METHOD OF MOMENT FOR THIN WIRE

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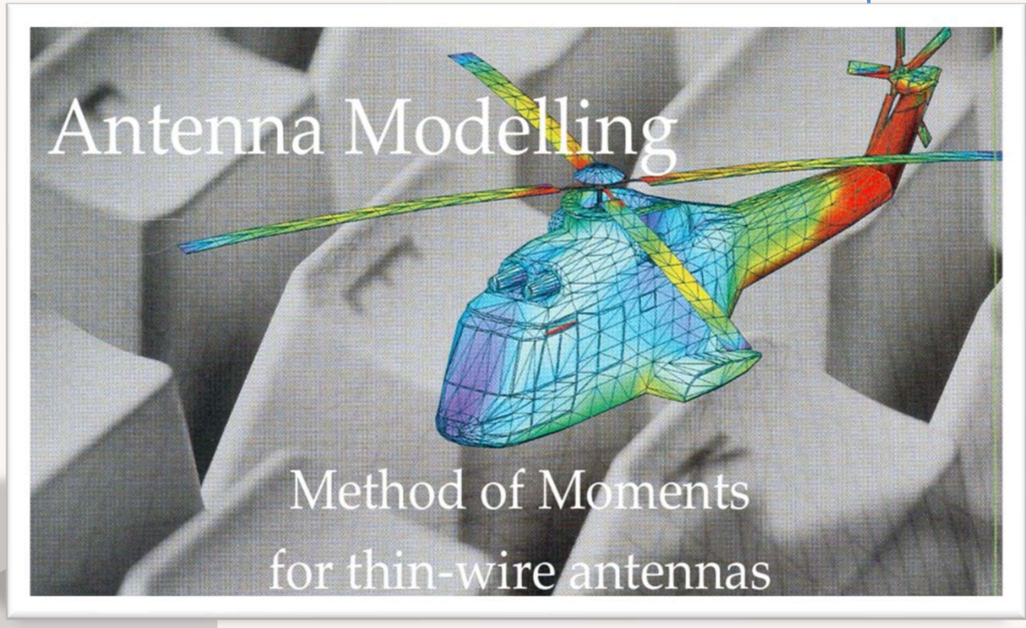

