWMA2 with $h=3.75, r=0.1$							
ARL	163	35.0	12.4	7.40	5.34	4.22	3.52
SRL	152	24.0	5.55	2.41	1.42	0.96	0.74

For the out-of-control case, in Table 5.1 we used the assumption that the shift has occurred prior to the application of charts, while in Table 5.2 we used the assumption that the shift occurs after $v-1(=14)$ samples. As before, ARL and SRL (or ADRL and SDRL) in these tables are, respectively, the average and the standard deviation of at least 6000 simulated run lengths. The relative standard errors of the estimated ARL and ADRL are smaller than 0.01. In addition, the following remarks apply to these tables.

1. We let the reference values of all these charts be 0.5. For the parameter r of the MEWMA1 and MEWMA2 chart, $r = 0.1$ is recommended by Lowry, Woodall, Champ and Rigdon (1992). Our simulation study shows that for the MEWMA1, with $r = 0.1$, the in-control SRL is larger than the in-control ARL (see Table 5.1). For example, when $p = 2$, $h = \sqrt{7.88}$ and $r = 0.1$, the in-control ARL and SRL are 132 and 135. As pointed out by Chan and Zhang (1996a), this results in a higher chance that the run length takes a small value.

2. Note that the PPCUSUM, FPCUSUM, MEWMA1 and MEWMA2 and MC1 charts are directionally invariant (see Pignatiello and Runger (1990); Lowry, Woodall, Champ and Rigdon (1992)). Hence, without loss of generality, we assume that $\Sigma_0 = I_{p\times p}$ and $\mu - \mu_0$ is of the form $(\delta, \ldots, \delta)$. For $p = 2, 5$, we let $d = \sqrt{n}d(\mu, \mu_0) = 0.5, 1.0,\ 1.5, 2.0,\ 2.5, 3.0$. For convenience, we assume $n = 1$ although the results hold for any integer n.

3. To compare different control procedures fairly, we need to calibrate each procedure so that the in-control ARL is approximately the same for all the procedures (see Pignatiello and Runger (1990)).

When $p = 2$, several points emerge from Table 5.1. **(i)** For the initial shift μ_1 with $0.5 \le d(\mu_1, \mu_0) \le 2$, the PPCUSUM produces slightly larger ARL than the MC1 chart. **(ii)** The MEWMA1 chart with $r = 0.1$ is slightly quicker than the PPCUSUM chart in detecting initial shifts in terms of ARL. However, the in-control SRL of that MEWMA1 chart is larger than that of the PPCUSUM chart. If we let $r = 0.4$ so that the in-control $SRL = 128$ is smaller than the in-control $ARL = 132$, then the PPCUSUM chart performs better than the MEWMA1 chart. **(iii)** The MEWMA2 chart with $r = 0.1$ performs slightly better than the PPCUSUM chart when $0.5 \le d(\mu, \mu_0) \le 1.0$, while the latter performs better when $1.5 \le d(\mu, \mu_0) \le 3.0$. **(iv)** Table 5.2 indicates that when $p = 2$ the PPCUSUM chart has better ADRL and SDRL performances than the competing charts.

For $p = 5$, similar conclusions can be drawn from Tables 5.1 and 5.2. The performance of the FPCUSUM chart has been shown to be better than PPCUSUM chart in some cases by Ngai and Zhang (1998).

Table 5.2. ADRL and SDRL comparison when $v = 15$.

$p = 2$						
d	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h = 5, k = 0.5$						
ADRL	26.1	8.45	4.84	3.44	2.74	2.26
SDRL	22.7	5.16	2.25	1.37	0.95	0.72
MC1 with $h = 4.33, k = 0.5$						
ADRL	26.5	8.81	5.22	3.78	3.00	2.56
SDRL	22.2	5.00	2.48	1.59	1.18	0.92
MEWMA1 with $h = \sqrt{7.88}, r = 0.1$						
ADRL	24.4	9.29	5.65	4.12	3.30	2.75
SDRL	17.0	4.25	4.12	1.20	0.85	0.65
MEWMA1 with $h = 3.077, r = 0.4$						
ADRL	40.8	11.1	5.18	3.23	2.42	1.93
SDRL	38.5	8.65	3.12	1.52	0.95	0.69
MEWMA2 with $h = 2.763, r = 0.1$						
ADRL	23.6	9.15	5.64	4.10	3.31	2.75
SDRL	16.2	4.08	1.97	1.18	0.85	0.64
$p = 5$						
d	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h = 8, k = 0.5$						
ADRL	33.7	11.1	6.60	4.71	3.69	3.06
SDRL	27.9	5.80	2.70	1.65	1.18	0.94
MC1 with $h = 6.55, k = 0.5$						
ADRL	37.0	12.8	7.69	5.55	4.37	3.67
SDRL	30.1	7.16	3.59	2.35	1.74	1.41
MEWMA1 with $h = \sqrt{14.3}, r = 0.1$						
ADRL	35.9	12.6	7.39	5.32	4.20	3.50
SDRL	26.0	5.64	2.42	1.43	0.96	0.73
MEWMA1 with $h = \sqrt{15.97}, r = 0.4$						
ADRL	66.3	18.1	7.41	4.33	3.06	2.41
SDRL	63.8	15.2	4.81	2.15	1.21	0.82
MEWMA2 with $h = 3.75, r = 0.1$						
ADRL	34.5	12.4	7.37	5.33	4.24	3.53
SDRL	24.6	5.46	2.42	1.41	0.98	0.72

Generally speaking, we recommend using the MCUSUM, MC1, MEWMA1 and MEWMA2 charts for detecting initial shifts and the PPCUSUM and FP-CUSUM charts for detecting delayed shifts. The following examples highlight these conclusions.

In a manufacturing process, raw material can have a large effect on the quality of products. Abnormalities in some characteristics of the raw material may cause production upsets and it is important to use control charts in this context. Prusinoski (1979) and Sultan (1986) have applied Hotelling's T^2-charts. We consider this situation again by using PPCUSUM, FPCUSUM, MC1 and MEWMA1 charts.

Example 5.1. Two quality characteristics of steel sections are monitored: $x_1 =$ Brinell hardness and $x_2 =$ tensile strength measured in Kg/mm^2 (see Sultan (1986); Chan and Li (1995)). There are 30 samples of size 1 with the first 25 from the original shipment and the next 5 from a new shipment.

Case 1. To illustrate the situation that the process mean is out-of-control at the beginning of the process, we take the first 25 observations as the in-control ones and use the PPCUSUM, FPCUSUM, MC1 and MEWMA1 charts to test 5 observations from the new shipment. Take the PPCUSUM chart as an example, the procedure is as follows.

Step 1. Estimate the target values by the sample mean and covariance matrix based on 25 observations from the original shipment:

$$\mu_0 = \begin{pmatrix} 177 \\ 53 \end{pmatrix} \text{ and } \Sigma_0 = \begin{pmatrix} 324 & 65 \\ 65 & 20 \end{pmatrix}.$$

Step 2. Suppose $d(\mu_1, \mu_0) = 1$ where μ_1 denotes the out-of-control value of the process mean of interest. Choose the reference value $k = 0.5$. By Table 3.1, choose the control limit $h = 5$ in the PPCUSUM chart so that the in-control ARL is about 133.

Step 3. Transform the ith sample mean, calculate C_i and plot it against i in the PPCUSUM chart (see Figure 1) and check whether it falls outside the decision intervals. It turns out that the PPCUSUM chart gives an out-of-control signal at the 4th sample from the new shipment.

Similarly, choose $h = 4.75$ and $k = 0.5$ in the MC1 chart; $h = \sqrt{8.79}$ and $r = 0.1$ in the MEWMA1 chart so that they have approximately the same in-control ARL of 133 as the PPCUSUM chart. Calculate and plot the statistics $MC1_i, NZ_i$ against i in the MC1, MEWMA1 charts, respectively (the figures are available from the authors). Then the MEWMA1 chart gives an out-of-control signal at the 4th sample. No out-of-control signal is generated from the MC1 chart.

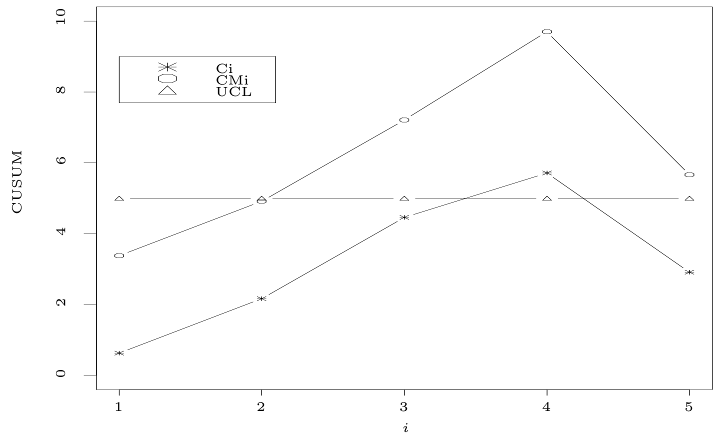

Figure 1. PPCUSUM and FPCUSUM charts for Example 5.1 (Case 1).