Picture token from the course slides

Summary

Introduction

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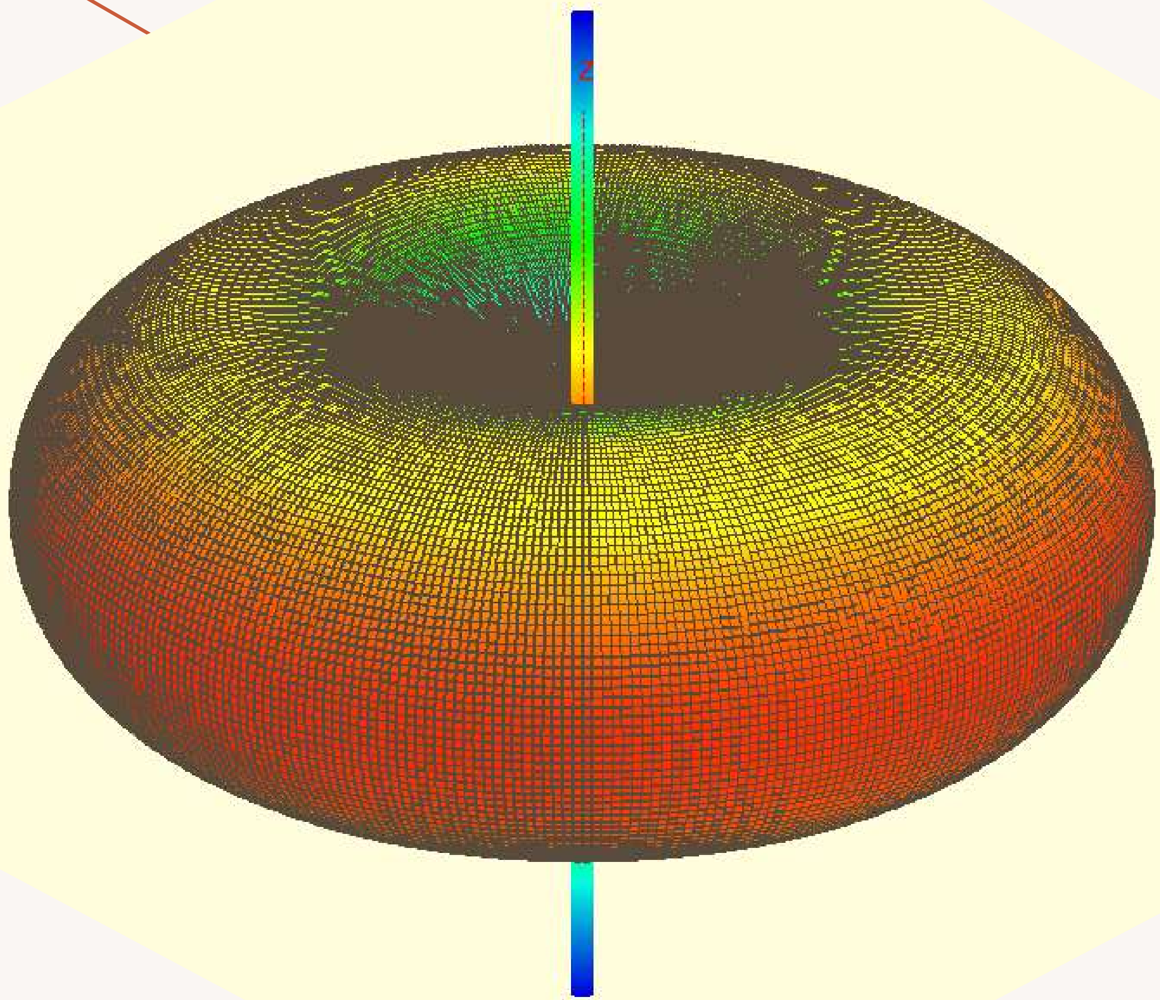
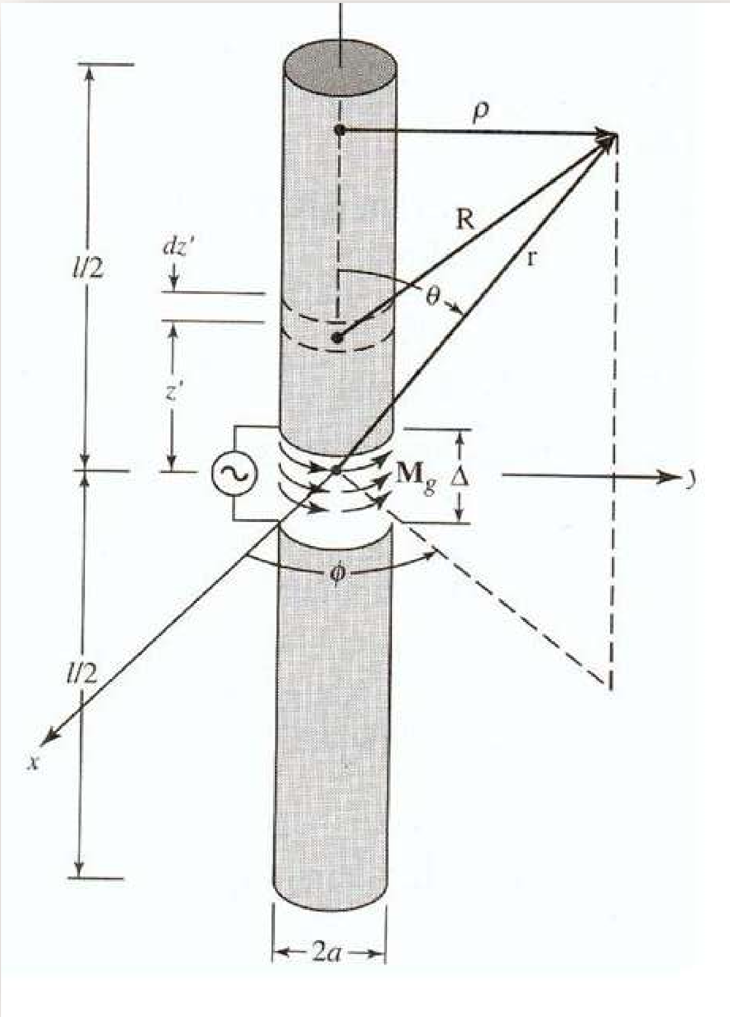
Introduction

The *Method of Moments (MoM)* is a numerical technique used to solve integral equations in electromagnetics, particularly for antenna analysis. It transforms continuous equations into solvable linear systems, ideal for modeling thin-wire antennas. MoM calculates important parameters like current distribution and input impedance, essential for antenna design. It is especially valuable for complex structures where analytical solutions are not possible.



simulation of a thin wire antenna on Feko



Context and Objectives

CONTEXT :

- APPLICATION OF THE METHOD OF MOMENTS TO SOLVE INTEGRAL EQUATIONS
- ANTENNA MODELING AND NUMERICAL SIMULATIONS

OBJECTIVES :

- USE THE METHOD OF MOMENTS TO ANALYZE THE CURRENT DISTRIBUTION ON A THIN-WIRE ANTENNA
- SOLVE THE INTEGRAL EQUATION USING TWO CONFIGURATION AND COMPARE THE RESULTS WITH THOSE OBTAINED FROM NEC

Key Principles Of the Method of Moments

Discretization

Divide the structure into small segments and Approximate the current distribution with basis functions

```
N = [7, 17, 27];
nb_of_segments = N(m);
Zmn = zeros(nb_of_segments, nb_of_segments);
delta = l / nb_of_segments;
```

Impedance Matrix Z{mn}

Rows: Observation points (testing functions).
Columns: Current contributions (basis functions).

$$\begin{pmatrix} Z_{11} & \cdots & Z_{1N} \\ \vdots & \ddots & \vdots \\ Z_{N1} & \cdots & Z_{NN} \end{pmatrix}$$

Integral Equation

Formulate the integral equation using Green's function.

$$Z_{mn} = \frac{j}{\omega\varepsilon} \int_{(n-1)\Delta}^{n\Delta} \left(k^2 + \frac{\partial^2}{\partial z^2}\right) G(R)\, dz'$$

$$\frac{j}{\omega\varepsilon_0} \frac{k}{sin(k\Delta)} \int_{z_m-\Delta/2}^{z_m+\Delta/2} [G(R_{n+1}) + G(R_{n-1}) - 2G(R_n)\cos(k\Delta)]\, dz$$

Testing Functions

Apply **test functions** to convert the integral equation into a set of linear algebraic equations

$$\delta(z - z_a)$$

$$g_n(z) = u\left[z - \left(z_n - \frac{\Delta}{2}\right)\right] - u\left[z - \left(z_n + \frac{\Delta}{2}\right)\right]$$