Note that if we use the four charts to make retrospective test for the samples from the original shipment, then we find that the first sample is out-of-control and the last 24 are in-control.

Case 2. For an illustration of a process mean that is out-of-control after the process mean has been in-control for some time, we change the assumption used in Case 1. Here we do not know whether the last 24 samples come from an in-control process, but we know the values of μ_0 and Σ_0 :

$$\mu_0 = \begin{pmatrix} 177 \\ 53 \end{pmatrix} \text{ and } \Sigma_0 = \begin{pmatrix} 324 & 65 \\ 65 & 20 \end{pmatrix}.$$

We apply the PPCUSUM chart with $h = 5, k = 0.5$, MC1 with $h = 4.75, k = 0.5$ and MEWMA1 with $h = \sqrt{8.79}, r = 0.1$. The data set is composed of the last 24 samples from the original shipment and 5 samples from the new shipment. The PPCUSUM gives an out-of-control signal at the 4th sample from the new shipment (see Figure 2), while no out-of-control signal is presented by the MC1 and MEWMA1 charts (the figures are available from the authors). This shows the PPCUSUM is more effective than the MC1 and MEWMA1 in detecting a late shift in mean.

The last example is used to show how to estimate the shift time.

Example 5.2. We apply the PPCUSUM (with $h = 5, k = 0.5$), defined in Example 5.1 to a data set given by Crosier (1988). This data set is composed of the samples of size 1 from the normal distribution with unit variances and a correlation coefficient of 0.5. The process mean is in control for the first five samples and then shifts to $(1,2)^T$ for the last five samples. In this example μ_0 is zero.

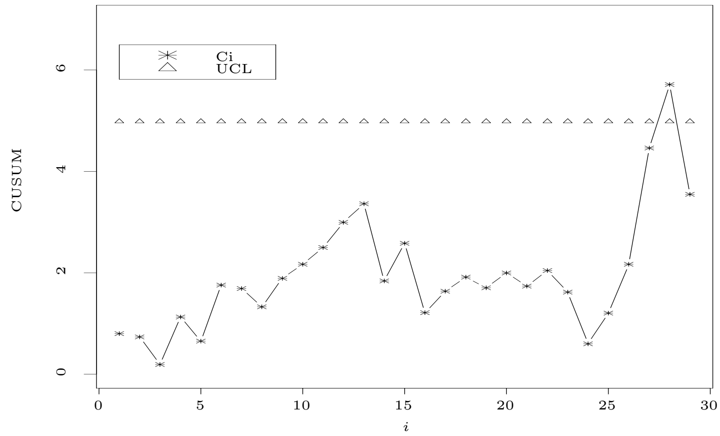

Figure 2. PPCUSUM chart for Example 5.1 (Case 2).

The PPCUSUM chart gives the out-of-control signal at the 10th sample (see Figure 3).

We project the standardized versions $\bar{y}_i, 1 \le i \le 10$ to the estimated optimal direction of the shift, $\hat{a}_0^T = (-0.039023, 0.9992383)$. Then $i_3 = 5$ is the first i such that $S_{ui}^{\hat{a}_0} = 0$ (see the definition in Section 2). So the estimated shift-time is 6.

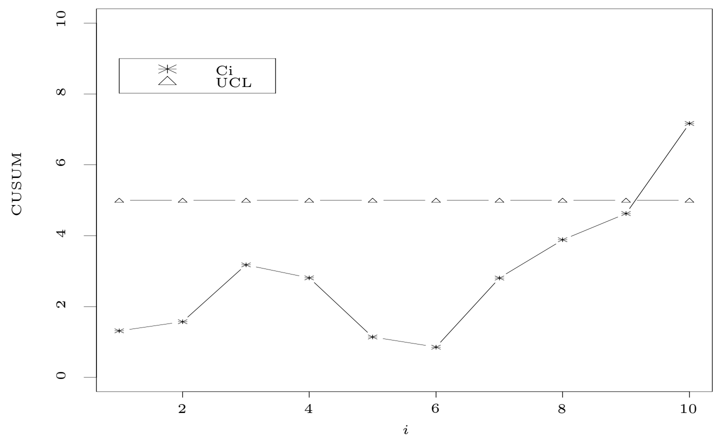

Figure 3. PPCUSUM chart for Example 5.2.