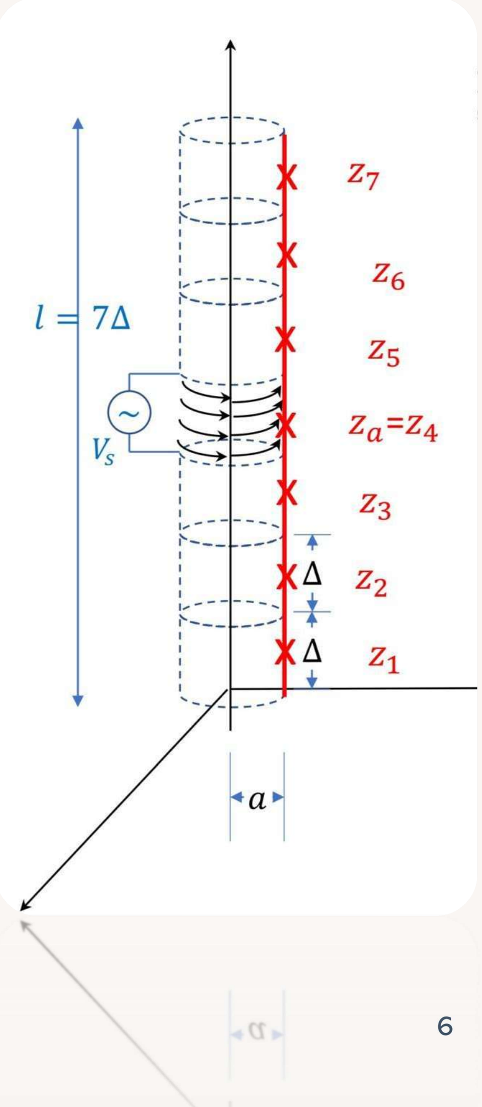
Solution

$$I = Z^{-1} \times V$$

```
I_n = Zmn \ V_m;
```

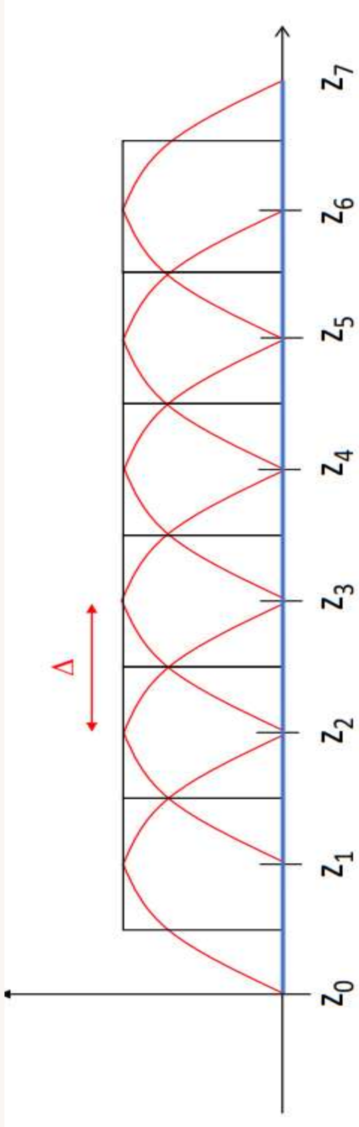
First using piecewise continuous basis functions and point-matching testing

```
figure;
hold on;
for m = 1:3

    nb_of_segments = N(m);                                  % Select number of basis functions (segments)
    Zmn = zeros(nb_of_segments, nb_of_segments);                  % Initialize impedance matrix
    delta = 1 / nb_of_segments;                             % Length of each segment
    legend_labels{m} = ['Segments: ', num2str(nb_of_segments)];
    % Fill impedance matrix Zmn
    for j = 1:nb_of_segments
        for i = 1:nb_of_segments
            zm = (i - 0.5) * delta;                          % Midpoint of the segment
            R = sqrt(a^2 + (zm - z).^2);                     % Distance between observation and source points

            % Define integrand for impedance calculation using point-matching

            func = inline((1j / (2 * pi * frq * e0)) * ...
                (exp(-1i * k * R) / (4 * pi * R^5) * (-R^2 * (1 + 1i * k * R) - ...
                (1i * k * R * (zm - z).^2 - (1 + 1i * k * R) * (3 + 1i * k * R) * (zm - z).^2)) + ...
                k^2 * exp(-1i * k * R) / (4 * pi * R)));

            Zmn(j, i) = quad(func, (j-1) * delta, j * delta);          % Numerical integration over segment
        end
    end

    % Define excitation voltage vector and solve for current distribution I_n
    V_m = zeros(nb_of_segments, 1);                               % Initialize voltage vector
    V_m((nb_of_segments + 1) / 2) = 1 / delta;                    % Excitation at the center segment

    I_n = Zmn \ V_m;                                 % Solve for current distribution I_n
    z_N = linspace(delta / 2, 1 - delta / 2, nb_of_segments);    % Segment midpoints for plotting

    % Store total impedance by taking the reciprocal of center segment current
    impedance_values(m) = 1 / I_n((nb_of_segments + 1) / 2);
```



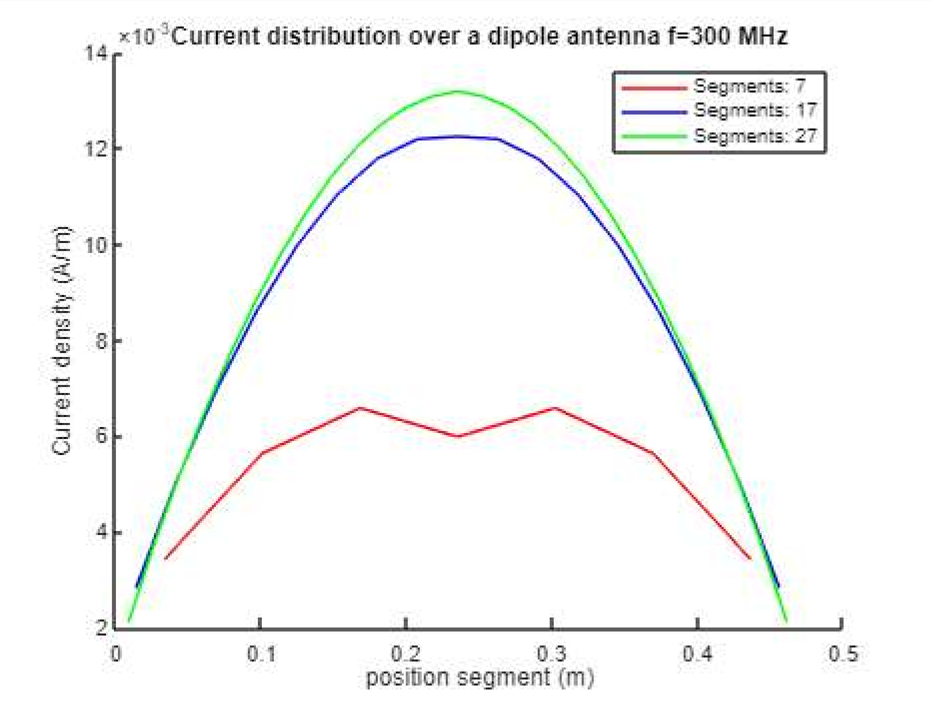
Second using piecewise sinusoidal basis functions and piecewise continuous test functions

```
figure;
hold on;
for m = 1:3
    nb_of_segments = N(m);                              % Number of segments
    Zmn2 = zeros(nb_of_segments, nb_of_segments);              % Initialize impedance matrix
    delta = l / (nb_of_segments + 1);                   % Segment length with sinusoidal basis
    legend_labels{m} = ['Segments: ', num2str(nb_of_segments)];
    % Fill impedance matrix Zmn2
    for j = 1:nb_of_segments
        for i = 1:nb_of_segments
            zm = i * delta;                 % Center of each segment
            zm_1 = zm - delta;
            zm_2 = zm + delta;

            % Calculate distances for sinusoidal basis function points
            R = sqrt(a^2 + (zm - z).^2);
            R_1 = sqrt(a^2 + (zm_1 - z).^2);
            R_2 = sqrt(a^2 + (zm_2 - z).^2);

            % Define integrand for sinusoidal basis function configuration
            func2 = inline((1j / (2 * pi * frq * e0)) * (k / sin(delta * k)) * (...
                (exp(-1j * k * R_2) / (4 * pi * R_2)) + ...
                (exp(-1j * k * R_1) / (4 * pi * R_1)) - ...
                (2 * (exp(-1j * k * R) / (4 * pi * R)) * cos(k * delta))));

            Zmn2(j, i) = quad(func2, (j - 0.5) * delta, (j + 0.5) * delta);  % Numerical integration over segment
        end
    end

    % Define excitation vector and solve for current distribution I_n2
    V_m2 = zeros(nb_of_segments, 1);
    V_m2((nb_of_segments + 1) / 2) = 1;                 % Center excitation

    I_n2 = Zmn2 \ V_m2;                         % Solve for current distribution
    z_N = linspace(delta, l - delta, nb_of_segments);    % Segment midpoints for plotting
```