6. Inertia Problems

The "inertia" problem was originally studied by Yashchin (1987) for the univariate EWMA chart. Lowry, Woodall, Champ and Rigdon (1992) pointed out that the MC1, MEWMA1, MEWMA2 and MCUSUM charts can have "inertia" in reacting to shifts in the process mean relative to the univariate CUSUM chart. They presented a worst-case inertia existing in the MEWMA1 and MEWMA2 charts. In contrast, it is easy to see that the PPCUSUM and FPCUSUM charts can avoid this problem. In the same paper, Lowry et al. suggested that the χ^2-chart should always be used in conjunction with the MEWMA chart to help to prevent such delays. However, as noted by them, the use of the χ^2-chart does not completely solve the inertia problem. When inertia has built up, a sequence of relatively large shifts from the target value that does not trigger the χ^2- chart may still not result in out-of-control signal from the MCUSUM, MC1, MEWMA1 and MEWMA2 charts. Simulation results displayed in Table 6.1 show that for **Model 1,** described in the next paragraph, the PPCUSUM chart can be more effective than the MC1, MEWMA1, MEWMA2, the combined χ^2-MEWMA1 and the combined χ^2-MEWMA2 charts against moderate inertia (see Appendix 1 for the definitions of these charts and note that the rough rule of thumb of Lowry et al. (1992) is used here in designing χ^2-MEWMA1 and χ^2-MEWMA2 charts). For the alternative normal model described below, Table 6.2 shows that the PPCUSUM chart performs better than MC1, MEWMA1 and MEWMA2 charts. Ngai and Zhang (1998) showed that similar results hold for the FPCUSUM chart. For the alternative normal model with some small means, the FPCUSUM chart can be superior to the combined χ^2-MEWMA1 and χ^2-MEWMA2 charts.

In Table 6.1 we adopt a special model, called **Model 1**. In this model, we let the sample size be 1. Let $y_i = (0,0)^T$, for $1 \leq i \leq 17$, $y_{18} = (-2.8, -0.5)^T$ and $y_{19} = (-1.5, -1.5)^T$ for $p = 2$; and let $y_i = (0,0,0,0,0)^T$, $1 \leq i \leq 17$, and $y_{18} = (-2.8, -0.5, -0.5, -0.5, -0.5)^T$ and $y_{19} = (-1.5, -1.5, -1.5, -1.5, -1.5)^T$ for $p = 5$. Assume that $\mu_0 = 0$. Let $y_i, 20 \leq i < \infty$ be samples from $N(\mu, I_{2\times2})$ for $p = 2$ and $N(\mu, I_{5\times5})$ for $p = 5$, where μ is of the form $(\delta, \ldots, \delta)$. We assume that the process mean is in-control during the time period from 1 to 19. Note the last two observations are located in the third quadrant (all components are negative). However a shift to the first quadrant occurs after period 19. Applying the MEWMA chart $\{y_i, 1 \leq i < \infty\}$, we find the 18th and 19th values of the MEWMA statistics are well below the centerline. Since the components of μ are positive, we will wait a longer time to find the MEWMA statistic over the upper control limit than when y_{18}, y_{19} and μ are located in the first quadrant. This is a prototype of the inertia model suggested by Yashchin (1987) and Lowry, Woodall, Champ and Rigdon (1992).

Remark. Shifts of the form $(\delta, \ldots, \delta)^T$ are considered in Table 6.1. Here, $\delta = d/\sqrt{p}$ and $d = 0.5, 1.0, 1.5, 2.0, 2.5$ and 3.0. Each pair of ARL and SRL values in Table 6.1 is obtained from 6000 replications.

Table 6.1. ARL and SRL comparison under *model* 1 for the following charts of the same in-control ARL approximately and $\delta = d/\sqrt{p}$.