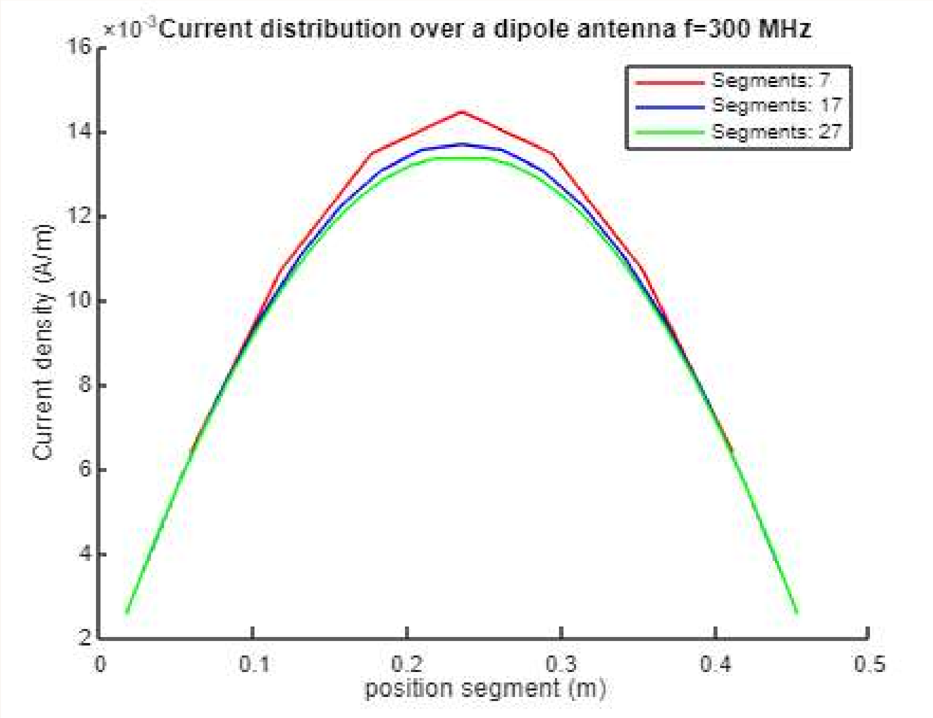


Output results

First Method



Second Method



Comparison using nec software

Number_of_Segments	Impedance_Value	Method
"7"	"122.6052-113.6914i"	"Method 1"
"17"	"81.00956-10.11419i"	"Method 1"
"27"	"75.50314+6.395627i"	"Method 1"
"--"	"-----------------"	"--------"
"7"	"68.9659+5.56679i"	"Method 2"
"17"	"73.12-1.0705i"	"Method 2"
"27"	"74.8893-3.44255i"	"Method 2"