$p = 2$						
d	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h = 5, k = 0.5$						
ARL-19	26.7	9.26	5.38	3.83	2.98	2.51
SRL	21.9	4.85	2.11	1.24	0.83	0.64
MC1 with $h = 4.33, k = 0.5$						
ARL-19	27.8	11.8	7.79	5.90	4.77	4.02
SRL	23.2	5.43	2.47	1.50	1.06	0.80
MEWMA1 with $h = \sqrt{7.88}, r = 0.1$						
ARL-19	28.8	12.3	7.93	5.86	4.72	3.99
SRL	17.1	4.52	2.23	1.36	0.98	0.75
χ^2-MEWMA1 with $h = 2.823, h_1 = 4, r = 0.1$						
ARL-19	29.1	12.3	7.16	5.58	4.18	3.14
SRL	17.6	4.67	2.43	1.75	1.57	1.44
MEWMA2 with $h = 2.763, r = 0.1$						
ARL-19	27.9	12.2	7.85	5.83	4.68	3.95
SRL	16.6	4.42	2.17	1.34	0.95	0.75
χ^2-MEWMA2 with $h = 2.78, h_1 = 4, r = 0.1$						
ARL-19	28.1	12.1	7.68	5.52	4.15	3.13
SRL	16.9	4.60	2.41	1.73	1.54	1.42
$p = 5$						
d	0.5	1.0	1.5	2.0	2.5	3.0
PPCUSUM with $h = 8, k = 0.5$						
ARL-19	36.2	12.5	7.45	5.36	4.21	3.50
SRL	26.9	5.59	2.40	1.44	0.98	0.73
MC1 with $h = 6.55, k = 0.5$						
ARL-19	43.2	17.4	10.9	7.97	6.29	5.23
SRL	31.7	6.86	3.10	1.81	1.25	0.92
MEWMA1 with $h = \sqrt{14.3}, r = 0.1$						
ARL-19	40.8	16.1	10.0	7.39	5.90	4.94
SRL	26.2	5.79	2.67	1.59	1.11	0.85
χ^2-MEWMA1 with $h = 3.8, h_1 = 4.83, r = 0.1$						
ARL-19	41.2	16.1	9.95	7.21	5.54	4.26
SRL	26.7	5.97	3.84	1.96	1.67	1.61
MEWMA2 with $h = 3.75, r = 0.1$						
ARL-19	39.5	15.9	9.95	7.34	5.87	4.93
SRL	25.1	5.69	2.64	1.58	1.11	0.84
χ^2-MEWMA2 with $h = 3.77, h_1 = 4.8, r = 0.1$						
ARL-19	40.0	15.9	9.85	7.14	5.48	4.22
SRL	25.7	5.85	2.83	1.95	1.71	1.63

7. Conclusions

Two new charts, namely PPCUSUM and FPCUSUM, are developed for detecting mean shifts in a multivariate normal distribution. It is shown that these charts are directionally invariant. The PPCUSUM chart is a natural multivariate extension of the univariate CUSUM chart because it reduces to the two-sided CUSUM chart when the dimension p is equal to 1. Simulations indicate that the new charts have slightly superior SRL, ADRL and SDRL performances than the MC1 and MEWMA1 charts, while performing a little worse in terms of ARL. The main advantage of the new charts is that they can be more effective in avoiding the inertia problem and coping with delayed shifts than the MC1, MEWMA1, MEWMA2, the combined χ^2-MEWMA1 and the combined χ^2-MEWMA2 charts. On the whole, the new charts seem promising when online computers are used during production.

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Appendix 1. Brief definitions of the MC1 and MEWMA charts

The MC1 chart: Let $l_0 = 0, l_1 = 1$ and, for $i = 1, 2, \ldots,$

$$D_i = \sum_{i-l_i+1}^{i} y_j, MC_i = \max\{0, ||D_i|| - kl_i\},$$
$$l_i = \begin{cases} l_{i-1} + 1, MC_{i-1} > 0, \\ \quad 1, \qquad \text{otherwise.} \end{cases}$$

An out-of-control signal is given as soon as $MC_i > h$, where $k > 0$ and $h > 0$.

The MEWMA1 chart (using the exact covariance of the MEWMA statistic): Let $Z_0 = 0$ and $Z_i = ry_i + (1-r)Z_{i-1}, \quad 1 \le i < \infty$,where $0 < r < 1$ is a constant. An out-of-control signal is given as soon as $NZ_i = ||Z_i|| \left(\frac{2-r}{r(1-(1-r)^{2i})}\right)^{1/2} > h$.

The MEWMA2 chart (using the asymptotic covariance of the MEWMA statistic): replace NZ_i in MEWMA1 chart by $NZ_i = ||Z_i|| \left(\frac{2-r}{r}\right)^{1/2}$.

The combined χ^2-MEWMA1 chart: Let h_1^2 be the upper control limit of the χ^2- chart. Let Z_i and NZ_i be those used in the MEWMA1 chart. Then, the combined χ^2-MEWMA1 chart gives an out-of-control signal as soon as $||Z_i|| > h_1$ or $NZ_i > h$.

Similarly, we can define the combined χ^2-MEWMA2 chart.

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